

Is Molecular Rectification Caused by Asymmetric Electrode Couplings or by a Molecular Bias Drop?

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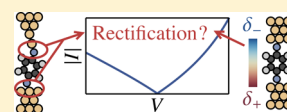
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^{||}Supporting Information

ABSTRACT: We investigate possible causes of molecular rectification in electrode–molecule–electrode junctions. By using a simple model and simulated conductance histograms, we show that a molecular bias drop is responsible for rectification; conversely, asymmetric molecule–electrode couplings do not directly result in rectification. Instead, the degree of coupling (a)symmetry can be observed in the line shapes of the conductance histograms used to experimentally assess the current–voltage properties of such molecular junctions. More coupling asymmetry leads to less positively skewed histogram peaks.



1. INTRODUCTION

Understanding how single molecules conduct electric current when connected to electrodes is interesting for both fundamental and applied reasons.^{1–5} For instance, electron dynamics change when the system is driven away from equilibrium, and the ability to control electric current on molecular time and length scales may lead to improved photovoltaics, thermoelectrics, and sensors. However, tools from conventional electronics are not readily applicable to single molecules, which have an inherently quantum mechanical nature. Unlike traditional systems, molecules have a discrete number of conduction channels through which electric current can flow.^{6,7} Each channel has a conductance G between 0 and $G_0 \equiv 2e^2/h$, and in this sense, a molecule exhibits quantized conductance.

Both theoretical and experimental investigations have explored the ramifications of quantized conductance over the last 20 years. Let us consider two examples. First, the molecule–electrode interfaces are critically important,^{8–25} as often evidenced through the use of various chemical linker groups. It is also possible that one molecule can bind to the electrodes in several ways, each of which results in a different conductance.^{10,13,16,19,20,26–29} Second, current–voltage profiles are usually non-Ohmic; that is, the conductance (alternatively, the resistance) changes with the applied bias, V .^{3,30–33} In this case, both the current, I , and the differential conductance, $G \equiv dI/dV$, help to quantify the electrical response properties of the molecular junction.³¹

This non-Ohmic behavior suggests that molecules can function as rectifiers,^{2,24,31,34–38} where the electric current through the molecular junction is different for positive and negative biases. Indeed, rectification has been reported through junctions containing either a single molecule or many molecules^{24,39–42} (and the rectification properties can be quite different due to cooperative effects between mole-

cules^{43,44}). Many of these studies used asymmetric molecules,^{2,24,34,40,41,45} where the molecule either possesses a permanent electric dipole or uses different linking groups to bind the two electrodes; however, rectification has also been demonstrated in symmetric molecules.^{2,37,39,42}

Regardless of the molecular symmetry, two mechanisms have been suggested for rectification. First, some of the applied bias drops across the molecule,^{24,36,37,45} indicating that the electrode–molecule–electrode junction has a (permanent or induced) dipole. In effect, the molecular channel energies change with the bias: A positive (negative) bias might bring a channel closer to resonance, thereby increasing the current, whereas a negative (positive) bias would push the channel away from resonance and decrease the current. Second, molecular asymmetry results in a molecular channel that is coupled differently to the two electrodes. Should such asymmetric couplings lead to an asymmetric electric potential profile along the molecule, rectification will be observed.^{31,36,37,45}

At first glance, it may appear that the latter mechanism is inconsistent because some symmetric molecules have displayed rectification. However, common experimental techniques for measuring molecular conductance are complicated by geometric uncontrollability and irreproducibility.^{9–12,15,16,27,29,32,33,42,46–60} Even though the molecule may be symmetric, a scanning tunneling microscope-based break junction experiment (for example) cannot determine the geometrical details of the measured junction,^{9,12,16,50–53,57,58} let alone reliably create a perfectly symmetric junction. In all likelihood, the two electrodes will have different shapes or surface features, thus causing the couplings to be asymmetric. 79

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As another consequence of this experimental uncertainty, the electronic properties of molecules are statistically assessed from many (typically thousands or more) measurements by compiling the data into a histogram.^{9,11,12,18,24,29,33,42,46–48,51,52,54,56,58,59,61–66} A conductance histogram peak not only reports the most probable conductance, for example, through the molecular junction,^{9,12,15,32,46,47,57,61,67} but the statistics of the data (the peak's line shape) provide additional insight into molecular conductance.^{11,14,15,29,32,48,49,57,60,64,65,67–69} For example, the zero-bias conductance histogram peak for transport through a molecule is positively skewed;⁶⁴ that is, the histogram peak has a longer tail to higher conductances than to smaller conductances.

