
Master Thesis

The Use of Loaded Antennas as Scattering Relays for Post-Cellular Networks with Closely Spaced Terminals

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Declaration of Originality

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Contents

Preface	v
Abstract	vii
Notations, Acronyms and Abbreviations	ix
List of Figures	xi
1 Introduction	1
1.1 Motivation and Goals	1
1.2 State of the Art	2
1.3 Outline	2
2 System	3
2.1 Receiver Circuit Description	4
2.1.1 Multiport Networks	4
2.1.2 Receiver Blocks	5
2.1.3 Transfer Function of the Receiver	6
2.1.4 Port Reduction	7
2.1.5 Signal Covariance Matrix	8
2.2 Noise Description	8
2.2.1 Antenna Noise	8
2.2.2 LNA Noise	9
2.2.3 Downstream Noise	9
2.2.4 Noise Coupling	10
2.2.5 Noise Covariance Matrix	10
3 Analytical Gradient	13
3.1 Signal Gradients	13
3.2 Noise Gradients	14
3.2.1 Antenna Noise Gradient	14
3.2.2 LNA Noise Gradient	15
3.2.3 Downstream Noise Gradient	15
3.2.4 Noise Gradient	15
4 Problem Statement and Solver Methods	17
4.1 Utility Function	17
4.1.1 Convexity of the Utility Function	17
4.1.2 Dependency on the Input Power	18
4.1.3 Difference of Local Optima	18
4.1.4 Complexity of the Problem	18
4.2 Gradient Search	19
4.2.1 Choice of Initial Values	19

4.2.2	Adaptive Step Size	20
4.2.3	Conjugate Gradient	20
4.3	Heuristic Optimization Algorithms	22
4.3.1	Simulated Annealing	22
4.3.2	GlobalSearch	22
4.4	Further Algorithm Improvements	24
4.4.1	Optimization of the Interference Function	24
4.4.2	Post Refinement of GlobalSearch by Gradient Search	25
4.4.3	Stepwise Relay Improvements	25
5	Results	27
6	Conclusion and Outlook	29
6.1	Conclusion	29
6.2	Future Work	29
	References	31

Preface

I would like to thank Professor Wittneben for giving me the opportunity to work on this semester thesis at the Wireless Communication Group. Further, I want to thank Yahia Hassan, my supervisor, for his help and guidance throughout the thesis.

Abstract

In nowadays wireless networks there are mainly two factors which limit the achievable transmission rates - fading and interference. When multiple-input-multiple-output MIMO systems mostly reduce the impact of fading, the problem of interference has not yet been solved satisfyingly. The most common methods to address the problem of interference are protocols based on schemes like time-division- (TDMA), frequency-division- (FDMA), or code-division multiple access (CDMA). The solution - but in the same time also the downside of those schemes - is the unique allocation of a specific time and/or frequency slot for a single user, hence the blocking of all other users. This effect appears in the pre-logarithmic disproportional factor of the achievable sum rate.

In this thesis we address the problem of interference by the use of passive relay antennas (short: passive relays), i.e. antennas with only passive, lossless impedances attached, which interact with the receiving antennas only by the effect of coupling. By the choice of the passive elements the effect of the coupling can be changed and by using multiple passive relays, the coupling can be used to increase the signal- and reduce the interference power at the receivers.

In the following, a new description of the system will be derived, which allows an easier, more elegant way of analyzing the effect of coupling. Different solver methods are analyzed and discussed in order to find an optimal choice of the relay impedances. As some of the solvers are based on the method of gradient search, the analytical gradient will be derived. For the gradient search method, a variety of initial value choices will be compared to each other. Further, the results are evaluated and compared to previously known methods.

Notations, Acronyms and Abbreviations

Symbols

\mathbf{x}	Vector
$\mathbf{x}[i]$ or \mathbf{x}_i	i-th element of vector \mathbf{x}
$\ \mathbf{x}\ $	2-norm of vector \mathbf{x}
$\ \mathbf{x}\ _p$	LP-norm of vector \mathbf{x}
\mathbf{X}	Matrix
\mathbf{X}^T	transpose of matrix \mathbf{X}
\mathbf{X}^{-1}	inverse of matrix \mathbf{X}

Indices

x, y, z	x, y, z axis
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Acronyms and Abbreviations

ETH	Eidgenössische Technische Hochschule
WCG	Wireless Communicatoin Group
UWB	Ultra-Wide Band
HMT	Human Motion Tracking
CIR	Channel Impulse Response
(N)LOS	(Non) Line of Sight
DSSS	Direct-Sequence Spread Spectrum
(A,T,TD)oA	(Angle,Time,Time Difference) of Arrival
ML	Maximum Likelihood
SDP	Semidefinite Programming
CDF	Cummulative Distribution Function
PDF	Probability Density Function
RMSE	Root Mean Squared Error

List of Figures

2.1	Overview of the system.	3
2.2	A twoport network ("www.antennamagnus.com").	4
2.3	The receiver, with the coupling network \mathbf{Z}_C , the SP-matching network \mathbf{Z}_M , the LNA, and loads attached to the LNA.	5
2.4	Port reduction on a network with one load connected to the last port.	7
2.5	An example of the matrix shaping function. On the left the matrix shaped by $\Gamma(\cdot)$ on the right the matrix shaped by $\Gamma(\cdot)^{-1}$. The submatrices \mathbf{A} , \mathbf{B} and \mathbf{C} are square matrices and lie on the diagonal.	11
4.1	The utility function for different imaginary relay impedances for two relays and two different channel realizations.	18
4.2	The sum rate over the gradient search iterations.	19
4.3	Comparison of the number of initial values used.	19
4.4	Schematic of the adaptive step size algorithm.	20
4.5	A comparison of the convergence of gradient descent with optimal step size (in green) and conjugate vector (in red) for minimizing a quadratic function associated with a given linear system[7].	20
4.6	Comparison between Steepest Ascent, Polak-Ribière, and Fletcher-Reeves.	21
4.7	Comparison of the Simulated Annealing algorithm for different number of initializations and the results from "Polak-Ribière" and "Steepest Ascent" gradient searches.	22
4.8	Schematics of the GlobalSearch and MultiStart algorithms [12].	23
4.9	Performance of the GlobalSearch algorithm in comparison to the previous results.	23
4.10	Comparison of the number of initial values used.	24
4.11	Comparison of the number of initial values used.	25

Chapter 1

Introduction

Future wireless networks are assumed to be of higher density at the receiver side, as more and more devices with access to the internet are appearing on the market, and more and more types of devices are staffed with modules which are able to connect to the internet (internet of things IoT). Examples for such scenarios are cellular networks in an urban area, wireless networks in public spaces as concert halls, or sensor networks.

Such networks mainly suffer from two rate limiting effects: fading and interference. When the effect of fading is mostly solved by MIMO-techniques, the interference is the remaining bottleneck to achieve high data rates. Current methods to overcome interference are protocols like TDMA, FDMA or CDMA. The downside of those protocols is the unique allocation of a user to a specific time or frequency slot, hence the blocking of all other users for this period. More advanced techniques use multiple antennas as in the case of fading, to achieve multiple observations of the incoming signals and therefore the ability to zeroforce the interfering signals. This, however, is only possible, if the number of antennas per receiver is larger than the number of interfering signal streams ?? - in high density networks nearly impossible, or very expensive as the size of the receiver structure grows rapidly.

In this thesis the use of passive relays, i.e. antennae with pure imaginary impedances attached is introduced. As they are placed very closely (in terms of wavelengths) around the receivers, they interact by the effect of coupling with the receivers. By the choice of the impedances, the strength of coupling can be changed. With an increasing number of such passive relays, the coupling can be used to steer the signal towards the receiver and to amplify the signal and block the interference.

1.1 Motivation and Goals

The achievable rate of a connection pair is proportional to the signal to noise and interference ratio (SINR) (??). In a high power region the effect of noise can be neglected as the interference is the main diminishing factor. Therefore an interference-free connection is the main goal.

As the method of using passive relays only by their coupling is a new way of addressing the problem of interference, this thesis will look more into the achievable improvements than into the feasibility of the method. To achieve the highest possible rate for any realization, the shape of the problem will be analyzed and different solver methods will be introduced and compared.

1.2 State of the Art

TBD...

Nossekk Ivrlac Hassan Wittneben

1.3 Outline

In the following chapter the whole system will be described. The effect of coupling will be analyzed and the overall transfer function - from the transmitter to the receiver will be stated. All the noise sources appearing in the systems and their transfer function will be shown and derived. The transfer functions will be splitted up into smaller, simpler (sub-) functions, so that the effect of coupling and the interference can be better described and analyzed.

As one of the solver methods is gradient search, in the third chapter the analytical gradients will be derived - for the signal, the interference and the noise parts.

Last, the different solver methods will be evaluated. The results will be compared versus the number of relays used in a setup, versus different types of placings of the relays and versus different numbers of connection pairs. Additionally theoretical limits of the setups will be derived and the results will be compared to current protocols which overcome interference (TDMA).

Chapter 2

System

In the following the system will be described. Figure 2.1 gives an overview of it. The transmitters on the left are assumed to be widely spaced, so that they experience no coupling among each other. The signals are transmitted over a spatial interference channel, therefore a transmitted signal reaches every receiver.

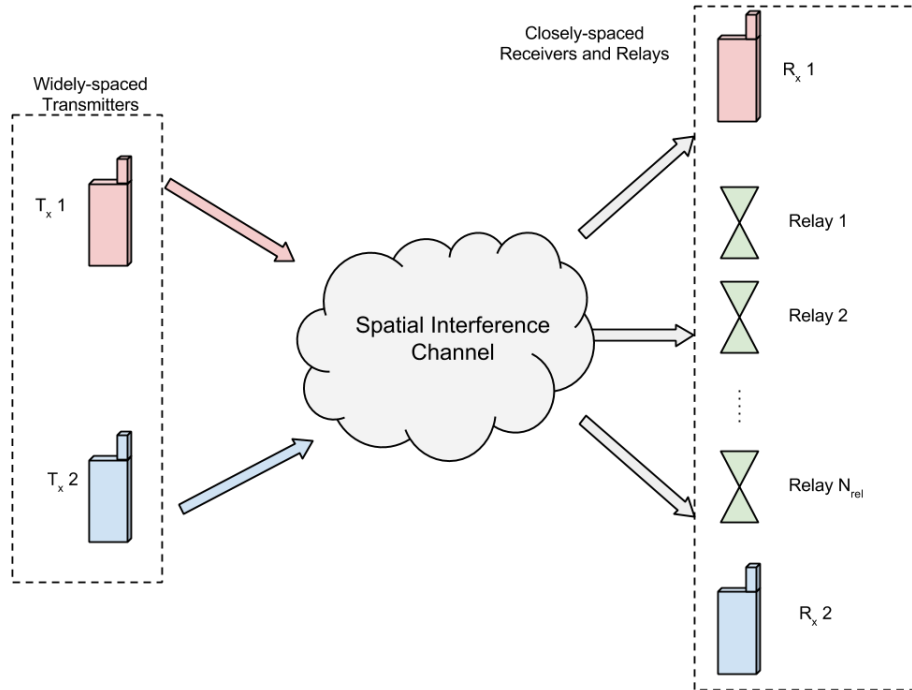


Figure 2.1: Overview of the system.

