



Kartverket

# A Simple VLBI Analysis

Geir Arne Hjelle

July 12, 2016

# Part I

## Background

# Who am I?

- ▶ Geir Arne Hjelle
- ▶ Ph.D. in Complex Analysis from Norwegian University of Science and Technology (NTNU) in Trondheim, Norway
- ▶ Post.doc at Washington University in St. Louis
- ▶ Currently working as a space geodesy researcher at the Norwegian Mapping Authority (Kartverket)

# Who am I?

- ▶ Geir Arne Hjelle
- ▶ Ph.D. in Complex Analysis from Norwegian University of Science and Technology (NTNU) in Trondheim, Norway
- ▶ Post.doc at Washington University in St. Louis
- ▶ Currently working as a space geodesy researcher at the Norwegian Mapping Authority (Kartverket)

At Kartverket I work in a small team where we are developing both mathematical and physical models for better positioning, as well as developing software to carry out the analysis.

# What are we doing?

We seek to contribute to *better global reference frames*.

# What are we doing?

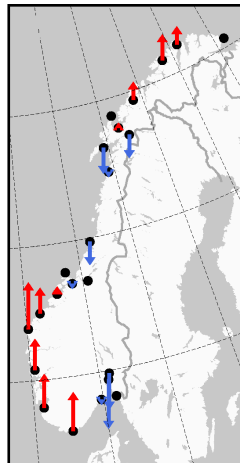
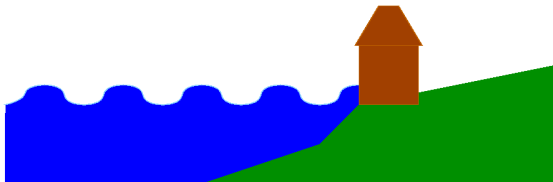
We seek to contribute to *better global reference frames*.

A reference frame is simply put a coordinate system. A global reference frame is necessary in order to compare positions at different places and at different times.



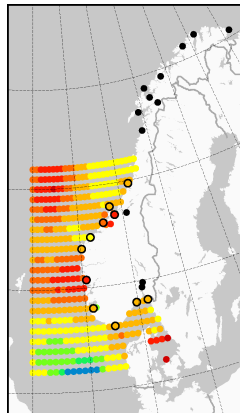
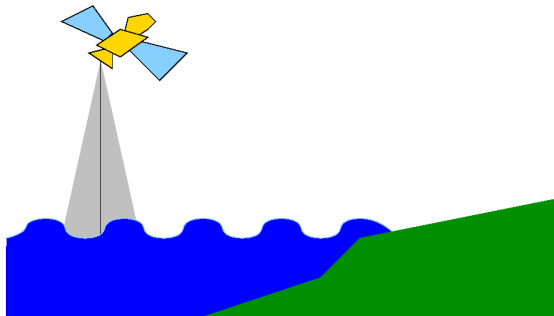
# Example – Measuring sea level

Traditionally sea level has been measured at fixed tide gauges.



## Example – Measuring sea level

Traditionally sea level has been measured at fixed tide gauges. Recently we can also measure sea level using altimetry satellites.





# Reference frames

Global reference frames are used to compare positions at different places and at different times.

# Reference frames

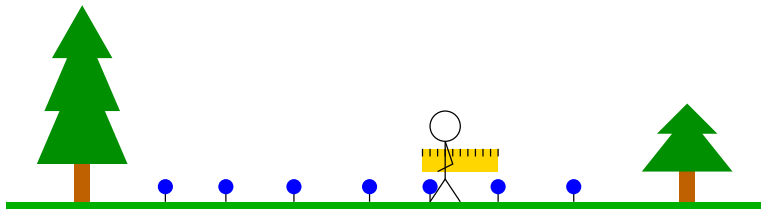
Global reference frames are used to compare positions at different places and at different times.



# Reference frames

Global reference frames are used to compare positions at different places and at different times.

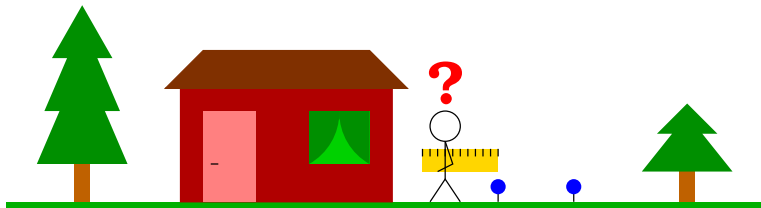
- ▶ The reference frames consists of *reference points* other points can be measured against.