In this work we employ a model similar to that of ref 24 to investigate the independent effects of a molecular bias drop and of asymmetric electrode couplings on electron transport properties. We find that asymmetric electrode couplings are *not* responsible, on their own, for rectification, whereas a molecular bias drop always results in rectification. We then develop and use a computational framework for simulating conductance histograms to show that the statistics in the histogram peak provide a signature for asymmetric molecule–electrode couplings. Although our results show that only a molecular bias drop is directly responsible for rectification, asymmetric electrode coupling is still encoded in experimental data.

The layout of this paper is as follows. We begin by introducing Landauer–Büttiker theory for electron transport and by developing our theory of conductance histograms in section 2. Section 3 then presents the main results, showing that a molecular bias drop causes rectification and that asymmetric electrode couplings are evident in the skewness of a conductance histogram peak. Finally, we summarize and conclude in section 4.

2. ELECTRON TRANSPORT AND CONDUCTANCE HISTOGRAMS

In this section we discuss pertinent aspects of electron transport theory (section 2.1), introduce our model system for investigating the mechanism of molecular rectification, and finally describe our framework for simulating conductance histograms (section 2.2).

2.1. Landauer–Büttiker Theory. Single electron transport theories typically employ scattering theory to describe electron dynamics.³ When we limit our attention to elastic, coherent scattering under steady-state conditions, we obtain the Landauer–Büttiker formalism,^{6,70} which is often used to describe electric current through molecules. Conduction channels are central to this formalism, and each channel has a probability of transmitting an electron with energy E from one electrode to the other. The sum of such transmission probabilities over all channels yields the transmission function, $T(E)$, to which each channel usually contributes a Lorentzian-shaped component.

All of the transport quantities we seek to understand build upon the transmission function. Consider the electric current,³

$$I(V) = \frac{2e}{h} \int_{-\infty}^{\infty} dE T(E; V) [f_L(E; V) - f_R(E; V)]$$

where f_L (f_R) is the Fermi function of the left (right) electrode. Note that the Fermi functions and the transmission function generally depend on the applied bias. Because tunneling

behavior is reasonably insensitive to temperature, it is convenient to work in the limit of zero temperature, where the Fermi functions become step functions. Then,

$$I(V) = \frac{2e}{h} \int_{E_F - eV/2}^{E_F + eV/2} dE T(E; V) \quad (1)$$

where E_F is the Fermi energy of the electrode–molecule–electrode junction. Finally, we obtain an expression for the differential conductance by combining its definition with eq 1,

$$\begin{aligned} G(V) &\equiv \frac{d}{dV} I(V) \\ &= \frac{2e^2}{h} \frac{1}{2} [T(E_F + eV/2; V) + T(E_F - eV/2; V)] \\ &\quad + \frac{2e}{h} \int_{E_F - eV/2}^{E_F + eV/2} dE \frac{\partial}{\partial V} T(E; V) \end{aligned} \quad (2)$$

Equations 1 and 2 show how to calculate electron transport properties through an electrode–molecule–electrode junction, up to obtaining the transmission function. There are three key components that lead to $T(E)$: (i) the Hamiltonian of the isolated molecule (channel), \mathbf{H} ; (ii) a self-energy, $\Sigma_L(E)$, describing how the isolated molecule couples to the left electrode; and (iii) a similar self-energy, $\Sigma_R(E)$, for the right electrode. Note that the self-energies are non-Hermitian operators that essentially encapsulate open-system boundary conditions. From these,³