Besides the receivers, there are passive relays on the right side. The relays and the receivers are closely spaced, therefore the channels between a transmitter and the receivers and relays are spatially correlated and the elements on the receiver side experience coupling among each other.

2.1 Receiver Circuit Description

As mentioned in Section 1.2, the idea of describing the receiver circuitry is based on the work of [1]. To do so, we represent each receiver block (shown in Figure 2.3) by n-ports. A short overview on 2-ports (simplified n-ports) is given in the following.

2.1.1 Multiport Networks

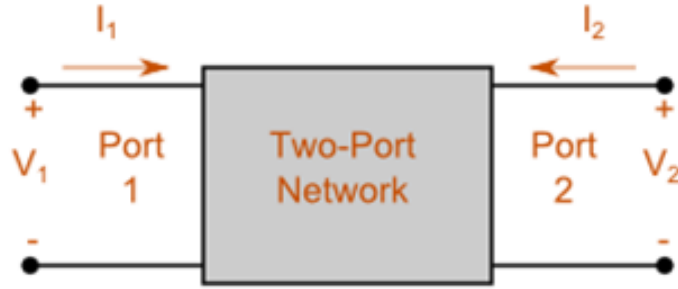


Figure 2.2: A twoport network ("www.antennamagnus.com").

In the following a 2-port network will be analyzed. Later this can be easily extended to a multiport network.

Figure 2.5 shows a 2-port network. The twoport network can be represented by its impedance matrix \mathbf{Z} . The elements Z_{ij} of the matrix are defined in the following way:

$$Z_{ij} = \frac{V_i}{I_j}, \quad (2.1)$$

for the currents $I_l = 0, \quad l \neq j.$

Therefore the 2-port's input/output relations can be written as

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \mathbf{Z} \cdot \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \quad (2.2)$$

For a multiport network, elements V_1, V_2, I_1 and I_2 become vectors, and the elements Z_{ij} , $i, j \in \{1, 2\}$ become submatrices. Looking from the left into the network with a load R_L attached to port two, the equivalent input impedance becomes

$$\mathbf{Z}_{in_1} = \mathbf{Z}_{11} - \mathbf{Z}_{21} \cdot (\mathbf{Z}_{22} + R_L \cdot \mathbf{I})^{-1} \cdot \mathbf{Z}_{12}. \quad (2.3)$$

With no load attached on port two, the equivalent input impedance becomes

$$\mathbf{Z}_{in_1} = \mathbf{Z}_{11}. \quad (2.4)$$

Last, having port one short-circuited and looking from port two into the network, the equivalent network impedance becomes

$$\mathbf{Z}_{in_2} = \mathbf{Z}_{22} - \mathbf{Z}_{12} \cdot (\mathbf{Z}_{11})^{-1} \cdot \mathbf{Z}_{21}. \quad (2.5)$$

For reciprocal (multiport-)networks (RLC -Networks, containing only passive elements), $\mathbf{Z}_{12} = \mathbf{Z}_{21}^T$.

2.1.2 Receiver Blocks

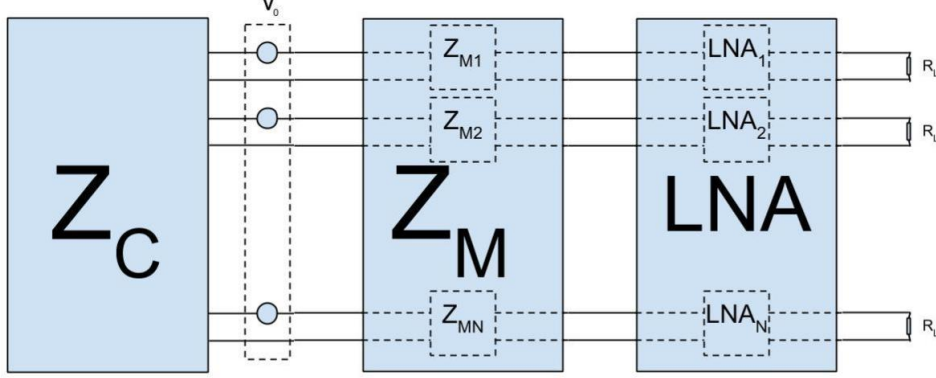


Figure 2.3: The receiver, with the coupling network \mathbf{Z}_C , the SP-matching network \mathbf{Z}_M , the LNA, and loads attached to the LNA.

The receiver (as shown in Figure 2.3) consists of three blocks. From left to right:

1. the coupling network (\mathbf{Z}_C),
2. the matching network (\mathbf{Z}_M), and
3. the low-noise-amplifier (\mathbf{LNA}).

All these blocks can be described in multiport networks. In the following each block will be discussed.

The Coupling Network

The coupling network comes from the fact, that compact antenna arrays are used. The strength of the coupling between two antennas depends on the spacing between the antennas. In the following a uniform spacing of the antennas is assumed. As the effect of coupling from one antenna to another is the same like the reverse, the impedance matrix \mathbf{Z}_C becomes symmetric.

The Matching Network

Behind the each receiving antenna a matching network placed, to improve the performance of the receiver. For complexity and bandwidth reasons ("WCNC - Paper"), single-port (SP) matching is assumed. The matching network has the form of

$$\mathbf{Z}_M = \begin{bmatrix} \mathbf{Z}_{M11} & \mathbf{Z}_{M12} \\ \mathbf{Z}_{M21} & \mathbf{Z}_{M22} \end{bmatrix}. \quad (2.6)$$

For a matching network to be lossless it must be pure imaginary and symmetric[1]. Because we assume SP matching the submatrices become diagonal, with the additional property of $\mathbf{Z}_{M12} = \mathbf{Z}_{M21}^T \implies \mathbf{Z}_{M12} = \mathbf{Z}_{M21}$.

The Low-Noise-Amplifier

In the LNA-block the received signal after the matching network gets amplified. As in the matching network the LNA can be represented in the following way

$$\mathbf{LNA} = \begin{bmatrix} \mathbf{c} & \mathbf{d} \\ \mathbf{e} & \mathbf{g} \end{bmatrix}. \quad (2.7)$$

As each branch has it's own LNA, the submatrices \mathbf{c} , \mathbf{e} and \mathbf{g} are again diagonal. Additionally, matrix \mathbf{d} is an all-zeros matrix if the unilateral assumption (*the input of the LNA is not affected by the output of the LNA*) is applied.

2.1.3 Transfer Function of the Receiver

The main interest lies in the transfer function of the input voltages \mathbf{v}_0 to the voltage measured at the loads (in Figure 2.3) \mathbf{v}_L . In the following the transferfunction over each block of the receiver will be derived. We use the termination: \mathbf{v}_i is the voltage on the left of the block i (i.e. v_M is the voltage on the left ports of the matching network). Additionally, we see the left ports as input ports and the right ports as output ports of each block.

For the derivation of the transferfunction we need four equivalent impedance matrices, namely

1. \mathbf{Z}_{eqM_1} , the impedance matrix looking from the left into the matching network,
2. \mathbf{Z}_{eqM_2} , the impedance matrix looking from the right into the matching network,
3. \mathbf{Z}_{eqLNA_1} , the impedance matrix looking from the left into the LNA, and
4. \mathbf{Z}_{eqLNA_2} , the impedance matrix looking from the right into the LNA.

To calculate \mathbf{Z}_{eqM_1} and \mathbf{Z}_{eqLNA_1} , we use Equation (2.3) and get the following

$$\mathbf{Z}_{eqM_1} = \mathbf{Z}_{M11} - \mathbf{Z}_{M21} \cdot (\mathbf{Z}_{M22} + \mathbf{Z}_{eqLNA_1})^{-1} \cdot \mathbf{Z}_{M12}, \quad (2.8)$$

$$\mathbf{Z}_{eqLNA_1} = \mathbf{c} - \mathbf{e} \cdot (R_L \mathbf{I}_{N_r} + \mathbf{g})^{-1} \cdot \mathbf{d} = \mathbf{c}. \quad (2.9)$$

As we step through the receiver blocks from left to right, we always have the parallel voltages applied to each receiver block on the left. Therefore the equivalent input impedances \mathbf{Z}_{eqM_2} and \mathbf{Z}_{eqLNA_2} can be derived using Equation (2.4) and lead to the following

$$\mathbf{Z}_{eqM_2} = \mathbf{Z}_{M22} - \mathbf{Z}_{M12} \cdot (\mathbf{Z}_{M11})^{-1} \cdot \mathbf{Z}_{M21}, \quad (2.10)$$

$$\mathbf{Z}_{eqLNA_2} = \mathbf{g} - \mathbf{d} \cdot (\mathbf{c})^{-1} \cdot \mathbf{e} = \mathbf{g}. \quad (2.11)$$

To get the parallel input voltage on the left of the matching network, we use the principle of a voltage divider as

$$\mathbf{v}_M = \mathbf{Z}_{eqM_1} \cdot (\mathbf{Z}_{eqM_1} + \mathbf{Z}_C)^{-1} \cdot \mathbf{v}_0 \quad (2.12)$$

To transfer the voltages from the left ports to the right ports of each block we proceed for each block in the following:

1. Calculate the input currents,
2. transfer the input currents to the output voltages in series, and
3. calculate the output voltages in parallel, by a voltage divider.

