"My own brain is to me the most unaccountable of machinery - always buzzing, humming, soaring roaring diving, and then buried in mud. And why? What's this passion for?"

-Virginia Woolf.

Abstract

This work of thesis deals with the study of brain dynamics using the method of the boolean networks (BNs), an useful way to model biological systems defined by a set of nodes that can take only two values (1 and 0) and a list of Boolean functions. The status of the nodes in a BN changes each time step and it's related to the status of the nodes connected to it and to the Boolean function that is assigned.

In the first part of this thesis the main notions useful in understanding the entire document are explained, while in the second part there is the illustration of the whole simulation with the respective results. The cognitive process chosen for the simulation is called "Action-Execution". When the brain has to face a process like this, at a time $t = 0$ a certain number of areas are activated but, after an amount of time, some areas turn off and others turn on: there is a chaotic evolution involving these networks. A program has been written in Wolfram Mathematica that allows to study the temporal evolution of the BNs and it ends with the extraction of 12 circuits that relate various areas between them.

Contents

Chapter 1

Brain modeling and Boolean networks

1.1 An introduction to the Boolean Networks

A complex system is an arrangement made of a great number of elements related to each other with complicate relationships and interconnections. These types of systems need to be modeled due to their complexity and, currently, there are a lot of ways of working with these entities. The type of modeling that is analyzed in this document is the one that foresees the use of the Boolean Networks (BNs) but, before getting into the explanations, it is necessary to introduce some definitions.

A complex system is made by a lot of elements, each of them is called *node* and pairs of them are connected to each other by a link called *edge*. A set of edges and nodes can form a simple graph (a mathematical representation of a real-world network), but the situation gets more complicated when the description of the system goes deeper. An edge is called *directed* when it is oriented from the start to the end, otherwise it is *undirected*. Directed graphs contain only directed edges, undirected graphs contain only undirected edges. There is also another distinction that is important to do, it is the one that concerns the edges (again): there are the ones that are associated with a weight (it can be, for example, the measure of the length of a path or the capacity of a line) and the binary ones (the weights are binarized). One of the simplest ways to represent graphs is the connection matrix. It can define the topology of the graph by associating nodes to the matrix rows and edges to the columns. When talking about directed graphs, it is important to talk about the indegree and outdegree, which are respectively the number of incoming and outcoming edges.

Given these few preliminary definitions, it can be explained what BNs are: they are particular types of networks which consist of a triplet made of a graph G, a set S and a function f.

- The graph can be written as $G = \{V, E\}$, where $V = \{v_1...v_n\}$ (n stands for the number of elements v called vertices) and $E = \{\varepsilon_1...\varepsilon_s\}$ is a set containing the edges between the vertices. $\varepsilon_i \equiv \varepsilon_{jk} \equiv v_j \to v_k$. Each vertex can be 0 or 1: in other words, it can take one of the boolean values.
- $S = \{0, 1\}$;
- f is a function that follows an evolution rule $f : C \to C$, where C represents the space of all possible configurations. The values of the states of all the nodes give the configuration C and its cardinality is equal to 2^n.

The initial state of the system is fundamental: its critical setting will produce different emerging configurations of the system.

If you see the symbol I_i^i, you are in front of the *input neighbourhood* (its cardinality is called in-degree), the set of all nodes v_k. Similarly, the symbol I_i^0 is called *output neighbour* (its cardinality is called out-degree). $I_i = I_i^i \cup I_i^0$. BN is a type of dynamic network because its configuration $c \in C$ is dependent from time.

Given the status of each node $s_i(t+1)$, it can be defined as determined only by the value of the states of the nodes of the input neighbour. Mathematically speaking, $s_i(t+1) = f_i(s_j(t), s_k(t)...s_i(t))$, where each s represents the state of the node belonging to the input neighbour. This equation is given by a table T_i called table of rules.

The evolution rule of the BN is defined as the set T of the T_i: $T = \{T_1...T_n\}$. Then the number of rules contained in T will be $n_T = \sum_{i=1}^{n} id_i$. It follows automatically that all the possible combinations of inputs are given by 2^{n_T}. At a certain time t, the state of a node is given by $x_i(t)$. After each iteration, the next state will be

$$x_i(t+1) = b_i(x_{i1}(t)...x_{jk}(t)),$$

where the last two subscripts j and k are referred to the states of the nodes related to i.

The update of the states of the nodes of the network happens simultaneously for every state and this process runs until the reaching of a stable point. In this way it is possible to explore the connection between dynamics and brain network structure. The connection of each node of the BN to every other input node is defined by the connectome and, after choosing an initial state, the sums of the values of the input regions can be calculated and, if at step t the ith node is connected to a number of a switched node that is within a given range, it will be switched on at time $t + 1$ and the activation gets on, otherwise the node will be turned off.

It is important to distinguish six types of boolean networks:

- The Asynchronous Random Boolean Networks(ARBNs): they follow a not-deterministic method. At each step a randomly selected node is chosen and it is updated with probability $1/N$.

- The Classic Random Boolean Networks (CRBNs): they are characterized by a deterministic updating: all nodes are updated simultaneously in the passage from time t to time $t + 1$.

- The Deterministic Asynchronous Random Boolean Networks(DARBNs) follow a deterministic method where two parameters p and q are assigned to each node (it is necessary to choose a maximum value for p and q), with $q < p$. Each node will be updated in the r-th step if $q = r(\mod p)$.

- The Generalized Deterministic Asynchronous Random Boolean Networks (DGARBNs) follows a deterministic rule, the same as DARBN's, but if more nodes meet the updating condition, they are updated simultaneously.
- The Probabilistic Boolean Networks (PBN) are updated, as the name suggests, randomly.
- The Generalized Asynchronous Random Boolean Networks (GARBNs) follow a non-deterministic method. After choosing a number g of nodes, at each step, only those are updated.

1.2 The brain as a complex system

As said before, the complex systems are arrangements made of a great number of elements related to each other with complicate relationships and interconnections. The brain represents perfectly these entities due to its complexity. It contains a lot of interconnected neurons (the information messengers of the nervous system) that constitute networks. These connections make up the connectome of the brain, that is built mapping where neurons are connected through synapses (the spaces between two neurons, where they can communicate).

The connectome is very important for the comprehension of dynamic brain data because it shows how each neuron is connected and how each of them interacts with the other. Neurons form a real and complex network, so it's fundamental to study the brain through the net-science for trying to understand its behaviour. In this document the mapping of the connectome is obtained from a file taken from Brainmap.org, a site that gives maps of many human brains. This mapping is possible thanks to the Broadmann theory, a way of seeing the brain as a set of different areas. The anatomist Korbinian Brodmann said that, considering the connections and the types of neurons, one can distinguishes 52 types of Broadmann areas that cover the primary somatosensory cortex, the primary motor cortex, the somatosensory association cortex, the premotor and supplementary motor cortex, the dorsolateral/anterior prefrontal cortex, the anterior prefrontal cortex, and primary visual and audio cortexes and additional areas. These regions have different functions, as one can see in picture 1 of the Appendix.

The first step for the mapping process was downloading the activation maps for the cognitive function of Action-Execution by Brainmap.org . The major functional networks of the brain were already identified, representative of a big part of all functional activation studies performed on more than 7000 functional maps, derived from different experimental fMRI (functional magnetic resonance imaging) conditions, summarized using ICA (this acronym stands for independent component analysis. It is a data-driven approach used for finding independent patterns in multivariate data stored in the BrainMap.org data repository.) based on coordinated positions of activation peaks. It is necessary a conversion from Brodmann areas to the corresponding node number in the connectome model. For doing this operation it is useful the Talaraich repository, a coordinate system that allows the location of any point of the brain to be located with reference to an atlas published by the doctors Jean Talairach. This frame of reference is based on a series of deformations which allow any type of brain to be realigned to the model brain described by these two scientists in their atlas. This conversion is shown in the second picture of the Appendix.

The connections between the various Broadmann areas are given by a connection matrix 82×82 (as shown in the table) whose values are between 0 and 1. The connectome modeled on these areas assumes this form:

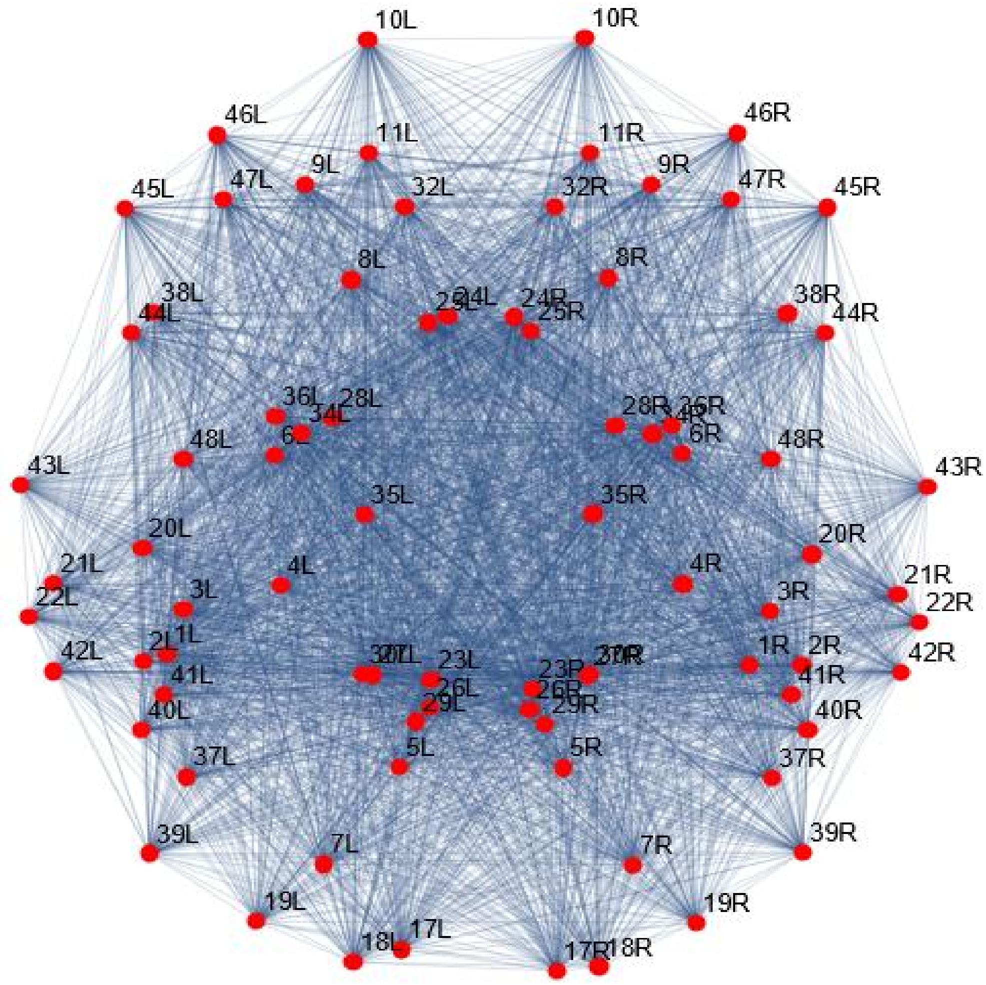
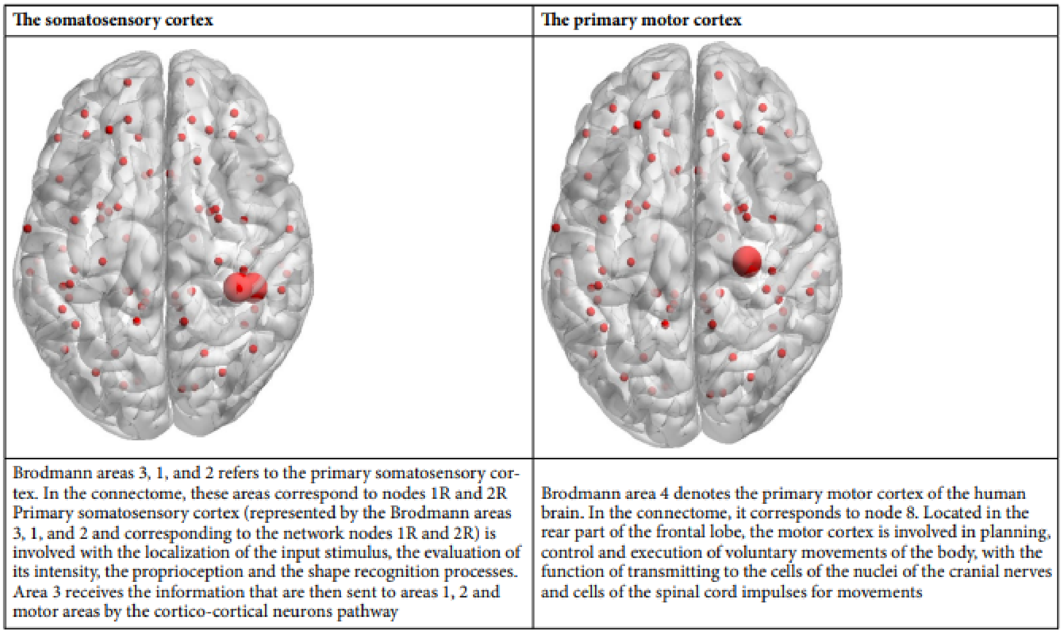


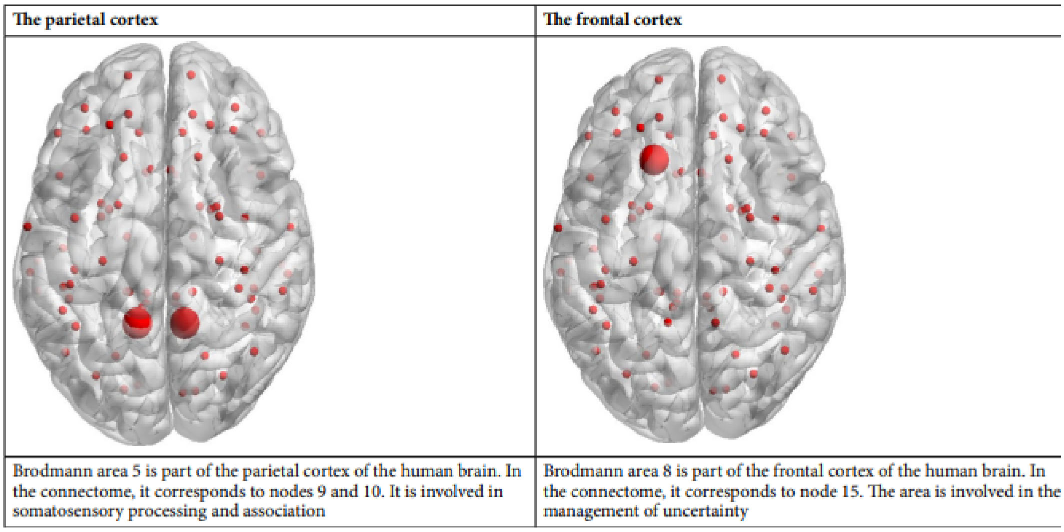
Figure 1.1

Now, before the explanation of the steps of the simulation, let's have a look to the functions of the main Broadmann areas (the following tables are taken from the article "Modelling brain dynamics by Boolean networks" published on Scientific Reports).



The somatosensory cortex	The primary motor cortex
Brodmann areas 3, 1, and 2 refers to the primary somatosensory cortex. In the connectome, these areas correspond to nodes 1R and 2R Primary somatosensory cortex (represented by the Brodmann areas 3, 1, and 2 and corresponding to the network nodes 1R and 2R) is involved with the localization of the input stimulus, the evaluation of its intensity, the proprioception and the shape recognition processes. Area 3 receives the information that are then sent to areas 1, 2 and motor areas by the cortico-cortical neurons pathway	Brodmann area 4 denotes the primary motor cortex of the human brain. In the connectome, it corresponds to node 8. Located in the rear part of the frontal lobe, the motor cortex is involved in planning, control and execution of voluntary movements of the body, with the function of transmitting to the cells of the nuclei of the cranial nerves and cells of the spinal cord impulses for movements

Figure 1.2



The parietal cortex	The frontal cortex
Brodmann area 5 is part of the parietal cortex of the human brain. In the connectome, it corresponds to nodes 9 and 10. It is involved in somatosensory processing and association	Brodmann area 8 is part of the frontal cortex of the human brain. In the connectome, it corresponds to node 15. The area is involved in the management of uncertainty

Figure 1.3

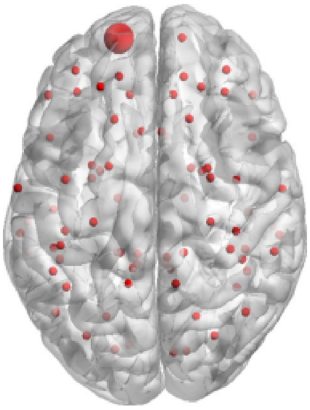
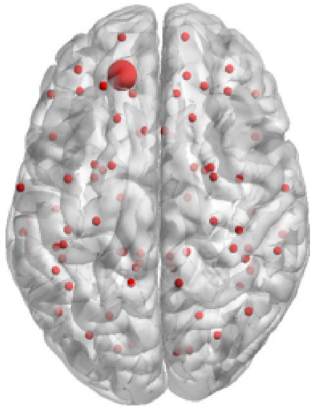
The anterior portion of the prefrontal cortex	**The frontal cortex**
	
Brodmann area 10 is the most anterior portion of the prefrontal cortex of the human brain. In the connectome, it corresponds to node 19. Brodmann area 10, the largest area of the human brain and the most unknown, is involved in strategic processes in memory recall and various executive functions	Brodmann area 11 is part of the frontal cortex in the human brain. In the connectome, it corresponds to node 21. It is involved in decision making and processing rewards, planning, encoding new information into long-term memory, and reasoning

Figure 1.4

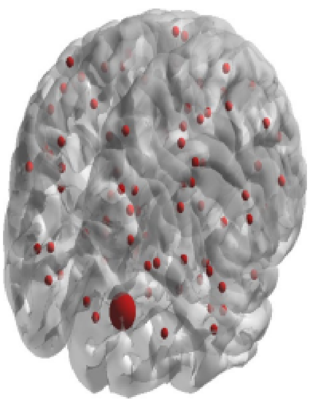
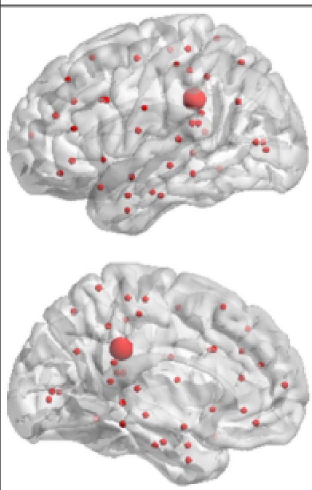
The extra striate cortex	**The posterior cingulate cortex**
	
Brodmann area 19 is part of the occipital lobe cortex in the human brain. In the connectome, it corresponds to node 28. In humans with normal sight, extrastriate cortex is a visual association area, with feature-extracting, shape recognition, attentional, and multimodal integrating functions	Brodmann area 23 corresponds to some portion of the posterior cingulate cortex. In the connectome, it corresponds to node 35. It communicates with various brain networks simultaneously and is involved in various functions such as human awareness, pain, and episodic memory retrieval

Figure 1.5

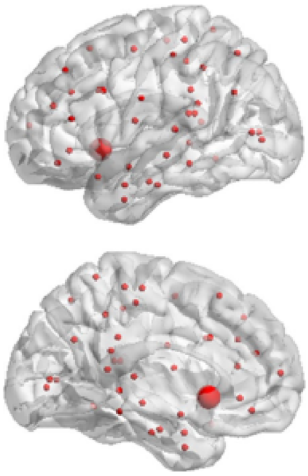
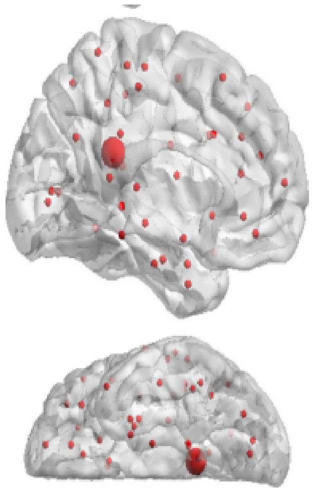
The subgenual area of the cerebral cortex	The retrosplenial region of the cerebral cortex
	
Brodmann area 25 corresponds to the subgenual area of the human cerebral cortex. In the connectome, it corresponds to node 39. Extremely rich in serotonin transporters, this area is involved in vast networks comprising areas like hypothalamus and brain stem, which affect appetite and sleep; the amygdala and insula, which affect the mood and anxiety; the hippocampus, which plays an important role in memory formation; and some parts of the frontal cortex responsible for self-esteem	Ectosplenial area 26. It is a cytoarchitecturally defined portion of the retrosplenial region of the cerebral cortex. In the connectome, it corresponds to node 41. This area is bounded externally by the granular retrolimbic area 29

Figure 1.6

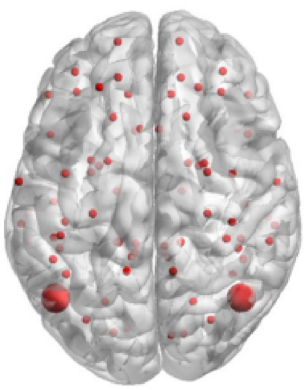
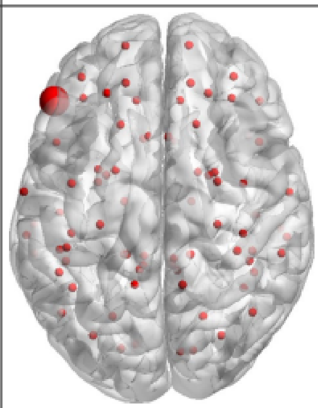
The parietal area of the cerebral cortex	The Broca's area of the frontal cerebral cortex
	

Figure 1.7

1.3 How BNs can model a complex system

The boolean networks contain a set of n variables and each of them represents one component of the modeled system. The value of a variable describes the current state of the designated component and can assume two values: 0 or 1. These states represent an approximation able to describe the qualitative behaviour of the system.

BNs can be considered as a directed graph where each regulatory component is represented by one node of the graph. Their interactions are represented by directed edges between the components. Time is considered as discrete, because, at each time t, a new state of the network is updated by applying the defined Boolean functions. Due to the semplicity of the model, BNs are perfect for studying complex dynamic behaviour in biological systems: their application can unravel the mechanisms regulating the properties of the system.

Returning to the mapping of the brain, it is necessary to obtain an adjacency matrix, as mentioned before. For this scope, a certain value x called threshold for the values of the matrix is introduced, which, below x, they will be set equal to 0 and, above it, they will be set equal to 1. Defined $\sigma_i = (s_{1i}(t) + ... + s_{ik}(t))$ as the sum of the states in a neighbourhood, then the updating function will be:

$$f(t+1) = \begin{cases} 1 & \text{if } a < \sigma_i < b \\ 0 & \text{otherwise} \end{cases}$$

If one varies the threshold of the matrix and a and b, a deep investigation of brain dynamics at the global level of the systems can be obtained. For obtaining the connection matrix, it is sufficient to apply one of the threshold levels after obtaining the patterns. For identifying which dynamics have correlations above that threshold and consequently identify which nodes are correlated with each other, one has to calculate these correlations and to use a threshold value z. This method allows for the identification of the nodes which are connected and is able to give the visualization of the overall circuit.

At the beginning of the simulation, an initial state is chosen through the assignment of a Boolean variable to each node. In local dynamics, the interactions of each node and of its neighbours will be investigated, while, in the global dynamics, the dynamics of all the nodes that belong to the function under examination will be analyzed. The BNs method allows to understand the interaction among the brain regions, to explore the dynamics that lead to the creation of emerging networks and also to establish an interaction time. The third point of the appendix shows the Talaraich coordinates of the brain areas.

1.3.1 RBNs: a short story

The first neural network was modeled in 1943 by a neurophysiologist called Warren McCulloch and a mathematician called Walter Pitts. They wanted to describe how neurons work in the brain, so they decided to model the first neural network ever using electrical circuits.

Donald Hebb, in 1949, wrote a work in which emerged the fact that neural pathways are strengthened each time they are used: the connection between two nerves is enhanced if they fire at the same time and this discovery opened a way to the study of the human way of learning.

During the years important discoveries about networks were done, but the sensational idea of modelling with the BNs arrived in 1969. Stuart Kauffman is the father of this model that works with genetic regulatory networks, which he called random Boolean networks (RBNs, the first types of the boolean networks ever). These ones can either be ‘on’ or ‘off, there is no a third choice.

Networks are made of N genes and each of them can regulate K other genes. The inventor noticed that these RBNs spontaneously displayed some rather stable, ordered behaviour in randomly generated networks. A lot of biologists feared that RBNs were oversimplified, that they didn’t capture the complex subtleties of the real world biology. The activity of just a single gene is regulated by many factors and this translates to a high K value in the random boolean network model. K varies also for different genes,but this isn’t the case in RBNs, in which K is fixed for every gene and is usually low. Despite the first critics of the biologists, scientists from all parts of the world soon realized that RBNs are an idealized Boolean abstraction model of many networks studied in biology that captures some important aspects of real biologic world. Ian Stewart, a scientist, said: "It is not useful to criticize these models on the grounds of what they leave out: the test is what they predict, and how well it fits the appropriate aspects of reality." From 1969 on, new types of BNs were invented and quite accurate biological models have been made.

Chapter 2

The three steps of the simulation

2.1 The first step of the simulation: mapping the brain

The first step is mapping the brain. It is possible thanks to the maps offered by `BrainMap.org`, a sites that offers brain maps based on the region of interest (ROI). When the download of the activation maps of the chosen cognitive function is done and after having extracted the brain activation points, these ones are mapped with the corresponding nodes in the brain connectome. The way for obtaining the brain connectome is the use of the classic methods related to the acquisition of DTI and T1-weighted MRI, the division of the brain's white matter, the creation of white matter tractography and the division into ROI, followed in many sofware applications. The degree of an ROI allows the definition of the role that each module plays as a source of connections with other region of interest improving the computational dynamics by using the boolean networks. A connectome, as already said, is the mathematical representation of the brain through nodes and edges. Highly connected nodes are hubs, that connect primarily with other nodes in the same community (in this case it is called provincial hub) or with nodes that belong to different communities (connector hub).

An 82-connectome model is used as the parcellization method, based on Brodmann areas, mapping the ICAs of the behavioural domain on the regions of interest of the Brodmann organization. This allows to identify the network node corresponding to the functional activation areas for each category of the chosen function. Then, the reference areas in the 82-connectome model is obtained. Areas are connected by an adjacency matrix of 82 rows and 82 columns, whose values are comprised between 0 and 1. On average, each network node is strongly connected with the other nodes (with an average connection value of 0.51 and a standard deviation of 0.288 for each node.) If the nodes in a network are highly connected, the system doesn't exhibit complex behaviours and it goes on fixed-point behaviour or dies immediately, therefore in this simulation it is decided to set the nodes' weight values at 0.87.

2.2 The second step: modeling with BNs

As a requirement for the model, there is obviously the congruence of the variables with the brain data and there must be the possibility to parametrize them explicitly, quantitatively and qualitatively. The BNs in the random form are the right choice for this model, here each node has two possible states: "0" (inhibition state) and "1" (activation state). At each time step, the status of the nodes in a Boolean network changes in relation to the status of the nodes connected to it and to the Boolean function that is assigned. The possible states are 2^N, where N is the number of the possible states of a BN: this number represents the cardinality of the state space. BNs have a rule table for their development, that, as already said, can be totalistic (BNs might be the same for all nodes of the network), deterministic or non-deterministic.

This model is able to explore systematically the relationship between brain network structure and dynamics that might, otherwise, be impossible. Each node of a Boolean network is connected randomly to a set of K input nodes, in the case of this document the connections are given by the connectome. After having chosen an initial state, it's necessary to calculate the sums of the values of the input regions because the Boolean functions associated with the node will not be used. Following this procedure, updating will take place in this way: if at step t, the i-th node is connected to a number of a switched node that is within a given range, then it will be switched on at time $t + 1$, otherwise it will be turned off.