$$T(E) = \text{Tr}[\mathbf{G}(E)\mathbf{\Gamma}_L(E)\mathbf{G}^\dagger(E)\mathbf{\Gamma}_R(E)] \quad (3a)$$

where

$$\mathbf{G}(E) = [\mathbf{E}\mathbf{I} - \mathbf{H} - \Sigma_L(E) - \Sigma_R(E)]^{-1} \quad (3b)$$

is the molecular Green function⁷¹ (as modified by the electrodes), \mathbf{I} is the identity operator, and

$$\mathbf{\Gamma}_{L/R}(E) = i[\Sigma_{L/R}(E) - \Sigma_{L/R}^\dagger(E)] \quad (3c)$$

is the spectral density for coupling to the left/right electrode. We write all of these operators as matrices in the following discussion, where, for simplicity, we assume an orthonormal basis set.

Our model junction consists of a single channel that couples asymmetrically to the electrodes and drops bias. The bias drop is reflected in the bias-dependent level energy. Mathematically, $\mathbf{H} = [\varepsilon + aeV]$, $\Sigma_L(E) = [-i\Gamma_L/2]$, and $\Sigma_R(E) = [-i\Gamma_R/2]$, where ε is the channel's energy level, a is the strength of the bias drop across the channel, and $\Gamma_L > 0$ is the channel-left electrode coupling element, and likewise for $\Gamma_R > 0$. Using eq 3,

$$T(E) = \frac{4\Gamma_L\Gamma_R}{4(E - \varepsilon - aeV)^2 + (\Gamma_L + \Gamma_R)^2} \quad (4a)$$

is Lorentzian, as expected. Then, from eqs 1 and 2,

$$\begin{aligned} I(V) &= \frac{4e\Gamma_L\Gamma_R}{h(\Gamma_L + \Gamma_R)} \left[\arctan\left(\frac{2[E_F - \varepsilon + (1/2 - a)eV]}{\Gamma_L + \Gamma_R}\right) \right. \\ &\quad \left. - \arctan\left(\frac{2[E_F - \varepsilon - (1/2 + a)eV]}{\Gamma_L + \Gamma_R}\right) \right] \end{aligned} \quad (4b)$$

is the current and

$$G(V) = \frac{2e^2}{h} \left[\frac{4(1/2 - a)\Gamma_L\Gamma_R}{4[E_F - \varepsilon + (1/2 - a)eV]^2 + (\Gamma_L + \Gamma_R)^2} + \frac{4(1/2 + a)\Gamma_L\Gamma_R}{4[E_F - \varepsilon - (1/2 + a)eV]^2 + (\Gamma_L + \Gamma_R)^2} \right] \quad (4c)$$

177

178 is the differential conductance. Note that this model is closely
179 related to that of ref 24; we consider the single-channel
180 equivalent of that two-channel model. Finally, we use the ratio

$$\xi \equiv \frac{\max(\Gamma_L, \Gamma_R)}{\min(\Gamma_L, \Gamma_R)}$$

181 to quantify the degree of asymmetric coupling.

182 2.2. Conductance Histograms as Probability Density

183 **Functions.** Our theory for conductance histograms begins
184 with the idea that the various model parameters (e.g., ε , a , Γ_L ,
185 and Γ_R) behind electron transport are random variables.^{64,65,67}

186 Figure 1 depicts this concept for a generic model junction. In

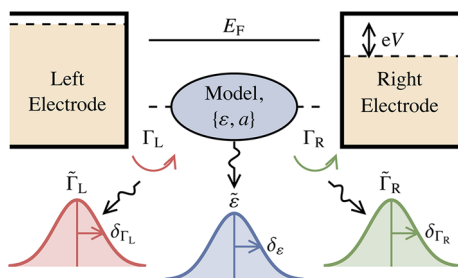


Figure 1. Schematic of our framework for simulating conductance histograms. A model system is placed between two electrodes and a bias (V) is applied across the junction. Our model system (blue) depends on the channel energy (ε) and the bias drop (a). The conduction channel in the model system couples to the left (right) electrode with Γ_L (Γ_R), and the Fermi energies of the two electrodes (dashed lines) are offset from the junction Fermi energy (E_F) by the bias. When constructing histograms, we assume that each of these physical parameters is an independent random variable. As depicted by the normal distributions, each conductance “measurement” samples from the probability distributions of these parameters.

