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Positioning and Location-Based Services

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EXERCISE 1

RF TRANSFORMATION

REPORT

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Introduction

The objective of this exercise is to compute the geodetic coordinates (latitude, longitude and geodetic height) and the relative standard deviations of four points located in the outskirts of Como, starting from the Global Cartesian coordinates and covariance of coordinates and velocity of the city of Como that can be downloaded from the EUREF Permanent GNSS Network website.

The starting data that was provided consisted of:

- Coordinates of the baseline Como – Brunate, Brunate being one of the four points to calculate the coordinates of;
- Coordinates of the baseline Brunate – st.1, with station 1 being one of the four points to calculate the coordinates of;
- The Local Level (LL) coordinates of stations 1, 2 and 3;
- The accuracies of the coordinates of all three points;
- The components of the vertical deflection in Brunate, csi and eta;
- Chosen epoch for calculations.

This report will roughly follow the working flow described in the suggested steps, for each one giving a brief description of the actions undertaken and the results obtained.

Step 1 – Download of data and transformation of the coordinates to the chosen epoque.

The first thing that was needed to proceed with the computations was to download the starting data, i.e., the cartesian coordinates of Como in the geocentric ITRS, from the EPN website. The chosen epoch for our calculations is Feb 2nd, 2019.

In particular, we downloaded the IGS14 (ITRF14) estimates, in the time interval 043/2017 – 261/2021, which contains our chosen epoch. Both the coordinates and the velocities in this time interval are given with starting time 001/2010;

therefore, we will need to find the value of these coordinates nine years after. This is a fairly simple process that can be accomplished by applying the classic mechanics formula:

$$X\_t = x\_t0 + v*\Delta t$$

In this case, x_t and x_t0 are tridimensional, thus making them a vector of three coordinates. The same happens with the velocities.

Δt can be easily calculated knowing that t_0 is 2010 and t can be expressed as:

$$t = 2019 + (31+2)/365$$

and then subtracting t_0 from t.

$$\Delta t = 9.090410958904158$$

This results in the following coordinates of Como at present epoch:

[4398306 .076, 704150.112, 4550154.835]

STEP 2 - Find the coordinates of Brunate at epoch t

The geocentric coordinates of Brunate at t = 2/2/19 were computed by summing the coordinates of Como to the Como-Brunate baseline (i.e., the difference of coordinates in meters between the two points).

The baseline is:

[-1040.168; -72.970; 1631.398]

When added to the previously calculated Como coordinates, we obtain the coordinates at t of Brunate:

[4397265.908, 704077.142, 4551786.233]

STEP 3 - Compute the geodetic coordinates (phi, lambda, h) of Brunate

We then computed the geodetic coordinates (phi, lambda, h) of Brunate, using the provided function Cart2Geod:

$$[45819 \quad 9.096 \quad 738.116]$$

And then we converted them into radians:

$$[0.799694072132946\ 0.158769365429904\ 12.882558610557918]$$

STEP 4 - Compute the GC of st. 1 wrt Brun

We then computed the geocentric cartesian coordinates for point P1, which were computed by summing the P1-Brunate baseline and the previously calculated Brunate coordinates. The baseline is:

$$[-51.130, 76.749, 38.681]$$

And it results in:

$$[4397214.778, 704153.891, 4551824.914]$$

STEP 5 – Brunate P1 LC coordinates

In order to calculate the Local Cartesian (LC) coordinates, the Geocentric Cartesian ones that were previously calculated need to be converted into geodetic first – in order to obtain the latitude and longitude (phi, lambda).

We obtain the following results for lambda and phi (in rad):

$$[0.7996940721164912\ 0.15876936542990405]$$

In order to convert the GC coordinates to LC, we need to compute the rotation matrix R0.

$$\mathbf{R}_0 = \begin{bmatrix} -\sin\lambda_0 & \cos\lambda_0 & 0 \\ -\sin\varphi_0\cos\lambda_0 & -\sin\varphi_0\sin\lambda_0 & \cos\varphi_0 \\ \cos\varphi_0\cos\lambda_0 & \cos\varphi_0\sin\lambda_0 & \sin\varphi_0 \end{bmatrix}$$

The rotation matrix must be multiplied (dot product) by the GC coordinates, in order to obtain the LC coordinates. The coordinates of P1 in ITRF Geocentric cartesian are:

[4397214.77828 704153.8906274 4551824.9138126]

And the LC coordinates of P1 with respect to Brunate:

[83.86751209 54.46213629 1.0108359]

STEP 6 – P1 LC to LL coordinates

Now we need to compute the rotation matrices Rx(-ξ) and Ry (η).