2.3 The last step: the dynamic analysis

After one has obtained the connections between the various Broadmann areas, it is realized a connection matrix 82×82 whose values are between 0 and 1 and, after the managing of all these variables, it is possible to get data on the local interactions among brain areas. To transform this matrix into an adjacency matrix capable of modeling the system as a not fully connected graph, a threshold x for the values of the matrix has been introduced which, as already explained, below they will be set equal to 0 while above the threshold they will be set equal to 1. If a value of 0.8 for the treshold is chosen, it is possible to increase this value by 0.01 every row of the matrix, thus fluctuating values (which correspond to diferent values between the nodes) should be visualized. Then, by modifying the threshold of the x matrix and the values of a and b, a deep investigation of brain dynamics at the global level of the systems is achieved. After having the pattern and applying one of the above-mentioned threshold levels, a connection matrix is obtained which highlights the emerging synchronized circuits. The single nodes are gonna show dynamics that can be correlated with the dynamics of other nodes. The calculation of these correlations and the use of a threshold value z can identify which dynamics have correlations above that threshold and consequently identify which nodes are correlated with each other. The method allows the circuits extraction, the identification of the nodes which are connected in the network and the involved connected Broadmann areas which have emerged from the dynamics. A lot of visualizations techniques allow to get the visualization circuit by circuit, and the overall circuit. The analysis of each circuit and of its dynamic behaviour can be made. As mentioned before, a Boolean network is a graph with N nodes where, a Boolean variable representing the state of the node (1 or 0; ON or OFF) and the associated function determines the evolution rule associated with it, are assigned at each node. To start the simulation, an initial state, assigning to each node a Boolean variable, is chosen. The dynamics provide for the transition of the state of each node based on the Boolean function that is assigned to it and the Boolean variables of the nodes to which it is connected. The system behaviour is then emergent and unpredictable. Both the local and the global dynamics will be analysed. In local dynamics, the interactions of each node and of its neighbours will be deepened. In the global dynamics, the dynamics of all the nodes that belong to the behaviour under examination will be studied.

This method is useful because it considers the interactions of each brain region with other regions, discovers the rhythms of interaction among the regions, where these exist, or if the interactions are purely random. It explores also the dynamics that lead to the creation of emerging networks that are structured precisely according to the process that is taking place. It allows also to establish an interaction time, taking care to detect whether emergencies are the result of a latent synchronization process, or whether there are submodules of the brain that synchronize and whether these submodules are in some way coordinated with each other. Another problem that is analyzed in the global dynamics behaviour is the stability of the emerged circuits. Obtaining the local dynamic circuits, it is possible to analyze the global dynamic taking as a reference the following example. In an earlier evolution of BN, given 10 circuits, some important values are considered: $a = 1$ and $b = 4$, $x > 0.9$ and a correlation threshold $z > 0.87$. These are network parameters and threshold values respectively. These circuits are considered as a reference configuration to which all the other configurations compare, obtained by changing the parameters a, b and x. In this way, an application in a four-dimensional space $R \times N \times N \times R$, the identification of the space C of all the circuits is possible. The two thresholds vary in the space $\mathbb{R}$ of real numbers, even if the values of the connection weights matrix and the threshold values are conventionally rounded to the second decimal (varying between 0 and 1). These discrete spaces are denoted with H and K. Thus, an application f that is achieved and takes 4 numbers (integers — a and b, and two numbers between 0 and 1 at the second decimal digit) as an input and returns the circuits organization in the four-dimensional space. The cognitive function chosen is "action-execution", in order to repeat the simulation of the article "Modelling brain dynamics by Boolean networks".

A picture of the main brain activation points for Action–Execution, archived in the `BrainMap.org` database, can be seen in the appendix (it's the third point of the list).

Chapter 3

The results of the steps

After the individuation of the activation points, the results can be simulated through Wolfram Mathematica. Then, the activation of the useful brain areas is followed by the running of the simulation: a lot of steps are visualized and, after each step, one can discover the activated and non-activated areas. This is possible thanks to the colour rule, a rule that consists in the assignation of a colour to the active areas and to the non-active ones (in the 4th point of the appendix there is an example of this rule).

In order to see how a measure of how the performance of a brain area is related to the performance of another area, the correlation matrix can be examined (even if this definition was already explained, a refresh is always useful: a correlation matrix is a square matrix showing the correlation coefficients between two variables). Two areas are synchronized when the correlation between these ones is equal to 1. If one inserts a threshold z and selects the pairs of areas that have a correlation higher than z, the groups of areas that have fairly similar behaviour can be identified and called brain circuits. Wolfram Mathematica is not the only software used for the simulation of brain dynamics, but it allows to build all the needed graphics for the data analysis, such as the intersection array and correlation matrix for representing relationships among the emerged circuits, the spatio-temporal pattern of evolution of the BN system or even the time-series of the evolution showing the periodic behaviour of the system.

3.1 The results of the first step

The decision for this project has been to repeat the simulation made in the article “Modelling brain dynamics by Boolean networks”, using entirely Wolfram Mathematica.

The first active nodes for a generic action-execution cognitive process are the following:

2, 4, 8, 9, 10, 15, 19, 21, 28, 35, 39, 41, 63, 64, 75, 79.

These are related to specific Broadmann areas.

- Node 2 corresponds to the Primary somesthetic cortex, it is an area of the telencephalic cortex located in the parietal lobe in the brain and responsible for the reception of sensory stimuli of touch.
- Node 4 is the one that corresponds to the motor cortex, a brain region involved in planning, controlling, and executing voluntary body movements.
- Node 8 deals with the frontal visual fields, so it is responsible for the voluntary eye movements.
- Node 9 is the dorsolateral prefrontal cortex, the centre of ”executive control” of our brain.
- Node 10 is the anterior prefrontal cortex, the most rostral part of the superior and middle frontal gyrus.
- Node 15 represents the temporal lobe, one of the four major lobes of the cerebral cortex in the mammalian brain.
- Node 19 is called “associative area of vision” because it is involved in the analysis, recognition and interpretation of images processed in the primary visual cortex.
- Node 21 is the middle temporal gyrus, one of the temporal lobes. It plays a key role in understanding spoken and written language.

- Node 28 is the entorhinal cortex, a structure connected to different areas of the brain. Its role is to do a bridge between specific brain regions.
- Node 35 is the perirhinal cortex, an area associated with memory and recognition.
- Node 39, related with the angular gyrus, is a region of the brain involved in a number of processes related to language, number processing and spatial cognition, memory recall, attention and theory of mind.
- Node 41 is the auditory cortex, it receives auditory information.
- Nodes 63 and 64 in the connectome correspond to Brodmann area 39, this one is divided in more nodes in the connectome.
- Node 75 corresponds to area 45, a region active in semantic tasks.
- Node 79 is related with area 47, a region involved in the processing of syntax in oral and sign languages, musical syntax, and semantic aspects of language.

After importing in Wolfram Mathematica the coordinates of the human connectome, it is possible to create the adjacency matrix thanks to the command “MatrixPlot”, where the matrix represents the position of the points but in a matrix form, it is useful to determine the connection between the various brain structures. The result can be seen in the fifth point of the appendix.

By plotting the coordinates as a graph plot, one can obtain the desired graph. The result is shown in the following picture 3.1.

3.2 The results of the second step

Step 2 corresponds to the running of the simulation and the 16 Broadmann areas named in the previous subsection are activated. The graph shown in picture 3.1 helps to understand which areas are activated and which are the ones that stay turned off and, by modifying the a and b parameters related

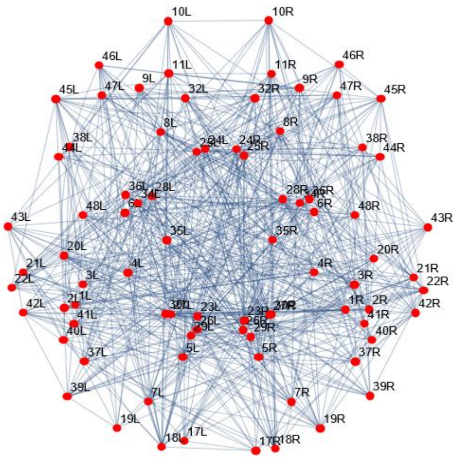


Figure 3.1

with the evolution rule, it is possible to build a matrix that shows the evolution of the system. It is trivial to say that the behaviour of the networks changes step by step, but how does it change? Here is presented an example of the evolution of the system after the 400th step (the maximum is given by 500) to let you see what happens after running through a big number of paces.

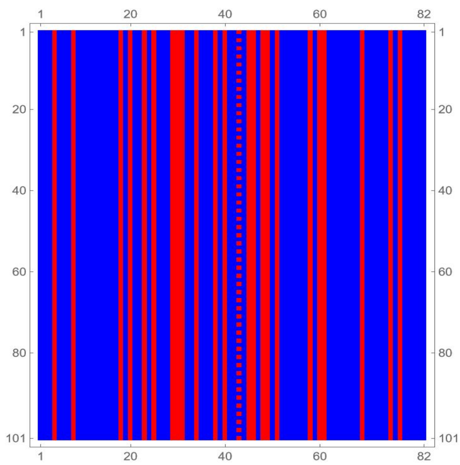


Figure 3.2: This picture represents a simulation made with $a = 1$, $b = 6$, $x = 0.9$ and $z = 0.87$.

One can immediately notice that the behaviour doesn't change almost at all with time. That is because, as already explained before, these kind of networks start their activity in a chaotic way that ends with a periodic stability. A lot of simulation have been done with different parameters, but the simulation that is more similar to the one of the reference article has the following parameters: $x = 0.88$ and $z = 0.87$. If compared precisely with the results of the article, it is evident that the progress of the switching on and off of the various areas is a bit different. But, globally, from $a = 4$ and $b = 4$ on, one can visualize nothing in both this document and the article. Varying a and b, the result is the following:

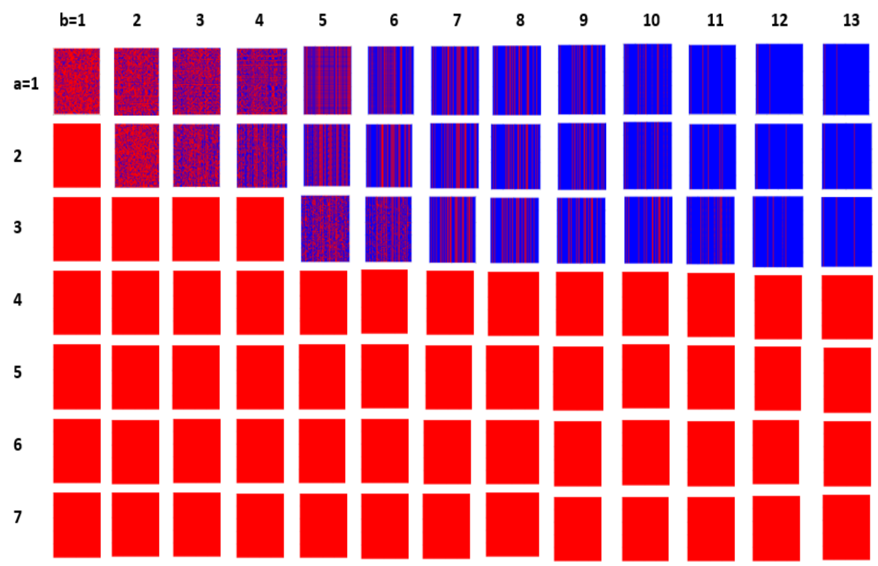


Figure 3.3

A zoom of every picture that is in this collage can be seen in the appendix (it's in the sixth point of the list).

As already said, updating takes place in this way: if at a step t, the i-th node is connected to a number of a switched nodes that is within a given range, it is switched on at time $t+1$: otherwise it is turned off. Despite the big number of simulations made with different parameters, the most similar to the reference article has $x = 0.88$ and $z = 0.9$. In the seventh point of the appendix the results of the simulations with the parameters are shown. 12 circuits have emerged from the attempt with $x = 0.88$ and $z = 0.9$. After their intersection, it is necessary to put them in a matrix to see their correlation.

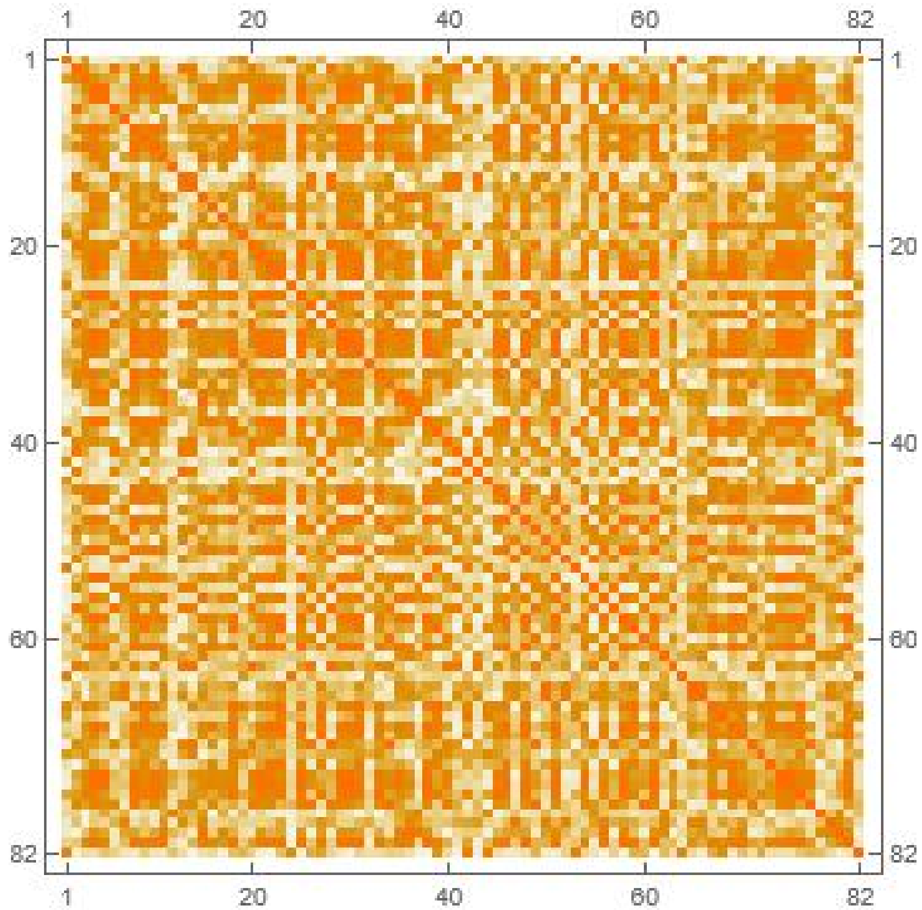


Figure 3.4

3.3 The results of the third step

It is important to highlight the complex emerging dynamics and the presence of structures in the simulations carried out so, if a simulation with initial conditions like these is taken,

network threshold:

$$x > 0.9,$$

correlation threshold:

$$z > 0.87$$

the system can be analyzed in depth due to the choice of these parameters that reproduce a sufficiently complex but not chaotic behavior.

Nodes always turned on and off have been removed, because they don't take part in the dynamics that is going on if not in a subtle way. The correlation matrix provides with a measure of how the performance of a brain area is related to the performance of another area and this value is between 0 and 1. When the correlation between the two areas is equal to 1, the two ones are perfectly synchronized. By changing the parameters and the correlation threshold, different results have been obtained, even if none of them is identical to the reference article.

Extracting the circuits from the simulation, that's the result.

	Connectome nodes	**Broadmann areas**
Circuit 1	{15,36}	{8L,23R}
Circuit 2	{27,70}	{19L,42R}
Circuit 3	{18,31}	{9R,21L}
Circuit 4	{38,61}	{24R,38L}
Circuit 5	{39,67}	{25L,41L}
Circuit 6	{33,45}	{22L,28L}
Circuit 7	{46,69}	{28R,42L}
Circuit 8	{50,66}	{30R,40R}
Circuit 9	{33,76}	{22L,45R}
Circuit 10	{18,31,67}	{9R,21L,41L}
Circuit 11	{33,45,76}	{22L,28L,45R}
Circuit 12	{18,39,67}	{9R,25L,41L}

The first circuit is made by a correlation between the Broadmann areas called 8L and the 23R. The first one has an important role in controlling visual and eye movements. The other one's function is receive the olfactory sensory stimulus.

The second circuit is made by 19L and 42R and the functions of these areas are the following:

- 19L it is one of the visual cortices and is involved in the creation of images.
- 42R works with the perception of sound.

The third circuit is made by an area (9R) involved in strategic processes in memory recovery and another area involved in contemplating at a distance, the recognition of familiar faces, and the recognition of the meaning of words while reading.

The 4th circuit is made by a brain area that is particularly active in an individual subject to stress or in paranoid individuals. Studies on this area constitute an important starting point for understanding part of the neurobiological correlates of some mental disorders, such as:

-Schizophrenic behaviors;

-Obsessive-compulsive disorders;

-Paranoia and fixations;

-Dysexecutive Syndromes.

The other area of the circuit it is the most rostral part of the superior and middle temporal gyri in which specific sound frequencies are perceived.

The first area that compose the fifth circuit is the 25L. This region is extremely rich in serotonin transporters and is considered a coordinating region for a vast network involving areas that regulate many factors, including appetite and sleep. The second area included in the circuit is part of the auditory cortex.

Both areas of the sixth circuit have connections to vast regions of the cerebral cortex and have functions related to sound processing.

The 7th circuit is composed (again) by two areas involved in sound processing.

The region called 30R, which makes up the eighth circuit, presides over the coordination between sensitive affections and emotional elaborations. The other region, 40R, is part of the somatosensory associative cortex, which interprets tactile sensory data and is involved in the perception of the position of spaces and limbs. It takes also part in identifying other people's postures and gestures, and is therefore involved in the mirror neuron system (it's a system formed by neurons that allow us to explain physiologically our ability to be in relation to others).

The ninth circuit is composed of regions employed in linguistic comprehension and expression and in the elaboration of higher psychic activities.

Circuit 10 is formed by areas that have the function of recognition: they are the regions of the brain that recognize the familiar faces, the meaning of words during reading and the sounds.

Circuit 11 consists of three nodes corresponding to three different regions: the first one deals with the motor activity of the body, the second with hearing and the last with the elaboration and understanding of language.

The last one is a very interesting circuit, since it is composed of regions involved in strategy creation, planning, emotion control, attention, concentration, impulse self-control and other such functions. The second of these regions is also involved in the transport of seratonin.

The graphs with the extracted circuits can be seen in the 7th point of the appendix.

Finally, since the end of this simulation has come to an end, it is possible to observe how the development of the network looks as a function of time.

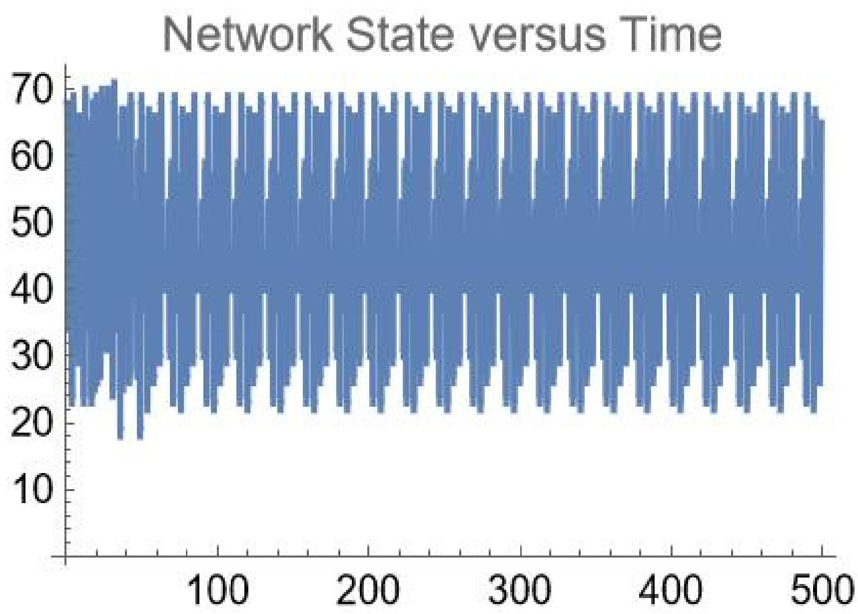


Figure 3.5

3.4 A brief deepening on the mathematica code

The simulation analyzed in this document was done with Wolfram Mathematica. After one has defined N nodes and K edges, it is fundamental to build a rule table for each node that specifies the state of the node at time $t+1$ step. In this demonstration, each node is randomly assigned both an initial state and a rule table. In network dynamics different values of N and K result and they can be seen in the plot of network state (defined as the Hamming distance between the network's present state and initial state) versus time and the power spectrum of this plot. If t varies, one can directly observe the real-time evolution of the network, which appears as a system of blinking lights. Once the activation points are known, it is possible to create a whole program that simulates the BNs behaviour. Following the previous code, for this aim it is necessary to apply the table rule for n elements, because it has to be applied for every element of the system. First of all, as already said, it is necessary to import the data for the Broadmann connectome for obtaining the coordinates of the nodes. After the selection of the labels, it is possible to create a vector that contains the number of vertices nv: in othis case it is equal to Length[connBroadmann] , where connBroadmann is the the vector containing the coordinates. "Length" is a command that gives as an output, trivially, the length of a certain vector. Then the first step is to find the adjacency matrix with connection $i \to j$ if $c_{ij} > x$.

After the choice of the x value, it is possible to create a matrix containing the coordinates using "Plot". Here's an example:

```
L1 = Table[ Table[If[ connBroadmann[[i, j]] > x, 1, 0], {j, 1, nv}],
{i, 1, nv}];
MatrixPlot[L1, PlotLegends → Automatic].
```

After plotting the adjacency matrix into graph form, it is necessary to set the initial conditions. If an area is active at a time $t = 0$, then its value will be 1, otherwise 0. After the set of the constants a, b for the evolution rule and the maximum time, it is possible to plot state evolution for $400 \leq t \leq 500$

or even for times below 400 steps.

The command is the following(the chosen colour rule has 0 for the areas that are off and 1 for the ones that are on.):

MatrixPlot[TimeStepsN[[400 ;;]], ColorRules → { 0 → Red , 1→ Blue}]. As mentioned before, in this simulation the vertices with trivial dynamics are removed.

trivial0 = ConstantArray[0, tmax];

trivial1 = ConstantArray[1, tmax];

trivInd = ; For[i = 1, i ≤ nv, i++,

If[HammingDistance[Transpose[TimeStepsN][[i]], trivial0] ≤ 10

or if

HammingDistance[Transpose[TimeStepsN][[i]], trivial1]≤ 10,

trivInd = Join[trivInd, i], trivInd = trivInd]].

Just as a reminder, the Hamming distance between two strings of equal length is the number of positions at which the corresponding symbols are different.

Then, the most important action is the extraction of the circuits. The duplicates are deleted and the intersection of the remaining values forms the circuit.

Appendix

1 - An explanatory picture of the Broadmann areas.

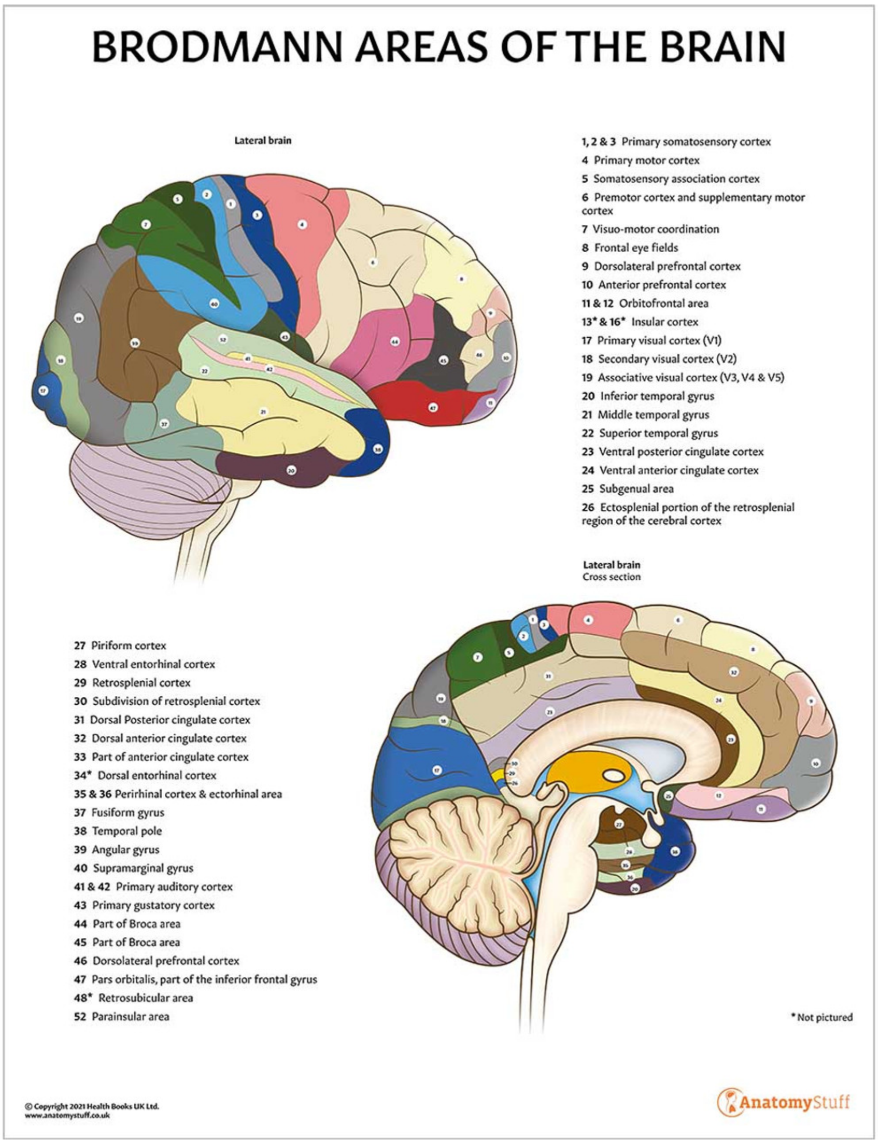


Figure 3.6: Picture 1.

2- The conversion of the brain areas into Talaraich coordinates with the respective nomenclature. The first values are associated with the first Braodmann area, the second values with the second area and so on.

$$
\begin{pmatrix}
-43.347 & -33.267 \\
36.017 & -34.877 \\
-46.592 & -34.283 \\
43.042 & -34.882 \\
-41.124 & -26.628 \\
38.726 & -26.779 \\
-27.911 & -22.877 \\
26.884 & -22.774 \\
-11.711 & -50.335 \\
10.565 & -50.376 \\
-28.722 & -3.387 \\
26.821 & -3.144 \\
-21.962 & -64.89 \\
20.152 & -64.938 \\
-18.278 & 22.911 \\
16.811 & 23.184 \\
-24.594 & 37.32 \\
22.509 & 37.266 \\
-16.038 & 59.012 \\
13.505 & 59.285 \\
-15.868 & 41.999 \\
14.287 & 41.912 \\
-11.367 & -77.659 \\
9.722 & -80.832 \\
-17.944 & -79.578 \\
15.445 & -80.241 \\
-31.112 & -73.468 \\
28.645 & -73.589 \\
-46.697 & -17.316 \\
44.482 & -18.356 \\
-58.701 & -22.674 \\
56.124 & -24.4 \\
-62.09 & -27.757 \\
59.074 & -28.558 \\
-7.35 & -37.121 \\
6.353 & -38.471 \\
-5.127 & 17.481 \\
3.935 & 17.297 \\
-7.702 & 16.525 \\
6.237 & 15.268 \\
-7.569 & -41.384
\end{pmatrix}
\begin{pmatrix}
1L \\ 1R \\ 2L \\ 2R \\ 3L \\ 3R \\ 4L \\ 4R \\ 5L \\ 5R \\ 6L \\ 6R \\ 7L \\ 7R \\ 8L \\ 8R \\ 9L \\ 9R \\ 10L \\ 10R \\ 11L \\ 11R \\ 17L \\ 17R \\ 18L \\ 18R \\ 19L \\ 19R \\ 20L \\ 20R \\ 21L \\ 21R \\ 22L \\ 22R \\ 23L \\ 23R \\ 24L \\ 24R \\ 25L \\ 25R \\ 26L
\end{pmatrix}
\qquad
\begin{pmatrix}
5.975 & -41.549 \\
-15.386 & -36.466 \\
13.866 & -36.568 \\
-20.746 & 2.123 \\
17.748 & 1.104 \\
-9.363 & -43.442 \\
8.147 & -43.753 \\
-16.732 & -36.233 \\
14.316 & -36.41 \\
-10.995 & 33.958 \\
9.428 & 33.952 \\
-25.029 & -0.127 \\
22.75 & -0.258 \\
-16.494 & -12.319 \\
14.662 & -12.248 \\
-28.581 & 2.414 \\
25.405 & 1.032 \\
-40.675 & -51.795 \\
39.008 & -51.835 \\
-45.029 & 18.096 \\
41.101 & 17.843 \\
-45.702 & -63.239 \\
43.234 & -63.01 \\
-46.866 & -44.705 \\
43.95 & -44.701 \\
-43.761 & -39.447 \\
41.665 & -39.463 \\
-58.73 & -35.916 \\
56.621 & -35.952 \\
-63.212 & -7.812 \\
60.264 & -8.114 \\
-48.196 & 14.902 \\
46.225 & 14.941 \\
-49.053 & 33.661 \\
46.494 & 33.754 \\
-36.422 & 44.641 \\
34.233 & 44.927 \\
-35.645 & 35.003 \\
33.515 & 34.987 \\
-41.123 & -4.011 \\
38.852 & -4.035
\end{pmatrix}
\begin{pmatrix}
26R \\ 27L \\ 27R \\ 28L \\ 28R \\ 29L \\ 29R \\ 30L \\ 30R \\ 32L \\ 32R \\ 34L \\ 34R \\ 35L \\ 35R \\ 36L \\ 36R \\ 37L \\ 37R \\ 38L \\ 38R \\ 39L \\ 39R \\ 40L \\ 40R \\ 41L \\ 41R \\ 42L \\ 42R \\ 43L \\ 43R \\ 44L \\ 44R \\ 45L \\ 45R \\ 46L \\ 46R \\ 47L \\ 47R \\ 48L \\ 48R
\end{pmatrix}
$$

3-Brain activation points during an action-execution process.