187 essence, we equate the experimental irreproducibility when
188 measuring conductance with stochasticity; each parameter has
189 an underlying probability distribution and every measurement
190 samples from these distributions. The conductance histogram
191 therefore reports the probability density function⁷² for the
192 conductance observable, which is directly determined by the
193 distributions of the model parameters.⁶⁵

194 This realization facilitates the simulation of conductance
195 histograms. Simply put, we emulate each conductance
196 measurement by using a random number generator to sample
197 from the model parameters’ distributions (and then compute
198 conductance as described in section 2.1). Unless otherwise
199 specified, we use normal distributions for each parameter.
200 Mimicking experiment, we then compile many simulated
201 conductance values into a histogram. By changing the
202 distributions of the parameters, we can investigate the
203 mechanism of molecular rectification. We implemented this
204 simulation framework for many model systems in C++11, and
205 our software, MolStat, is available open source.⁷³ Specific
206 MolStat input files for reproducing the histograms in section 3
207 are presented in the Supporting Information.

3. RESULTS AND DISCUSSION

208 Equipped with a model system that includes both asymmetric
209 electrode couplings and a molecular bias drop, we proceed to
210 investigate which effect leads to molecular rectification and how
211 these effects manifest themselves in experimental data. We first
212 examine asymmetric electrode couplings and then discuss a
213 molecular bias drop.

3.1. Asymmetric Electrode Couplings. We start by
214 examining the simplest system, where the molecular channel
215 couples symmetrically to both electrodes ($\xi = 1$) and does not
216 drop bias ($a = 0$). Mirroring ref 24, the molecular channel level
217 is at -3 eV, the Fermi energy is 0 eV, and $\Gamma_L = \Gamma_R = 0.1$ eV.
218 The current–voltage profile for this junction is displayed in
219 Figure 2b and is symmetric about $V = 0$; there is no
220 rectification. If we make the couplings less symmetric ($\xi = 2$)
221 by increasing Γ_R to 0.2 eV, we again observe a symmetric
222 current–voltage profile in Figure 2d. Finally, rectification
223 remains absent if the couplings are made even more
224 asymmetric, $\Gamma_R = 0.4$ eV (hence $\xi = 4$), in Figure 2e.

225 It seems doubtful from these simulations that asymmetry in
226 the molecule–electrode couplings is, on its own, responsible for
227 rectification. Before exploring the effects of a molecular bias
228 drop, however, we show that coupling asymmetry is instead
229 encoded in the statistics of experimental measurements.

230 To see this, we first consider the transmission spectra of the
231 same three junctions, which are shown in Figure 2a. As
232 expected, each transmission spectrum exhibits the characteristic
233 Lorentzian line shape for transport through a single channel.
234 When the coupling is symmetric (red line), the Lorentzian
235 peaks at 1, indicating the channel’s resonance energy, and
236 asymptotically decays for energies away from the resonance. As
237 the coupling becomes more asymmetric ($\xi = 2$ in green and $\xi =$
238 4 in blue), the peak decreases in magnitude and the tails decay
239 less rapidly. In all, the transmission spectrum becomes flatter as
240 the couplings become more asymmetric.

241 It has previously been shown^{64,65} that the line shape of a
242 conductance histogram peak reflects the shape of the
243 transmission function near the Fermi energy. The argument
244 is as follows. The fluctuations inherent to each experimental
245 measurement cause us to sample different points on the
246 transmission spectrum near E_F . For transport via nonresonant
247 tunneling (where E_F is in the Lorentzian tails), $T(E)$ rises with
248 E more quickly than it falls in this neighborhood.
249 Consequently, fluctuations are slightly more likely to increase
250 the transmission, resulting in a positively skewed histogram
251 peak.⁶⁴ Transmission spectra with flatter tails will be more
252 insulated from this effect; larger fluctuations will be required to
253 noticeably change the observed transmission. We should,
254 therefore, expect that junctions with increasingly asymmetric
255 couplings will yield less skewed conductance histogram peaks.