For the matching network, it follows:

$$\mathbf{i}_M = \mathbf{Z}_{M11}^{-1} \cdot \mathbf{v}_M, \quad (2.13)$$

$$\mathbf{v}_{LNA_series} = \mathbf{Z}_{M12} \cdot \mathbf{i}_M = \mathbf{Z}_{M12} \mathbf{Z}_{M11}^{-1} \cdot \mathbf{v}_M, \quad (2.14)$$

$$\begin{aligned} \mathbf{v}_{LNA} &= \mathbf{Z}_{eqLNA_1} (\mathbf{Z}_{eqLNA_1} + \mathbf{Z}_{eqM_2})^{-1} \cdot \mathbf{v}_{LNA_series} \\ &= \mathbf{Z}_{eqLNA_1} (\mathbf{Z}_{eqLNA_1} + \mathbf{Z}_{eqM_2})^{-1} \mathbf{Z}_{M12} \mathbf{Z}_{M11}^{-1} \cdot \mathbf{v}_M, \end{aligned} \quad (2.15)$$

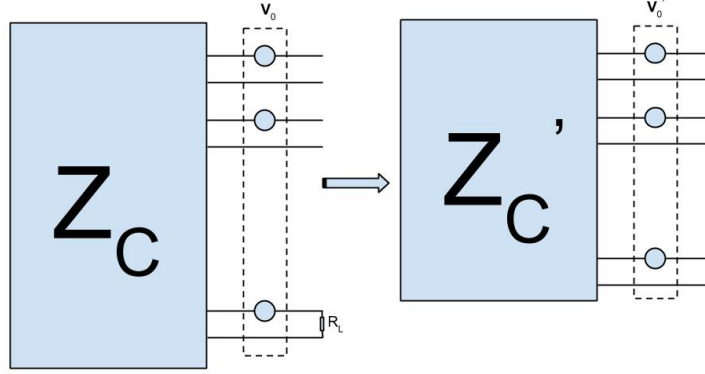


Figure 2.4: Port reduction on a network with one load connected to the last port.

and equivalent for the LNA block

$$\mathbf{v}_L = R_L \mathbf{I}_{N_R} (R_L \mathbf{I}_{N_R} + \mathbf{Z}_{LNA_2})^{-1} \mathbf{Z}_{LNA_{12}} \mathbf{Z}_{LNA_{11}}^{-1} \cdot \mathbf{v}_{LNA}. \quad (2.16)$$

Therefore we have three transfer functions to characterize our system. Denoting the transfer function from voltage \mathbf{v}_j to voltage \mathbf{v}_i as $\mathbf{H}_{i,j}$ (i.e. $\mathbf{v}_i = \mathbf{H}_{i,j} \cdot \mathbf{v}_j$), it follows

$$\mathbf{H}_{M,0'} = \mathbf{Z}_{eqM_1} \cdot (\mathbf{Z}_{eqM_1} + \mathbf{Z}_C)^{-1}, \quad (2.17)$$

$$\mathbf{H}_{LNA,M} = \mathbf{Z}_{eqLNA_1} (\mathbf{Z}_{eqLNA_1} + \mathbf{Z}_{eqM_2})^{-1} \mathbf{Z}_{M12} \mathbf{Z}_{M11}^{-1}, \quad \text{and} \quad (2.18)$$

$$\mathbf{H}_{L,LNA} = R_L \mathbf{I}_{N_R} (R_L \mathbf{I}_{N_R} + \mathbf{Z}_{eqLNA_2})^{-1} \mathbf{Z}_{LNA_{12}} \mathbf{Z}_{LNA_{11}}^{-1}. \quad (2.19)$$

And the overall transferfunction

$$\mathbf{H}_{L,0'} = \mathbf{H}_{L,LNA} \cdot \mathbf{H}_{LNA,M} \cdot \mathbf{H}_{M,0'}. \quad (2.20)$$

2.1.4 Port Reduction

In the following we describe the open circuit reduction of a system like in Figure 2.4. We do the port reduction, because we are interested in the signal only at the loads of some receivers. The signal picked up at relay antennae or not considered receivers contributes to the considered receiver by the coupling between the antennae, as described in Section 2.1.2. Passive antennas (e.g. relay antennas) can be modeled by connecting a load directly onto the coupling network as shown in Figure 2.4. Undesired receivers can be equivalently modelled, however in this case, the load corresponds to the equivalent input impedance, when looking from the left into the matching network (as in Equation (2.8)). In the following, \mathbf{v}_C denotes the voltages on the ports between the coupling matrix \mathbf{Z}_C and the voltage sources \mathbf{v}_0 , the same for the currents. As we are not interested in the input/output-relation of these passive antennas, a port reduction is performed. For the port reduction, the coupling matrix \mathbf{Z}_C can be represented by four submatrices

$$\mathbf{Z}_C = \begin{bmatrix} \mathbf{Z}_{OO} & \mathbf{Z}_{OL} \\ \mathbf{Z}_{LO} & \mathbf{Z}_{LL} \end{bmatrix}, \quad (2.21)$$

so that we get the system relations

$$\begin{bmatrix} \mathbf{v}_{CO} \\ \mathbf{v}_{CL} \end{bmatrix} = \begin{bmatrix} \mathbf{Z}_{OO} & \mathbf{Z}_{OL} \\ \mathbf{Z}_{LO} & \mathbf{Z}_{LL} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{i}_{CO} \\ \mathbf{i}_{CL} \end{bmatrix}. \quad (2.22)$$

The index "O" denotes hereby the open circuit ports, the index "L" the ports with a load attached. We assume, that there are N_R antennas, whereby the first N_i antennas are active, and the later ones are passive (and therefore modeled by a load). \mathbf{Z}_{pass} denotes in the following the $N_r - N_i$ loads representing the passive antennas. Therefore it is a diagonal matrix.

From Equation (2.22) and the property

$$\mathbf{v}_{CL} = \mathbf{v}_0[N_i + 1 : N_R] - \mathbf{Z}_{pass} \cdot \mathbf{i}_{CL} \quad \text{it follows,} \quad (2.23)$$

$$\mathbf{i}_{CL} = -(\mathbf{Z}_{pass} + \mathbf{Z}_{LL})^{-1} \mathbf{Z}_{LO} \cdot \mathbf{i}_{CO} - (\mathbf{Z}_{pass} + \mathbf{Z}_{LL})^{-1} \cdot \mathbf{v}_0[N_i + 1 : N_R] \quad (2.24)$$

and therefore,

$$\mathbf{v}_{CO} = (\mathbf{Z}_{OO} - \mathbf{Z}_{OL}(\mathbf{Z}_{pass} + \mathbf{Z}_{LL})^{-1} \mathbf{Z}_{LO}) \cdot \mathbf{i}_{CO} - \mathbf{Z}_{OL}(\mathbf{Z}_{pass} + \mathbf{Z}_{LL})^{-1} \cdot \mathbf{v}_0[N_i + 1 : N_R]. \quad (2.25)$$

With this port reduction we have the new coupling matrix and the new voltage input as

$$\mathbf{Z}'_C = \mathbf{Z}_{OO} - \mathbf{Z}_{OL}(\mathbf{Z}_{pass} + \mathbf{Z}_{LL})^{-1} \mathbf{Z}_{LO} \quad \text{and} \quad (2.26)$$

$$\mathbf{v}'_0 = \mathbf{H}_0 \cdot \mathbf{v}_0 \quad (2.27)$$

$$\text{with } \mathbf{H}_0 = [\mathbf{I}_{N_i} \quad -\mathbf{Z}_{OL}(\mathbf{Z}_{pass} + \mathbf{Z}_{LL})^{-1}],$$

and so we can reduce the system shown in Figure 2.4, to the system shown in Figure 2.3.

2.1.5 Signal Covariance Matrix

To calculate the achievable sum rate of the systems, we need to derive the signal covariance matrix, defined as $\mathbb{E}[\mathbf{v}_L \mathbf{v}_L^H]$. As in all the transfer functions from \mathbf{v}_0 to \mathbf{v}_L only impedance matrices appear, the covariance matrix becomes

$$\mathbf{K}_s = \mathbb{E}[\mathbf{v}_L \mathbf{v}_L^H] = \mathbf{H}_{L,0} \cdot \mathbb{E}[\mathbf{v}_0 \mathbf{v}_0^H] \cdot \mathbf{H}_{L,0}^H, \quad (2.28)$$

where $\mathbf{H}_{L,0}$ is the transfer function including any port reduction.

2.2 Noise Description

As mentioned in [2], there are four main noise sources in the receiver. In the following each noise source will be described and its transfer function towards the loads will be derived.

2.2.1 Antenna Noise

The antennas introduce two noise sources. The external noise \mathbf{n}_{ext} , collected from the radiation component of the antenna array and the noise generated by the losses in the antennas \mathbf{n}_l . From [3] it follows,

$$\mathbf{R}_{na} = \mathbb{E}[\mathbf{n}_{AR} \mathbf{n}_{AR}^H] = 4k_B B_W (T_{AE} \mathbb{R}\{\mathbf{Z}_{AR}\} + T_{AL} \mathbf{R}_{AR}), \quad (2.29)$$

with k_B the Boltzmann constant and B_W the bandwidth.

Transfer Function of the Antenna Noise

As the antenna noise is picked up by the antennas in the same way as the signal, the transfer function remains the same as the one derived in Section 2.1.3 for the signal, e.g.

$$\begin{aligned}\mathbf{H}_{L,0} &= \mathbf{H}_{L,LNA} \cdot \mathbf{H}_{LNA,M} \cdot \mathbf{H}_{M,0}, \quad \text{including port reduction} \\ \mathbf{H}_{L,0} &= \mathbf{H}_{L,LNA} \cdot \mathbf{H}_{LNA,M} \cdot \mathbf{H}_{M,0} \cdot \mathbf{H}_0.\end{aligned}\tag{2.30}$$

2.2.2 LNA Noise

The LNA introduces the third noise source. From the discussion in [1], the noise of the LNA is modeled by a series of voltages and parallel currents at the input of the LNA. The noise sources have the following statistical properties,

$$\begin{aligned}\mathbb{E}[\mathbf{i}_{LNA} \mathbf{i}_{LNA}^H] &= \beta \mathbf{I}_{N_r}, \\ \mathbb{E}[\mathbf{v}_{LNA} \mathbf{v}_{LNA}^H] &= \beta R_n^2 \mathbf{I}_{N_r}, \quad \text{and} \\ \mathbb{E}[\mathbf{v}_{LNA} \mathbf{i}_{LNA}^H] &= \rho \beta R_n \mathbf{I}_{N_r},\end{aligned}\tag{2.31}$$

with ρ and β as correlation coefficients.

Transfer Function of the LNA Noise

As written above, we have two noise sources, the serial voltage sources and the parallel current sources. First we will transfer the current source into a voltage source in series as we then can use the same transfer function for the voltage and the transferred current sources. To do so, we need the equivalent input impedances looking from the left into the LNA block and from the right into the matching network. The equivalent input impedance for the LNA network \mathbf{Z}_{eqLNA_1} was already derived in (2.9). To get the equivalent input impedance for the matching network looking from the right into it we use Equation (2.3) and get

$$\tilde{\mathbf{Z}}_{eqM_2} = \mathbf{Z}_{M22} - \mathbf{Z}_{M12} \cdot (\mathbf{Z}_{M11} + \mathbf{Z}_{C'})^{-1} \cdot \mathbf{Z}_{M21}.\tag{2.32}$$

Transferring the current source into a series of voltages gives us

$$\mathbf{v}_{LNA_c} = -\tilde{\mathbf{Z}}_{eqM_2} \mathbf{i}_{LNA},\tag{2.33}$$

and therefore a transfer function of

$$\mathbf{H}_{L,LNA_v} = R_L \mathbf{I}_{N_r} (R_L \mathbf{I}_{N_r} + \tilde{\mathbf{Z}}_{eqLNA_2})^{-1} \mathbf{e} \cdot (\mathbf{c} + \tilde{\mathbf{Z}}_{eqM_2})^{-1},\tag{2.34}$$

for the series voltages and

$$\mathbf{H}_{L,LNA_c} = -R_L \mathbf{I}_{N_r} (R_L \mathbf{I}_{N_r} + \tilde{\mathbf{Z}}_{eqLNA_2})^{-1} \mathbf{e} \cdot (\mathbf{c} + \tilde{\mathbf{Z}}_{eqM_2})^{-1} \tilde{\mathbf{Z}}_{eqM_2},\tag{2.35}$$

for the LNA noise currents.