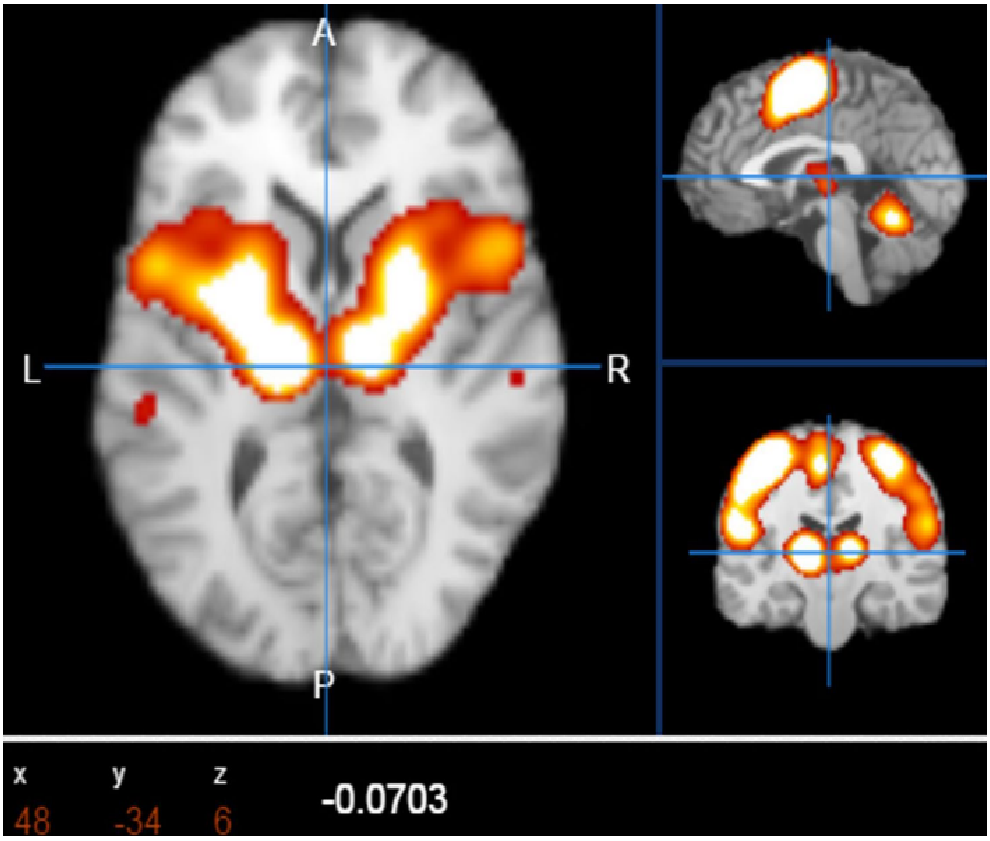


Figure 3.7

4-An example of the colour rule.

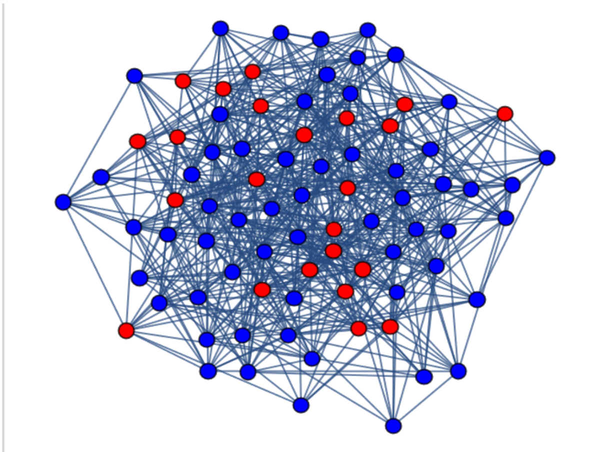


Figure 3.8: Red stands for 0, blue stands for 1.

5-The following picture represents the adjacency matrix.

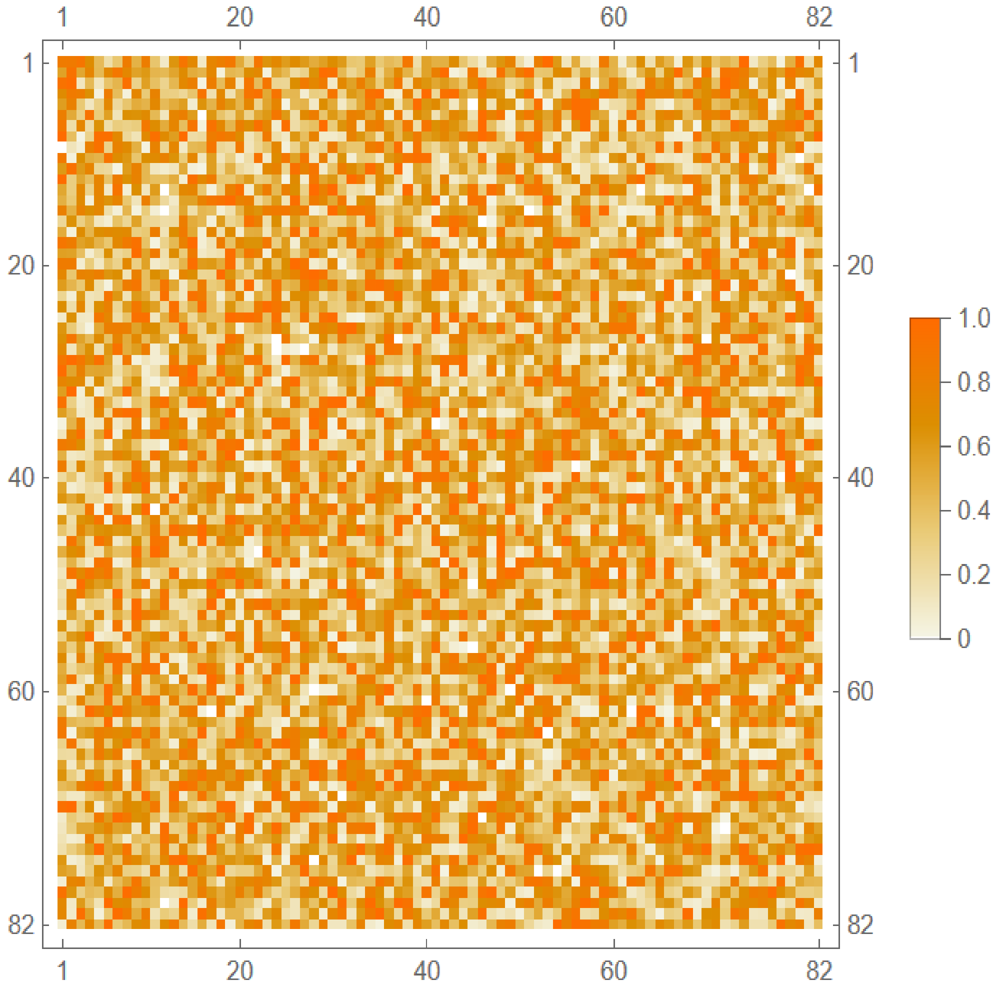


Figure 3.9

6-Here's a zoom of every picture of the collage called "Figure 3.3".

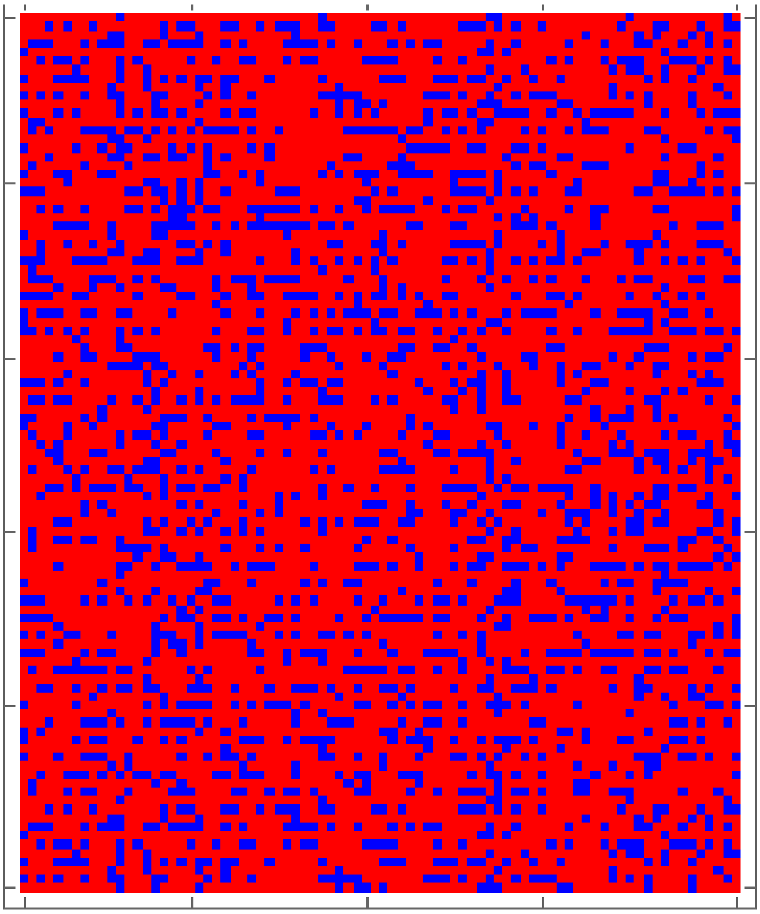


Figure 3.10: a=1 and b=1.

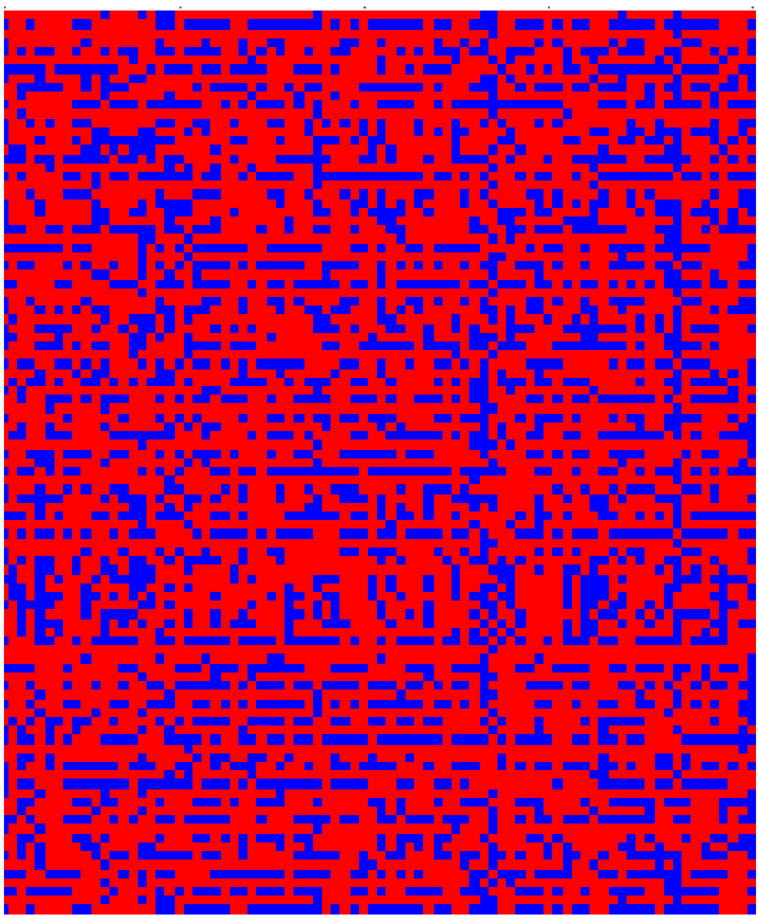


Figure 3.11: a=1 and b=2.

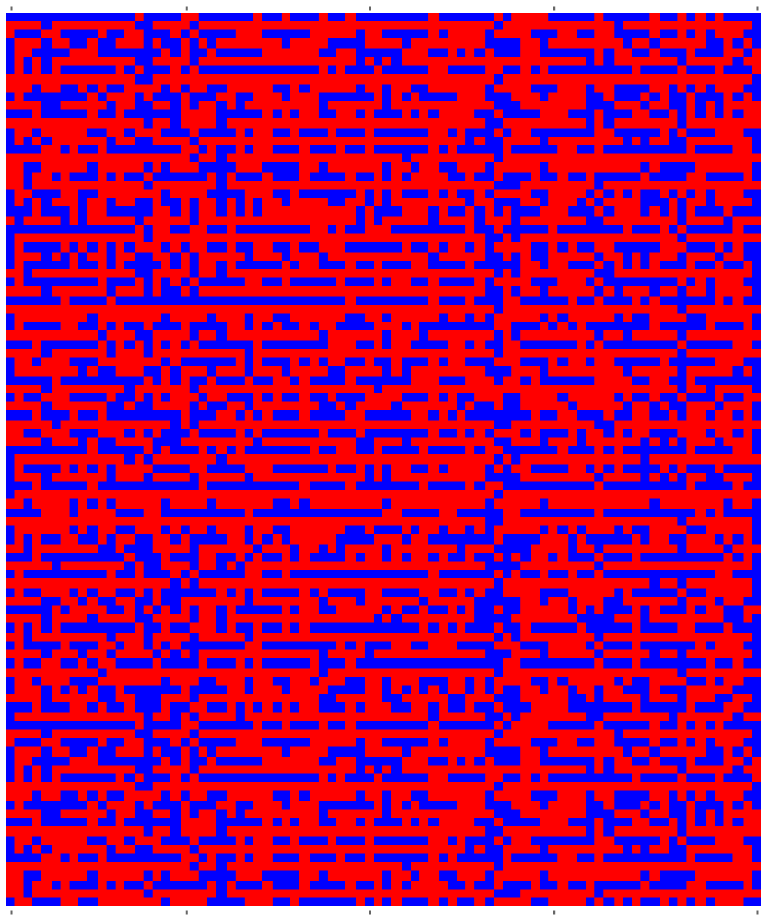


Figure 3.12: a=1 and b=3.

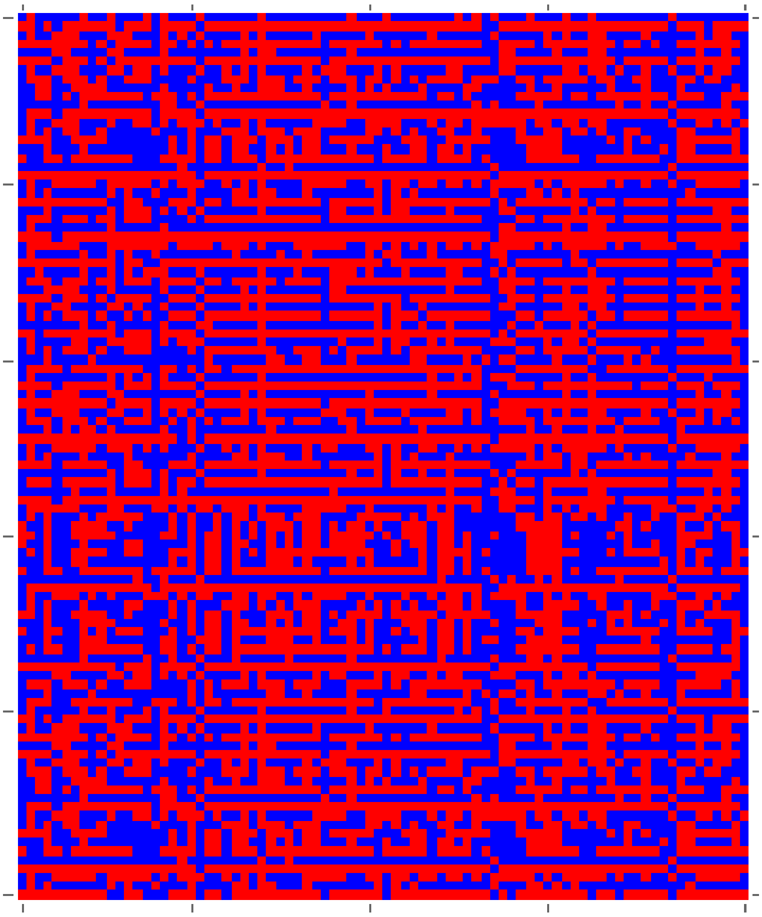


Figure 3.13: a=1 and b=4.

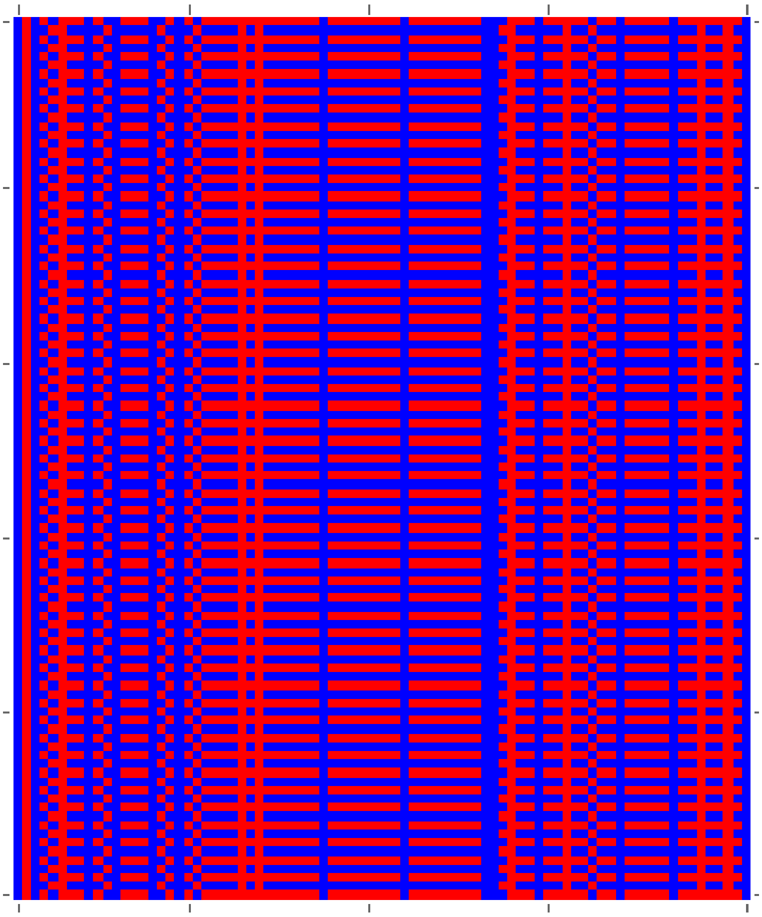


Figure 3.14: a=1 and b=5.

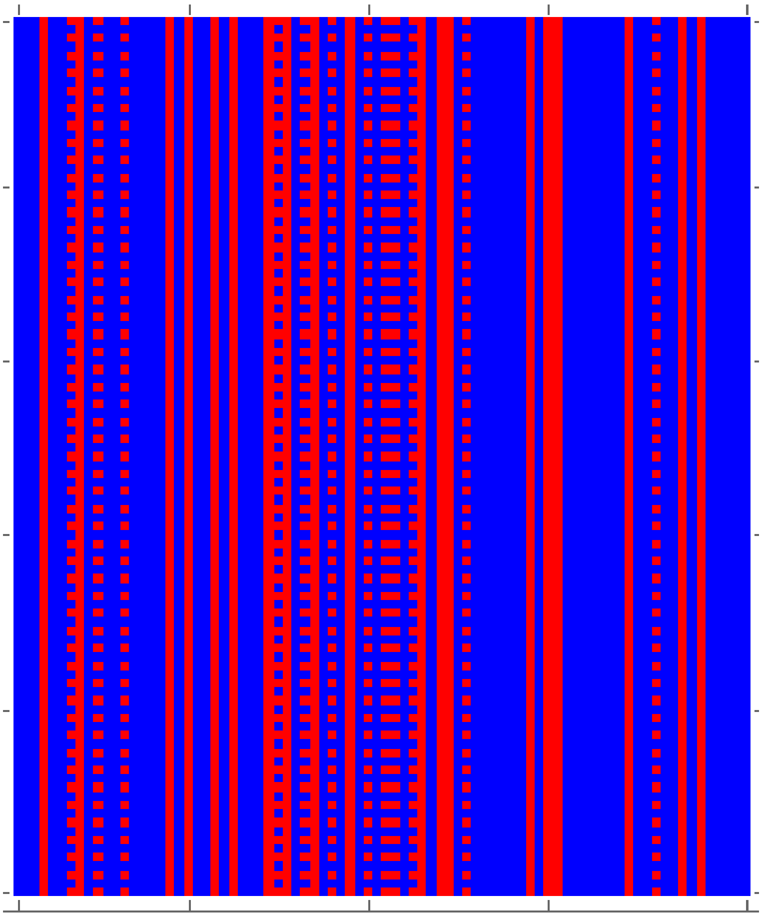


Figure 3.15: a=1 and b=6.

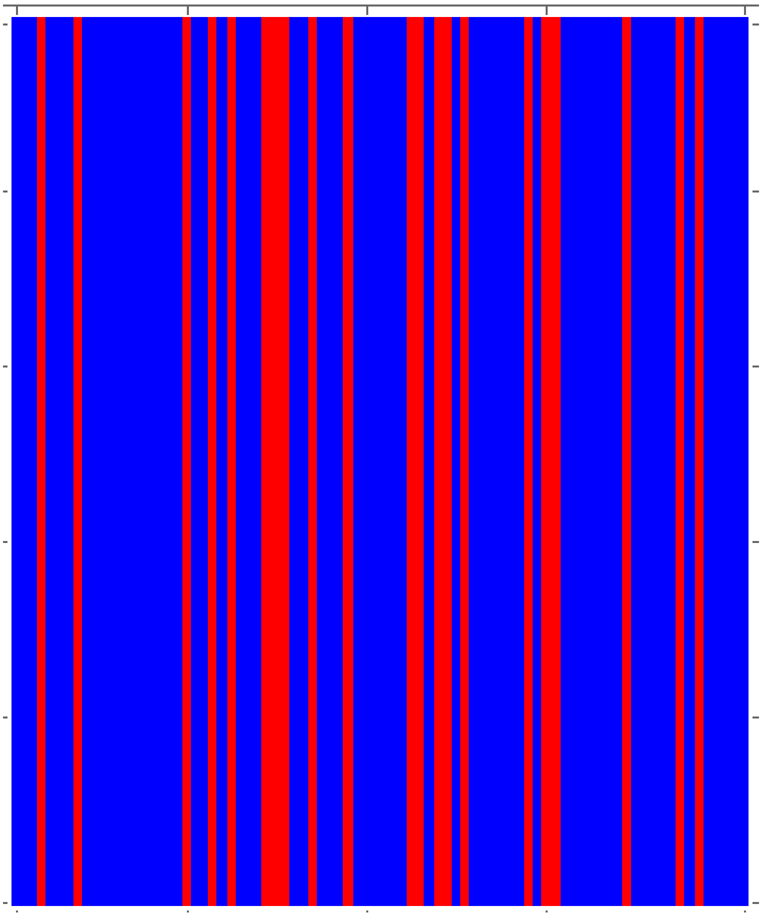


Figure 3.16: a=1 and b=7.

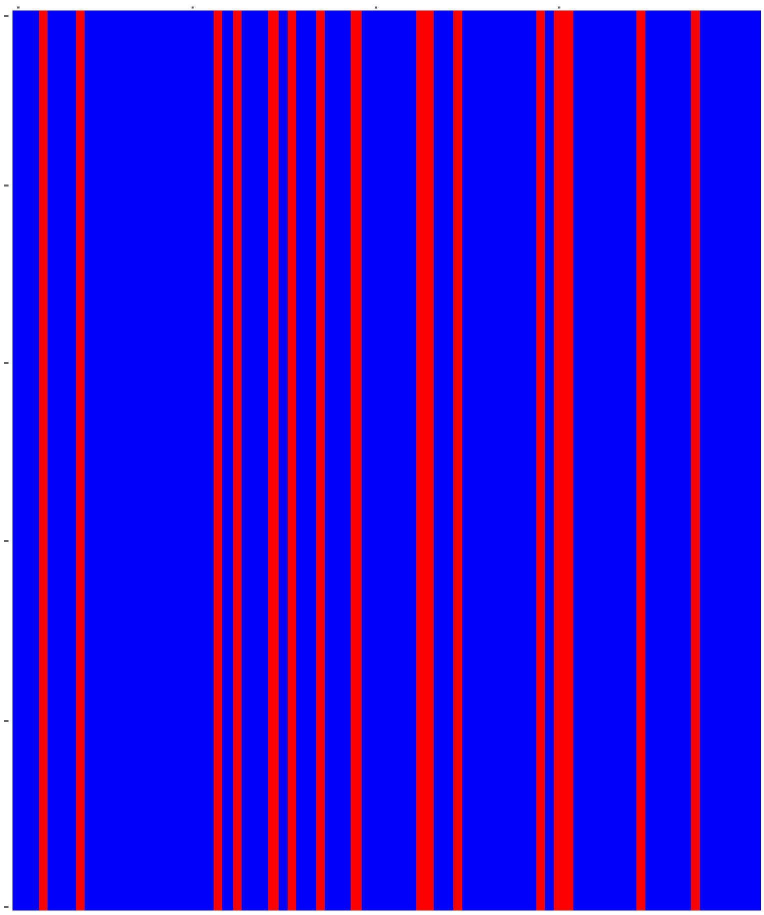


Figure 3.17: a=1 and b=8.

