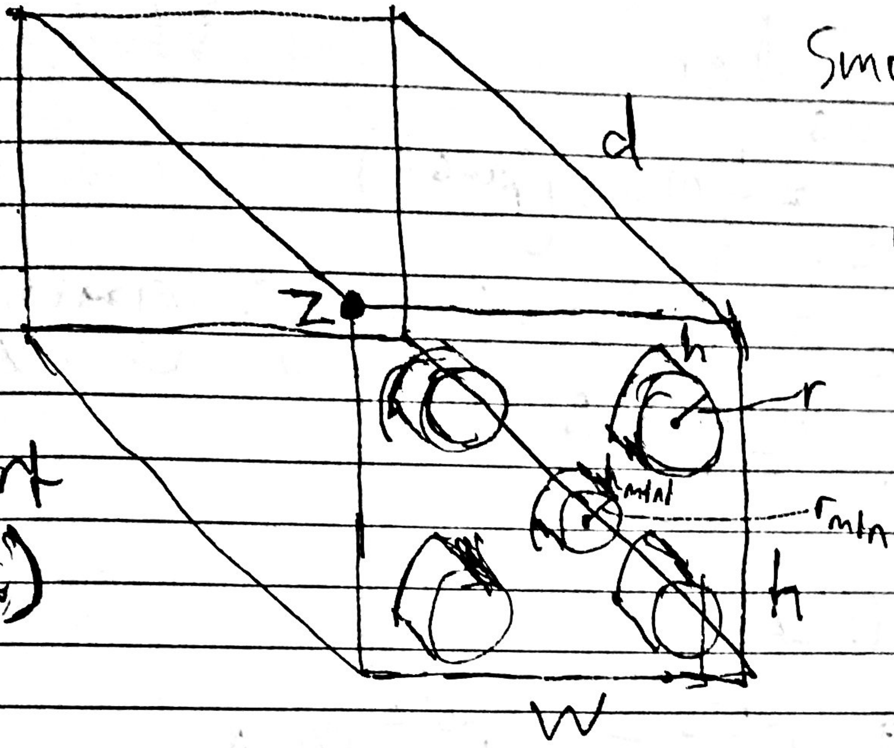
Spacecraft Body B

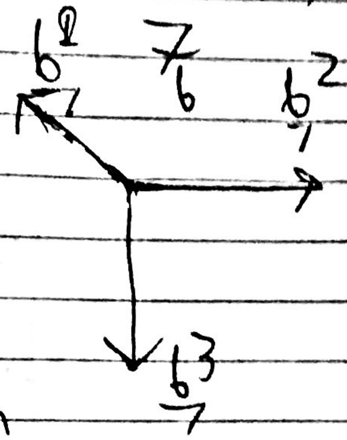
ISIS 16U Smallsat Chassis

I)

use $\vec{z}$ in b frame since it's "inertial" wrt the rigid body B

a)





b) and c) Define sub-bodies

B^1: Cuboid

B^{2-6}: Cylinders

* Spacecraft is 20 kg full. Use constant volumetric mass for now

Using axes above, princ. MOI's for sub-bodies about their centers of mass

Cuboid: $I_1^{B^1} = \frac{1}{12} m_{B^1}(w^2+h^2)$

$I_2^{B^1} = \frac{1}{12} m_{B^1}(d^2+h^2)$, $I_3^{B^1} = \frac{1}{12} m_{B^1}(d^2+w^2)$

$m_{sc} = 20$ kg

$\sigma = m_{sc}/V_{total}$

$\boxed{m_{B^i} = \sigma V^i}$

Cylinder

$$I_1^{B^{2-6}} = \frac{1}{2} m_{B^{2-6}} \left(r^{2-6}\right)^2$$

$$I_2^{B^{2-6}} = \frac{1}{12} m_{B^{2-6}} \left(3\left(r^{2-6}\right)^2 + \left(h^{2-6}\right)^2\right)$$

$$I_3^{B^{2-6}} = I_2^{B^{2-6}}$$

~~MOI~~

b) $$\underline{r}_b^{cz} = \frac{\sum \underline{r}_b^{c^i z} m_{B^i}}{\sum m_{B^i}}$$

$c^i \rightarrow$ center of masses of sub-bodies

$\underline{\rho}_b^{c^i c} = \underline{r}_b^{c^i z} - \underline{r}_b^{cz} \rightarrow$ get body vectors to sub-bodies wrt center of mass c resolved in body frame

c) $$\boxed{\underline{I}_b^{Bc} = \sum_{i=1}^{6} \underline{I}_b^{B^i c^i} - m_{B^i} \underline{\rho}_b^{c^i c \times} \underline{\rho}_b^{c^i c \times}}$$

* See scMOI.m for details

Test script shows $\underline{I}_b^{Bc}$ is symmetric, has positive eigenvalues, and is the principal MOI matrix

d) $\underline{x} = \begin{bmatrix} \underline{\epsilon} \\ \eta \\ \underline{\omega}_b^{ba} \end{bmatrix} = \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \\ \eta \\ \omega_{b1}^{ba} \\ \omega_{b2}^{ba} \\ \omega_{b3}^{ba} \end{bmatrix}$

$\underline{\underline{I}}_b^{Bc} \dot{\underline{\omega}}_b^{ba} = \underline{m}_b^{Bc} - \underline{\omega}_b^{ba\times} \underline{\underline{I}}_b^{Bc} \underline{\omega}_b^{ba}$

