Spacecraft Project Part 4: Coupled Orbital and Attitude Dynamics Simulation (Hand Calculations)

1. a) Forces:

~~$\underline{f}_a^B = -\frac{\mu m_B}{r^3}\underline{r}_a^{sp} + \frac{3\mu m_B J_2 R_e^2}{2r^5}\left(\left(\frac{5}{r^2}\left(\underline{r}_a^{sp\,T}\underline{1}_3\right)^2\underline{1}_3\right)\right)$~~

$\underline{f}_a^B = -\frac{\mu m_B}{r^3}\underline{r}_a^{sp} + \frac{3\mu m_B \bar{J}_2 R_e^2}{2r^5}\left(\left(\frac{5}{r^2}\left(\underline{r}_a^{sp\,T}\underline{1}_3\right)^2\underline{1}_3 - \underline{1}\right)\underline{r}_a^{sp} - 2\left(\underline{r}_a^{sp\,T}\underline{1}_3\right)\underline{1}_3\right)$

$\underline{T}_b^{BC} = \frac{3\mu}{r^5}\underline{r}_b^{sp\times}\underline{\bar{I}}_b^B\underline{r}_b^{sp}$

where $\underline{r}_b^{sp} = \underline{\underline{C}}_{ba}\underline{r}_a^{sp}$ and $r = \|\underline{r}_b^{sp}\|_2 = \sqrt{\underline{r}_a^{sp\,T}\underline{r}_a^{sp}}$

Find eqn of motion in first order form

$$\dot{x} = f(x)$$

$$x = \begin{bmatrix} \underline{r}_a^{sp} \\ \underline{v}_a^{sp/a} \\ \epsilon \\ \eta \\ \underline{\omega}_b^{ba} \end{bmatrix}$$

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$$\dot{X} = \begin{bmatrix} \underline{V}_a^{sp/a} \\ \underline{a}_a^{sp/a/a} \\ \dot{\epsilon} \\ \dot{\eta} \\ \underline{\dot{\omega}}_b^{ba} \end{bmatrix} = \begin{bmatrix} \underline{V}_a^{sp/a} \\ -\frac{\mu}{r^3}\underline{r}_a^{sp} + \frac{3\mu J_2 R_p^2}{2r^5}\left(\left(\frac{5}{r^2}\left(\underline{r}_a^{sp^T}\underline{1}_3\right)^2 \right.\right. \\ \Gamma(\epsilon,\eta)\begin{bmatrix}\underline{\omega}_b^{ba} \\ 0\end{bmatrix} \\ \underline{I}_b^{B^{-1}}\left(\frac{3M}{r^5}\underline{r}_b^{sp^X}\underline{I}_b^B\underline{r}_b^{sp}\right. \end{bmatrix}$$

$$\dot{X} = \begin{bmatrix} \underline{V}_a^{sp/a} \\ -\frac{\mu}{r^3}\underline{r}_a^{sp} + \frac{3\mu J_2 R_p^2}{2r^5}\left(\left(\frac{5}{r^2}\left(\underline{r}_a^{sp^T}\underline{1}_3\right)^2 - 1\right)\underline{r}_a^{sp} - 2\left(\underline{r}_a^{sp^T}\underline{1}_3\right)\underline{1}_3\right) \\ \Gamma_b^{ba}(\epsilon,\eta)\begin{bmatrix}\underline{\omega}_b^{ba} \\ 0\end{bmatrix} \\ \underline{I}_b^{B^{-1}}\left(\frac{3M}{r^5}\left(\underline{C}_{ba}\underline{r}_a^{sp}\right)^X\underline{I}_b^B\underline{C}_{ba}\underline{r}_a^{sp} - \underline{\omega}_b^{ba^X}\underline{I}_b^B\underline{\omega}_b^{ba}\right) \end{bmatrix}$$

where $r = \sqrt{\underline{r}_a^{sp^T}\underline{r}_a^{sp}} = \|\underline{r}_a^{sp}\|_2$

and $\underline{C}_{ba} = \underline{C}_{ba}(\epsilon,\eta)$

b) $E_{sp/a} = \frac{1}{2}m_s\underline{V}_a^{sp/a^T}\underline{V}_a^{sp/a} + \frac{1}{2}\underline{\omega}_b^{ba^T}\underline{I}_b^B\underline{\omega}_b^{ba}$

$$-\frac{\mu m_s}{r} + \frac{\mu m_s}{r^3}J_2 R_p^2\left(\frac{3}{2r^2}\left(\underline{r}_a^{sp^T}\underline{1}_3\right)^2 - \frac{1}{2}\right)$$

$$+\frac{1}{2}\frac{M}{r^3}\left(\frac{3}{r^2}\underline{r}_b^{sp^T}\underline{I}_b^B\underline{r}_b^{sp} - \text{trace}\left(\underline{I}_b^B\right)\right)$$

c) $\underline{m}_b^T = 1\times10^{-4}[1\ 1\ 0]$ A·m^2 placeholder residual magnetic dipole

$$\underline{\dot{\omega}}_b^{ba} = \underline{I}_b^{-1}\left(\frac{3M}{r^5}\left(\underline{C}_{ba}\underline{r}_a^{sp}\right)^X\underline{I}_b^B\underline{C}_{ba}\underline{r}_a^{sp} + \underline{m}_b^X\underline{C}_{ba}\underline{b}_a - \underline{\omega}_b^{ba^X}\underline{I}_b^B\underline{\omega}_b^{ba}\right)$$

d) if $\underline{C}_{ba} = [\underline{C}_1 \ \underline{C}_2 \ \underline{C}_3]$ where $\underline{C}_1 = \begin{bmatrix} \underline{C}_{11} \\ \underline{C}_{21} \\ \underline{C}_{31} \end{bmatrix}$ ...

$$\dot{X} = \begin{bmatrix} \underline{V}_a^{sp/a} \\ -\frac{\mu}{r^3}\underline{r}_a^{sp} + \frac{3\mu J_2 R_p^2}{2r^5}\left(\left(\frac{5}{r^2}\left(\underline{r}_a^{sp^T}\underline{1}_3\right)^2 - 1\right)\underline{r}_a^{sp} - 2\left(\underline{r}_a^{sp^T}\underline{1}_3\right)\underline{1}_3\right) \\ -\underline{\tilde{W}}_b^{ba^X}\underline{C}_1 \\ -\underline{\tilde{W}}_b^{ba^X}\underline{C}_2 \\ -\underline{\tilde{W}}_b^{ba^X}\underline{C}_3 \\ \underline{I}_b^{B^{-1}}\left(\frac{3\mu}{r^5}\left(\underline{C}_{ba}\underline{r}_a^{sp}\right)^X \underline{I}_b^B \underline{C}_{ba}\underline{r}_a^{sp} + m_b^X \underline{C}_{ba}\underline{b}_a\right) \end{bmatrix}$$

*Go to page 611 of SARC by James R. Wertz for magnetic spin-stabilization techniques.