256 Our conductance histogram simulation procedure (section
257 2.2) will now be used to demonstrate this effect. We assume ε ,
258 Γ_L , and Γ_R are random variables with normal distributions and
259 proceed to simulate 1 million conductance “measurements” for
260 each system. The average values of each parameter are taken to
261 be the values used in the respective transmission spectra and
262 current–voltage profiles. Full details on simulating these
263 histograms can be found in the Supporting Information.

264 To start, Figure 2b shows a histogram where $\Gamma_L = \Gamma_R$ in every
265 “measurement,” ensuring symmetric coupling. The histogram
266 has a bowl shape that is symmetric about $V = 0$ V, which is
267 consistent with ref 42. In contrast, Figure 2c shows a histogram
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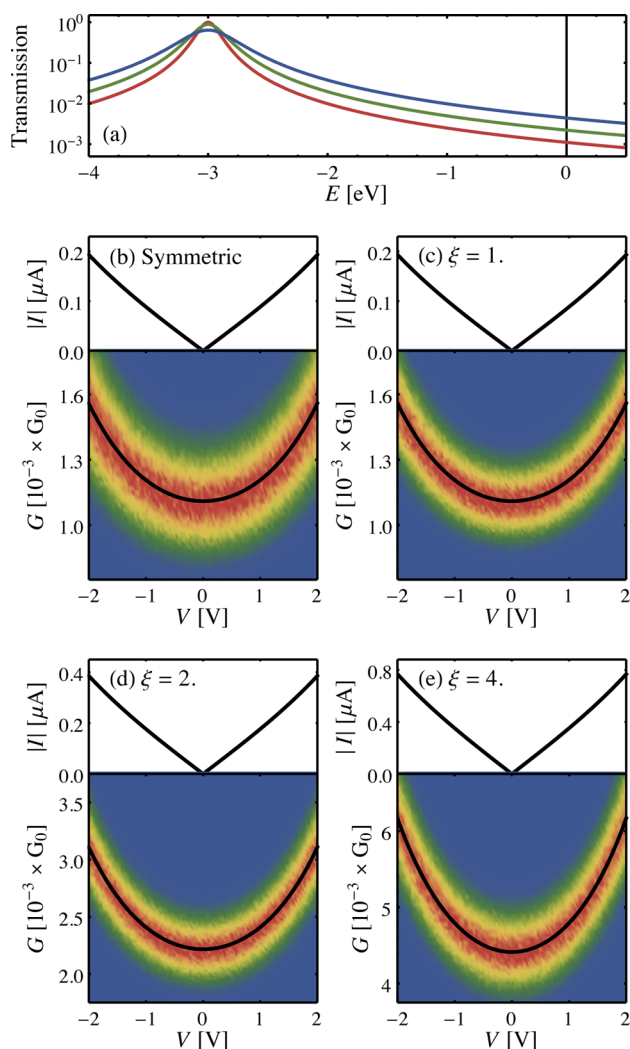


Figure 2. Electron transport properties for molecules that do not drop bias ($a = 0$). (a) Transmission spectra of channels with $\varepsilon = -3$ eV and varying degrees of coupling asymmetry. Red, $\xi = 1$; green, $\xi = 2$; blue, $\xi = 4$. The vertical line shows the Fermi energy ($E_F = 0$ eV) used for calculating current and conductance. (b)–(e) Simulated current–voltage profile (top) and conductance profile (black line, bottom) for the above channels. The bottom panel also displays a simulated conductance histogram for each channel. Red (blue) indicates a large (small) probability of observing the conductance; the absolute scale is arbitrary. In (b), $\Gamma_L = \Gamma_R$ in every sample; in (c)–(e), Γ_L and Γ_R are independently drawn from their distributions. In all cases, the current–voltage profile is symmetric about $V = 0$ V; asymmetric coupling does not result in rectification.

with the same average parameters, but where Γ_L and Γ_R are independently chosen from the same distribution. The couplings are the same, on average, but are likely to be slightly different from each other in any particular “measurement.” Most noticeably, the width of the histogram peak (at a given bias) is considerably smaller than that in Figure 2b. When $\Gamma_L = \Gamma_R$ for every “measurement,” smaller (larger) conductances become more probable because the electrode couplings are simultaneously small (large). The likelihood of having simultaneously small (large) couplings decreases when the two couplings are independent, resulting in a narrower histogram peak.