Both csi and eta values were given to us in degrees and were then converted to radians.

ξ: 10.23 η: 9.5 -> ξ: 4.6057299705405916e-05 η: 4.959643957750554e-05

$$\mathbf{R}_x(-\xi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\xi) & -\sin(\xi) \\ 0 & \sin(\xi) & \cos(\xi) \end{bmatrix}$$

$$\mathbf{R}_y(\eta) = \begin{bmatrix} \cos(\eta) & 0 & -\sin(\eta) \\ 0 & 1 & 0 \\ \sin(\eta) & 0 & \cos(\eta) \end{bmatrix}$$

$$\mathbf{R}_z(\alpha) = \begin{bmatrix} \cos(\alpha) & \sin(\alpha) & 0 \\ -\sin(\alpha) & \cos(\alpha) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

After having computed the matrices, we apply the following formula to obtain the LL coordinates.

$$\mathbf{R}_y(\eta)\mathbf{R}_x(-\xi)\mathbf{x}_{P1,LC}$$

Thus obtaining P1 in Local Level:

[9.99993529e+01 1.45981724e-15 1.01739974e+00]

The value alpha that was seen in the figure above, was calculated with an arctangent of the x, y values of the LL coordinates. This result is given in radians, and we need to convert it in degrees. Thus, we obtain a value for alpha:

[32.9989]

STEP 7 – P2 & P3 LL to LC coordinates

We are given the LL coordinates of all three points (P1, P2, P3). In order to "go back" in the calculations to LC we need to invert the formulas and calculate Rt as:

$$R^T = (R_z(\alpha) \bullet R_y(\eta) \bullet R_x(-\xi))^T$$

And then we can implement it in the formulas, by multiplying its value by the LL coordinates, thus obtaining the LC coordinates of P2 and P3:

P2_LC = [20.80142582 114.8578569 1.99334541]

P3_LC = [-26.41338406 -5.22895756 -2.99852415]

Step 8 – P2 & P3 LC TO GC

We now calculate the transpose of R0 in order to perform the same inverse operation on the LC coordinates – getting thusly the GC coordinates:

P2_GC = [4397182.65774637 704084.87818572 4551867.70976854]

P3_GC = [4397271.72359928 704051.32293165 4551780.43824507]

STEP 9 – ITRF to ETRF conversion

Finally, we convert the ITRF coordinates back to ETRF by using the EPN website. We obtain the following coordinates:

Como_ETRF = [4398306.508, 704149.561, 4550154.503]

Brunate_ETRF = [4397266.340 704076.591, 4551785.901]

P1_ETRF = [4397215.210 704153.340, 4551824.582]

P2_ETRF = [4397183.089, 704084.328, 4551867.378]

P3_ETRF = [4397272.155, 704050.773, 4551780.106]

We then convert these coordinates from cartesian to geodetic, giving the following results (in degrees):

Como_ETRF_geod: [45.802, 9.096, 292.291]

Brun_ETRF_geod: [45.819, 9.097, 738.115]

P1_ETRF_geod: [45.820, 9.098, 739.126]

P2_ETRF_geod: [45.820, 9.097, 740.109]

P3_ETRF_geod: [45.819, 9.096, 735.116]

Finally, we convert these from decimal degrees to sexadecimal degrees, in order to get the geographical coordinates, i.e., degrees, minutes, and seconds:

[point] : [lat] [lon] [h]

Como_geo: 45° 48' 7.77981'' 9° 5' 44.22525'' 292.2907

Brun_geo: 45° 49' 8.72747'' 9° 5' 48.50407'' 738.1145

P1_geo: 45° 49' 10.49124'' 9° 5' 52.38864'' 739.1261

P2_geo: 45° 49' 12.44721'' 9° 5' 49.46756'' 740.1090

P3_geo: 45° 49' 8.55813'' 9° 5' 47.28067'' 735.1161

STEP 10 – LL to LC accuracy propagation

In order to estimate the accuracy of the coordinates, we need to compute the standard deviation of the LL to LC transformation.

