Stats101-Generate instances of Sine Waves

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1 Problem Description

An oscillator is transmitting a noisy sine-wave $x[n]$, where,

$$x[n] = 100\cos\{\frac{37\pi n}{11} + \Phi\} + 25\epsilon[n]$$

where, $\Phi \sim Uniform\{-\pi, \pi\}$ and $\epsilon[n] \overset{iid}{\sim} \mathcal{N}(0,1)$, $0 \leq n \leq 1024$ ($\mathcal{N}(0,1)$, stands for a Gaussian distribution with mean 0 and variance 1). During normal operating conditions the signal received by the receiver is given by the above expression. Once in a while due to some random factors, $x[n]$ gets corrupted and it is received by the receiver as $\hat{x}[n]$. The expression for $\hat{x}[n]$ is given by,

$$\hat{x}[n] = 100\cos\{\frac{37\pi n}{11} + \Phi_1\} + \alpha\mathcal{A}[n]$$

where $\Phi_1 \sim Uniform\{-\pi, \pi\}$ $\alpha \sim Uniform\{20, 25\}$ and $\mathcal{A}[n]$ is an additive noise that can take 2 of the following forms with equal probability:

- $\mathcal{A}[n] \overset{iid}{\sim} 0.3\mathcal{N}(0, 0.25) + 0.3\mathcal{N}(0, 0.50) + 0.4\mathcal{N}(0, 0.75), \forall n$
- The statistics of $\mathcal{A}[n]$ are defined as

$$\mathbb{E}[\mathcal{A}[n]] = 0, \forall n$$

$$\mathbb{E}[\mathcal{A}[n]\mathcal{A}[n+k]] = \frac{1}{2}\{|k+1|^{1.4} - 2|k|^{1.4} + |k-1|^{1.4}\}, \forall k \geq 0$$

The Class of anomalous sine-waves are given by $\hat{x}[n]$. Generate 100 instances of Sine-waves during normal operating mode, $x[n]$ and corrupted Sine-waves, $\hat{x}[n]$.Write a python code to support your answer.