We now increase the degree of asymmetry in the couplings to $\xi = 2$ and $\xi = 4$ and show the resulting histograms in Figure

2d,e. When $\xi = 2$, asymmetric couplings are more prevalent in any particular “measurement,” but there is still a reasonable chance that $\Gamma_L \approx \Gamma_R$ for some samples. It is unlikely that $\Gamma_L \approx \Gamma_R$ when $\xi = 4$. Unsurprisingly, both histograms are qualitatively similar to those already discussed.

Finally, Figure 3a shows the skewness of the histogram peaks in Figure 2 as a function of the bias; that is, each skewness

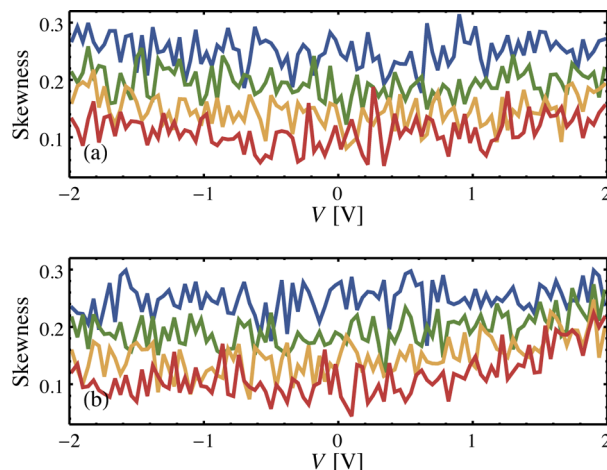


Figure 3. Skewness of the histogram peak as a function of the bias for the conductance histograms in (a) Figure 2 and (b) Figure 4. In both panels: blue is a junction where $\Gamma_L = \Gamma_R$ for every sample [panel (b) in Figures 2 and 4], green is $\xi = 1$, where Γ_L and Γ_R are the same, on average [panel (c) in Figures 2 and 4]; yellow is $\xi = 2$ [panel (d) in Figures 2 and 4]; and red is $\xi = 4$ [panel (e) in Figures 2 and 4]. As coupling asymmetry increases ($\xi \rightarrow \infty$), the skewness of the histogram peak decreases (on average).

reports the statistics from a vertical slice of a voltage-dependent conductance histogram. As expected,⁶⁴ the histogram peaks are positively skewed. Although there is some fluctuation in the skewness from one bias to another, it is apparent that the skewness decreases, on average, as ξ increases. Coupling asymmetry, while not responsible for rectification in the current–voltage profiles, can be seen in the skewness of conductance histogram peaks.

3.2. Molecular Bias Drop. We now turn to the impact of a molecular bias drop ($a \neq 0$) on the molecule’s electron transport properties. Mirroring the previous discussion on coupling asymmetry, Figure 4b shows the current–voltage profile for a molecule that symmetrically couples to the electrodes and drops bias ($a = 0.15$). Because both $a > 0$ and $E_F > \varepsilon$, positive biases move the molecular channel closer to resonance; see Figure 4a, whereas negative biases push it further away. Changing to either $a < 0$ or $E_F < \varepsilon$ would lead to the opposite behavior. Consequently, the current increases more rapidly for positive biases and rectification is observed (albeit weak in this example system).

Our data suggests, therefore, that a molecular bias drop will lead to rectification. Putting all of these results together, Figure 4d,e shows the current–voltage profiles for molecules that couple asymmetrically to the electrodes and drop bias. As observed in ref 24, these systems still exhibit rectification. Finally, Figure 3b confirms that the skewnesses of the associated histogram peaks also reflect the coupling asymmetry.