# Reference frames

Global reference frames are used to compare positions at different places and at different times.

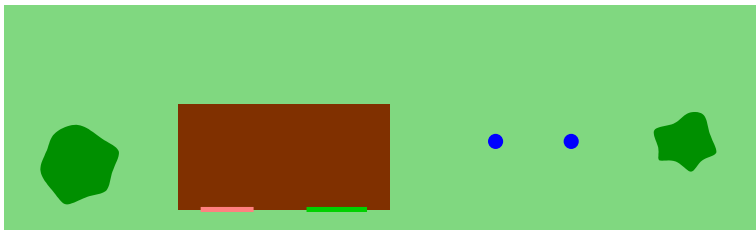
- ▶ The reference frames consists of *reference points* other points can be measured against.
- ▶ One challenge is that we can not always simply measure in a straight line.



# Reference frames

Global reference frames are used to compare positions at different places and at different times.

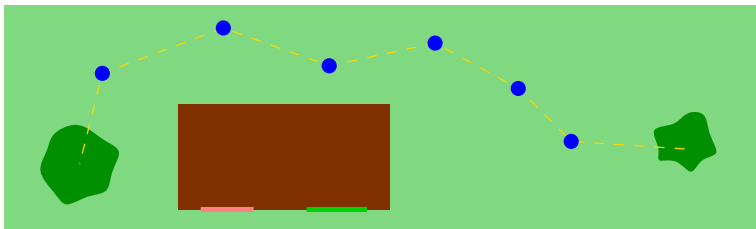
- ▶ The reference frames consists of *reference points* other points can be measured against.
- ▶ One challenge is that we can not always simply measure in a straight line.



# Reference frames

Global reference frames are used to compare positions at different places and at different times.

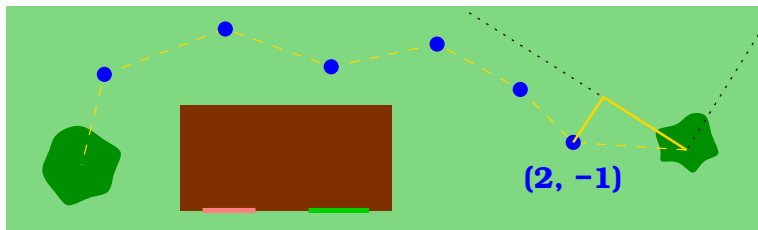
- ▶ The reference frames consists of *reference points* other points can be measured against.
- ▶ One challenge is that we can not always simply measure in a straight line.



# Reference frames

Global reference frames are used to compare positions at different places and at different times.

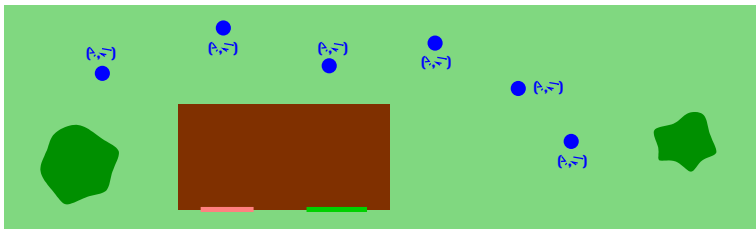
- ▶ The reference frames consists of *reference points* other points can be measured against.
- ▶ One challenge is that we can not always simply measure in a straight line.
- ▶ Using calculations we can give each point a coordinate.



# Reference frames

Global reference frames are used to compare positions at different places and at different times.

- ▶ The reference frames consists of *reference points* other points can be measured against.
- ▶ One challenge is that we can not always simply measure in a straight line.
- ▶ Using calculations we can give each point a coordinate.