2.2.3 Downstream Noise

The last noise source is the downstream noise, generated by all the circuitry after the LNA [4] and modeled by voltage sources $\tilde{\mathbf{v}}_n$ in series to the loads. With the statistical property

$$\mathbb{E}[\tilde{\mathbf{v}}_n \tilde{\mathbf{v}}_n^H] = \psi \mathbf{I}_{N_r}.\tag{2.36}$$

Transfer Function of the Downstream Noise

For the transfer function of the downstream noise we need a simple voltage divider of the loads and the equivalent input impedance $\tilde{\mathbf{Z}}_{eqLNA_2}$ looking from the right into the LNA block. Note, that this is NOT the same input impedance as in (2.11). Therefore the transfer function becomes

$$\mathbf{H}_{L,n} = R_L \mathbf{I}_{N_r} (R_L \mathbf{I}_{N_r} + \tilde{\mathbf{Z}}_{eqLNA_2})^{-1}, \quad (2.37)$$

with

$$\tilde{\mathbf{Z}}_{eqLNA_2} = \mathbf{g} - \mathbf{d} \cdot (\mathbf{c} + \tilde{\mathbf{Z}}_{eqM_2})^{-1} \cdot \mathbf{e} = \mathbf{g}. \quad (2.38)$$

However, under the unilateral assumption the input impedance reduces to $\tilde{\mathbf{Z}}_{eqLNA_2} = \mathbf{g}$.

2.2.4 Noise Coupling

So far we transferred the noise from it's source to the loads behind the LNA block. However additionally to the direct path, we need to take the noise of different receiver branches into account. Therefore we transfer the noise sources to series voltages next to the antenna voltages, so that we can use the transfer functions derived in Chapter 2.1.

As in the previous Section, we consider again the four noise types. For the antenna noise, we note, that we do not need to derive a transfer function. In a first step we transfer the LNA-current noise source and the downstream noise to the LNA-voltage noise source. Therefore the LNA-current noise source is multiplied by the equivalent input impedance of the LNA

$$\mathbf{v}_{LNAc} = \mathbf{Z}_{eqLNA_1} \mathbf{i}_{LNA}. \quad (2.39)$$

For the downstream noise we note, that under the unilateral assumption, the transferred noise is zero, else

$$\mathbf{v}_{LNA n} = \mathbf{d}(\mathbf{g} + R_L \mathbf{I}_{N_R})^{-1} \tilde{\mathbf{v}}_v. \quad (2.40)$$

Now we only need to find the transfer function for the LNA-voltage source $\mathbf{v}_{LNA v}$. The transfer function over the matching network is given by

$$\mathbf{H}_{0,LNA} = \mathbf{Z}_{M12}(\mathbf{Z}_{M22} + \mathbf{Z}_{eqLNA_1})^{-1}. \quad (2.41)$$

2.2.5 Noise Covariance Matrix

As in Chapter 2.1, we will derive the noise covariance matrix. To do so, we introduce the function $\Gamma(\cdot)$, which reshapes the matrices to our needs. For example Equation (2.41) must only be applied on the noise sources of the receivers, which shall be reduced, however a multiplication of the transfer matrix with the full noise vector is desired. Let further $\Gamma(\cdot)^{-1}$, denote the function, which shapes the matrices only to the branches which are not reduced. We note the following properties of the function:

$$\begin{aligned} \Gamma(\mathbf{A}) \cdot \Gamma(\mathbf{B}) &= \Gamma(\mathbf{A} \cdot \mathbf{B}), \\ \Gamma(\mathbf{A}^H) &= \Gamma(\mathbf{A})^H, \\ \Gamma(\mathbf{A}) \cdot \Gamma(\mathbf{B})^{-1} &= \mathbf{0}, \quad \text{and} \\ \forall \mathbf{A}, \mathbf{B}, \quad &\text{which form a valid matrix multiplication} \quad \mathbf{A} \cdot \mathbf{B}. \end{aligned} \quad (2.42)$$

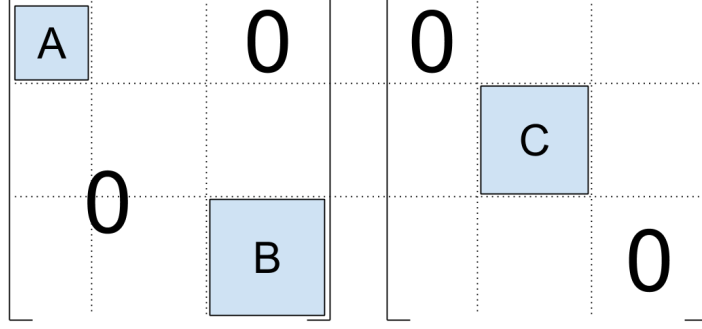


Figure 2.5: An example of the matrix shaping function. On the left the matrix shaped by $\Gamma(\cdot)$ on the right the matrix shaped by $\Gamma(\cdot)^{-1}$. The sub matrices **A**, **B** and **C** are square matrices and lie on the diagonal.

Note that the first two properties are also valid for $\Gamma(\cdot)^{-1}$. Additionally for diagonal matrices the following property holds $\Gamma(\mathbf{A}) + \Gamma(\mathbf{A})^{-1} = \mathbf{A}$.

As we have multiple noise sources, first we will describe the noise output \mathbf{u}_L for each noise source, i.e.:

$$\begin{aligned}
 \mathbf{u}_{AR} &= \mathbf{H}_{L,0} \cdot \mathbf{n}_{AR}, \\
 \mathbf{u}_{LNA_v} &= (\mathbf{H}_{L,0} \cdot \Gamma(\mathbf{H}_{0,LNA}) + \Gamma(\mathbf{H}_{L,LNA_v})^{-1}) \cdot \mathbf{v}_{LNA}, \\
 \mathbf{u}_{LNA_c} &= (\mathbf{H}_{L,0} \cdot \Gamma(\mathbf{H}_{0,LNA} \mathbf{Z}_{eqLNA_1}) + \Gamma(\mathbf{H}_{L,LNA_c})^{-1}) \cdot \mathbf{i}_{LNA}, \\
 \mathbf{u}_{\tilde{n}} &= (\mathbf{H}_{L,0} \cdot \Gamma(\mathbf{H}_{0,LNA} \cdot \mathbf{d}(\mathbf{g} + R_L \mathbf{I}_{N_R})^{-1}) + \Gamma(\mathbf{H}_{L,n})^{-1}) \cdot \tilde{\mathbf{v}}_n, \\
 \text{and therefore } \mathbf{u}_L &= \mathbf{u}_{AR} + \mathbf{u}_{LNA_v} + \mathbf{u}_{LNA_c} + \mathbf{u}_{\tilde{n}}.
 \end{aligned} \tag{2.43}$$

Because all the noise sources are uncorrelated, except for the LNA noise sources (Equation (??)), the noise covariance matrix can be written as:

$$\begin{aligned}
 \mathbf{K}_n = \mathbb{E}[\mathbf{u}_L \mathbf{u}_L^H] &= \mathbf{H}_{L,0} \mathbb{E}[\mathbf{n}_{AR} \mathbf{n}_{AR}^H] \mathbf{H}_{L,0}^H + \mathbb{E}[\mathbf{u}_{LNA_v} \mathbf{u}_{LNA_v}^H] + \mathbb{E}[\mathbf{u}_{LNA_c} \mathbf{u}_{LNA_c}^H] + \\
 &\quad \mathbb{E}[\mathbf{u}_{LNA_c} \mathbf{u}_{LNA_v}^H] + \mathbb{E}[\mathbf{u}_{LNA_v} \mathbf{u}_{LNA_c}^H] + \mathbb{E}[\mathbf{u}_{\tilde{n}} \mathbf{u}_{\tilde{n}}^H]
 \end{aligned} \tag{2.44}$$

For simplicity reasons, we will look in every summand separately in the following. As $-\mathbf{H}_{L,LNA_v} \cdot \tilde{\mathbf{Z}}_{eqM_2} = \mathbf{H}_{L,LNA_c}$ and using (2.29), (2.31), (2.36) and (2.42), it follows

$$\begin{aligned}
 \mathbf{K}_{n_{LNA_{vv}}} &= \mathbb{E} \left[(\mathbf{H}_{L,0} \cdot \Gamma(\mathbf{H}_{0,LNA}) + \Gamma(\mathbf{H}_{L,LNA_v})^{-1}) \cdot \mathbf{v}_{LNA} \mathbf{v}_{LNA}^H \cdot \right. \\
 &\quad \left. (\mathbf{H}_{L,0} \cdot \Gamma(\mathbf{H}_{0,LNA}) + \Gamma(\mathbf{H}_{L,LNA_v})^{-1})^H \right] \\
 &= (\mathbf{H}_{L,0} \cdot \Gamma(\mathbf{H}_{0,LNA}) + \Gamma(\mathbf{H}_{L,LNA_v})^{-1}) \cdot \mathbb{E}[\mathbf{v}_{LNA} \mathbf{v}_{LNA}^H] \cdot \\
 &\quad (\mathbf{H}_{L,0} \cdot \Gamma(\mathbf{H}_{0,LNA}) + \Gamma(\mathbf{H}_{L,LNA_v})^{-1})^H \\
 &= \beta R_n^2 \left(\mathbf{H}_{L,0} \cdot \Gamma(\mathbf{H}_{0,LNA} \mathbf{H}_{0,LNA}^H) \mathbf{H}_{0,LNA}^H + \Gamma(\mathbf{H}_{L,LNA_v} \mathbf{H}_{L,LNA_v}^H)^{-1} \right),
 \end{aligned} \tag{2.45}$$

for the LNA-voltage source and

$$\begin{aligned}
 \mathbf{K}_{n_{LNA_{cc}}} &= \beta \left(\mathbf{H}_{L,0} \cdot \Gamma(\mathbf{H}_{0,LNA} \mathbf{Z}_{eqLNA_1} \mathbf{Z}_{eqLNA_1}^H \mathbf{H}_{0,LNA}^H) \mathbf{H}_{0,LNA}^H + \right. \\
 &\quad \left. \Gamma(\mathbf{H}_{L,LNA_v} \tilde{\mathbf{Z}}_{eqM_2} \tilde{\mathbf{Z}}_{eqM_2}^H \mathbf{H}_{L,LNA_v}^H)^{-1} \right),
 \end{aligned} \tag{2.46}$$