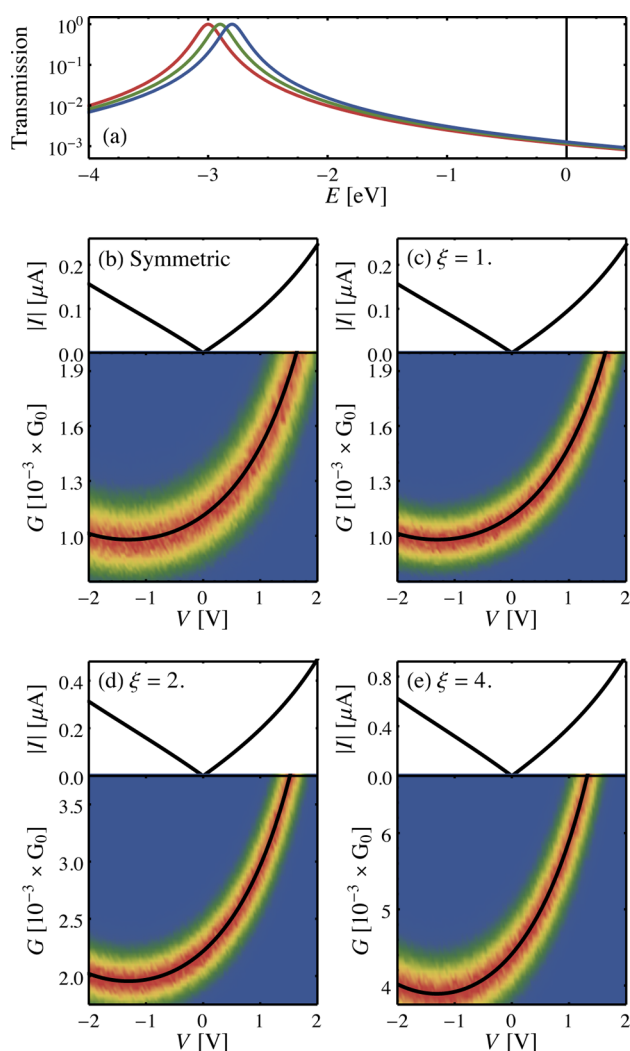


Figure 4. Electron transport properties for molecules that drop bias ($a \neq 0$). Apart from taking $a = 0.15$, all parameters are the same as in Figure 2 and the panels are similarly arranged. (a) Transmission spectrum for a channel that couples symmetrically to both electrodes and drops bias at 0 V (red), 1 V (green), and 2 V (blue). Positive (negative) biases shift the resonance to more positive (negative) energies such that the current–voltage profiles are not symmetric about $V = 0$ V. In all cases, the molecular bias drop across results in rectification.

4. CONCLUSIONS

In this work we investigated the cause of rectification in molecular junctions. Previous works have suggested two mechanisms: (i) a bias drop across the molecule and/or (ii) asymmetric couplings between the molecule and the electrodes. However, none of these studies examined the independent effects of these mechanisms, making it difficult to conclusively infer the cause of rectification. Our model, which is similar to those used before, showed that a molecular bias drop, and *not* asymmetry in the electrode couplings, is responsible for rectification. Instead, asymmetric electrode couplings lead to less positively skewed peaks in the experimental conductance histogram. This last point accentuates the high information content of conductance histogram line shapes. We end this discussion by noting that asymmetric electrode couplings may indirectly lead to rectification, even though they are not directly responsible for it. A bias drop across the

molecule can be attributed to a (permanent or induced) dipole in the molecular junction. It is probable that the different linker groups used to produce asymmetric couplings may also lead to different induced dipoles with an applied bias. The change in induced dipole from one system to the next might then change the rectification ratio of the junction, giving the illusion that asymmetric couplings lead to rectification.

■ ASSOCIATED CONTENT

Supporting Information

The MolStat⁷³ input files used to generate the histograms analyzed in Figures 2 and 4. This material is available free of charge via the Internet at <http://pubs.acs.org/>.

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Notes

The authors declare no competing financial interest.

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