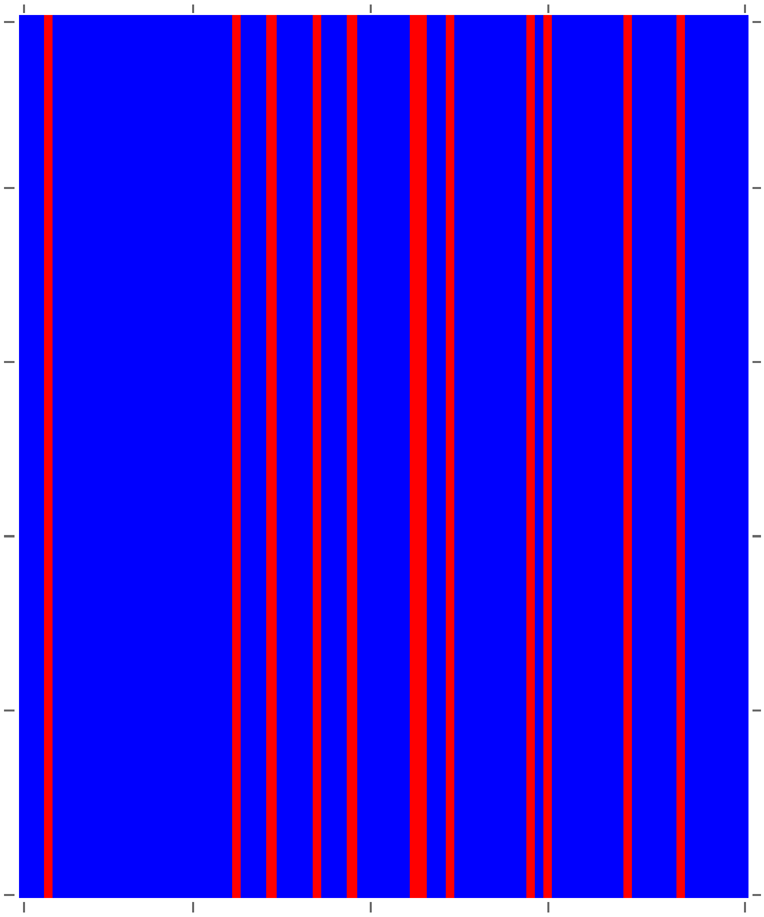


Figure 3.18: a=1 and b=9.

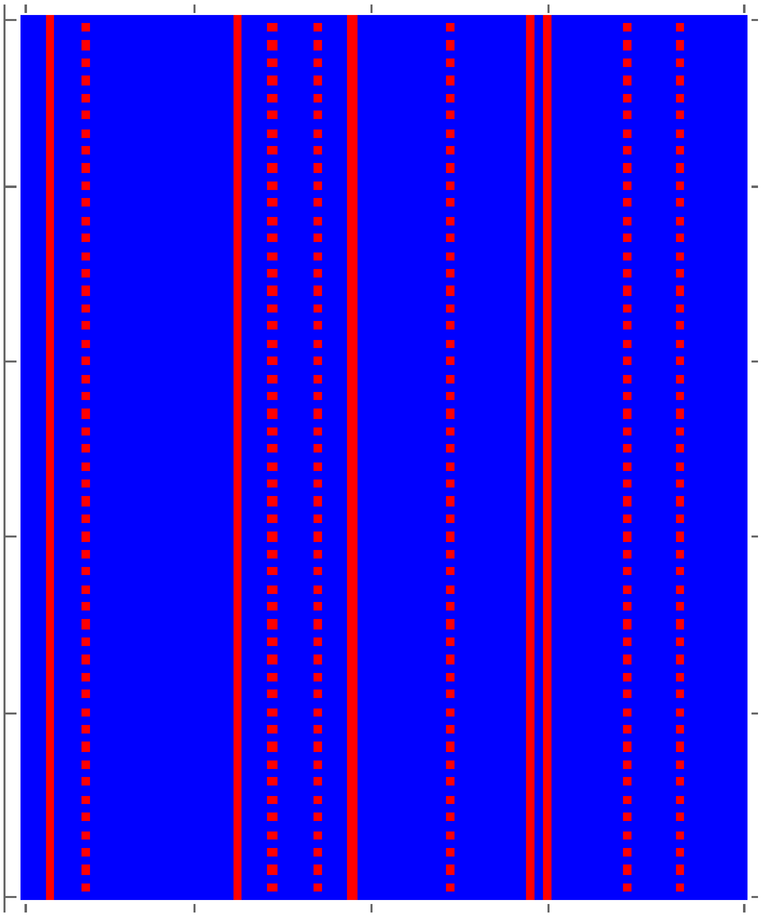


Figure 3.19: a=1 and b=10.

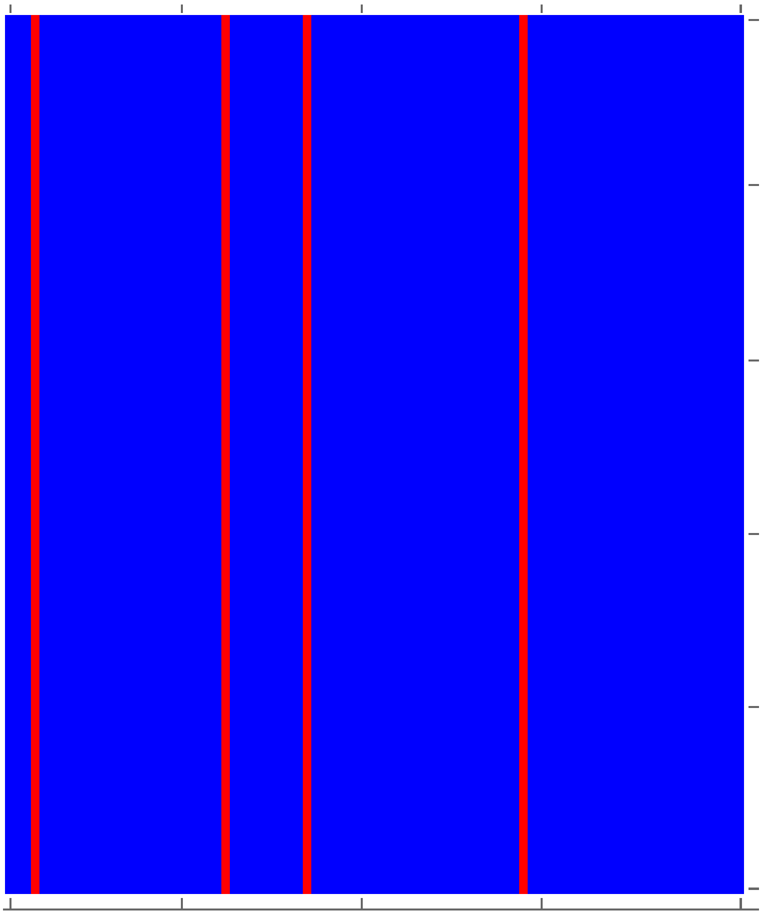


Figure 3.20: a=1 and b=11.

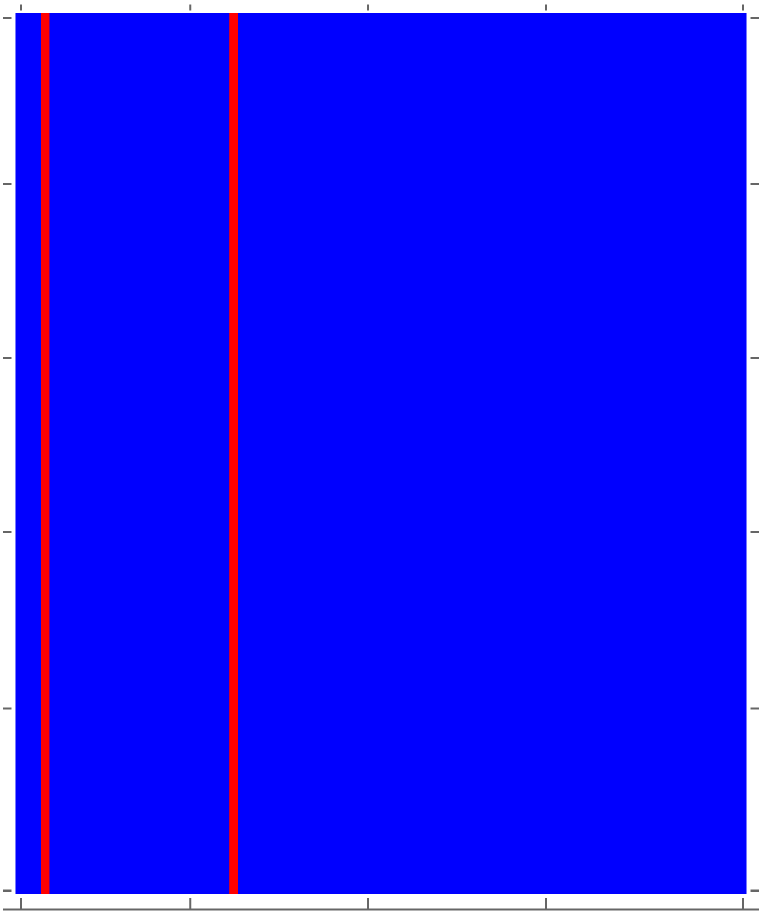


Figure 3.21: a=1 and b=12.

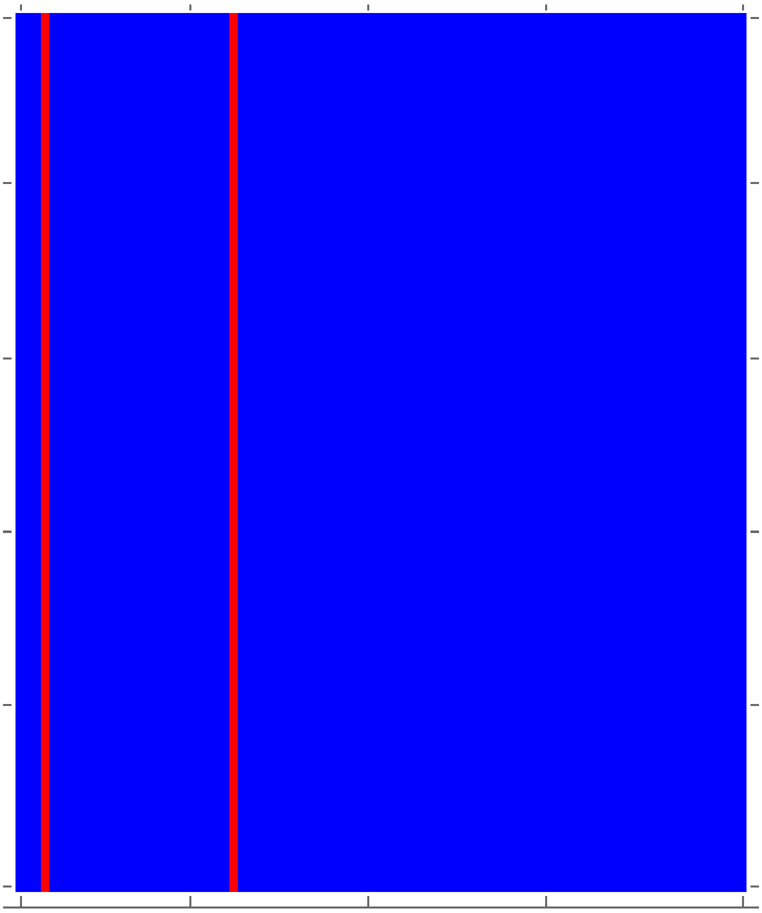


Figure 3.22: a=1 and b=13.

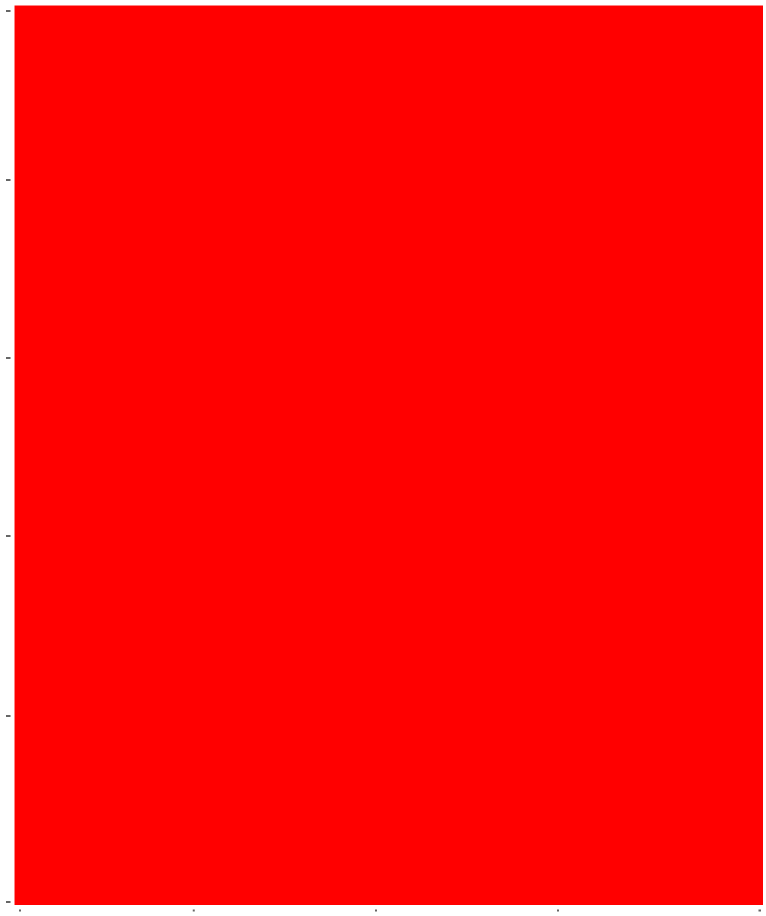


Figure 3.23: a=2 and b=1. This result is equal in a lot of cases, as you can see in figure 3.7 .

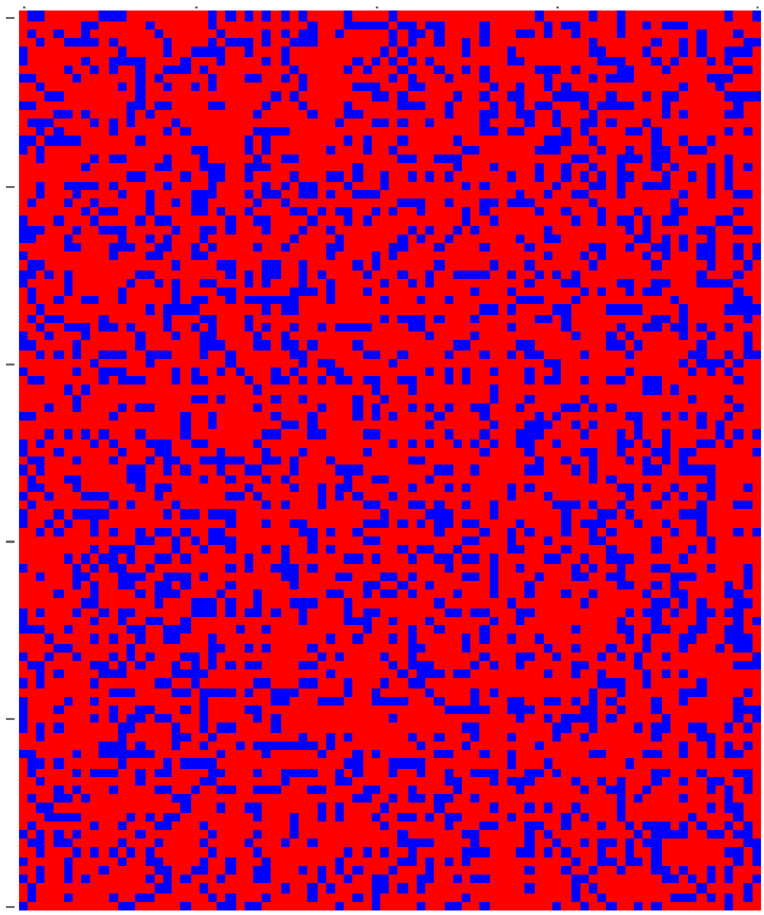


Figure 3.24: a=2 and b=2.

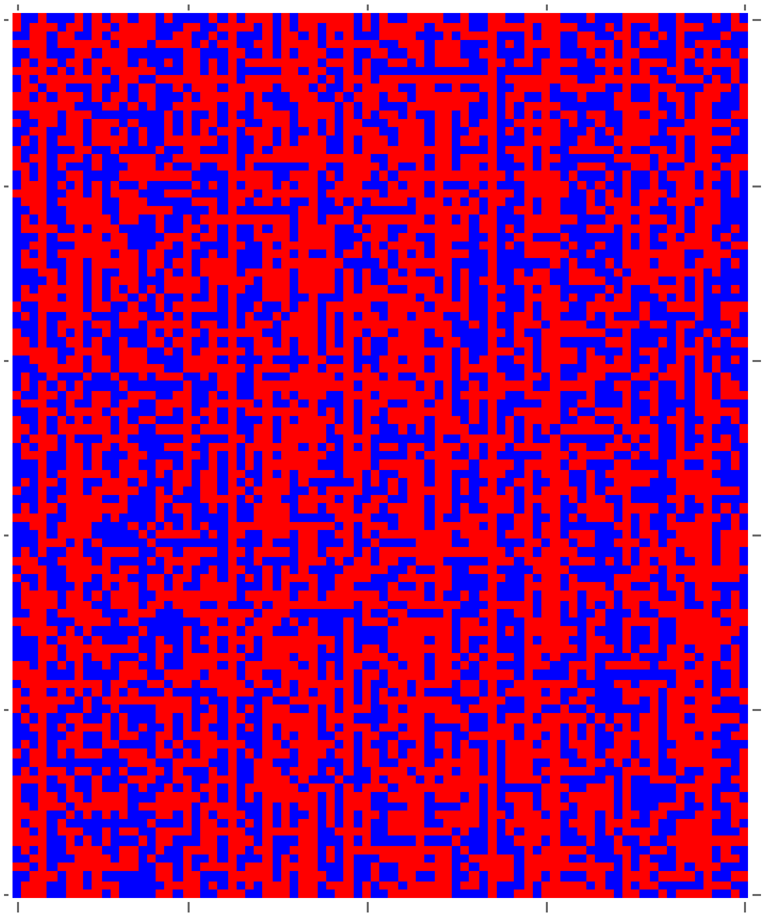


Figure 3.25: a=2 and b=3.

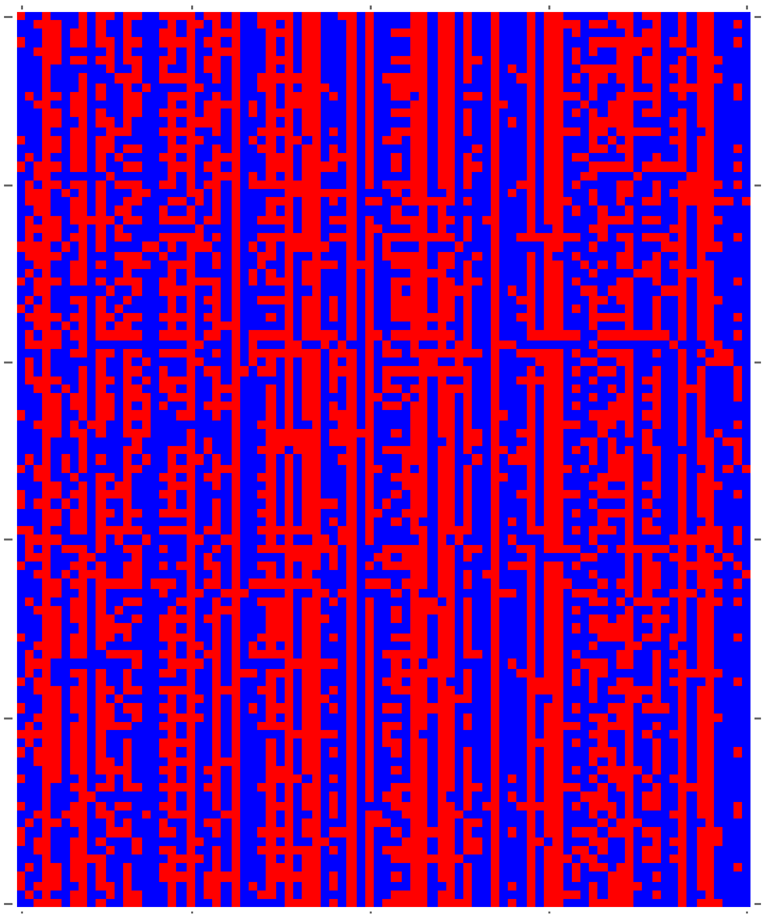


Figure 3.26: a=2 and b=4.

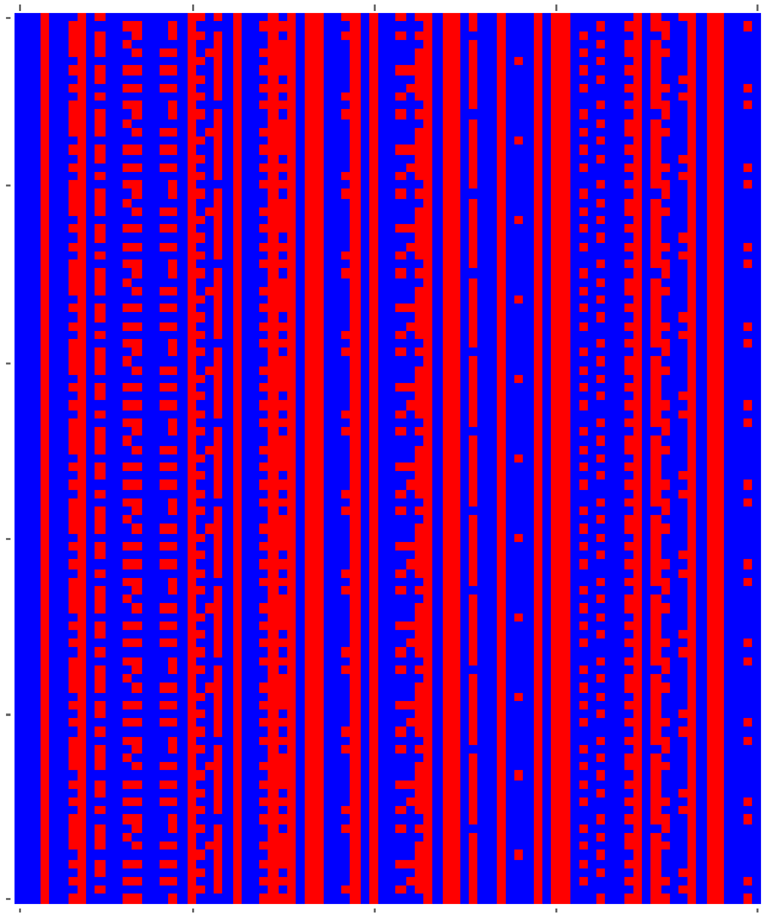


Figure 3.27: a=2 and b=5.

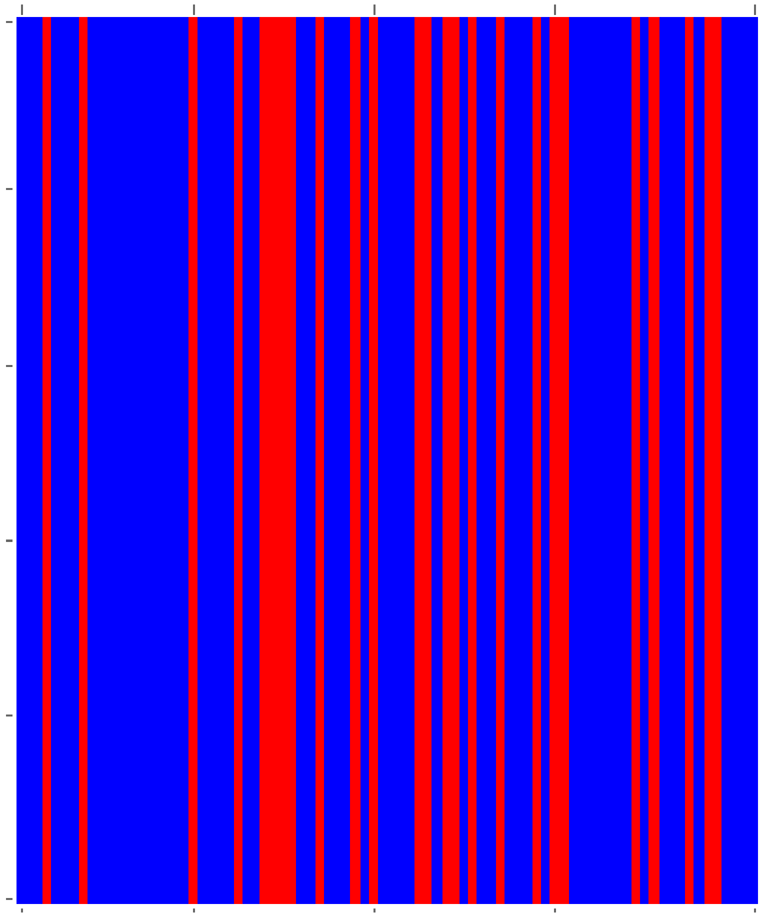


Figure 3.28: a=2 and b=6.

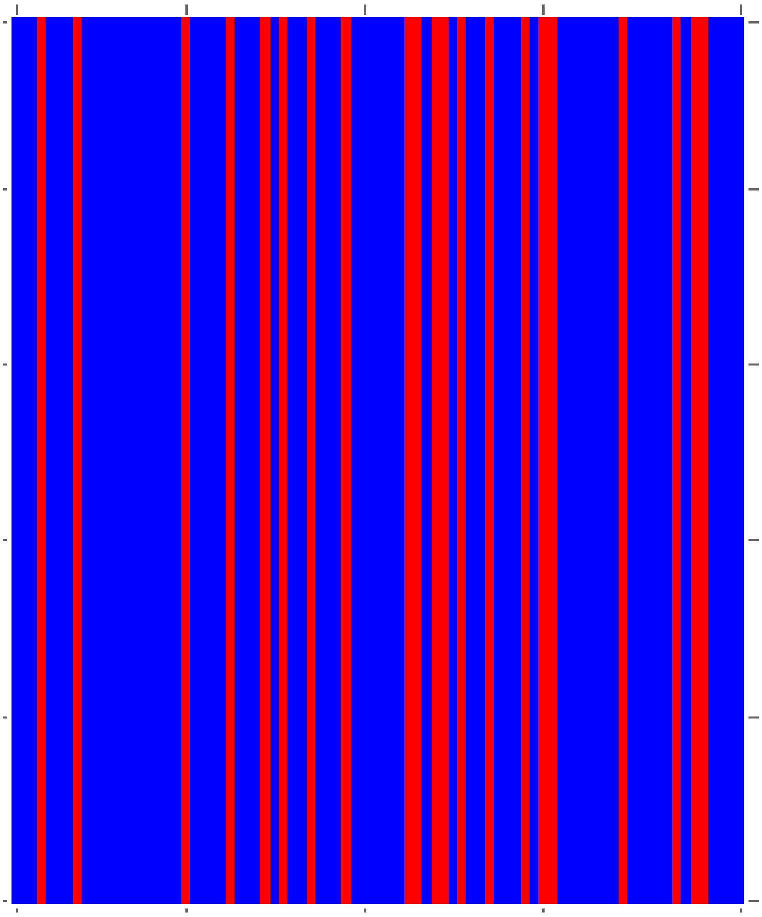


Figure 3.29: a=2 and b=7.

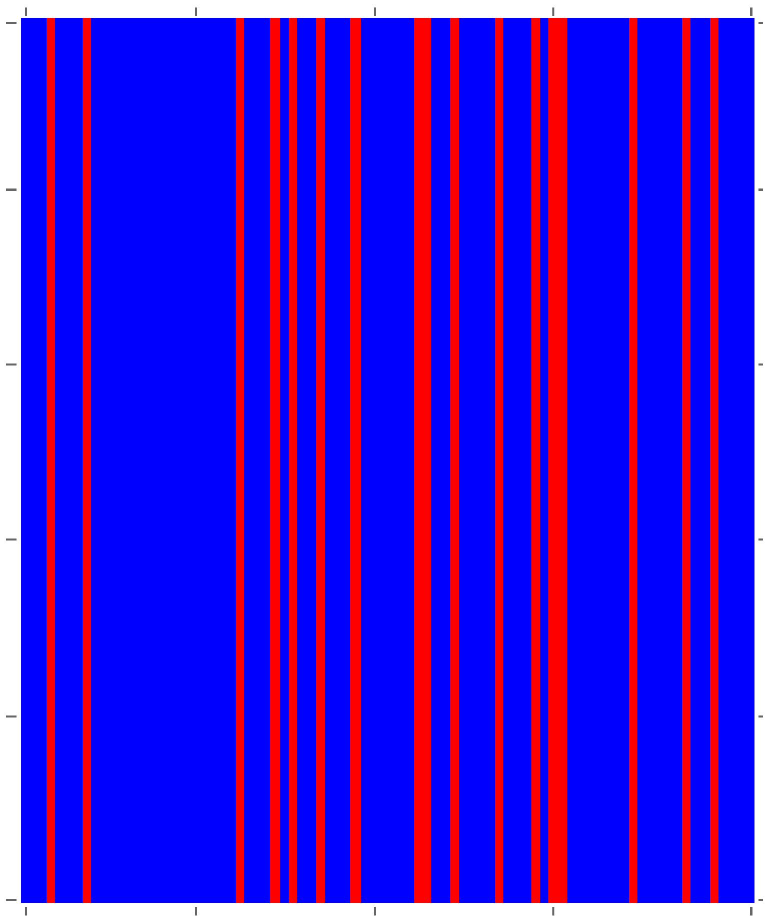


Figure 3.30: a=2 and b=8.