We retrace the same steps that were performed for the previous 9 steps, but this time, starting from standard deviation values and not coordinates and velocities.

We have as input data the covariance matrix of the Como-Brunate baseline, of the Brunate-P1 baseline (both for the GC coordinates) and the covariance matrix of all three points for the LL coordinates.

In the first step, we retrieve the variances of the Como GC coordinates and velocities at t0 from the EPN website:

C_Como_GC_2010 =[0.001, 0, 0], [0, 0.001, 0], [0, 0, 0.001]

Cv_Como_GC_2010 =[0.0001, 0, 0], [0, 0.0001, 0], [0, 0, 0.0001]

We compute the values for the Como covariance matrix at present epoch:

[[0.00826456 0.   0.   ]
 [0.   0.00826456 0.   ]
 [0.   0.   0.00826456]]

We then add these results to the covariance baseline matrix of Como-Brunate and we obtain the covariance matrix at epoch t of Brunate:

[[8.26655714e-03 5.00000000e-07 5.00000000e-07]
 [5.00000000e-07 8.26555714e-03 5.00000000e-07]
 [5.00000000e-07 5.00000000e-07 8.26655714e-03]]

After this, following the same steps as before, we compute the LC coordinates of P1, with respect to Brunate, by first computing P1 in GC coordinates and then transforming it to geodetic coordinates. The P1 covariance matrix at present epoch is:

[[8.26805714e-03 8.00000000e-07 8.00000000e-07]
 [8.00000000e-07 8.26655714e-03 7.00000000e-07]
 [8.00000000e-07 7.00000000e-07 8.26855714e-03]]

We then transform P1 to LC coordinates with respect to Brunate, using the GCtoLC function and obtain the covariance matrix in LC of P1 with respect to Brunate:

[[-1.30641611e-03 8.16245885e-03 5.64713283e-07]

[-5.85433559e-03 -9.37362131e-04 5.76192771e-03]

[5.69041316e-03 9.11913308e-04 5.93036483e-03]]

This way, we compute the covariance matrixes in LC coordinates of Brunate, P2, and P3, using the same rotation matrices and values that were found during steps 1-9. This last operation allows us to obtain:

Covariance matrix in LC of Brunate:

[[2.69523768e-06 6.02983884e-08 5.19612177e-07]

[6.02983884e-08 3.32126014e-06 -8.19063209e-08]

[5.19612177e-07 -8.19063209e-08 4.46256932e-06]]

Covariance matrix in LC of P1:

[[3.61406720e-06 1.64657088e-08 7.71447208e-07]

[1.64657088e-08 4.77814523e-06 1.18050371e-07]

[7.71447208e-07 1.18050371e-07 6.58685471e-06]]

Covariance matrix in LC of P2 & P3:

[[1.00032868e-02 -2.24786828e-07 -8.88061326e-09]

[-2.24786828e-07 1.00030714e-02 8.51776415e-07]

[-8.88061327e-09 8.51776415e-07 2.25041208e-02]]

As the last step in this procedure, in order to propagate the accuracies from the Local Cartesian to the geodetic coordinates, we will extract the diagonal elements of these covariances matrices, and square them, in order to obtain the standard deviation in (E, N, U) from LL to LC of the points Brunate, P1 and P2 (=P3).

As such, we are left with the following results:

$(E, N, U)_{\text{Brunate}} = (0.0016417179061260067, 0.0018224324787861443, 0.002112479424520763)$

$(E, N, U)_{\text{P1}} = (0.001901070015443014, 0.0021858968938193828, 0.002566486841584663)$

$(E, N, U)_{\text{P2,P3}} = (0.10001643257373374, 0.10001535606624355, 0.15001373548214958)$

The results are given in metres [m].

This concludes our analysis and transformation calculations.