for the LNA-current source. For the cross terms it follows

$$\begin{aligned}
\mathbf{K}_{n_{LNA_{vc}}} &= \rho\beta R_n \left(\mathbf{H}_{L,0} \cdot \Gamma(\mathbf{H}_{0,LNA} \mathbf{Z}_{eqLNA_1}^H \mathbf{H}_{0,LNA}^H) \mathbf{H}_{L,0}^H + \Gamma(\mathbf{H}_{L,LNA_v} \tilde{\mathbf{Z}}_{eqM_2}^H \mathbf{H}_{L,LNA_v}^H)^{-1} \right) \\
&\quad + \rho^* \beta R_n \left(\mathbf{H}_{L,0} \cdot \Gamma(\mathbf{H}_{0,LNA} \mathbf{Z}_{eqLNA_1} \mathbf{H}_{0,LNA}^H) \mathbf{H}_{L,0}^H + \Gamma(\mathbf{H}_{L,LNA_v} \tilde{\mathbf{Z}}_{eqM_2} \mathbf{H}_{L,LNA_v}^H)^{-1} \right) \\
&= 2\beta R_n \left(\mathbf{H}_{L,0} \cdot \Gamma(\mathbf{H}_{0,LNA} \mathbb{R}\{\rho^* \mathbf{Z}_{eqLNA_1}\} \mathbf{H}_{0,LNA}^H) \mathbf{H}_{L,0}^H + \right. \\
&\quad \left. \Gamma(\mathbf{H}_{L,LNA_v} \mathbb{R}\{\rho^* \tilde{\mathbf{Z}}_{eqM_2}\} \mathbf{H}_{L,LNA_v}^H)^{-1} \right), \tag{2.47}
\end{aligned}$$

and for the downstream noise

$$\begin{aligned}
\mathbf{K}_{n_{\tilde{n}\tilde{n}}} &= \psi(\mathbf{H}_{L,0} \cdot \Gamma(\mathbf{H}_{0,\tilde{n}}) + \Gamma(\mathbf{H}_{L,\tilde{n}})^{-1}) \cdot (\Gamma(\mathbf{H}_{0,\tilde{n}})^H \cdot \mathbf{H}_{L,0}^H + \Gamma(\mathbf{H}_{L,\tilde{n}}^H)^{-1}) \\
&= \psi(\mathbf{H}_{L,0} \cdot \Gamma(\mathbf{H}_{0,\tilde{n}} \mathbf{H}_{0,\tilde{n}}^H) \cdot \mathbf{H}_{L,0}^H + \Gamma(\mathbf{H}_{L,\tilde{n}} \mathbf{H}_{L,\tilde{n}}^H)^{-1}). \tag{2.48}
\end{aligned}$$

With these results we can form the final noise covariance matrix as

$$\mathbf{K}_n = \mathbf{H}_{L,0} \mathbb{E}[\mathbf{n}_{AR} \mathbf{n}_{AR}^H] \mathbf{H}_{L,0}^H + \mathbf{K}_{n_{LNA_{vv}}} + \mathbf{K}_{n_{LNA_{cc}}} + \mathbf{K}_{n_{LNA_{vc}}} + \mathbf{K}_{n_{\tilde{n}\tilde{n}}}. \tag{2.49}$$

Chapter 3

Analytical Gradient

In the following the derivation of the gradients will be given. In the first section the signal gradients will be derived, in the later the noise gradients. The gradients are derived to improve the achievable sum rate function of the system. It is given by

$$\begin{aligned} r &= \log_2(\det(\mathbf{K}_s \mathbf{K}_n^{-1} + \mathbf{I}_{N_r})) \\ &= \log_2(\det(\mathbf{K}_s + \mathbf{K}_n^{-1})) - \log_2(\det(\mathbf{K}_n)), \end{aligned} \quad (3.1)$$

where \mathbf{K}_s and \mathbf{K}_n denote the signal and noise covariance matrices derived in (??) and (3.16).

We are interested in the gradient of the achievable sum rate with respect to the matching network and the passive antenna loads. Using the matrix relations from [5] the gradient becomes

$$\frac{\partial r}{\partial \mathbf{Z}_{l,ij}} = \frac{1}{\ln 2} \text{Tr} \left((\mathbf{K}_s + \mathbf{K}_n)^{-1} \left(\frac{\partial \mathbf{K}_s}{\partial \mathbf{Z}_{l,ij}} + \frac{\partial \mathbf{K}_n}{\partial \mathbf{Z}_{l,ij}} \right) - \mathbf{K}_n^{-1} \frac{\partial \mathbf{K}_n}{\partial \mathbf{Z}_{l,ij}} \right), \quad (3.2)$$

with $l \in \{11, 12, 22, pass\}$ denoting the sub-matrices of the matching network or the passive antenna loads and i, j denoting the element in the i -th row and j -th column. In the following the gradient is split up in the signal and the noise part. The derivatives for each covariance matrix will be derived.

3.1 Signal Gradients

For the signal covariance matrix, we see from Equation (??), that we can take the derivative of each sub transfer function and place them together afterwards by the chain rule. First of all, we check which sub transfer functions are affected by the gradient. We see that for the matching network sub matrices, we need to take the derivative of the transfer functions $\mathbf{H}_{LNA,M}$ and $\mathbf{H}_{M,0}$. For the passive antenna loads \mathbf{Z}_{pass} obviously \mathbf{H}_0 for the port reduction is affected. Additionally $\mathbf{H}_{M,0}$ is affected because it contains $\mathbf{Z}_{C'}$.

For the following \mathbf{J}_{ij} denotes the single entry matrix, with respect to the i -th row and the j -th column. It follows for the port reduction transfer function

$$\frac{\partial \mathbf{H}_0}{\partial \mathbf{Z}_{pass,ij}} = [\mathbf{0}_{N_i} \quad \mathbf{Z}_{OL}(\mathbf{Z}_{pass} + \mathbf{Z}_{LL})^{-1} \mathbf{J}_{ij}(\mathbf{Z}_{pass} + \mathbf{Z}_{LL})^{-1}] \quad (3.3)$$

with $\mathbf{0}_{N_i}$ the N_i by N_i all-zeros matrix. For the voltage divider before the matching

network it follows,

$$\frac{\partial \mathbf{H}_{M,0}}{\partial \mathbf{Z}_{11,ij}} = \mathbf{J}_{ij}(\mathbf{Z}_{eqM_1} + \mathbf{Z}_{C'})^{-1} - \mathbf{Z}_{eqM_1}(\mathbf{Z}_{eqM_1} + \mathbf{Z}_{C'})^{-1} \mathbf{J}_{ij}(\mathbf{Z}_{eqM_1} + \mathbf{Z}_{C'})^{-1}, \quad (3.4)$$

$$\begin{aligned} \text{with } \frac{\partial \mathbf{Z}_{eqM_1}}{\partial \mathbf{Z}_{M12}} &= \mathbf{J}_{ij}^T(\mathbf{Z}_{M22} + \mathbf{Z}_{eqLNA_1})^{-1} \mathbf{Z}_{M12} + \mathbf{Z}_{M12}^T(\mathbf{Z}_{M22} + \mathbf{Z}_{eqLNA_1})^{-1} \mathbf{J}_{ij}, \\ \frac{\partial \mathbf{H}_{M,0}}{\partial \mathbf{Z}_{12,ij}} &= \frac{\partial \mathbf{Z}_{eqM_1}}{\partial \mathbf{Z}_{M12}}(\mathbf{Z}_{eqM_1} + \mathbf{Z}_{C'})^{-1} - \mathbf{Z}_{eqM_1}(\mathbf{Z}_{eqM_1} + \mathbf{Z}_{C'})^{-1} \frac{\partial \mathbf{Z}_{eqM_1}}{\partial \mathbf{Z}_{M12}}(\mathbf{Z}_{eqM_1} + \mathbf{Z}_{C'})^{-1}, \end{aligned} \quad (3.5)$$

$$\begin{aligned} \text{with } \frac{\partial \mathbf{Z}_{eqM_1}}{\partial \mathbf{Z}_{M22}} &= \mathbf{Z}_{M12}^T(\mathbf{Z}_{M22} + \mathbf{Z}_{eqLNA_1})^{-1} \mathbf{J}_{ij}^T(\mathbf{Z}_{M22} + \mathbf{Z}_{eqLNA_1})^{-1} \mathbf{Z}_{M12}, \\ \frac{\partial \mathbf{H}_{M,0}}{\partial \mathbf{Z}_{22,ij}} &= \frac{\partial \mathbf{Z}_{eqM_1}}{\partial \mathbf{Z}_{M22}}(\mathbf{Z}_{eqM_1} + \mathbf{Z}_{C'})^{-1} - \mathbf{Z}_{eqM_1}(\mathbf{Z}_{eqM_1} + \mathbf{Z}_{C'})^{-1} \frac{\partial \mathbf{Z}_{eqM_1}}{\partial \mathbf{Z}_{M22}}(\mathbf{Z}_{eqM_1} + \mathbf{Z}_{C'})^{-1}, \end{aligned} \quad (3.6)$$

$$\begin{aligned} \text{and with } \frac{\partial \mathbf{Z}_{C'}}{\partial \mathbf{Z}_{pass,ij}} &= \mathbf{Z}_{OL}(\mathbf{Z}_{pass} + \mathbf{Z}_{LL})^{-1} \mathbf{J}_{ij}(\mathbf{Z}_{pass} + \mathbf{Z}_{LL})^{-1} \mathbf{Z}_{LO} \\ \frac{\partial \mathbf{H}_{M,0}}{\partial \mathbf{Z}_{pass,ij}} &= -\mathbf{Z}_{eqM_1}(\mathbf{Z}_{eqM_1} + \mathbf{Z}_{C'})^{-1} \frac{\partial \mathbf{Z}_{C'}}{\partial \mathbf{Z}_{pass,ij}}(\mathbf{Z}_{eqM_1} + \mathbf{Z}_{C'})^{-1}. \end{aligned} \quad (3.7)$$

Last, the transfer function over the matching network is affected as follows

$$\frac{\partial \mathbf{H}_{LNA,M}}{\partial \mathbf{Z}_{11,ij}} = -\mathbf{H}_{LNA,M} \mathbf{J}_{ij} \mathbf{Z}_{M11}^{-1} \mathbf{Z}_{M12}(\mathbf{Z}_{eqLNA_1} + \mathbf{Z}_{eqM_2})^{-1} \mathbf{Z}_{M12} \mathbf{Z}_{M11}^{-1} - \mathbf{H}_{LNA,M} \mathbf{J}_{ij} \mathbf{Z}_{M11}^{-1} \quad (3.8)$$

$$\begin{aligned} \frac{\partial \mathbf{H}_{LNA,M}}{\partial \mathbf{Z}_{12,ij}} &= \mathbf{Z}_{eqLNA_1}(\mathbf{Z}_{eqLNA_1} + \mathbf{Z}_{eqM_2})^{-1} (-\mathbf{J}_{ij}^T \mathbf{Z}_{M11}^{-1} \mathbf{Z}_{M12} - \mathbf{Z}_{M12}^T \mathbf{Z}_{M11}^{-1} \mathbf{J}_{ij}) \cdot \\ &\quad (\mathbf{Z}_{eqLNA_1} + \mathbf{Z}_{eqM_2})^{-1} \mathbf{Z}_{M12} \mathbf{Z}_{M11}^{-1} + \mathbf{Z}_{eqLNA_1}(\mathbf{Z}_{eqLNA_1} + \mathbf{Z}_{eqM_2})^{-1} \mathbf{J}_{ij} \mathbf{Z}_{M11}^{-1}, \end{aligned} \quad (3.9)$$

$$\frac{\partial \mathbf{H}_{LNA,M}}{\partial \mathbf{Z}_{22,ij}} = \mathbf{Z}_{eqLNA_1}(\mathbf{Z}_{eqLNA_1} + \mathbf{Z}_{eqM_2})^{-1} \mathbf{J}_{ij}(\mathbf{Z}_{eqLNA_1} + \mathbf{Z}_{eqM_2})^{-1} \mathbf{Z}_{M12} \mathbf{Z}_{M11}^{-1} \quad (3.10)$$

With these derivations, we can apply the chain rule on the transfer function in (??) to get $\frac{\partial \mathbf{K}_s}{\partial \mathbf{Z}_{l,ij}}$.