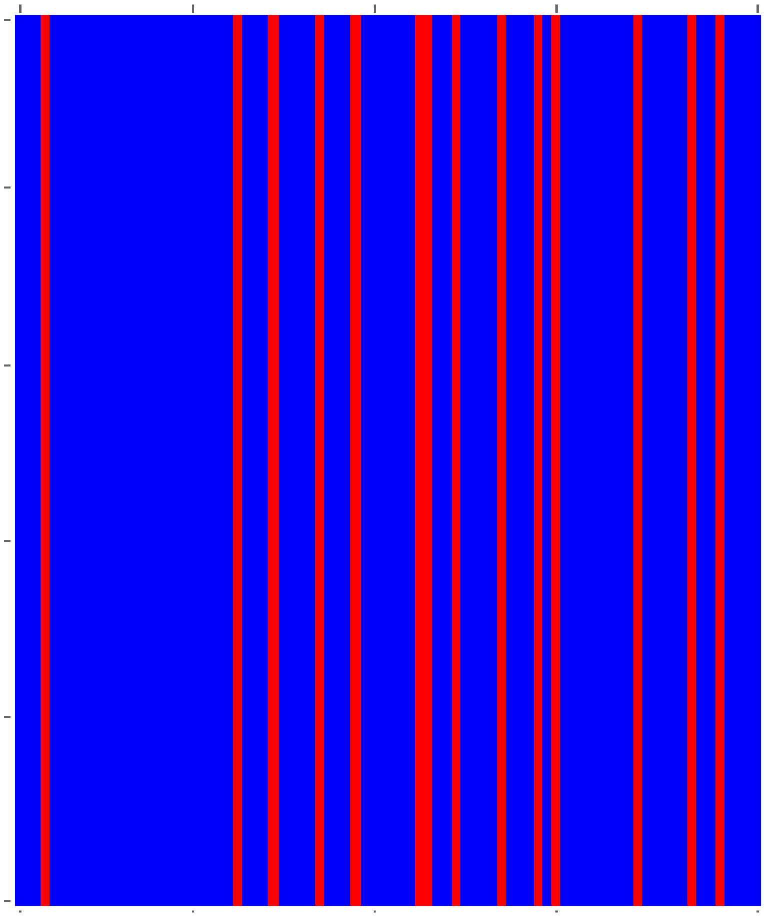


Figure 3.31: a=2 and b=9.

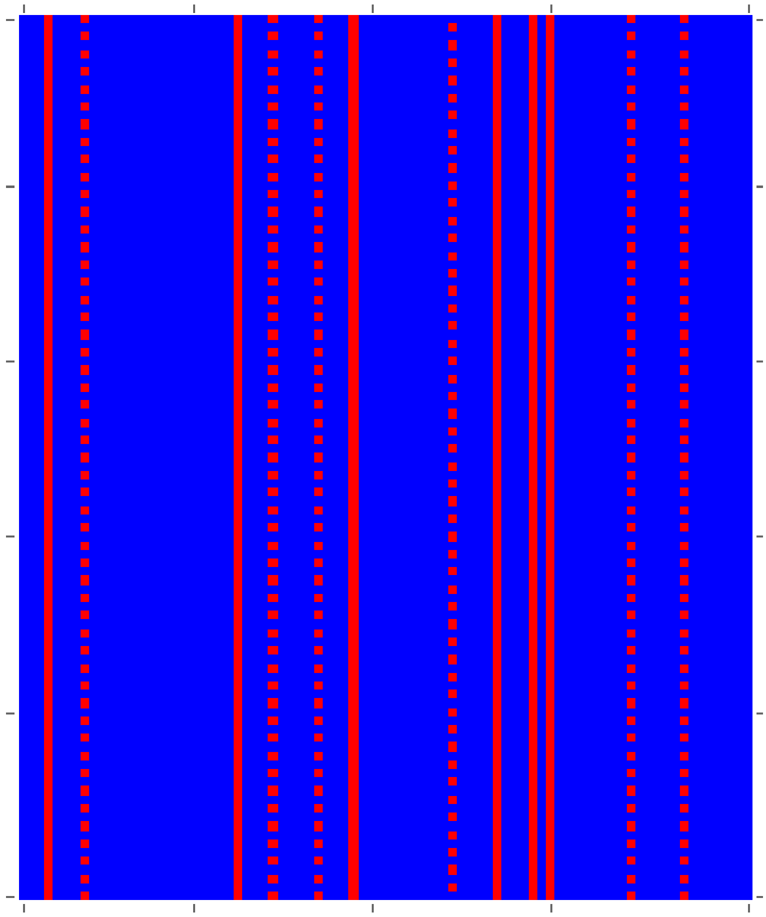


Figure 3.32: a=2 and b=10.

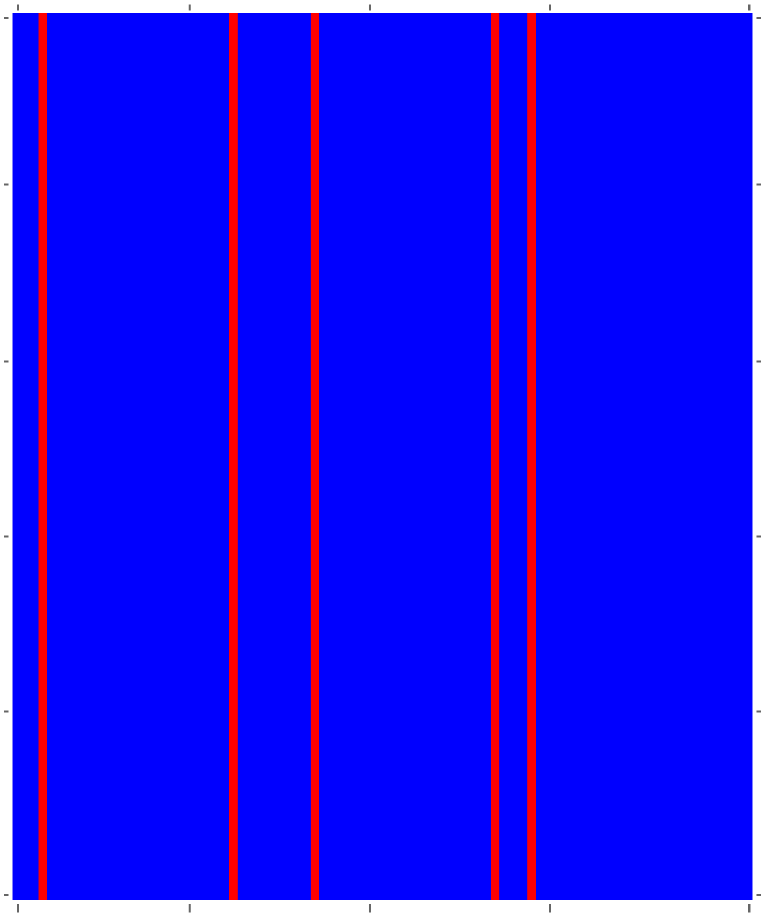


Figure 3.33: a=2 and b=11.

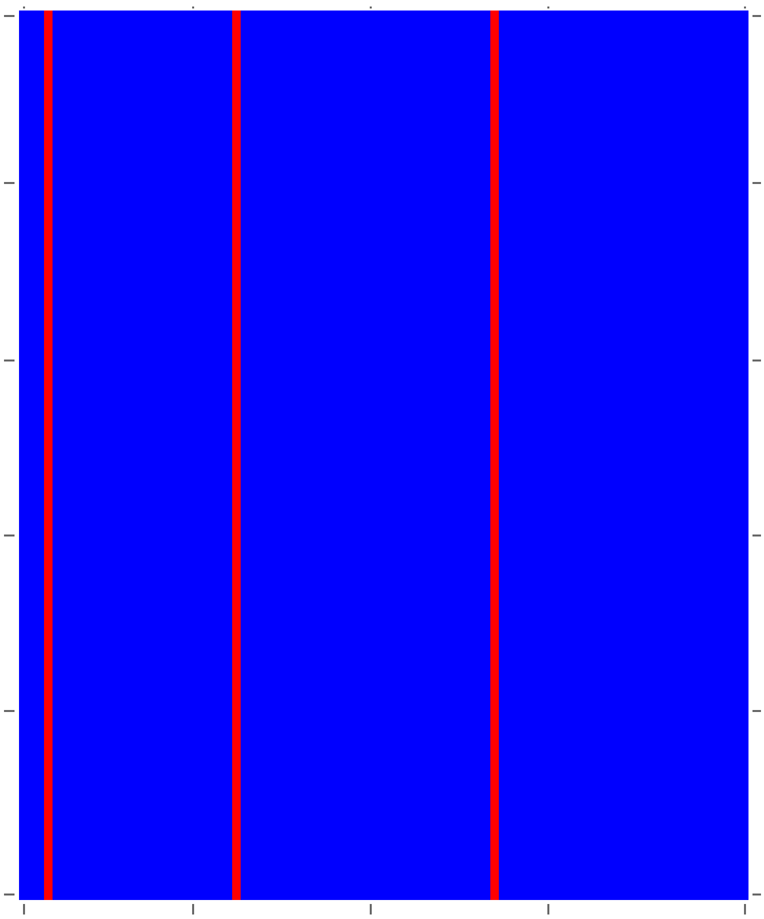


Figure 3.34: a=2 and b=12.

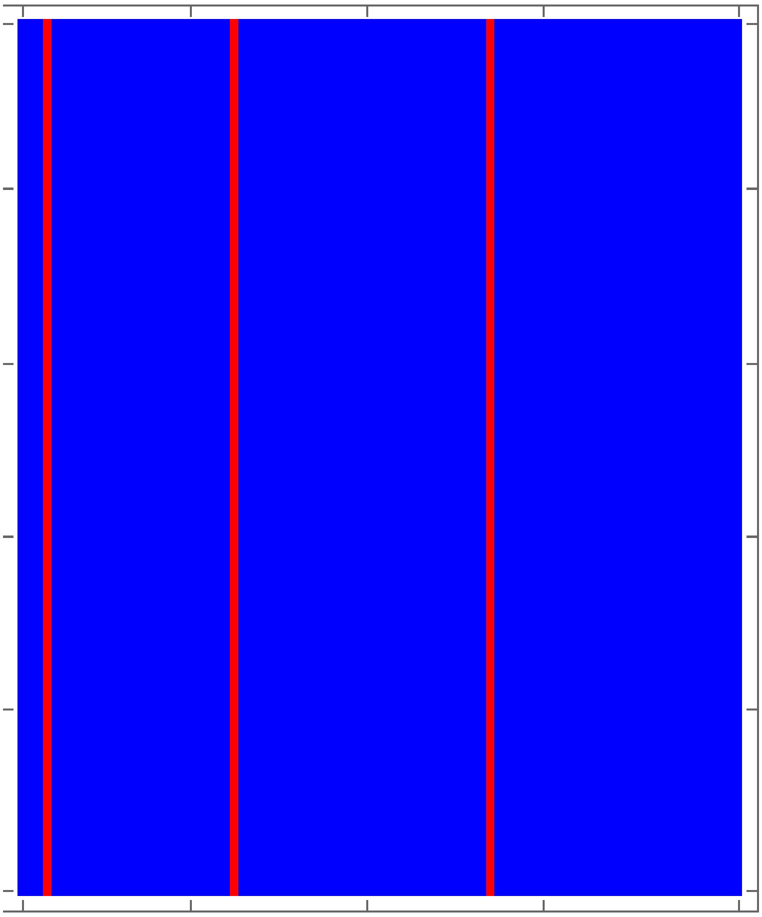


Figure 3.35: a=2 and b=13.

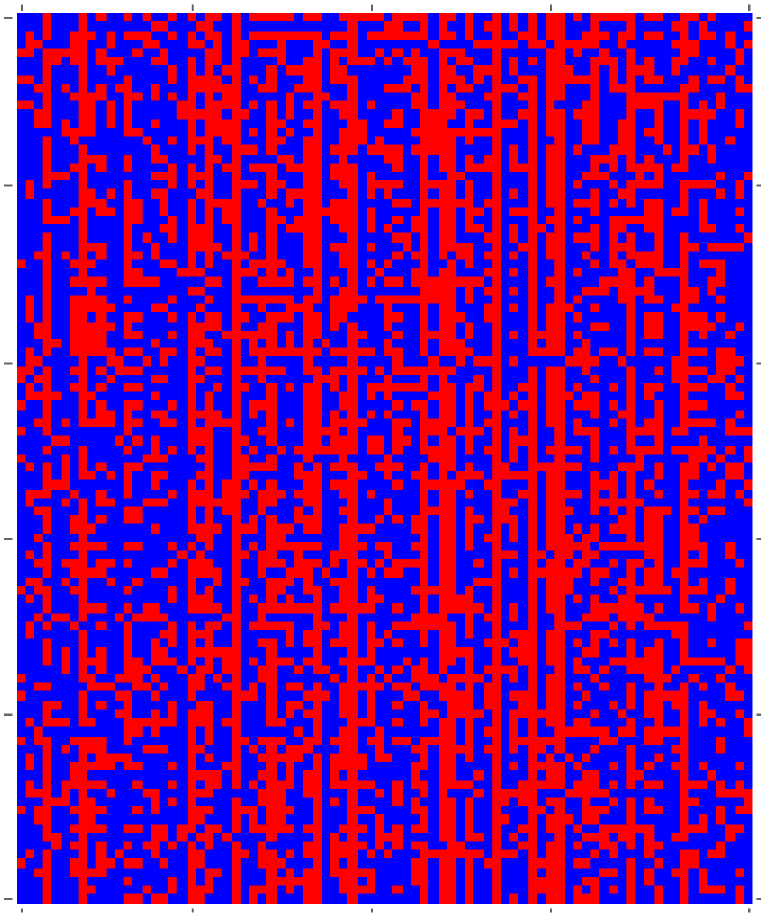


Figure 3.36: a=3 and b=5.

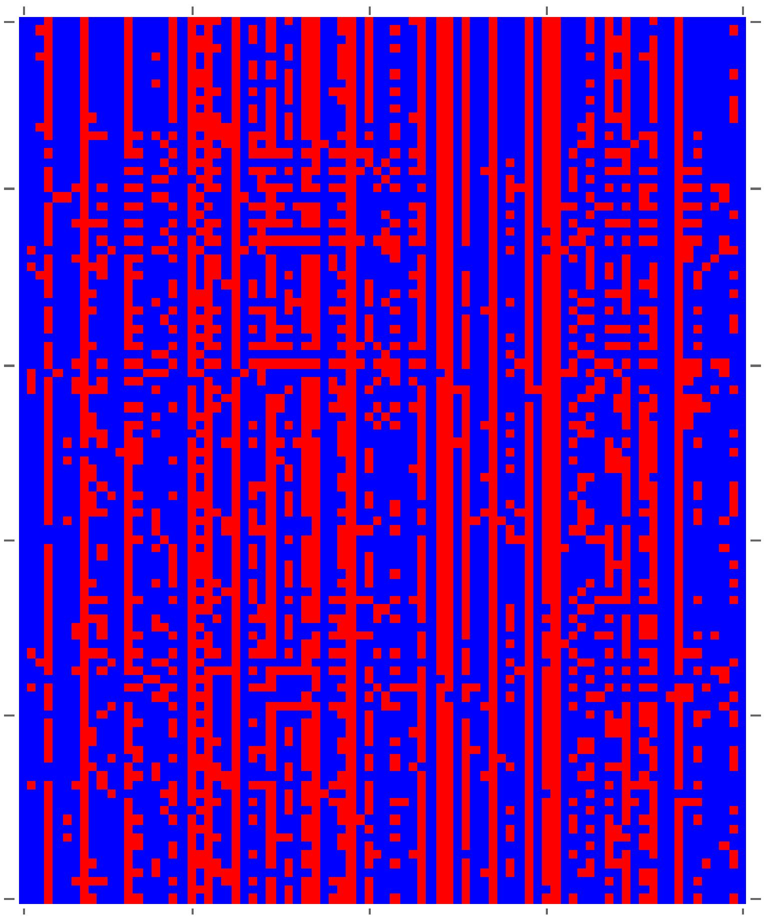


Figure 3.37: a=3 and b=6.

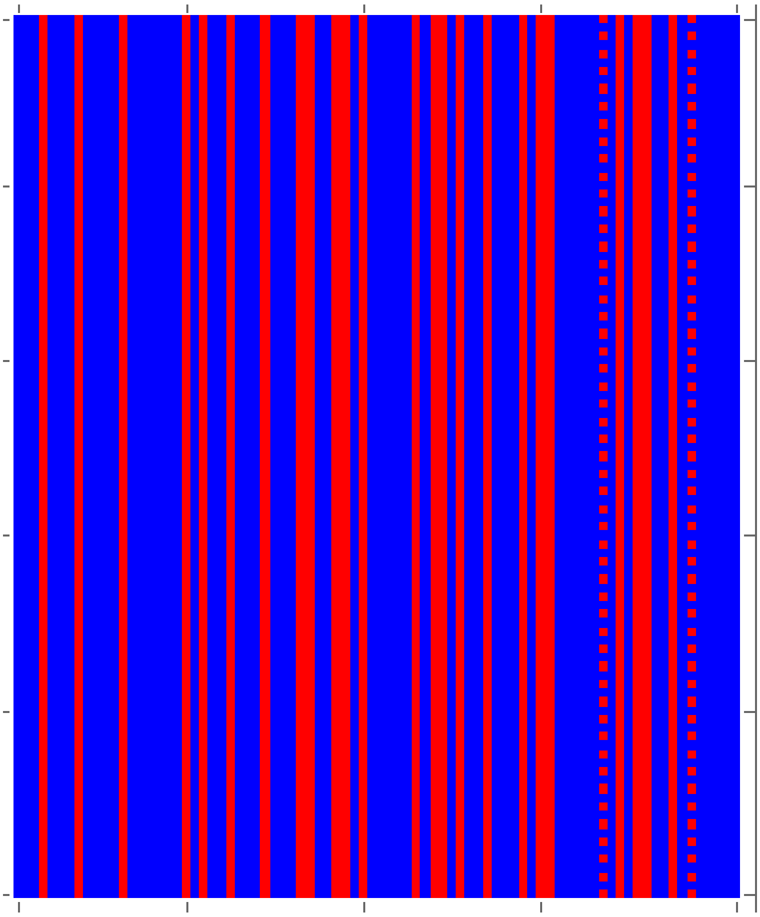


Figure 3.38: a=3 and b=7.

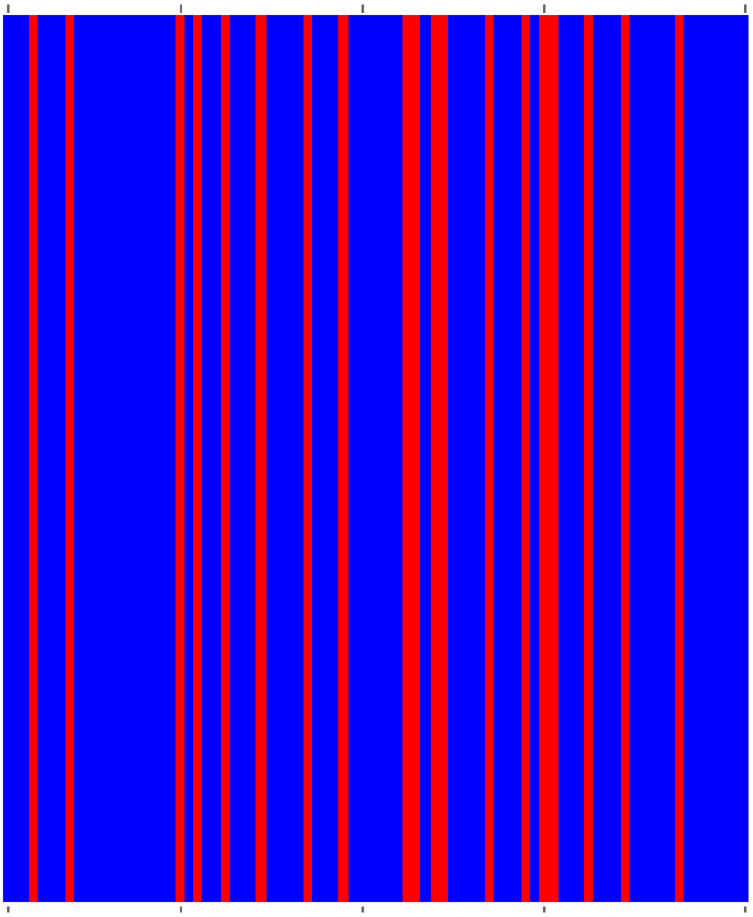


Figure 3.39: a=3 and b=8.

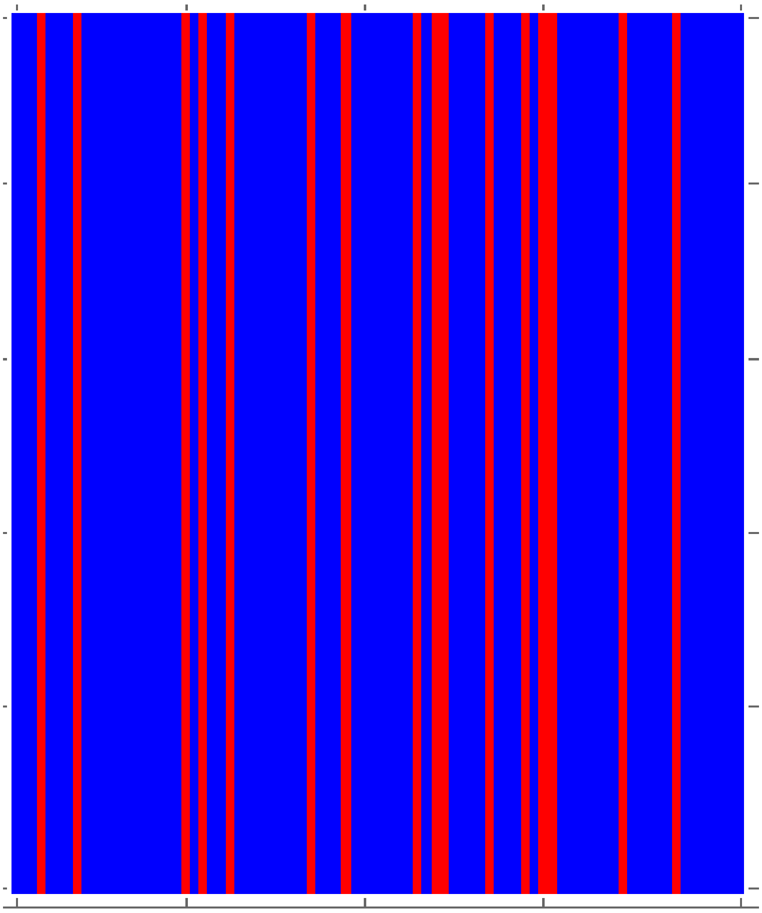


Figure 3.40: a=3 and b=9.

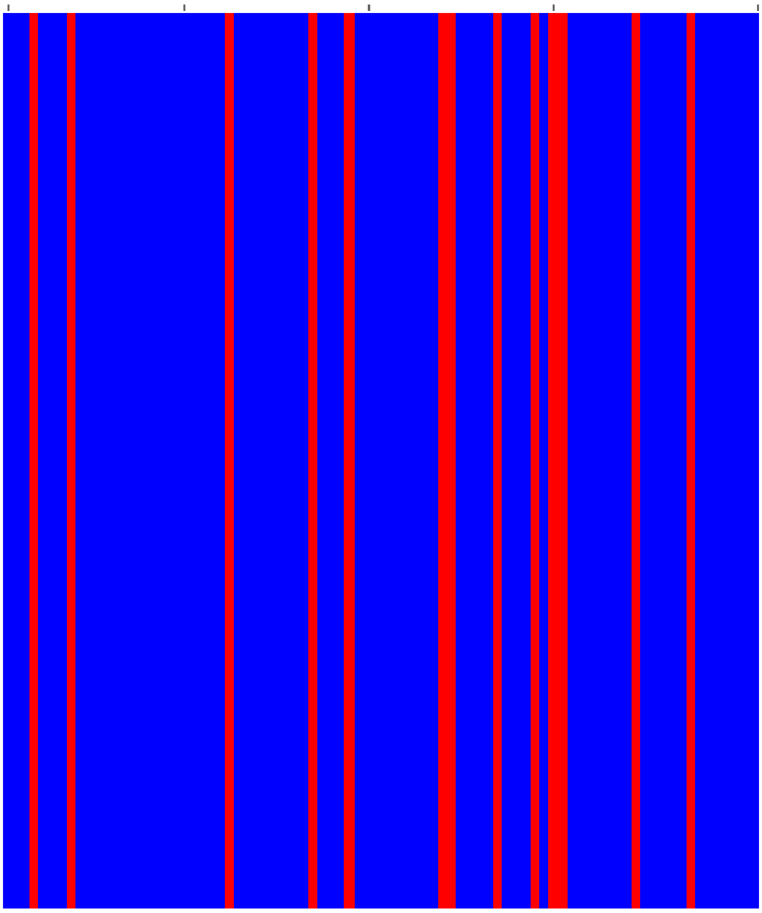


Figure 3.41: a=3 and b=10.

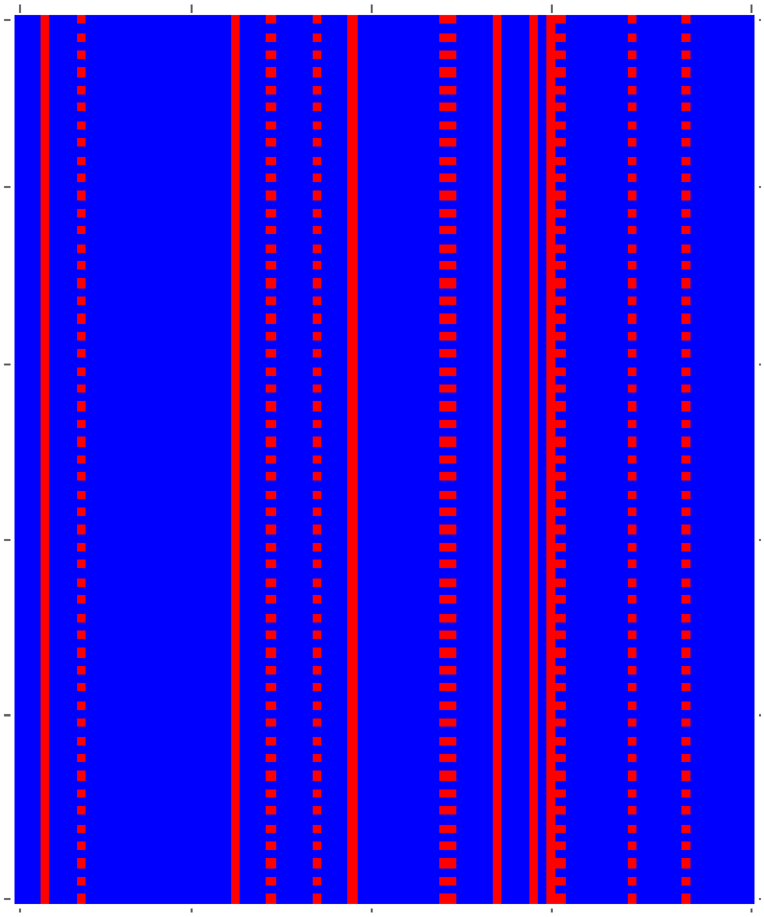


Figure 3.42: a=3 and b=12.