$\dot{\underline{\omega}}_b^{ba} = \underline{\underline{I}}_b^{Bc^{-1}} \underline{m}_b^{Bc} - \underline{\underline{I}}_b^{Bc^{-1}} \underline{\omega}_b^{ba\times} \underline{\underline{I}}_b^{Bc} \underline{\omega}_b^{ba}$

$\cancel{\dot{\underline{x}} = \begin{bmatrix} \dot{\underline{\epsilon}} \\ \dot{\eta} \\ \dot{\underline{\omega}}_b^{ba} \end{bmatrix}} = \boxed{\dot{\underline{\omega}}_b^{ba} = \underline{\underline{I}}_b^{Bc^{-1}} \left(\underline{m}_b^{Bc} - \underline{\omega}_b^{ba\times} \underline{\underline{I}}_b^{Bc} \underline{\omega}_b^{ba} \right)}$

$$\begin{bmatrix} \dot{\underline{\epsilon}} \\ \dot{\eta} \end{bmatrix} = \frac{1}{2} \underbrace{\overset{4\times4}{\begin{bmatrix} \eta \underline{\underline{1}} + \underline{\epsilon}^{\times} & \underline{\epsilon} \\ -\underline{\epsilon}^{T} & \eta \end{bmatrix}}}_{\Gamma(\epsilon, \eta)} \overset{4\times1}{\begin{bmatrix} \underline{\omega}_b^{ba} \\ 0 \end{bmatrix}}$$

$$\boxed{\dot{\underline{x}} = \begin{bmatrix} \dot{\underline{\epsilon}} \\ \dot{\eta} \\ \dot{\underline{\omega}}_b^{ba} \end{bmatrix} = \begin{bmatrix} \frac{1}{2}\left(\eta \underline{\underline{1}} + \underline{\epsilon}^{\times}\right) \underline{\omega}_b^{ba} \\ -\frac{1}{2} \underline{\epsilon}^{T} \underline{\omega}_b^{ba} \\ \underline{\underline{I}}_b^{ba^{-1}} \left(\underline{m}_b^{Bc} - \underline{\omega}_b^{ba\times} \underline{\underline{I}}_b^{Bc} \underline{\omega}_b^{ba} \right) \end{bmatrix}}$$

Operationally will just use Gamma function that was written to assemble $\dot{x}$

e) $\boxed{T_{B\,w/a} = \frac{1}{2}\underline{\omega}_b^{ba^T}\underline{\underline{I}}_b^{Bc}\underline{\omega}_b^{ba}}$ ✓ 6/

f) Poisson's Eqn: $\underline{\dot{C}}_{ba} + \underline{\omega}_b^{ba^\times}\underline{C}_{ba} = \underline{0}$

$$\underline{\dot{C}}_{ba} = -\underline{\omega}_b^{ba^\times}\underline{C}_{ba}$$

vectorize

$$\begin{bmatrix}\underline{\dot{c}}_1^{ba} & \underline{\dot{c}}_2^{ba} & \underline{\dot{c}}_3^{ba}\end{bmatrix} \to \begin{bmatrix} 0 & \underline{\omega}_{b3}^{ba} & -\underline{\omega}_{b2}^{ba} \\ -\underline{\omega}_{b3}^{ba} & 0 & \underline{\omega}_{b1}^{ba} \\ \underline{\omega}_{b2}^{ba} & -\underline{\omega}_{b1}^{ba} & 0 \end{bmatrix} \begin{bmatrix}\underline{c}_1^{ba} & \underline{c}_2^{ba} & \underline{c}_3^{ba}\end{bmatrix}$$

* equiv to:

$$\underline{\dot{c}}_i^{ba} = -\underline{\omega}_b^{ba^\times}\underline{c}_i^{ba}$$

for $\underline{c}_i^{ba}$ if the matrix mult is done

$$\underline{\dot{c}}_1^{ba} = \begin{bmatrix} \cancel{0} \\ \underline{\omega}_{b3}^{ba} \\ \underline{\omega}_{b2}^{ba} \end{bmatrix}$$

$$\begin{bmatrix}\underline{\dot{c}}_1^{ba} & \underline{\dot{c}}_2^{ba} & \underline{\dot{c}}_3^{ba}\end{bmatrix} = \begin{bmatrix} 0 & \underline{\omega}_{b3}^{ba}\underline{c}_{12}^{ba} & -\underline{\omega}_{b2}^{ba}\underline{c}_{13}^{ba} \\ & & \end{bmatrix}$$

$$\cancel{\begin{bmatrix} \dot{\underline{c}}_1^{ba} & \dot{\underline{c}}_2^{ba} & \dot{\underline{c}}_3^{ba} \end{bmatrix}}$$

$$\boxed{\dot{X} = \begin{bmatrix} \dot{\underline{c}}_1^{ba} \\ \dot{\underline{c}}_2^{ba} \\ \dot{\underline{c}}_3^{ba} \\ \dot{\underline{\omega}}_b^{ba} \end{bmatrix} = \begin{bmatrix} -\underline{\omega}_b^{ba\times} \underline{c}_1^{ba} \\ -\underline{\omega}_b^{ba\times} \underline{c}_2^{ba} \\ -\underline{\omega}_b^{ba\times} \underline{c}_3^{ba} \\ \underline{I}_b^{Bc^{-1}} \left(\underline{m}_b^{Bc} - \underline{\omega}_b^{ba\times} \underline{I}_b^{Bc} \underline{\omega}_b^{ba} \right) \end{bmatrix}}$$