3.2 Noise Gradients

As written in Section ??, we have four noise sources. Because they sum up at the loads after the LNA, we can look at each noise transfer function separately and analyze it on its own.

3.2.1 Antenna Noise Gradient

As the antenna noise is picked up at the same place as the signal, its gradient is also the same as the signal covariance gradient. The formulas from (3.3) to (3.15) apply in the same manner to the antenna noise transfer functions. Therefore they are omitted here. We only note, that the transfer functions \mathbf{H}_0 , $\mathbf{H}_{M,0}$ and $\mathbf{H}_{LNA,M}$ are affected in the derivation of $\frac{\partial \mathbf{K}_n}{\partial \mathbf{Z}_{l,ij}}$, with $l \in \{11, 12, 22, pass\}$.

3.2.2 LNA Noise Gradient

Looking at the transfer function of the two LNA noise sources, we see that neither $\mathbf{H}_{L,LNA}$ nor \mathbf{Z}_{eqLNA_1} are affected by the derivations. Only $\tilde{\mathbf{Z}}_{eqM_2}$ and therefore \mathbf{H}_{L,LNA_v} have to be taken into account. It follows,

$$\frac{\partial \tilde{\mathbf{Z}}_{eqM_2}}{\partial \mathbf{Z}_{11,ij}} = \mathbf{Z}_{M12}(\mathbf{Z}_{M11} + \mathbf{Z}_{C'})^{-1} \mathbf{J}_{ij}(\mathbf{Z}_{M11} + \mathbf{Z}_{C'})^{-1}, \quad (3.11)$$

$$\frac{\partial \tilde{\mathbf{Z}}_{eqM_2}}{\partial \mathbf{Z}_{12,ij}} = -\mathbf{J}_{ij}(\mathbf{Z}_{M11} + \mathbf{Z}_{C'})^{-1} \mathbf{Z}_{M21} - \mathbf{Z}_{M12}(\mathbf{Z}_{M11} + \mathbf{Z}_{C'})^{-1} \mathbf{J}_{ij}^T, \quad (3.12)$$

$$\frac{\partial \tilde{\mathbf{Z}}_{eqM_2}}{\partial \mathbf{Z}_{22,ij}} = \mathbf{J}_{ij}, \quad (3.13)$$

and with $\frac{\partial \mathbf{Z}_{C'}}{\partial \mathbf{Z}_{pass,ij}} = \mathbf{Z}_{OL}(\mathbf{Z}_{pass} + \mathbf{Z}_{LL})^{-1} \mathbf{J}_{ij}(\mathbf{Z}_{pass} + \mathbf{Z}_{LL})^{-1} \mathbf{Z}_{LO},$

$$\frac{\partial \tilde{\mathbf{Z}}_{eqM_2}}{\partial \mathbf{Z}_{pass,ij}} = \mathbf{Z}_{M12}(\mathbf{Z}_{M11} + \mathbf{Z}_{C'})^{-1} \frac{\partial \mathbf{Z}_{C'}}{\partial \mathbf{Z}_{pass,ij}} (\mathbf{Z}_{M11} + \mathbf{Z}_{C'})^{-1} \mathbf{Z}_{M21}, \quad (3.14)$$

which results in

$$\frac{\mathbf{H}_{L,LNA_v}}{\partial \mathbf{Z}_{l,ij}} = -\mathbf{H}_{L,LNA} \mathbf{Z}_{eqLNA_1} (\mathbf{Z}_{eqLNA_1} + \tilde{\mathbf{Z}}_{eqM_2})^{-1} \frac{\partial \tilde{\mathbf{Z}}_{eqM_2}}{\partial \mathbf{Z}_{l,ij}} (\mathbf{Z}_{eqLNA_1} + \tilde{\mathbf{Z}}_{eqM_2})^{-1}. \quad (3.15)$$

3.2.3 Downstream Noise Gradient

Last, we see, that the transfer function of the downstream noise $\mathbf{H}_{L,\tilde{n}}$ is not affected by any of the derivations above under the unilateral assumption. Otherwise, $\tilde{\mathbf{Z}}_{eqLNA_2}$ is a function of the matching network and the passive antenna loads. It follows,

3.2.4 Noise Gradient

We can now write the gradient of the noise covariance matrix as

$$\begin{aligned} \frac{\partial \mathbf{K}_n}{\partial \mathbf{Z}_{l,ij}} &= \frac{\partial \mathbf{H}_{L,0}}{\partial \mathbf{Z}_{l,ij}} \mathbf{R}_{na} \mathbf{H}_{L,0}^H + \mathbf{H}_{L,0} \mathbf{R}_{na} \frac{\partial \mathbf{H}_{L,0}^H}{\partial \mathbf{Z}_{l,ij}} \\ &\quad + \frac{\partial \mathbf{H}_{L,LNA_v}}{\partial \mathbf{Z}_{l,ij}} \beta \left(R_N^2 \mathbf{I}_{N_r} - 2R_N \mathbb{R}\{\rho^* \tilde{\mathbf{Z}}_{eqM_2}\} + \tilde{\mathbf{Z}}_{eqM_2} \tilde{\mathbf{Z}}_{eqM_2}^H \right) \mathbf{H}_{L,LNA_v}^H + \\ &\quad \mathbf{H}_{L,LNA_v} \beta \left(2R_N \frac{\partial \mathbb{R}\{\rho^* \tilde{\mathbf{Z}}_{eqM_2}\}}{\partial \mathbf{Z}_{l,ij}} + \frac{\partial \tilde{\mathbf{Z}}_{eqM_2}}{\partial \mathbf{Z}_{l,ij}} \tilde{\mathbf{Z}}_{eqM_2}^H + \tilde{\mathbf{Z}}_{eqM_2} \left(\frac{\partial \tilde{\mathbf{Z}}_{eqM_2}}{\partial \mathbf{Z}_{l,ij}} \right)^H \right) \mathbf{H}_{L,LNA_v}^H + \\ &\quad \mathbf{H}_{L,LNA_v} \beta \left(R_N^2 \mathbf{I}_{N_r} - 2R_N \mathbb{R}\{\rho^* \tilde{\mathbf{Z}}_{eqM_2}\} + \tilde{\mathbf{Z}}_{eqM_2} \tilde{\mathbf{Z}}_{eqM_2}^H \right) \frac{\partial \mathbf{H}_{L,LNA_v}^H}{\partial \mathbf{Z}_{l,ij}}. \end{aligned} \quad (3.16)$$

Chapter 4

Problem Statement and Solver Methods

In the following, the problem to optimize will be introduced and analyzed. Although the thesis is not looking into finding the optimal solution, different solver strategies have to be introduced and compared to each other, as the utility function is not trivial and the differences are severe. Their advantages and disadvantages will be shown. If not specifically mentioned, we will look at a 2x2 MIMO system with one receive antenna per user and three relays, as shown in Figure ???. The relays in this system will be lossless, i.e. the impedances will be pure imaginary.

4.1 Utility Function

As the aim for wireless communication networks is to maximize the achievable rate, we take the formulas derived in Section ??, which describe the rates for each transmit-receive pair. In order to get an utility function from the achievable rates,

we stack them into the vector $\mathbf{r} = \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_N \end{bmatrix}$.

To optimize the rates, different approaches are possible.

- Optimize the minimum rate, i.e. $\max(\min(\mathbf{r}))$ (maxmin),
- Optimize the mean rate, i.e. $\max(\sum(\mathbf{r}))$ (maxsum), and
- Optimize the maximum rate, i.e. $\max(\max(\mathbf{r}))$ (maxmax).

This is done using the LP-norm ($\|\mathbf{r}\|_p$) [6]. By setting $p = -\text{Inf}$, the "maxmin" method is achieved. Setting $p = 1$, the "maxsum" method is applied and setting $p = +\text{Inf}$, the "maxmax" method is applied. For all values in between, the utility function "tends" to optimize the maximum and minimum value respectively.

In the following we restrict ourselves to the "maxsum" method. As the other methods easily can be applied and will lead to a similarly good result without loss of generality.

4.1.1 Convexity of the Utility Function

In the following the convexity of the utility function is analyzed. This is done, because the choice of the solver depends on the shape of the utility function (e.g. in

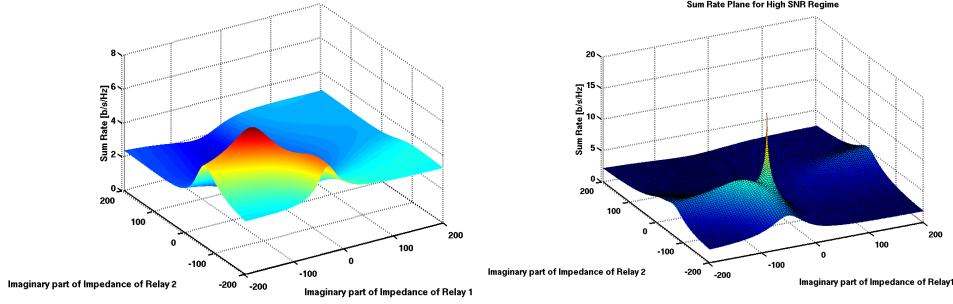


Figure 4.1: The utility function for different imaginary relay impedances for two relays and two different channel realizations.

case of a concave utility function, a normal gradient search is sufficient to provide good results).

To do so, we analyze the utility function at different input values for the passive relays and at different input power values. It is clear, that the function changes (heavily) for every channel realization. Figure 4.1 shows two different channel realizations for the same input power and the same domain of two passive impedances.

4.1.2 Dependency on the Input Power

We will analyze now the utility function for different input powers. In Figure ?? and Figure ?? we see the same channel realization for two different input powers. Figure ?? shows again the same channel realization at an input power close to the input power of Figure ??.

First we observe, that the optima in Figure ?? and Figure ?? lie at very different impedance values of the relays, therefore we can conclude, that an optimal solution for one input power does not necessarily lead to a good solution for a different input power solution.