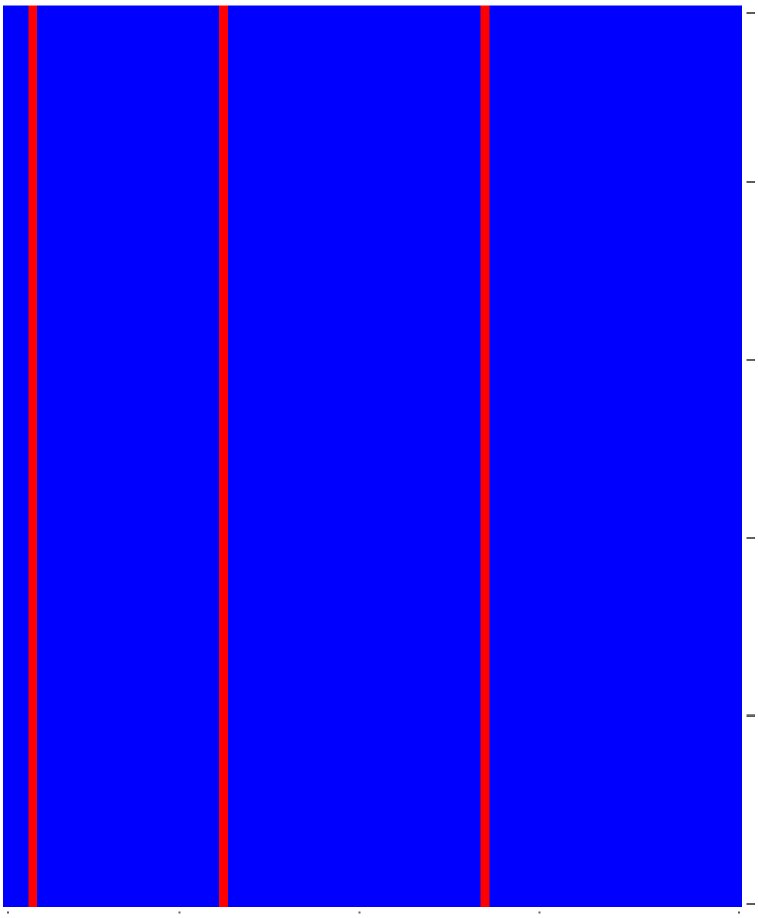


Figure 3.43: a=3 and b=13. The other results won't be showed because they are all red (off).

7- The results of the simulations are shown in the following pictures.

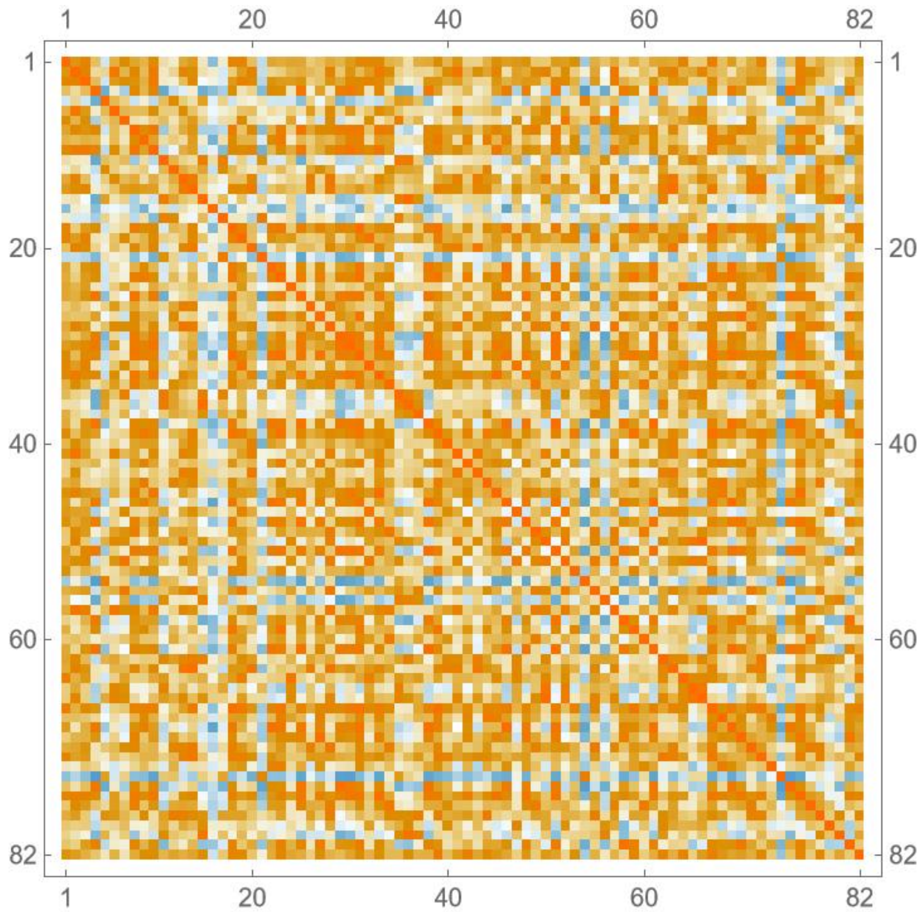


Figure 3.44: Correlation matrix among all the nodes that change over time. Here x=0.88 and z=0.87.

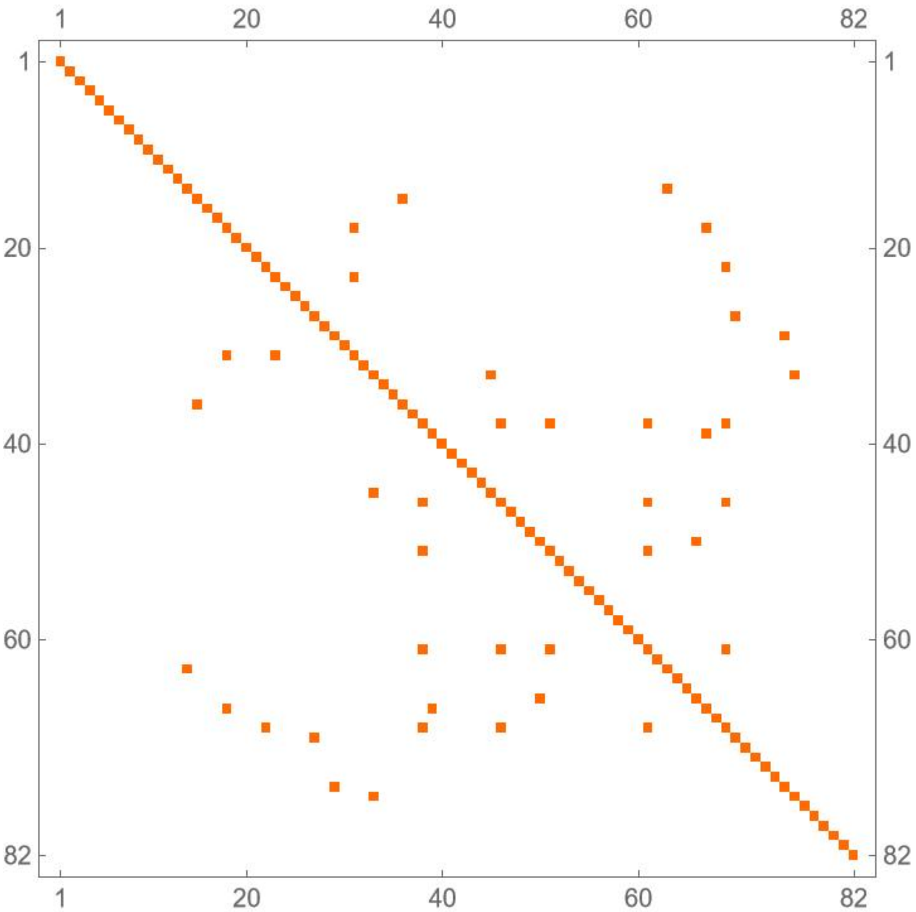


Figure 3.45: Correlation matrix among all the nodes and with the treshold. Here x=0.88 and z=0.87.

$$
\left(\begin{array}{cccccccccccccccc}
\{14, 63\} & \{\} & \{\} & \{\} & \{\} & \{\} & \{\} & \{\} & \{\} & \{\} & \{\} & \{\} & \{\} & \{\} & \{\} & \{\} \\
\{\} & \{15, 36\} & \{\} & \{\} & \{\} & \{\} & \{\} & \{\} & \{\} & \{\} & \{\} & \{\} & \{\} & \{\} & \{\} & \{\} \\
\{\} & \{\} & \{22, 69\} & \{\} & \{\} & \{\} & \{\} & \{\} & \{\} & \{\} & \{\} & \{\} & \{\} & \{\} & \{\} & \{69\} \\
\{\} & \{\} & \{\} & \{23, 31\} & \{\} & \{\} & \{\} & \{\} & \{\} & \{\} & \{31\} & \{23, 31\} & \{\} & \{\} & \{\} & \{\} \\
\{\} & \{\} & \{\} & \{\} & \{27, 70\} & \{\} & \{\} & \{\} & \{\} & \{\} & \{\} & \{\} & \{\} & \{\} & \{\} & \{\} \\
\{\} & \{\} & \{\} & \{\} & \{\} & \{29, 75\} & \{\} & \{\} & \{\} & \{\} & \{\} & \{\} & \{\} & \{\} & \{\} & \{\} \\
\{\} & \{\} & \{\} & \{\} & \{\} & \{\} & \{39, 67\} & \{\} & \{\} & \{\} & \{67\} & \{\} & \{\} & \{\} & \{39, 67\} & \{\} \\
\{\} & \{\} & \{\} & \{\} & \{\} & \{\} & \{\} & \{33, 45\} & \{\} & \{33\} & \{\} & \{\} & \{33, 45\} & \{\} & \{\} & \{\} \\
\{\} & \{\} & \{\} & \{\} & \{\} & \{\} & \{\} & \{\} & \{50, 66\} & \{\} & \{\} & \{\} & \{\} & \{\} & \{\} & \{\} \\
\{\} & \{\} & \{\} & \{\} & \{\} & \{\} & \{\} & \{33\} & \{\} & \{33, 76\} & \{\} & \{\} & \{33, 76\} & \{\} & \{\} & \{\} \\
\{\} & \{\} & \{\} & \{31\} & \{\} & \{\} & \{67\} & \{\} & \{\} & \{\} & \{18, 31, 67\} & \{18, 31\} & \{\} & \{\} & \{18, 67\} & \{\} \\
\{\} & \{\} & \{\} & \{23, 31\} & \{\} & \{\} & \{\} & \{\} & \{\} & \{\} & \{18, 31\} & \{18, 23, 31\} & \{\} & \{\} & \{18\} & \{\} \\
\{\} & \{\} & \{\} & \{\} & \{\} & \{\} & \{\} & \{33, 45\} & \{\} & \{33, 76\} & \{\} & \{\} & \{33, 45, 76\} & \{\} & \{\} & \{\} \\
\{\} & \{\} & \{\} & \{\} & \{\} & \{\} & \{\} & \{\} & \{\} & \{\} & \{\} & \{\} & \{\} & \{38, 51, 61\} & \{\} & \{38, 61\} \\
\{\} & \{\} & \{\} & \{\} & \{\} & \{\} & \{39, 67\} & \{\} & \{\} & \{\} & \{18, 67\} & \{18\} & \{\} & \{\} & \{18, 39, 67\} & \{\} \\
\{\} & \{\} & \{69\} & \{\} & \{\} & \{\} & \{\} & \{\} & \{\} & \{\} & \{\} & \{\} & \{\} & \{38, 61\} & \{\} & \{38, 46, 61, 69\}
\end{array}\right)
$$

Figure 3.46: A representation diagram of the intersection among the emerged circuits for the simulation with x=0.88 and z=0.87.

$$\begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4} \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & \frac{1}{2} & 0 & 0 & \frac{2}{3} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 1 & 0 & 0 & \frac{2}{3} & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 1 & \frac{2}{3} & 0 & 0 & \frac{2}{3} & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{2}{3} & 1 & 0 & 0 & \frac{1}{3} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & \frac{1}{2} \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 & 1 & 0 \\
0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{2}{3} & 0 & 1
\end{pmatrix}$$

Figure 3.47: An intersection array for representing relationships among the emerged circuits. x=0.88 and z=0.87.

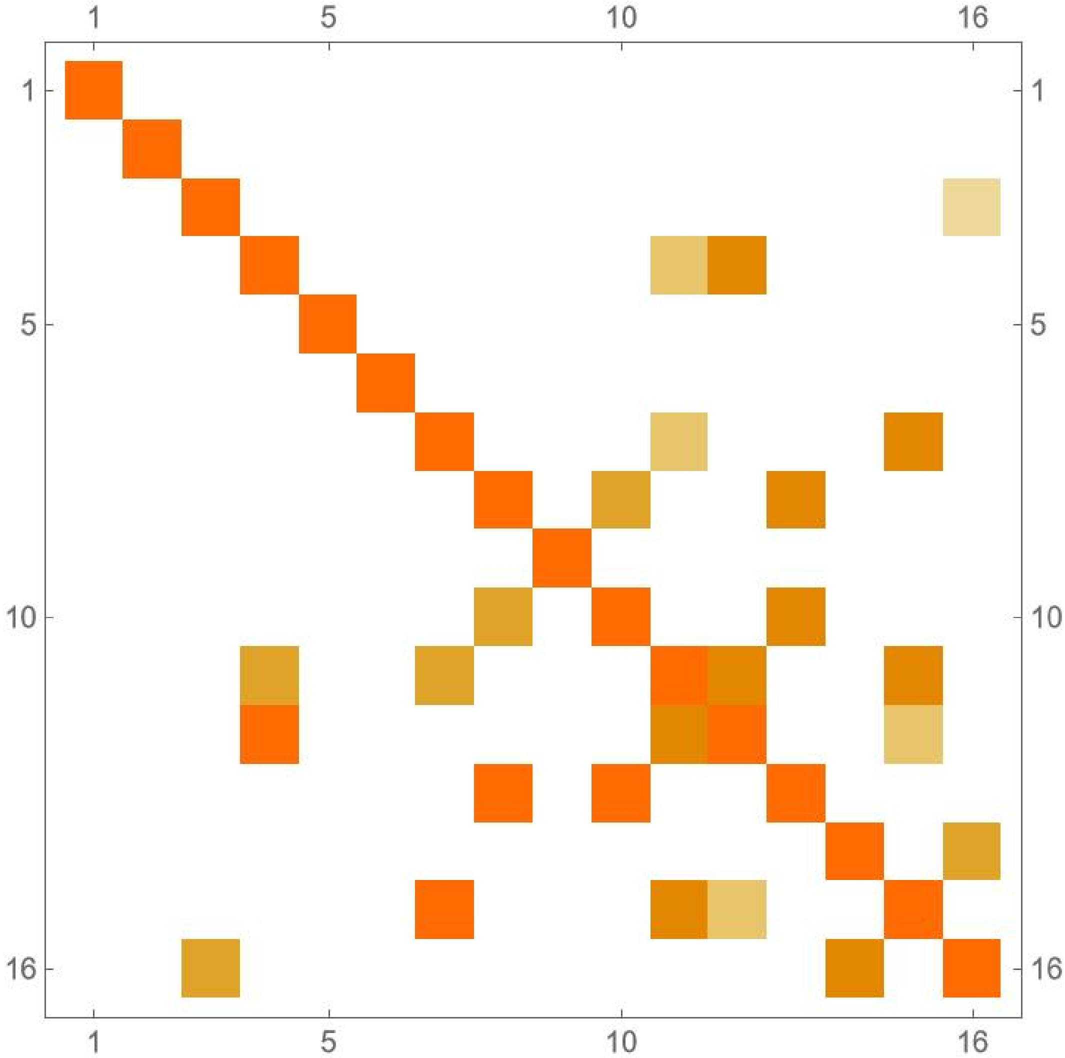


Figure 3.48: Correlation matrix for representing relationships among the emerged circuits. x=0.88 and z=0.87.

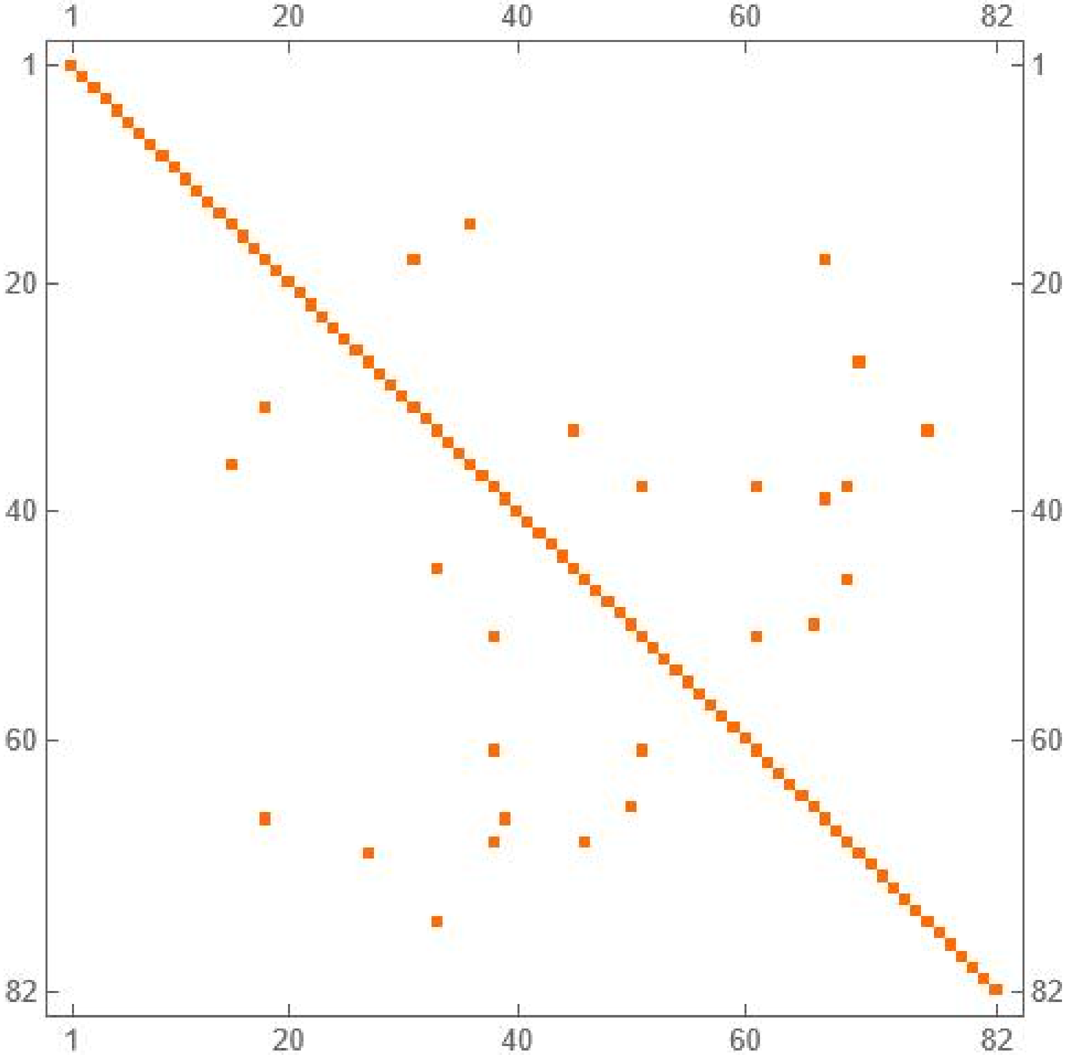


Figure 3.49: Correlation matrix among all the nodes that change over time. Here x=0.88 and z=0.89.

{14, 63}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{}
{}	{15, 36}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{}
{}	{}	{22, 69}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{69}	{69}	{22, 69}
{}	{}	{}	{23, 31}	{}	{}	{}	{}	{}	{}	{31}	{23, 31}	{}	{}	{}	{}	{}	{}
{}	{}	{}	{}	{27, 78}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{}
{}	{}	{}	{}	{}	{29, 75}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{}
{}	{}	{}	{}	{}	{}	{39, 67}	{}	{}	{}	{67}	{}	{}	{}	{39, 67}	{}	{}	{}
{}	{}	{}	{}	{}	{}	{}	{33, 45}	{}	{33}	{}	{}	{33, 45}	{}	{}	{}	{}	{}
{}	{}	{}	{}	{}	{}	{}	{}	{58, 66}	{}	{}	{}	{}	{}	{}	{}	{}	{}
{}	{}	{}	{}	{}	{}	{}	{33}	{}	{33, 76}	{}	{}	{33, 76}	{}	{}	{}	{}	{}
{}	{}	{}	{31}	{}	{}	{67}	{}	{}	{}	{18, 31, 67}	{18, 31}	{}	{}	{18, 67}	{}	{}	{}
{}	{}	{}	{23, 31}	{}	{}	{}	{}	{}	{}	{18, 31}	{18, 23, 31}	{}	{}	{18}	{}	{}	{}
{}	{}	{}	{}	{}	{}	{}	{33, 45}	{}	{33, 76}	{}	{}	{33, 45, 76}	{}	{}	{}	{}	{}
{}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{38, 51, 61}	{}	{38, 61}	{38, 51, 61}	{38, 61}
{}	{}	{}	{}	{}	{}	{39, 67}	{}	{}	{}	{18, 67}	{18}	{}	{}	{18, 39, 67}	{}	{}	{}
{}	{}	{69}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{38, 61}	{}	{38, 46, 61, 69}	{38, 46, 61, 69}	{38, 46, 61, 69}
{}	{}	{69}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{38, 51, 61}	{}	{38, 46, 61, 69}	{38, 46, 51, 61, 69}	{38, 46, 61, 69}
{}	{}	{22, 69}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{38, 61}	{}	{38, 46, 61, 69}	{38, 46, 61, 69}	{22, 38, 46, 61, 69}

Figure 3.50: A representation diagram of the intersection among the emerged circuits for the simulation with x=0.88 and z=0.89.

$$
\begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{5} & \frac{2}{5} \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & \frac{1}{2} & 0 & 0 & \frac{2}{3} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 1 & 0 & 0 & \frac{2}{3} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 1 & \frac{2}{3} & 0 & 0 & \frac{2}{3} & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{2}{3} & 1 & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & \frac{1}{2} & \frac{3}{5} & \frac{2}{5} \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{2}{3} & 0 & 1 & \frac{4}{5} & \frac{4}{5} \\
0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & \frac{4}{5} \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{2}{3} & 0 & 1 & \frac{4}{5} & 1
\end{pmatrix}
$$

Figure 3.51: An intersection array for representing relationships among the emerged circuits. x=0.88 and z=0.89.

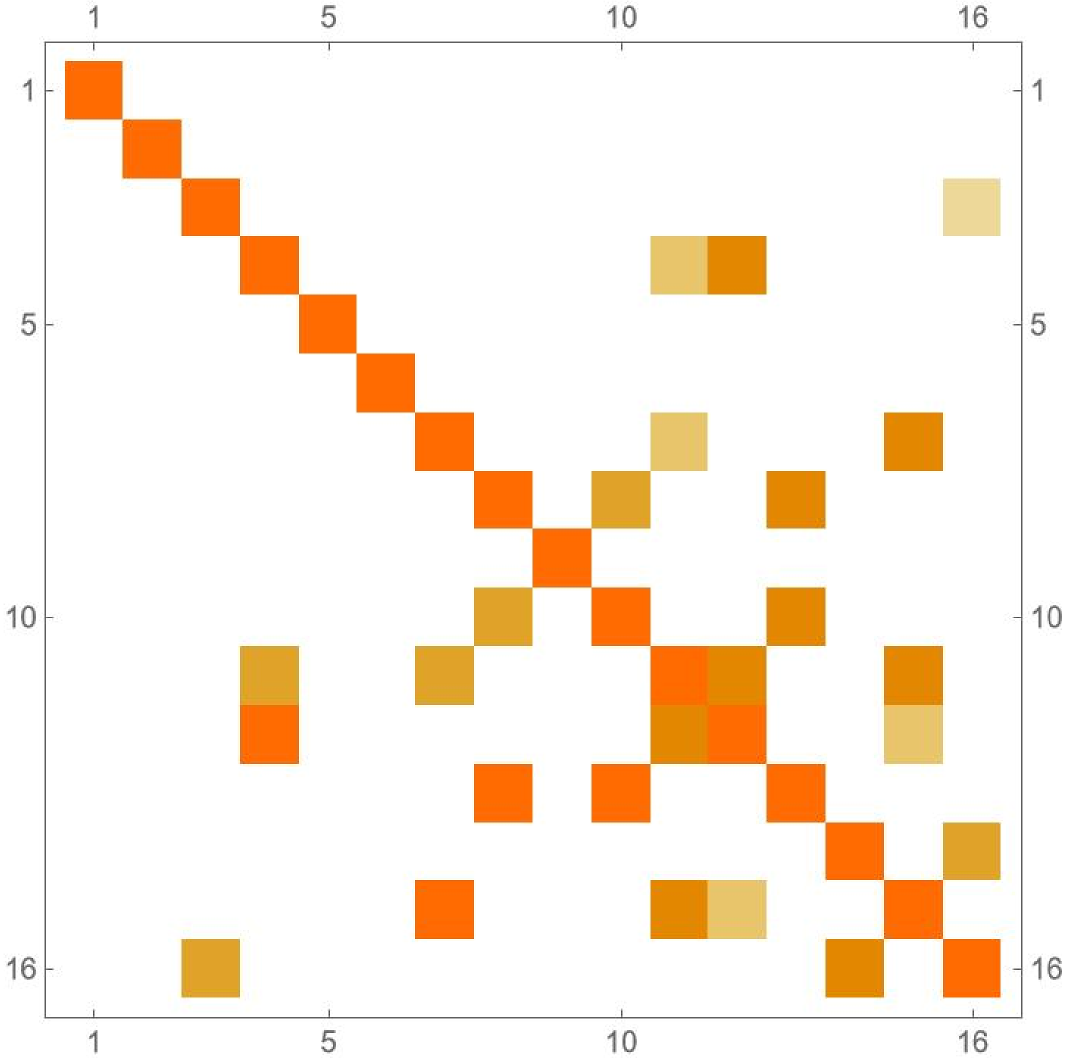


Figure 3.52: Correlation matrix for representing relationships among the emerged circuits. x=0.88 and z=0.89.

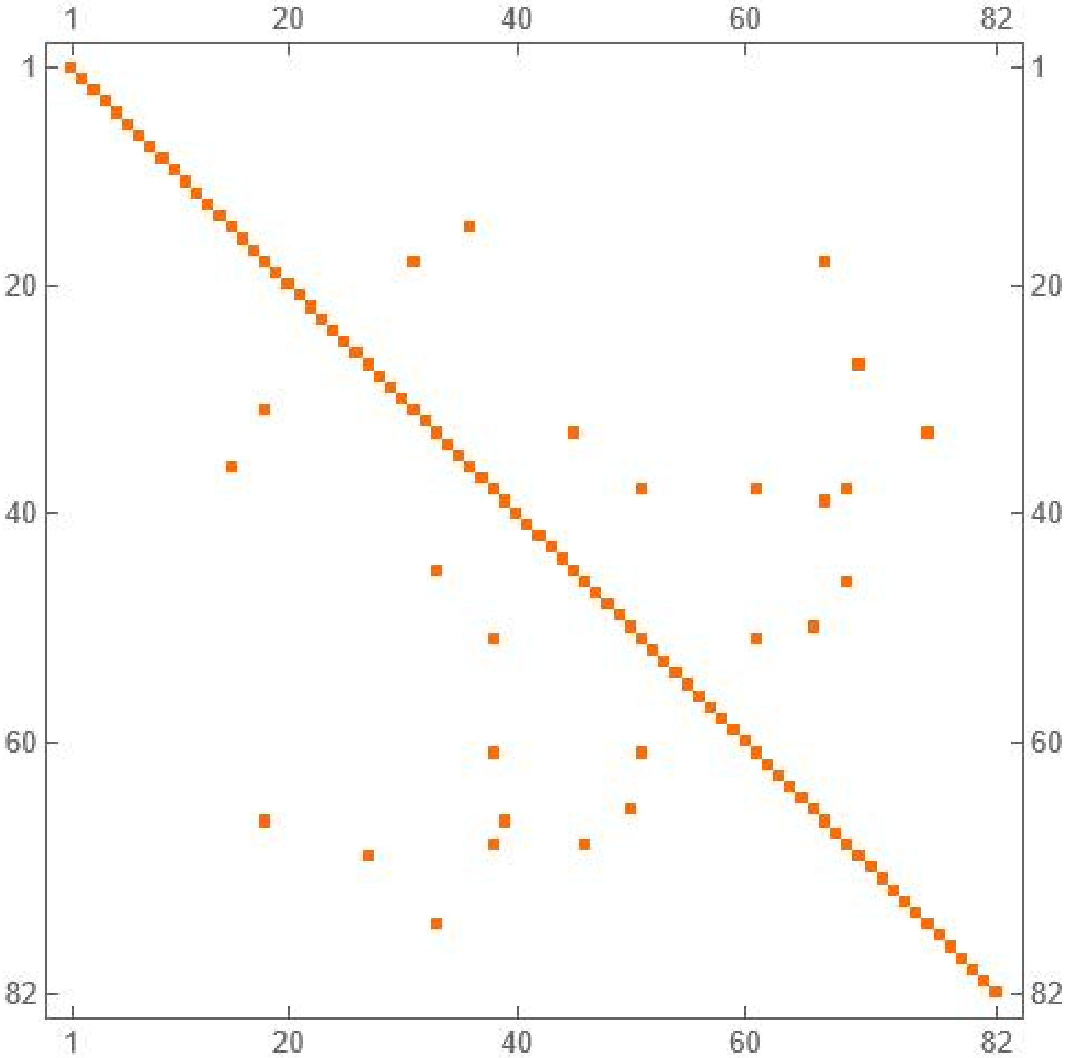


Figure 3.53: Correlation matrix among all the nodes that change over time. Here x=0.88 and z=0.88.

