Course Project
Part 1: Intro, Attitude Parameterization, and Kinematics

Due Date: February 21, 2020 at 1:25pm in Akerman Hall 225
Last Updated: February 7, 2020

Throughout this course you will in work in stages on a project that will ultimately lead to a complete numerical simulation of the orbital and attitude dynamics of a spacecraft, including an attitude determination and control system. Each part of the project will be submitted like the regular homework assignments at the beginning of class on their due date. The main differences between the project and the homework assignments is that the project will tend to involve more practical problems, each part of the project will build off of the previous parts, and it will typically involve more coding in `matlab`. Use this as an opportunity to use good programming practices, such as commenting your code and using multiple functions rather than a single piece of large code. Make sure to submit your `matlab` code for each stage of the project, unless it is stated otherwise.

The purpose of Part 1 of the project is to choose a spacecraft, choose an attitude parameterization and write `matlab` functions for this attitude parameterization that you will make use of later, and derive the kinematics of your chosen spacecraft. The following are the detailed tasks for Part 1 of the project:

1. **Spacecraft Selection**: You must choose a spacecraft that you will be interested in using for the duration of the project. List relevant information about your chosen spacecraft, including:

   (a) Approximate physical dimensions and mass. You should approximate any complex spacecraft geometries with simpler shapes, such as a cylinder or cube. This will simplify some of your future calculations.

   (b) Approximate information about the spacecraft's orbit (e.g., gravitational constant (μ), eccentricity (e), inclination (i), semimajor axis (a)).

   (c) Any information you can find regarding the sensors on-board the spacecraft (e.g., rate gyroscope, magnetometer, star tracker, etc.) and what properties do the sensors have (e.g., variance of measurement noise, bias, drift, etc.). We have not yet discussed spacecraft sensing in class, but do your best to find this information or come up with values that you think are reasonable. You can check pgs. 499–506 (Chapter 26.1) of [1] for more information.

   (d) Any information you can find regarding the actuators on-board the spacecraft (e.g., reaction wheel(s), momentum wheel(s), thrusters, magnetic torque rods, etc.) and what properties do the actuators have (e.g., maximum thrust, maximum angular momentum of reaction wheels, etc.). We have not yet discussed spacecraft actuation in class, but do your best to find this information or come up with values that you think are reasonable. You can check pgs. 506–511 (Chapter 26.2) of [1] for more information.

   You can choose from a variety of spacecraft, including CubeSats, GEO communications satellites, space stations, etc. You can even design your own spacecraft! For suggestions, you may want to check out one of the online spacecraft encyclopedias [2]. If you have any difficulty choosing your spacecraft or finding relevant information, speak to Prof. Caverly or the TA for guidance. Do not worry if you are unable to find all of the above information for your chosen spacecraft, as you will not lose marks for this. Find the information that you can, and try to think about plausible values for the information that you cannot. It may be a good idea to choose a spacecraft that orbits Earth, as future project parts will ask you to include Earth's magnetic field and the fact that Earth is not a perfect sphere.

Summary of Deliverables for Question 1

- Submit information regarding your chosen spacecraft's dimensions, orbit, sensors, and actuators.

2. **Attitude Parameterization**: You will investigate the use of the quaternion as an attitude parameterization.

   (a) Code a `matlab` function called `DCM2Quaternion` that computes the quaternion ($\boldsymbol{\epsilon}$ and η) given a DCM. You can use the provided codes `Quaternion2DCM`, which solve the reverse problem, as guides. See pgs. 25–27 (Chapter 1.3.4.1) of [1] for help with this.

   (b) Verify your `DCM2Quaternion` function by testing that the DCM

$$\mathbf{C}_{ba} = \begin{bmatrix} 0.8995 & 0.3870 & -0.2026 \\ -0.3201 & 0.8995 & 0.2974 \\ 0.2974 & -0.2026 & 0.9330 \end{bmatrix} \tag{1}$$

   returns

$$\mathbf{q} = \begin{bmatrix} \boldsymbol{\epsilon} \\ \eta \end{bmatrix} = \begin{bmatrix} 0.1294 \\ 0.1294 \\ 0.1830 \\ 0.9659 \end{bmatrix}.$$

   (c) Regardless of which attitude parameterization is used for computations, numerical simulation, attitude determination, or attitude control, it is often desirable to plot attitude results with Euler angles to obtain a better physical understanding of the spacecraft's attitude motion.

   Code a `matlab` function called `DCM2Euler321` that computes the Euler angles associated with a 3-2-1 rotation sequence given a DCM. See pgs. 22–23 (Chapter 1.3.3) of [1] for help with this.

   (d) Verify your `DCM2Euler321` function by testing that the DCM in Eq. (1) returns $\phi = 0.3086$ rad, $\theta = 0.2040$ rad, and $\psi = 0.4063$ rad, where $\mathbf{C}_{ba} = \mathbf{C}_1(\phi)\mathbf{C}_2(\theta)\mathbf{C}_3(\psi)$.

Summary of Deliverables for Question 2

- Submit a copy of the functions `DCM2Quaternion` and `DCM2Euler321`.
- Provide a `matlab` readout showing that you recover the expected results in parts (b) and (d).
  Hint: You may want to use `matlab`'s "Publish" function.

3. **Kinematics**: Solve the translational and attitude kinematics of the spacecraft in the following steps:

   (a) Follow the 5 steps to success in kinematics to determine $\underrightarrow{a}^{sw/a/a}$, the translational acceleration of the spacecraft relative to an unforced particle w with respect to an inertial frame $\mathcal{F}_a$. Start by drawing a diagram that shows the relevant reference frames and particles. Use cartesian coordinates to describe the position of the spacecraft as $\underrightarrow{r}^{sw} = \begin{bmatrix} x_a & y_a & z_a \end{bmatrix} \underrightarrow{\mathcal{F}}_a$.

   (b) Write out the kinematic relationship between $\boldsymbol{\omega}_b^{ba}$ and the quaternion rates. Specifically, obtain $\mathbf{S}_b^{ba}(\mathbf{q})$ such that $\boldsymbol{\omega}_b^{ba} = \mathbf{S}_b^{ba}(\mathbf{q})\dot{\mathbf{q}}$, where $\mathbf{q}$ is the quaternion. Code a `matlab` function called `SmatrixQuaternion` that computes $\mathbf{S}_b^{ba}(\mathbf{q})$.

(c) Write out the inverse kinematic relationship between $\boldsymbol{\omega}_b^{ba}$ and the quaternion rates. Specifically, obtain $\boldsymbol{\Gamma}_b^{ba}(\mathbf{q})$ such that $\dot{\mathbf{q}} = \boldsymbol{\Gamma}_b^{ba}(\mathbf{q})\boldsymbol{\omega}_b^{ba}$, where $\mathbf{q}$ is the quaternion. Code a `matlab` function called `GammaQuaternion` that computes $\boldsymbol{\Gamma}_b^{ba}(\mathbf{q})$.

Summary of Deliverables for Question 3

- Submit the work and any drawings you use to solve part (a).
- Submit a copy of the functions `SmatrixQuaternion` and `GammaQuaternion`.

References

[1] A. H. J. de Ruiter, C. J. Damaren, and J. R. Forbes, *Spacecraft Dynamics and Control: An Introduction*. West Sussex, UK: John Wiley & Sons, Ltd., 2013.

[2] G. D. Krebs, "Gunter's space page," 2019. [Online]. Available: https://space.skyrocket.de/directories/sat.htm