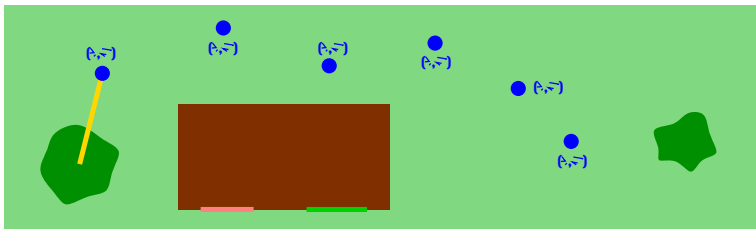




# Reference frames

Global reference frames are used to compare positions at different places and at different times.

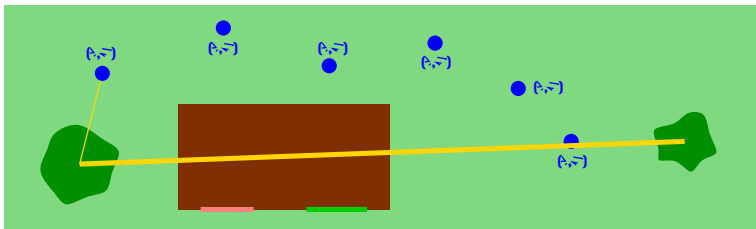
- ▶ The reference frames consists of *reference points* other points can be measured against.
- ▶ One challenge is that we can not always simply measure in a straight line.
- ▶ Using calculations we can give each point a coordinate.
- ▶ The coordinates can be used to find the total distance.



# Reference frames

Global reference frames are used to compare positions at different places and at different times.

- ▶ The reference frames consists of *reference points* other points can be measured against.
- ▶ One challenge is that we can not always simply measure in a straight line.
- ▶ Using calculations we can give each point a coordinate.
- ▶ The coordinates can be used to find the total distance.



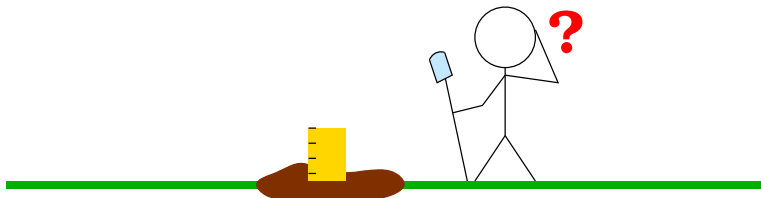
# Reference frames

One challenge with global reference frames is that we choose the origin as the center of the earth.

# Reference frames

One challenge with global reference frames is that we choose the origin as the center of the earth.

- ▶ We can not measure the reference points directly.



# Reference frames

One challenge with global reference frames is that we choose the origin as the center of the earth.

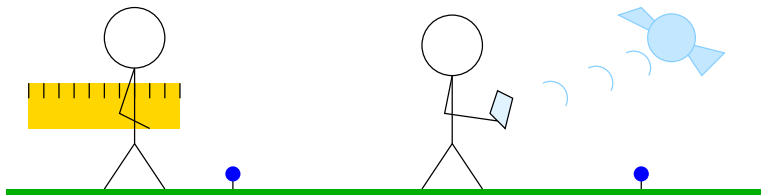
- ▶ We can not measure the reference points directly.
- ▶ Instead we *guess* at the coordinates of the reference points, and correct them later through indirect measures.
  - ▶ That is, we *estimate* the position.



# Reference frames

One challenge with global reference frames is that we choose the origin as the center of the earth.

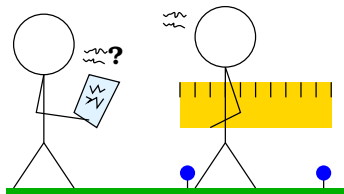
- ▶ We can not measure the reference points directly.
- ▶ Instead we *guess* at the coordinates of the reference points, and correct them later through indirect measures.
  - ▶ That is, we *estimate* the position.
- ▶ For the best possible corrections we *combine* different measuring techniques.



# Reference frames

Global reference frames are used to compare positions at different places and at different times.

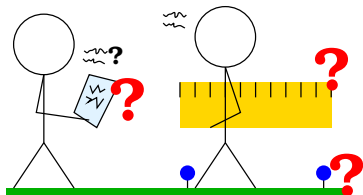
- Repeated measurements can give different results.



# Reference frames

Global reference frames are used to compare positions at different places and at different times.

- ▶ Repeated measurements can give different results.
- ▶ There are many sources for uncertainties.