{14, 63}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{}
{}	{15, 36}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{}
{}	{}	{22, 69}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{69}	{}	{}	{69}	{69}	{22, 69}
{}	{}	{}	{23, 31}	{}	{}	{}	{}	{}	{31}	{23, 31}	{}	{}	{}	{}	{}	{}	{}
{}	{}	{}	{}	{27, 78}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{}
{}	{}	{}	{}	{}	{39, 67}	{}	{}	{}	{67}	{}	{}	{}	{}	{39, 67}	{}	{}	{}
{}	{}	{}	{}	{}	{}	{33, 45}	{}	{33}	{}	{}	{33, 45}	{}	{}	{}	{}	{}	{}
{}	{}	{}	{}	{}	{}	{}	{58, 66}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{}
{}	{}	{}	{}	{}	{}	{33}	{}	{33, 76}	{}	{}	{33, 76}	{}	{}	{}	{}	{}	{}
{}	{}	{}	{31}	{}	{67}	{}	{}	{}	{18, 31, 67}	{18, 31}	{}	{}	{}	{18, 67}	{}	{}	{}
{}	{}	{}	{23, 31}	{}	{}	{}	{}	{}	{18, 31}	{18, 23, 31}	{}	{}	{}	{18}	{}	{}	{}
{}	{}	{}	{}	{}	{}	{33, 45}	{}	{33, 76}	{}	{}	{33, 45, 76}	{}	{}	{}	{}	{}	{}
{}	{}	{69}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{46, 61, 69}	{61}	{}	{61, 69}	{46, 61, 69}	{46, 61, 69}
{}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{61}	{38, 51, 61}	{}	{38, 51, 61}	{38, 51, 61}	{38, 61}
{}	{}	{}	{}	{}	{39, 67}	{}	{}	{}	{18, 67}	{18}	{}	{}	{}	{18, 39, 67}	{}	{}	{}
{}	{}	{69}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{61, 69}	{38, 51, 61}	{}	{38, 51, 61, 69}	{38, 51, 61, 69}	{38, 61, 69}
{}	{}	{69}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{46, 61, 69}	{38, 51, 61}	{}	{38, 51, 61, 69}	{38, 46, 51, 61, 69}	{38, 46, 61, 69}
{}	{}	{22, 69}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{46, 61, 69}	{38, 61}	{}	{38, 61, 69}	{38, 46, 61, 69}	{22, 38, 46, 61, 69}

Figure 3.54: A representation diagram of the intersection among the emerged circuits for the simulation with x=0.88 and z=0.88.

$$\begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{4} & \frac{1}{5} & \frac{2}{5} \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{2}{3} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & \frac{1}{2} & 0 & 0 & \frac{2}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 1 & 0 & 0 & \frac{2}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 1 & \frac{2}{3} & 0 & 0 & 0 & \frac{2}{3} & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & \frac{2}{3} & 1 & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & \frac{1}{3} & 0 & \frac{1}{2} & \frac{3}{5} & \frac{3}{5} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & 1 & 0 & \frac{3}{4} & \frac{3}{5} & \frac{2}{5} \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{2}{3} & 1 & 0 & 1 & \frac{4}{5} & \frac{3}{5} \\
0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & \frac{4}{5} \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & \frac{2}{3} & 0 & \frac{3}{4} & \frac{4}{5} & 1
\end{pmatrix}$$

Figure 3.55: An intersection array for representing relationships among the emerged circuits. x=0.88 and z=0.88.

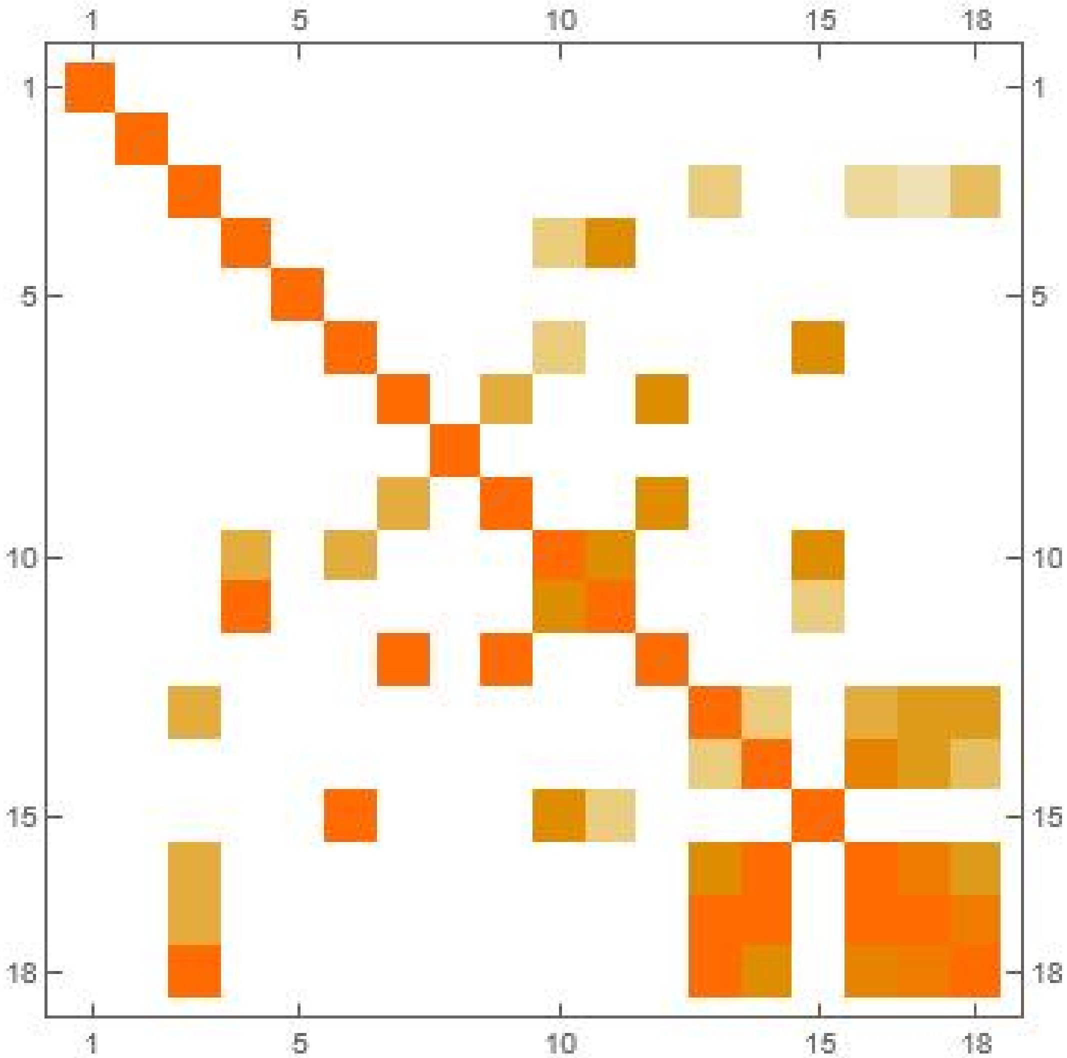


Figure 3.56: Correlation matrix for representing relationships among the emerged circuits. x=0.88 and z=0.88.

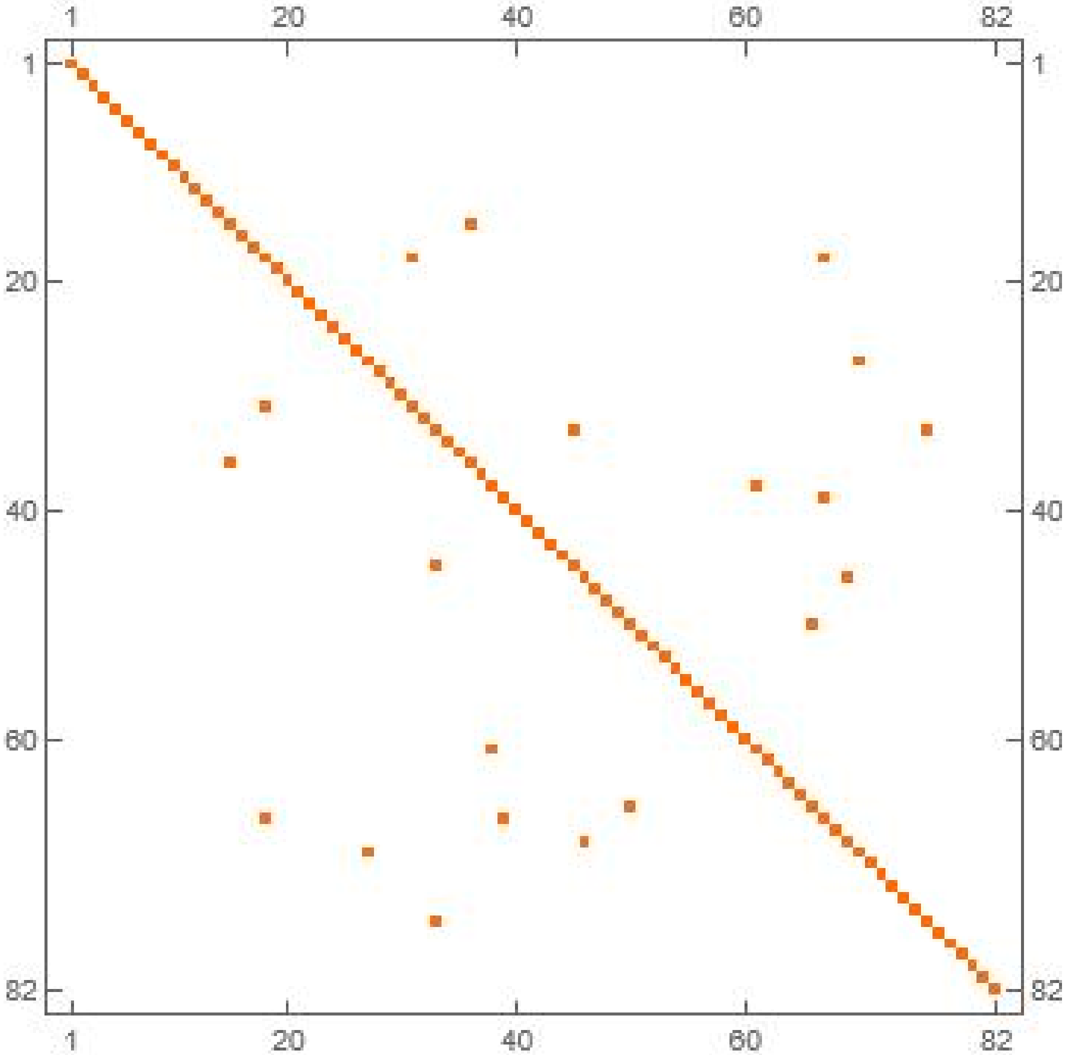


Figure 3.57: Correlation matrix among all the nodes that change over time. Here x=0.88 and z=0.9.

{14, 63}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{}
{}	{15, 36}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{}
{}	{}	{22, 69}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{69}	{69}	{22, 69}
{}	{}	{}	{23, 31}	{}	{}	{}	{}	{}	{}	{31}	{23, 31}	{}	{}	{}	{}	{}	{}
{}	{}	{}	{}	{27, 78}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{}
{}	{}	{}	{}	{}	{29, 75}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{}
{}	{}	{}	{}	{}	{}	{39, 67}	{}	{}	{}	{67}	{}	{}	{}	{39, 67}	{}	{}	{}
{}	{}	{}	{}	{}	{}	{}	{33, 45}	{}	{33}	{}	{}	{33, 45}	{}	{}	{}	{}	{}
{}	{}	{}	{}	{}	{}	{}	{}	{58, 66}	{}	{}	{}	{}	{}	{}	{}	{}	{}
{}	{}	{}	{}	{}	{}	{}	{33}	{}	{33, 76}	{}	{}	{33, 76}	{}	{}	{}	{}	{}
{}	{}	{}	{31}	{}	{}	{67}	{}	{}	{}	{18, 31, 67}	{18, 31}	{}	{}	{18, 67}	{}	{}	{}
{}	{}	{}	{23, 31}	{}	{}	{}	{}	{}	{}	{18, 31}	{18, 23, 31}	{}	{}	{18}	{}	{}	{}
{}	{}	{}	{}	{}	{}	{}	{33, 45}	{}	{33, 76}	{}	{}	{33, 45, 76}	{}	{}	{}	{}	{}
{}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{38, 51, 61}	{}	{38, 61}	{38, 51, 61}	{38, 61}
{}	{}	{}	{}	{}	{}	{39, 67}	{}	{}	{}	{18, 67}	{18}	{}	{}	{18, 39, 67}	{}	{}	{}
{}	{}	{69}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{38, 61}	{}	{38, 46, 61, 69}	{38, 46, 61, 69}	{38, 46, 61, 69}
{}	{}	{69}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{38, 51, 61}	{}	{38, 46, 61, 69}	{38, 46, 51, 61, 69}	{38, 46, 61, 69}
{}	{}	{22, 69}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{}	{38, 61}	{}	{38, 46, 61, 69}	{38, 46, 61, 69}	{22, 38, 46, 61, 69}

Figure 3.58: A representation diagram of the intersection among the emerged circuits for the simulation with x=0.88 and z=0.9.

$$
\begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{2}{3} & 0 & \frac{1}{3} \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & \frac{1}{3} & 0 & \frac{2}{3} \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & \frac{1}{2} & 0 & \frac{2}{3} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 1 & 0 & \frac{2}{3} & 0 \\
0 & 0 & 1 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 1 & 0 & \frac{2}{3} \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 \\
0 & 0 & \frac{1}{2} & 0 & 1 & 0 & 0 & 0 & 0 & \frac{2}{3} & 0 & 1
\end{pmatrix}
$$

Figure 3.59: An intersection array for representing relationships among the emerged circuits. x=0.88 and z=0.9.

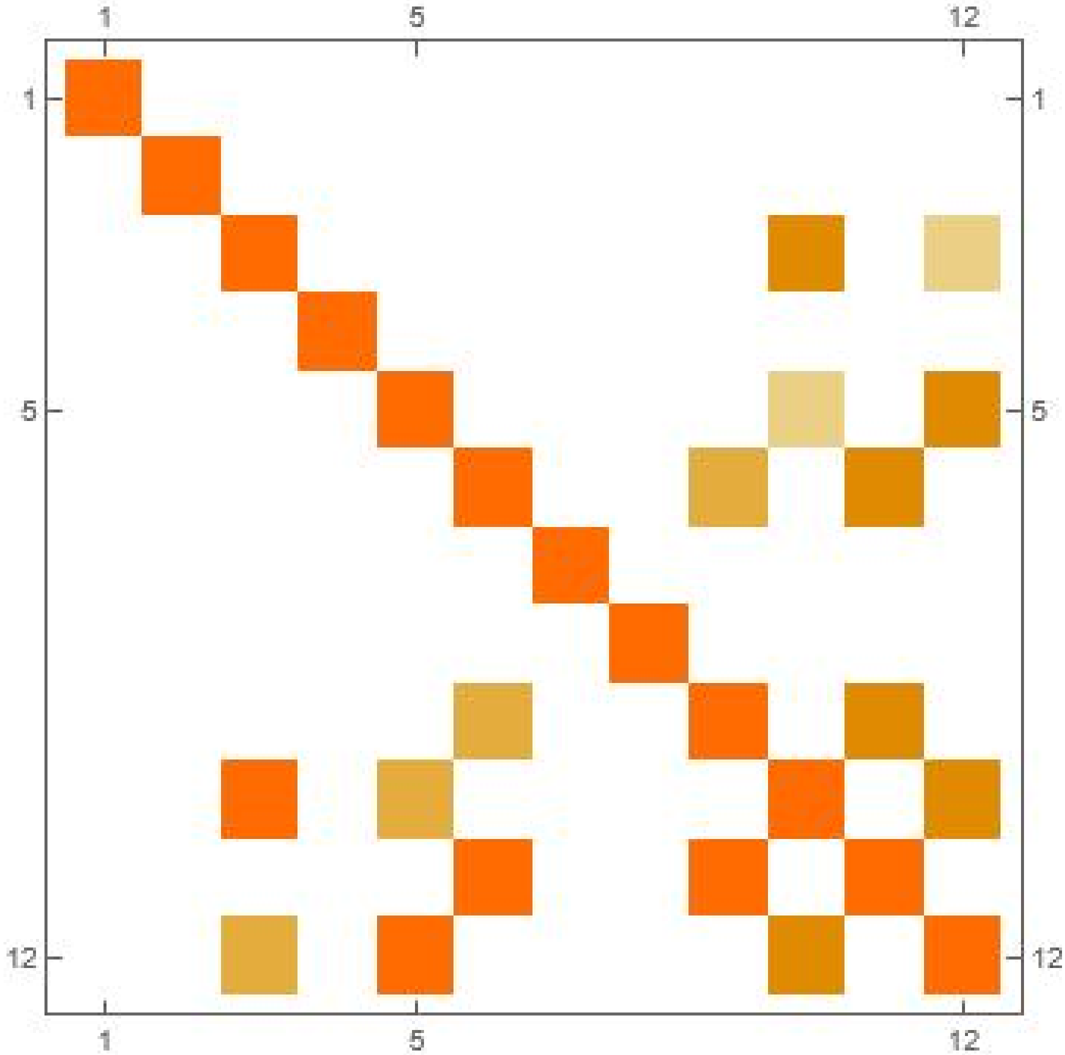


Figure 3.60: Correlation matrix for representing relationships among the emerged circuits. x=0.88 and z=0.9.

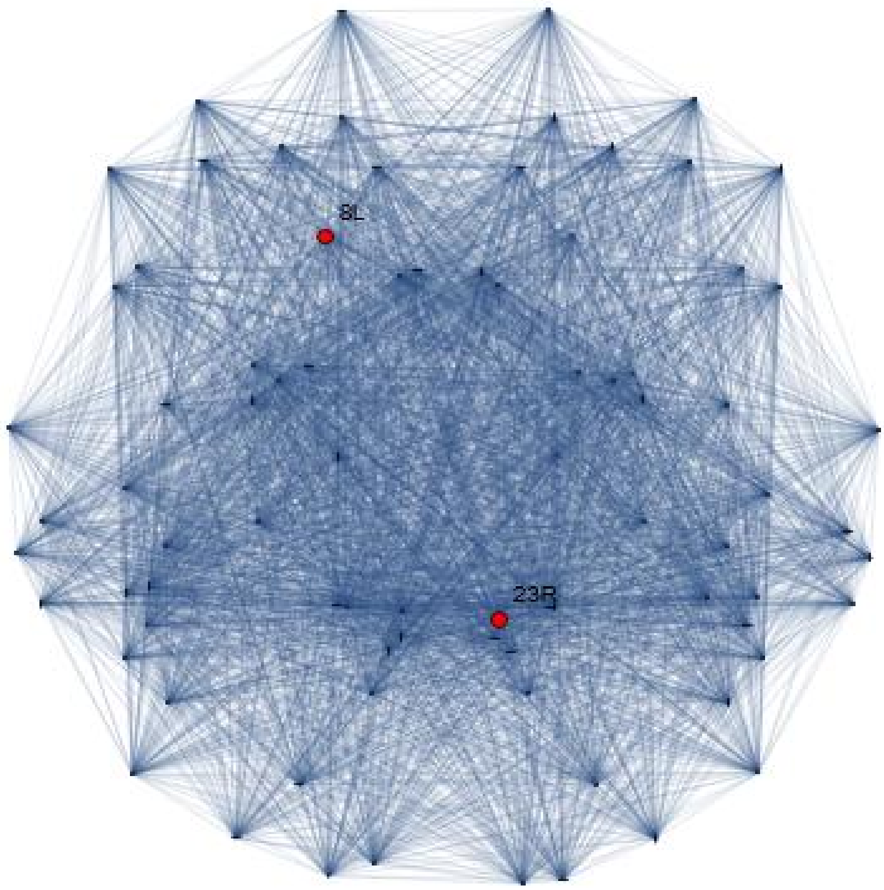


Figure 3.61: This is a graph with one of the extracted circuits. This is the first one.

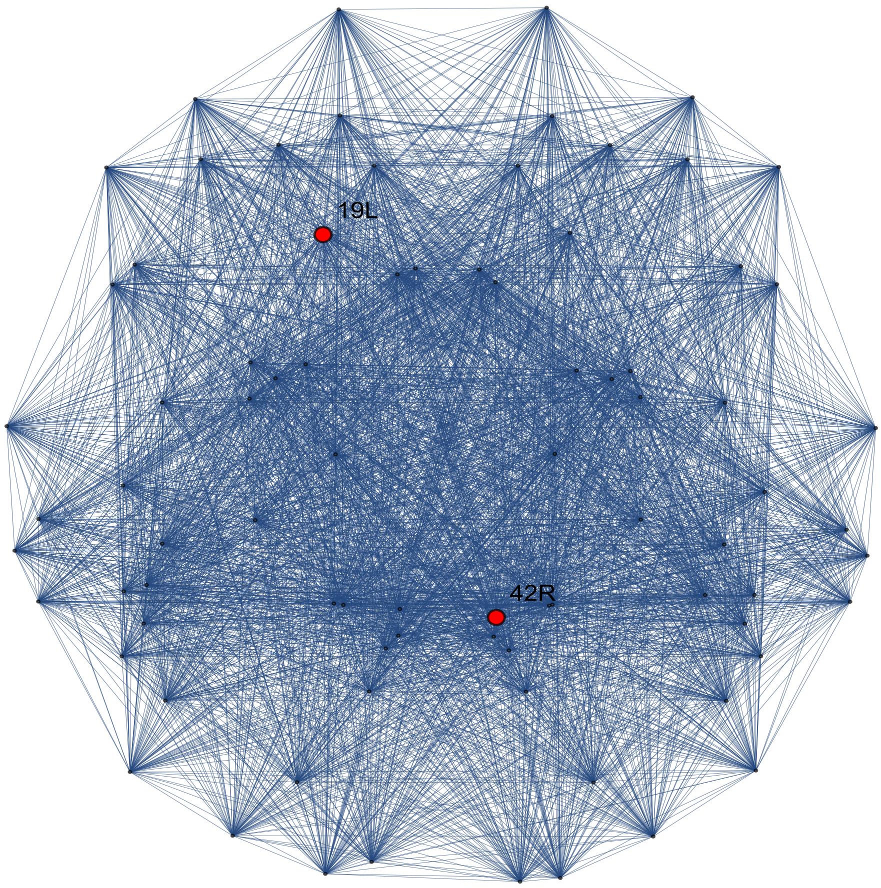


Figure 3.62: The graph of the 2nd circuit.

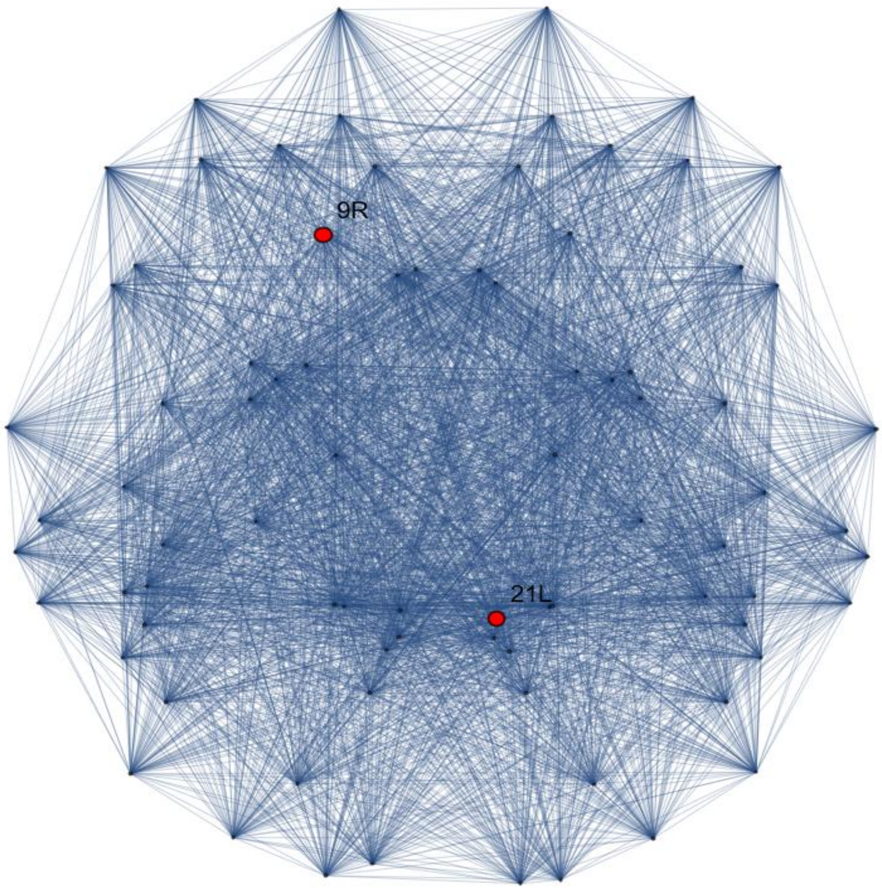


Figure 3.63: The graph of the 3rd circuit.

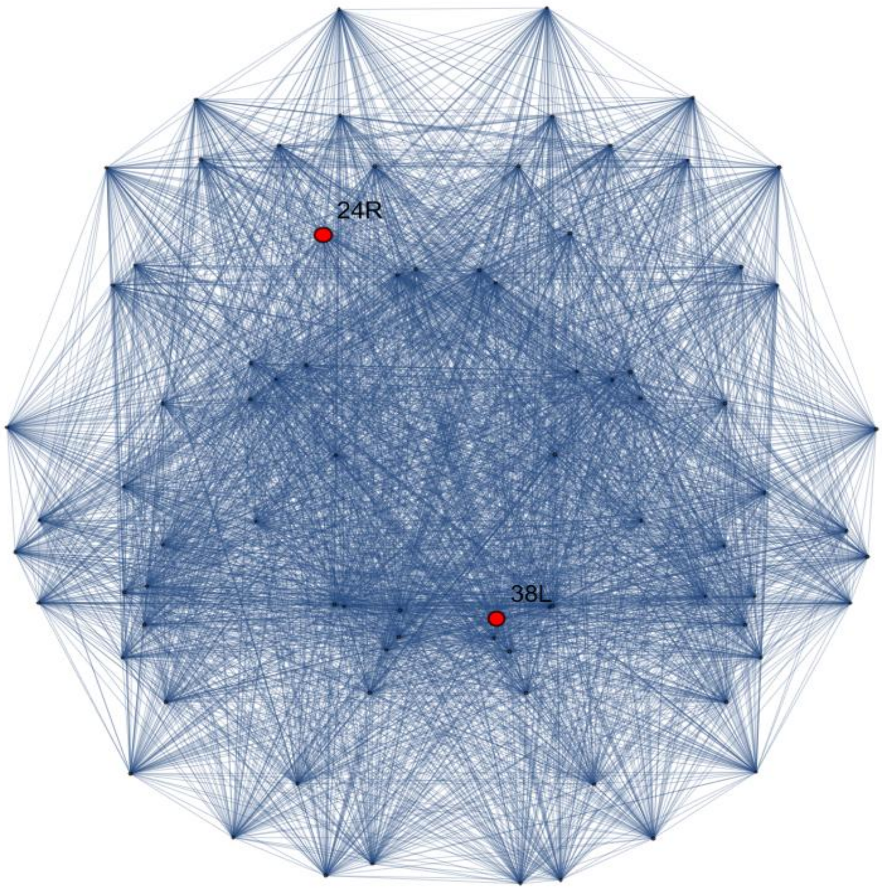


Figure 3.64: The graoh of the 4th circuit.