Next we analyze the optima of Figure ?? and Figure ??. Here we can see again, that using the values from Figure ?? in Figure ?? will not lead to the best solution, however it would serve as a good initial value for a gradient search method.

4.1.3 Difference of Local Optima

Last, we want to look at the local optima we can possibly run into. We see in Figure ??, that if we hit the local optimum at the left, the performance will be nearly the same as with the local optimum at the right. However looking at Figure ??, we see that the difference of the local optimum before the peak and the peak itself is severe. Therefore, we can not only try to run the optimization with one initialization, but instead, we need multiple initializations.

4.1.4 Complexity of the Problem

As we will discuss different solver approaches in the following Sections, a short description of the complexity of the problem is given. For each relay we use, we will have one variable (N_{Rel}), if we restrict the relays to the imaginary domain. Allowing the relay to become lossy and therefore the impedance complex, the number of variables is doubled. Further for each receiver branch, i.e. receiver antenna, we have a reciprocal (lossless) matching network (see Section ??), which has to be optimized.

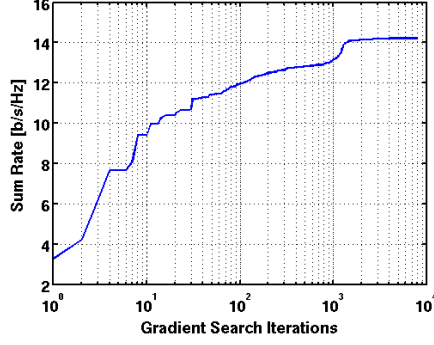


Figure 4.2: The sum rate over the gradient search iterations.

Therefore we have three elements per branch (in total: $3 \cdot N_R \cdot N_{R_x}$). For a four user MIMO system with two receive antennas and five lossy relays per user, this would lead to a problem of size $N_{\text{var}} = 2 \cdot 4 \cdot 5 + 3 \cdot 4 \cdot 2 = 64$. As mentioned in the beginning of this chapter, we look at a 2x2 MIMO system with one receive antenna and three lossless relays. The complexity of this system is therefore $N_{\text{var}} = 1 \cdot 2 \cdot 3 + 3 \cdot 2 \cdot 1 = 12$.

4.2 Gradient Search

Despite this large number of variables and the non-convexity of the utility function, the first approach remains a gradient search. Figure 4.2 shows the typical behaviour of the gradient search for a realization over the iteration steps. In the following, different methods are described, to improve the algorithm.

4.2.1 Choice of Initial Values

We try to overcome the non-pleasant properties of the problem with a larger number of initial guesses, so that the gradient search approach tends more towards a grid search optimization method. The gradient search routine itself is then used as a refinement step of the grid search.

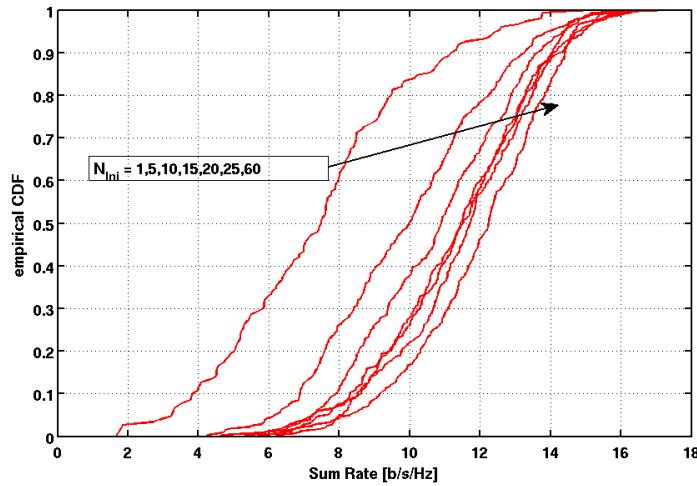


Figure 4.3: Comparison of the number of initial values used.

Figure 4.3 shows the empirical CDFs of the optimized sum rate, for different numbers of initializations. The initial values were drawn uniformly at random for $Z_{\text{Rel}}[i] \in [-600, 600]j$, $\forall i \in [1, N_{\text{Rel}}]$.

It is obvious and clear to see, that the larger the number of initializations, the better the result. However, even with 25 and 60 initializations, there improvement is still immense, which shows, that the number of initializations must be a lot larger than twice the input vector length. A good tradeoff between a decent optimization and a comparable small runtime of the optimization is achieved by a choice of four times the input vector length.

4.2.2 Adaptive Step Size

The performance of the gradient search routine is improved by the use of an adaptive step size. For each calculated gradient, the step taken into the direction of the gradient is increased until the new rate value is smaller than the previous calculated value as shown in Figure 4.4. Therefore the number of time-expensive gradient calculations is reduced immense.

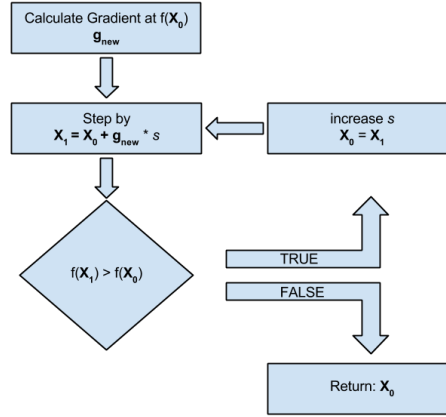


Figure 4.4: Schematic of the adaptive step size algorithm.

4.2.3 Conjugate Gradient

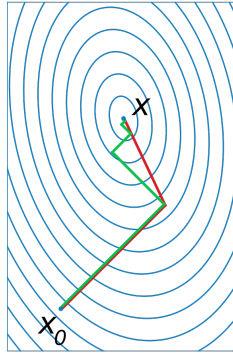


Figure 4.5: A comparison of the convergence of gradient descent with optimal step size (in green) and conjugate vector (in red) for minimizing a quadratic function associated with a given linear system[7].

As the gradient search performance still requires too many iterations, further improvements were made. The shape of such complex problems can - in some cases - have the shape of a crest, where, with a unpleasant choice of the initial value, a pure gradient search routine might jump around the optimum, without improving a lot. Conjugate gradient routines like "Fletcher-Reeves" or "Polak-Ribière" lead to a faster convergence, as they weight the gradient by the previous gradient and a weight factor β .

Like Polak-Ribière, the Fletcher-Reeves method updates the conjugate direction according to

$$\mathbf{s}_n = \mathbf{g}_n + \beta_n \cdot \mathbf{g}_{n-1}, \quad (4.1)$$

where \mathbf{g} denotes the gradient. The conjugate direction \mathbf{s} is then used to perform the conjugate gradient search. Polak-Ribière and Fletcher-Reeves differ in the way, they calculate the weight factor β .

Fletcher-Reeves

For the method of Fletcher-Reeves, β is calculated according to [8]

$$\beta_n^{FR} = \frac{\mathbf{g}_n^T \mathbf{g}_n}{\mathbf{g}_{n-1}^T \mathbf{g}_{n-1}}. \quad (4.2)$$

Polak-Ribière

For the method of Polak-Ribière, β is calculated according to [9]

$$\beta_n^{PR} = \frac{\mathbf{g}_n^T (\mathbf{g}_n - \mathbf{g}_{n-1})}{\mathbf{g}_{n-1}^T \mathbf{g}_{n-1}}. \quad (4.3)$$

In Figure 4.6 the pure gradient search (steepest ascent) method is compared to the methods of Fletcher Reeves and Polak-Ribière. The Figure shows once the optimization after 50 iterations (solid lines), to see, which method optimizes the problem the best after just a few steps, and at 500 iterations, to see which routine might get caught in the shape of the problem.

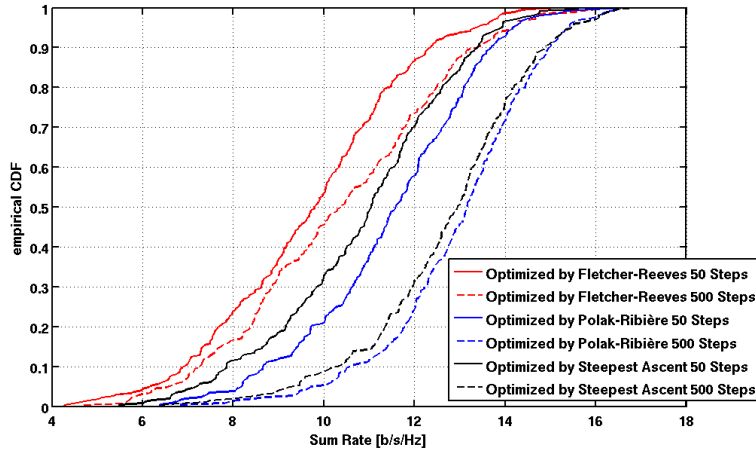


Figure 4.6: Comparison between Steepest Ascent, Polak-Ribière, and Fletcher-Reeves.

We can see, that Fletcher-Reeves is not well suited for our kind of optimization problem. Better is Polak-Ribière, which outperforms the standard gradient search method at 50 iterations and has a slightly better performance at 500 iterations.

4.3 Heuristic Optimization Algorithms

Due to the non convexity and the non trivial optimization problem, we further analyze (some) heuristic optimization methods. In the following three heuristic algorithms will be introduced and analyzed by their performance. They were chosen, because they already exist in the MATLAB library and do not require any strenuous implementation.

4.3.1 Simulated Annealing

Simulated Annealing was developed, as there is a deep and useful connection between statistical mechanics (the behavior of systems with many degrees of freedom in thermal equilibrium at a finite temperature) and multivariate or combinatorial optimization (finding the minimum of a given function depending on many parameters) [10]. In this thesis, Simulated Annealing was chosen as it is a method to solve optimization problems of multivariate optimization.

The Algorithm used in this thesis is a build-in function of MATLAB®. A description can be found on the MathWorks homepage [11].

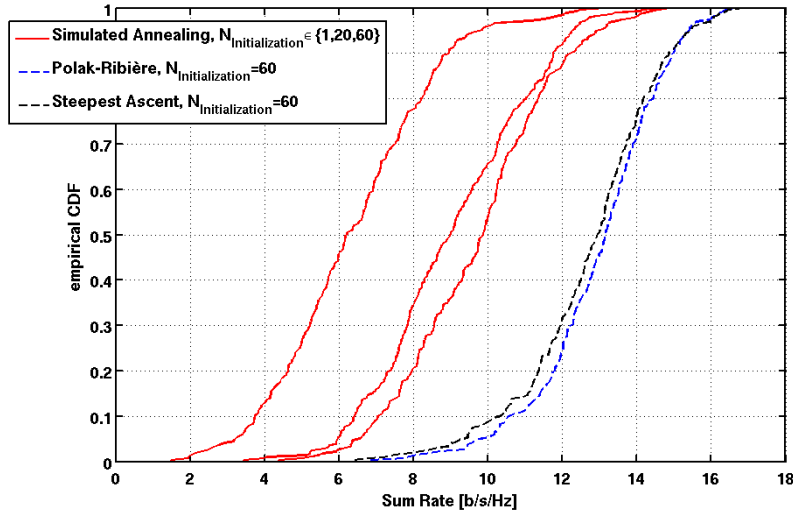


Figure 4.7: Comparison of the Simulated Annealing algorithm for different number of initializations and the results from "Polak-Ribière" and "Steepest Ascent" gradient searches.