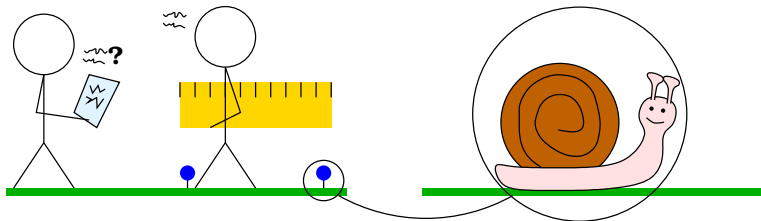




# Reference frames

Global reference frames are used to compare positions at different places and at different times.

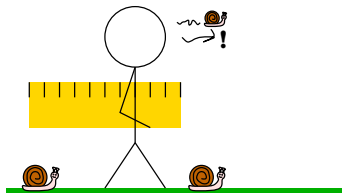
- ▶ Repeated measurements can give different results.
- ▶ There are many sources for uncertainties.
- ▶ If we assume that the reference points are moving



# Reference frames

Global reference frames are used to compare positions at different places and at different times.

- ▶ Repeated measurements can give different results.
- ▶ There are many sources for uncertainties.
- ▶ If we assume that the reference points are moving we can estimate velocities.



# Why are we doing this?

Global reference frames are necessary to

- ▶ monitor the climate over time,
- ▶ calculate precise satellite orbits, and
- ▶ take advantage of new technology within positioning.

# Why are we doing this?

Global reference frames are necessary to

- ▶ monitor the climate over time,
- ▶ calculate precise satellite orbits, and
- ▶ take advantage of new technology within positioning.

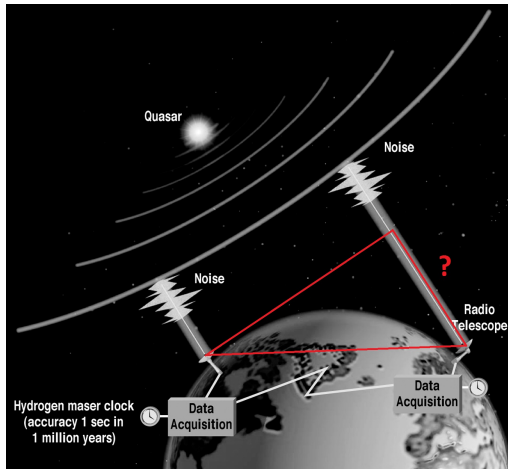
The four techniques mainly used in establishing a global reference frame are

- SLR Satellite Laser Ranging
- GNSS Global Navigation Satellite Systems (e.g. GPS)
- VLBI Very Long baseline Interferometry
- DORIS Doppler Orbitography and Radiopositioning Integrated by Satellite

## Part II

# VLBI - Very Long Baseline Interferometry

# VLBI – Observation



# VLBI – Model and residual

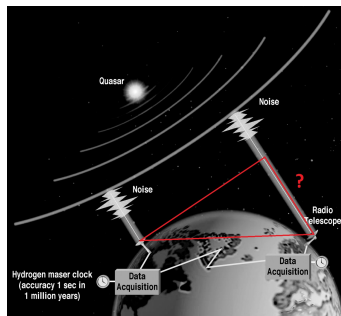
## Theoretical delay

$$\tau = \tau_{\text{geometric}} + \tau_{\text{grav}} + \tau_{\text{tropo}} + \tau_{\text{axisoffset}} \\ + \tau_{\text{thermdef}} + \tau_{\text{clock}} + \tau_{\text{cable}} + \tau_{\text{iono}}$$

## Residual

$$\text{residual} = \text{observation} - \text{model}$$

# The geometric delay



The geometric delay  $\tau_{\text{geometric}}$  can be calculated based on station positions and radio source coordinates,

$$\tau_{\text{geometric}} = \frac{K \cdot b}{c}.$$

Here  $K$  is the unit vector in the direction of the radio source, and  $b$  is the baseline vector, that is  $b = p_2 - p_1$ , the vector from station 1 to station 2.



# Part III

## Creating a simple analysis

# Software demo

Download the demo at

`https://github.com/gahjelle/vlbi\_demo`

Hit *Clone or Download* and choose *Download ZIP*.