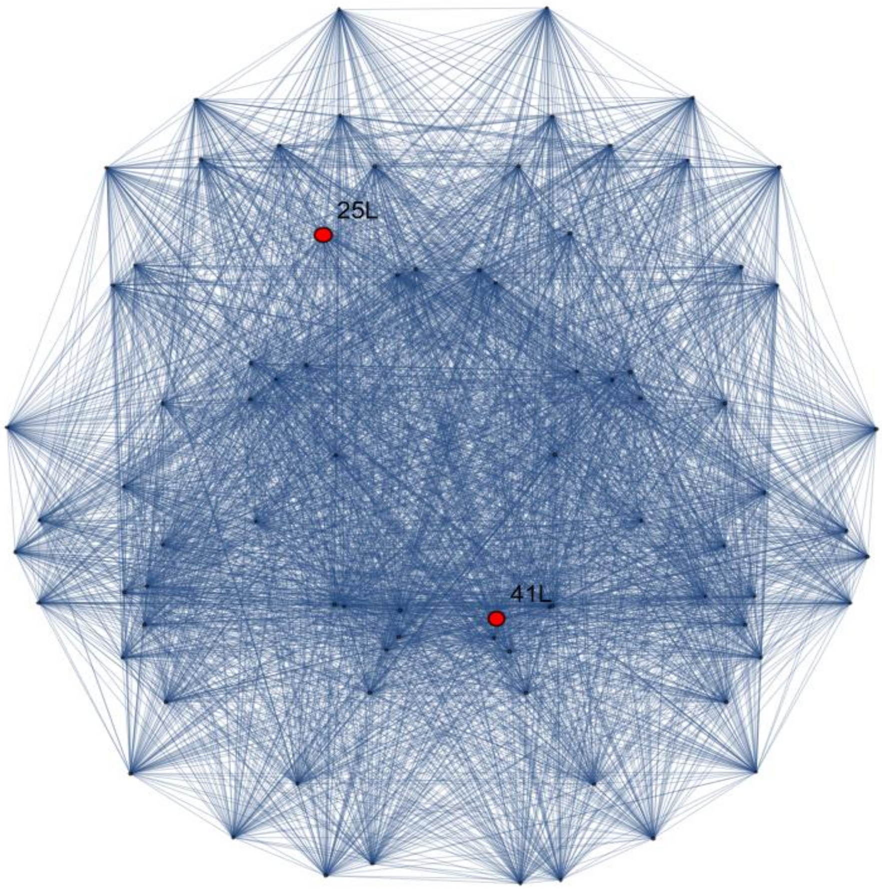


Figure 3.65: The graph of the 5th circuit.

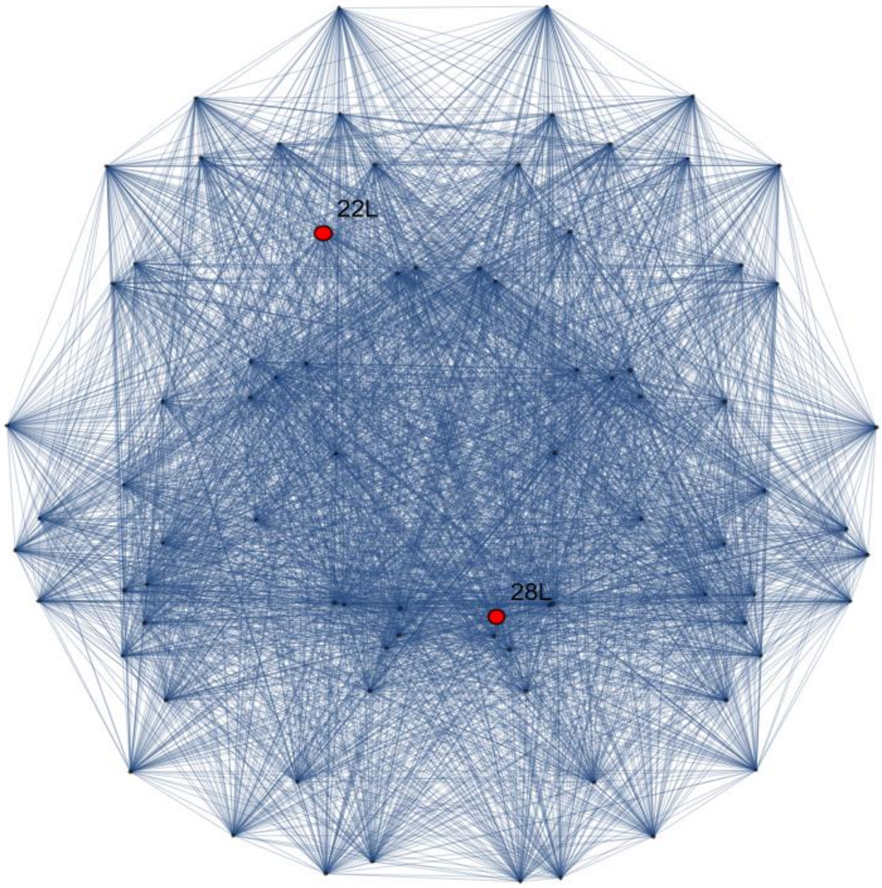


Figure 3.66: The graph of the 6th circuit.

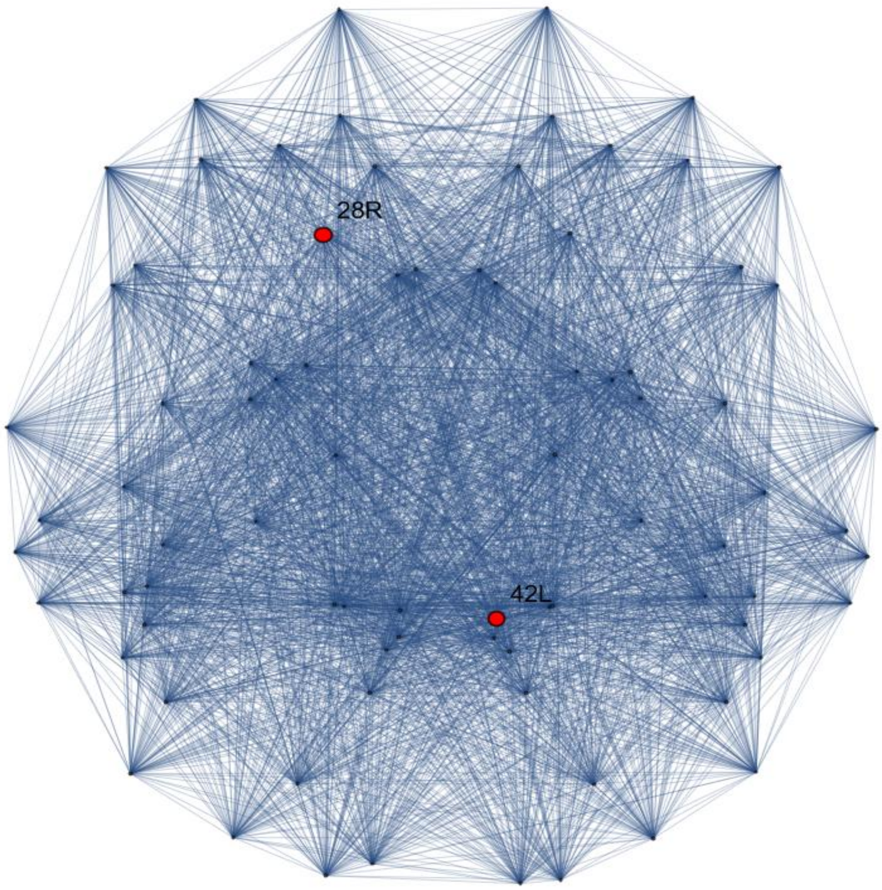


Figure 3.67: The graph of the 7th circuit.

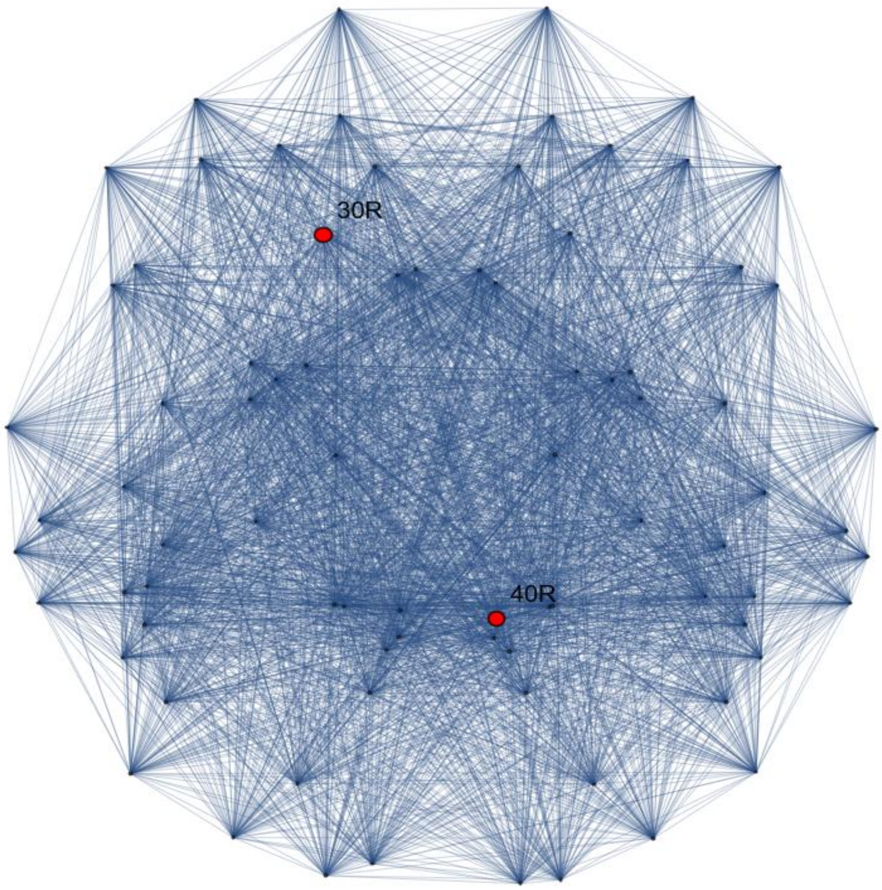


Figure 3.68: The graph of the 8th circuit.

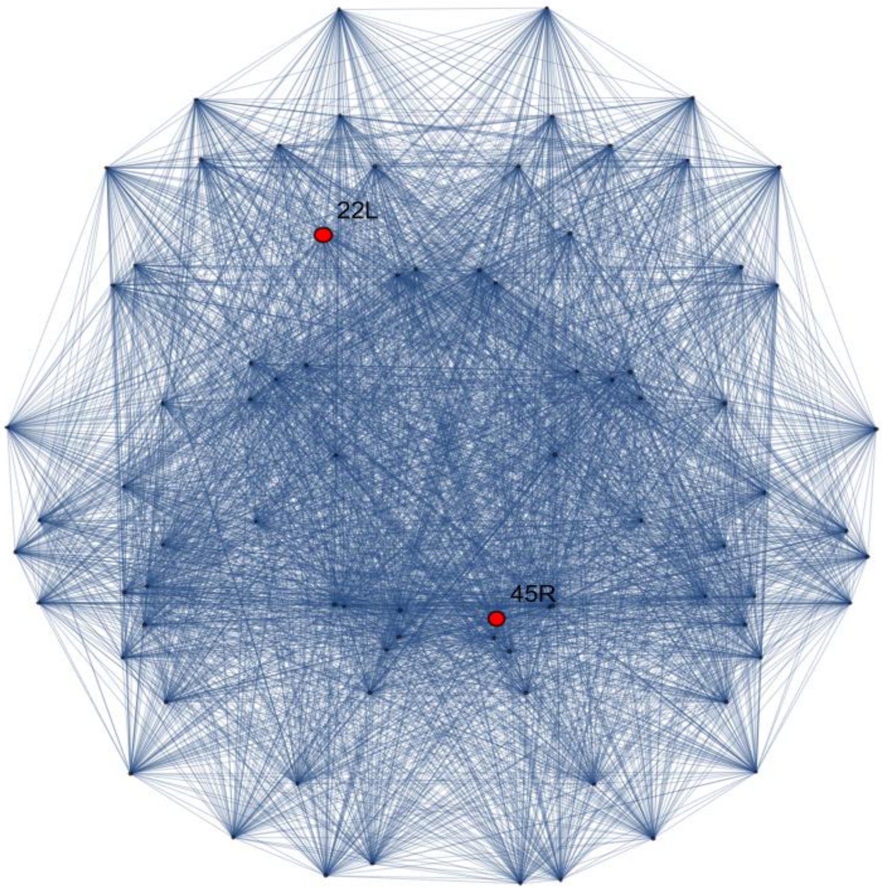


Figure 3.69: The graph of the 9th circuit.

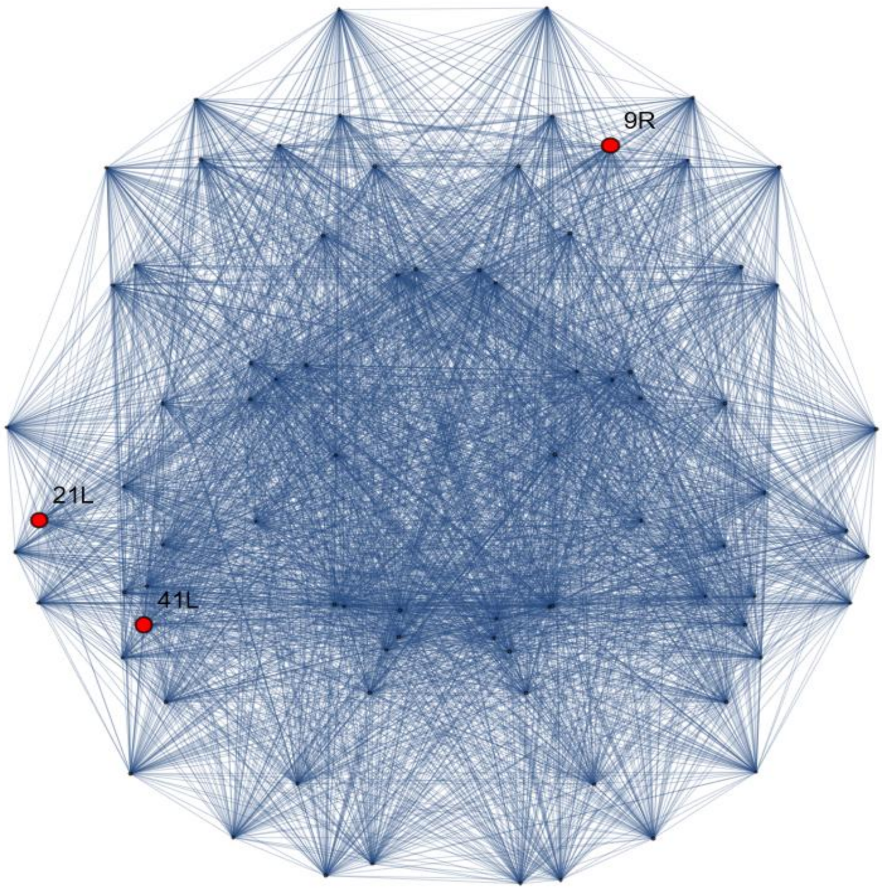


Figure 3.70: The graph of the 10th circuit.

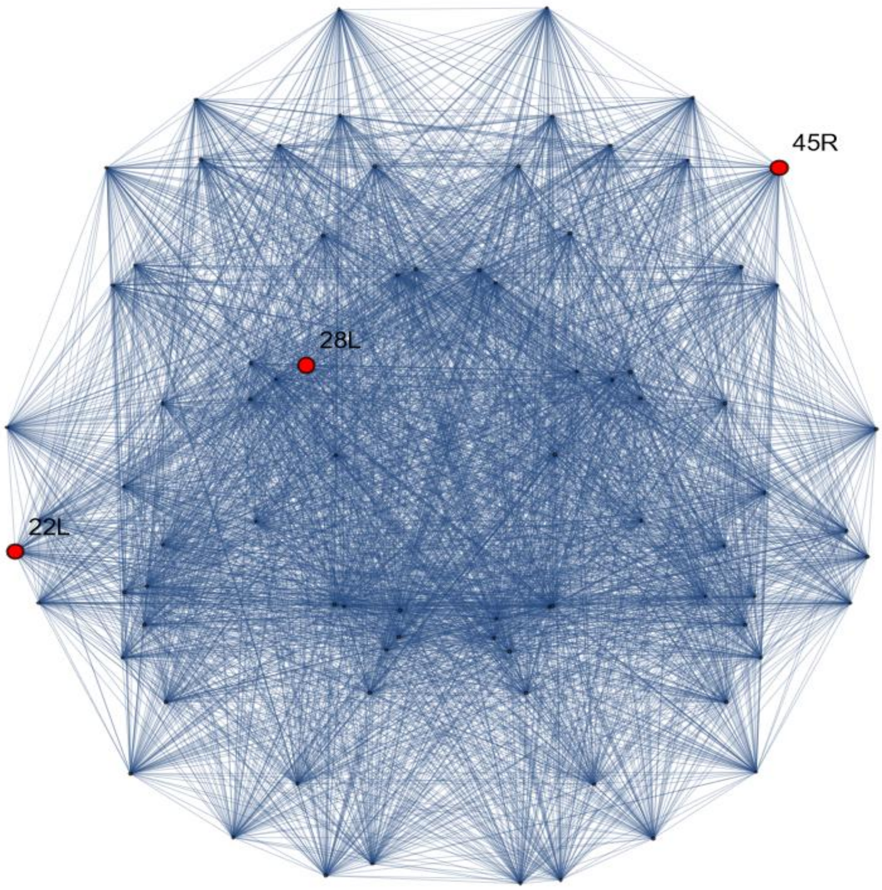


Figure 3.71: The graph of the 11th circuit.

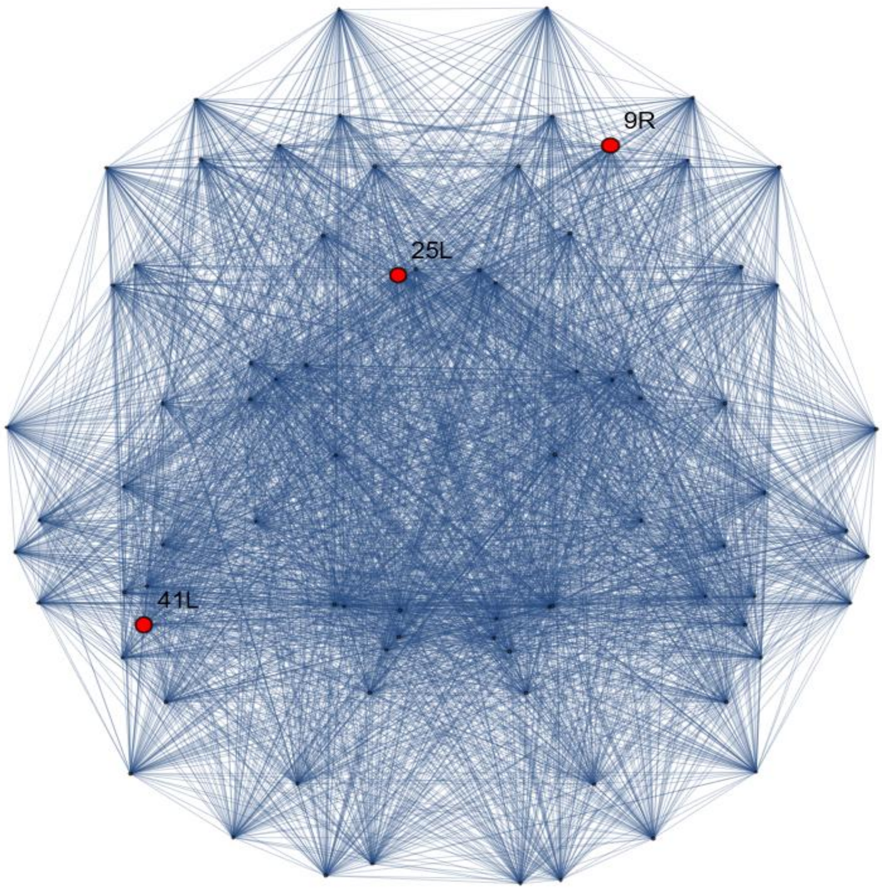


Figure 3.72: The graph of the 12th circuit.

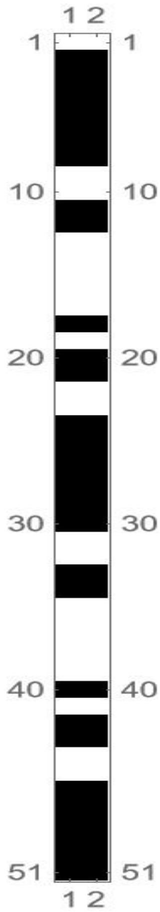


Figure 3.73: This is the circuit local evolution from the 450th step on. Black means off and white means on. The one in the picture is the 1st circuit.

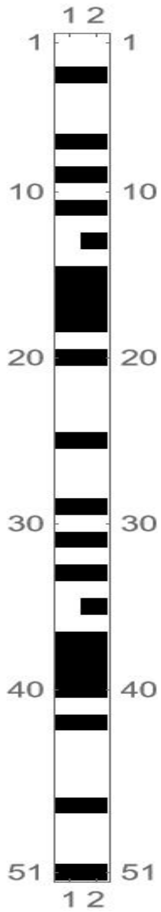


Figure 3.74: This is the circuit local evolution from the 450th step on. Black means off and white means on. The one in the picture is the 2nd circuit.

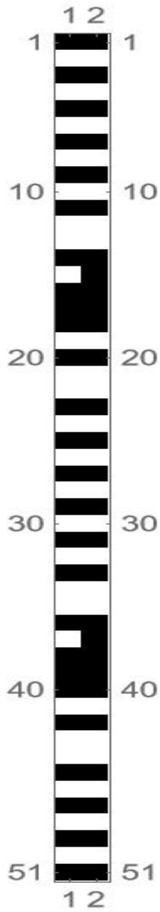


Figure 3.75: This is the circuit local evolution from the 450th step on. Black means off and white means on. The one in the picture is the 3rd circuit.

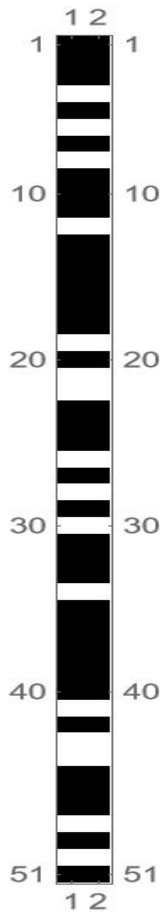


Figure 3.76: This is the circuit local evolution from the 450th step on. Black means off and white means on. The one in the picture is the 4th circuit.

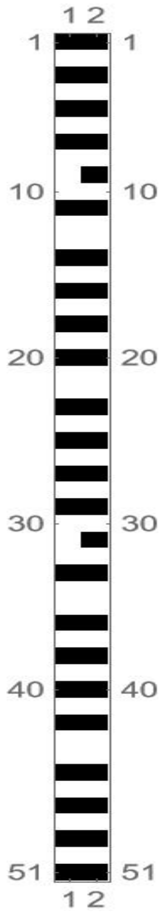


Figure 3.77: This is the circuit local evolution from the 450th step on. Black means off and white means on. The one in the picture is the 5th circuit.

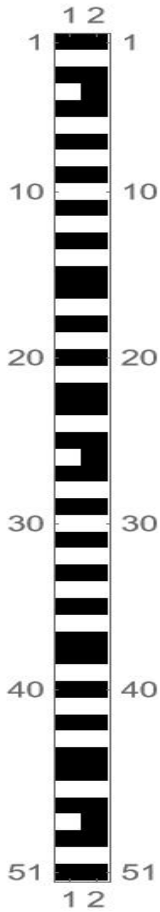


Figure 3.78: This is the circuit local evolution from the 450th step on. Black means off and white means on. The one in the picture is the 6th circuit.

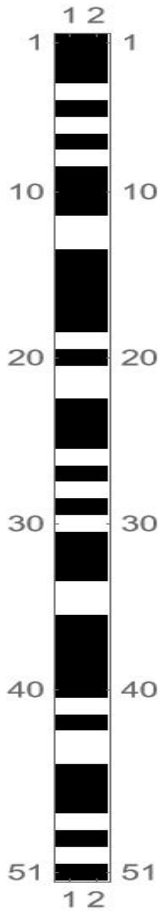


Figure 3.79: This is the circuit local evolution from the 450th step on. Black means off and white means on. The one in the picture is the 7th circuit.

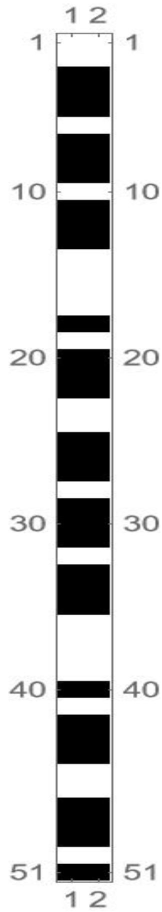


Figure 3.80: This is the circuit local evolution from the 450th step on. Black means off and white means on. The one in the picture is the 8th circuit.

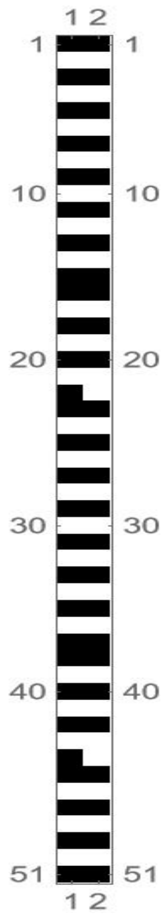


Figure 3.81: This is the circuit local evolution from the 450th step on. Black means off and white means on. The one in the picture is the 9th circuit.

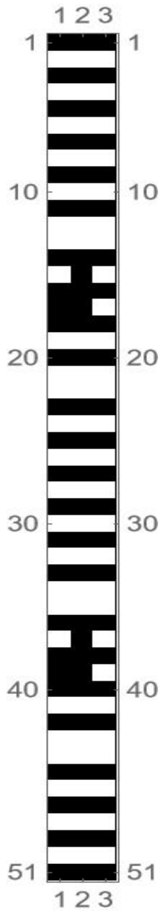


Figure 3.82: This is the circuit local evolution from the 450th step on. Black means off and white means on. The one in the picture is the 10th circuit.

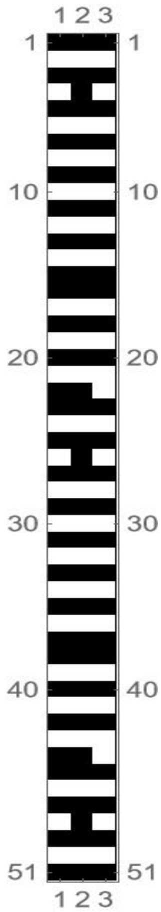


Figure 3.83: This is the circuit local evolution from the 450th step on. Black means off and white means on. The one in the picture is the 11th circuit.

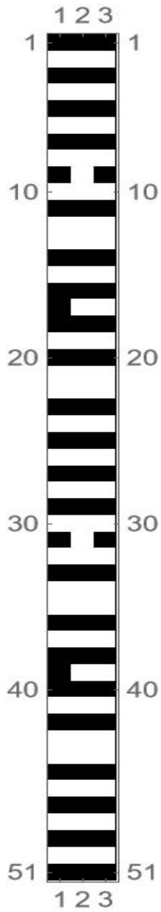


Figure 3.84: This is the circuit local evolution from the 450th step on. Black means off and white means on. The one in the picture is the 12th circuit.

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