Figure 4.7 shows the performance of Simulated Annealing algorithm dependent on the choice of numbers of initializations (red curves). Again we see, that the more initializations we have, the better the sum rate can be optimized. However, we also see, that even with 60 initializations, Simulated Annealing leads to a result, which is worse than the sum rates achieved by steepest ascent (black dashed curve) and Polak-Ribière (blue dashed curve).

4.3.2 GlobalSearch

The other two heuristic optimization algorithms are GlobalSearch (GS) and MultiStart (MS). They are very similar to each other, the only difference is the choice of the starting values. As MultiStart requires the number of initialization values,

GlobalSearch generates trial points on its own [12]. GlobalSearch - the optimization function, which we will analyze - is based on the MATLAB©built-in function *fmincon*. Figure 4.8 shows the schematics of GS and MS.

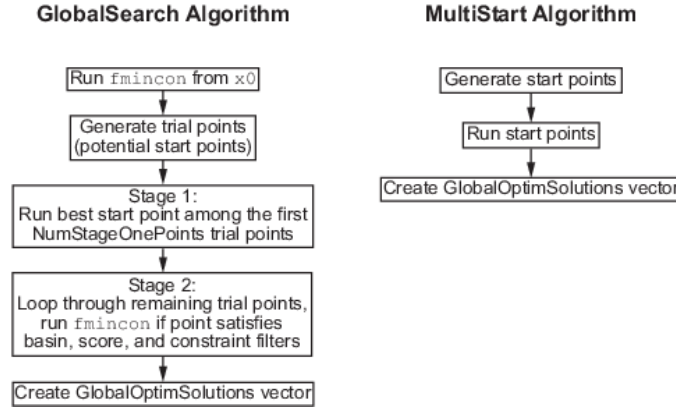


Figure 4.8: Schematics of the GlobalSearch and MultiStart algorithms [12].

Figure 4.9 shows the performance of GlobalSearch (red curve). We see, that it has almost the same performance as the conjugate gradient method by Polak-Ribière with 60 initializations (blue curve). GlobalSearch is less dependent on the initial value - for the relays it is chosen very large ($\mathbf{X}_{\text{Rel}} = 1000j$) and not at random, compared to the gradient search methods, to simulate an open circuited antenna and therefore no coupling.

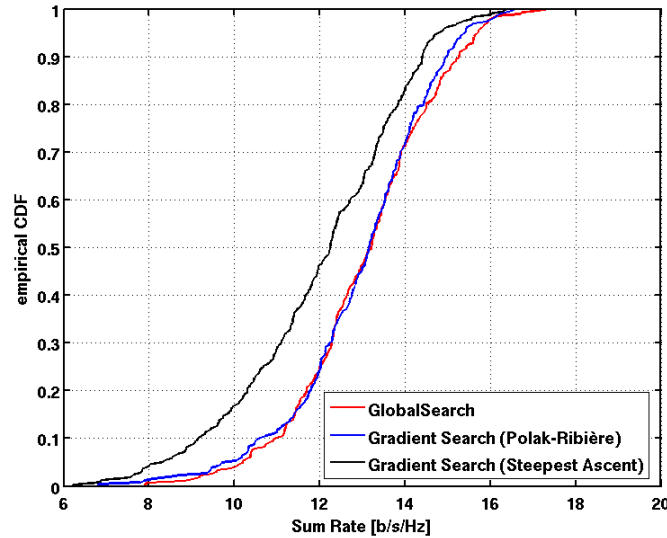


Figure 4.9: Performance of the GlobalSearch algorithm in comparison to the previous results.

4.4 Further Algorithm Improvements

4.4.1 Optimization of the Interference Function

A further approach to optimize the gradient search routine is a good initial guess. The problem of the sum rate maximization is reduced only to the description of the interference. Therefore the number of variables can be reduced by a factor of two third for the matching network, as the interference function

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only requires the input impedance of the matching network. This reduced problem has size of $N_{\text{var}} = 2 \cdot N_{\text{Rel}} + 2 \cdot N_{\text{R}} \cdot N_{\text{Rx}} = 2 \cdot 4 \cdot 5 + 2 \cdot 4 \cdot 2 = 56$, for the same settings as above. The factor 2 from the matching network variables comes from the fact, that with pure imaginary elements in the matching network, any complex value of the input impedance can be achieved.

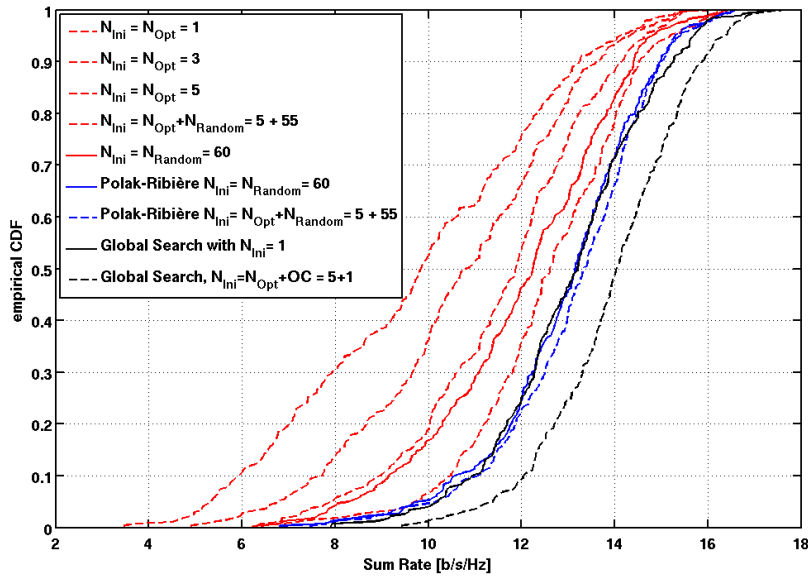


Figure 4.10: Comparison of the number of initial values used.

Figure 4.10 shows the performance of the steepest ascent algorithm, with the pre-optimized initial values (three red dotted curves on the left). We see, that using the five best pre-optimizations leads to a performance almost as good as initializing the algorithm with 60 random vectors (red solid curve). To push the optimization even further, we can combine these two techniques and take the best optimization from the five best pre-optimized initial values and 55 random initial values (red dotted curve on the right). This leads to a 0.5 [b/s/Hz] improvement at the mean and a 1.5 [b/s/Hz] improvement for the lowest 10% of the rates, than only considering random values.

Applying this to the method of Polak-Ribière (blue dotted curve), however, does not improve the results as much as with the steepest ascent method. A sum rate of only about 0.1 [b/s/Hz] higher is achieved by using pre-optimized initial values in combination with the method of Polak-Ribière.

Finally, this method is applied to the heuristic GlobalSearch routine. As mentioned in Section 4.3.2, GlobalSearch is less dependent on the initial value and therefore, the initial value is chose to be open circuited for the relays. With five optimized

initial values plus the standard open circuit initial value, an improvement of almost 1.0 [b/s/Hz] can be achieved (black dotted curve) compared to initializing only by the open circuited value (black solid curve).

4.4.2 Post Refinement of GlobalSearch by Gradient Search

Post refinement by gradient search

4.4.3 Stepwise Relay Improvements

In the previous Sections, we saw the performance of different methods for three relays per user. The number of relays was kept small for the analysis of the algorithms, so that complexity effects diminishing the performance were avoided. Increasing the number of relays might result in worse sum rates. Therefore we will have a look at the performance of the optimization and how it can be pushed in the following. For the results shown below, the number of relays were increased to seven, if not mentioned specifically.

In Figure 4.11 we see the performance of the GlobalSearch algorithm, optimizing a 2x2 MIMO system, with seven relays and one receiving antenna per user (red curve). The black curve shows the performance of the algorithm optimizing only three relays per user (c.f. Figure 4.9). Actually this should be a lower limit of the red curve, as one solution for seven relays is to optimize only three of them and open circuit the remaining four. However this is only the case for about 85% of the cases, the lowest 15% actually lead to a worse solution than only optimizing three relays. This shows, that the GlobalSearch optimization might not perform very good on larger problems, i.e. reducing the size of the problem results in a better sum rate.

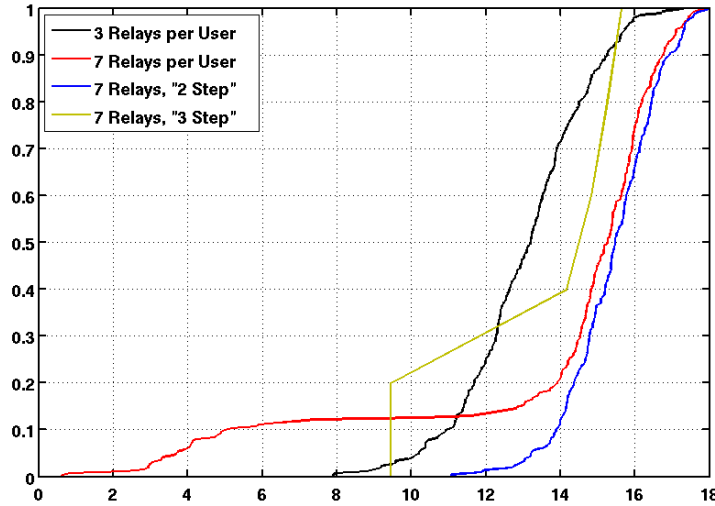


Figure 4.11: Comparison of the number of initial values used.

Therefore one approach to prevent GlobalSearch of running into a low rate, is to reduce the number of relays to optimize and increase them stepwise. The blue curve shows the result of the optimization, when GlobalSearch is allowed to optimize only two relays. Once finished, two additional relays are considered so that in total four relays per user are optimized. This is repeated until the total number of relays is

reached (in this case four times, $N_{\text{Rep}} = \lceil \frac{N_{\text{Relays}}}{N_{\text{Step}}} \rceil = \lceil \frac{7}{2} \rceil = 4$). This leads to a tradeoff between the precision of the routine and the run-time, as a smaller number of relay steps (N_{Step}) will lead (most probably) to a better result, but also to a larger number of repetitions and therefore it will take a lot longer ($\approx N_{\text{Rep}} * T_0$, for T_0 , the time for one optimization).

The yellow curve shows the optimization for a stepwise increment of $N_{\text{Step}} = 3$. Therefore $N_{\text{Rep}} = 3$ had to be taken.

Chapter 5

Results

In the following, the results of the thesis will be discussed. Different solver strategies will be introduced and compared to each other.

Chapter 6

Conclusion and Outlook

6.1 Conclusion

6.2 Future Work

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