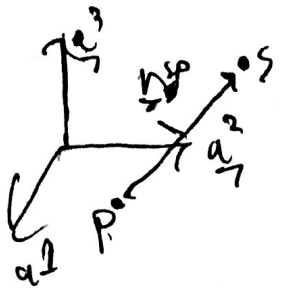
Project Part 2 | Spacecraft | Scratch Work

I a) $\vec{r}^{SP} = \vec{Z}_a^T \begin{bmatrix} x_a \\ y_a \\ z_a \end{bmatrix}$



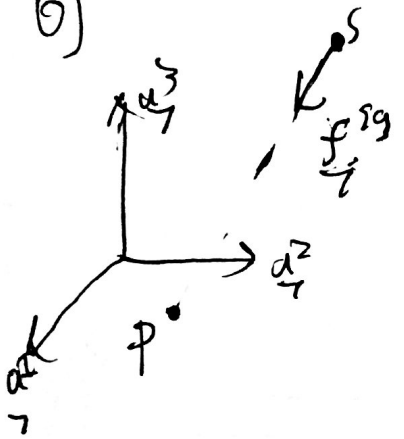
obtain $\vec{a}^{SP/a/a}$

$\vec{v}^{SP/a} = \vec{r}^{SP} \cdot a = \vec{Z}_a^T \cdot a \begin{bmatrix} x_a \\ y_a \\ z_a \end{bmatrix} + \vec{Z}_a^T \begin{bmatrix} \dot{x}_a \\ \dot{y}_a \\ \dot{z}_a \end{bmatrix}$

$\vec{a}^{SP/a/a} = \vec{v}^{SP/a} \cdot a = \vec{Z}_a^T \cdot a \begin{bmatrix} \dot{x}_a \\ \dot{y}_a \\ \dot{z}_a \end{bmatrix} + \vec{Z}_a^T \begin{bmatrix} \ddot{x}_a \\ \ddot{y}_a \\ \ddot{z}_a \end{bmatrix}$

$\vec{a}^{SP/a/a} = \vec{Z}_a^T \begin{bmatrix} \ddot{x}_a \\ \ddot{y}_a \\ \ddot{z}_a \end{bmatrix}$

b)



$\vec{f}^{Sg} = -\frac{\mu m_S}{\|\vec{r}^{SP}\|_2^3} \vec{r}^{SP} = \vec{Z}_a^T \left(-\frac{\mu m_S}{r^3} \vec{r}_a^{SP} \right)$

$m_S \vec{v}^{SP/a} \cdot a \rightarrow$ inertial

→ from $m_S \vec{v}^{SP/a}$

c) $\sum \vec{F} = \vec{f}^{Sg} = m_S \vec{a}^{SP/a/a}$

$\vec{a}^{SP/a/a} = \frac{\vec{Z}_a^T}{m_S} m_S \vec{a}_a^{SP/a/a} = \vec{Z}_a^T \left(-\frac{\mu m_S}{r^3} \vec{r}_a^{SP} \right)$

$\vec{a}^{SP/a/a} = -\frac{\mu}{r^3} \vec{r}_a^{SP}$

d $\dot{x} = f(x)$, let $x = \begin{bmatrix} \underset{\rightarrow}{r}^{sp} \\ \underset{\rightarrow}{r}^{sp/a} \end{bmatrix} = \begin{bmatrix} \underset{\rightarrow}{r}^{sp} \\ \underset{\rightarrow}{r}^{sp/a} \end{bmatrix}$

$$\dot{\underset{\rightarrow}{x}} = \begin{bmatrix} \underset{\rightarrow}{v}^{sp/a} \\ \underset{\rightarrow}{v}^{sp/a/a} \end{bmatrix} = \begin{bmatrix} \underset{\rightarrow}{v}^{sp/a} \\ \underset{\rightarrow}{a}^{sp/a/a} \end{bmatrix}.$$

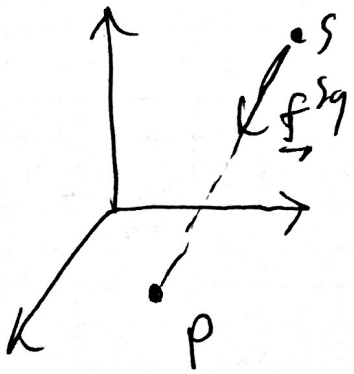
$$\dot{\underline{x}}_a = \begin{bmatrix} \underline{v}_a^{sp/a} \\ \underline{a}_a^{sp/a/a} \end{bmatrix} = \begin{bmatrix} \underline{v}_a^{sp/a} \\ -\frac{\mu}{r^3}\underline{r}_a^{sp} \end{bmatrix}$$

e) $E_{sp/a} = T_{sp/a} + V_{sp}$

$$= \frac{1}{2} m_s \underline{v}_a^{sp/a T} \underline{v}_a^{sp/a} - \frac{\mu m_s}{r}$$

f)

b)



$\mathcal{F}_a$ is inertial

c) $\sum \underline{f}^{sg} = m_s \underline{a}^{sp/a/a}$

$m_s \mathcal{F}_a^* \underline{a}^{sp/a/a} = \mathcal{F}_a \left[-\frac{\mu m_s}{r^3} \underline{r}_a^{sp} + \frac{3\mu m_s J_2 R_p^2}{2r^5} \left(\left(\frac{5}{r^2} \left(\underline{r}_a^{sp^T} \underline{1}_3 \right)^2 - 1 \right) - 2 \left(\underline{r}_a^{sp^T} \underline{1}_3 \right) \underline{1}_3 \right) \right]$

$$\underline{a}^{sp/a/a} = \frac{-\mu}{r^3} \underline{r}_a^{sp} + \frac{3\mu J_2 R_p^2}{2r^5} \left(\left(\frac{5}{r^2} \left(\underline{r}_a^{sp^T} \underline{1}_3 \right)^2 - 1 \right) - 2 \left(\underline{r}_a^{sp^T} \underline{1}_3 \right) \cdot \underline{1}_3 \right)$$

d) $X = \begin{bmatrix} \underline{r}_a^{sp} \\ \underline{r}_a^{sp \cdot a} \end{bmatrix} = \begin{bmatrix} \underline{r}_a^{sp} \\ \underline{v}_a^{sp/a} \end{bmatrix}$

$\dot{X} = \begin{bmatrix} \underline{r}_a^{sp \cdot a} \\ \underline{v}_a^{sp \cdot a} \end{bmatrix} = \begin{bmatrix} \underline{v}_a^{sp/a} \\ \underline{a}_a^{sp/a/a} \end{bmatrix}$

$\dot{X} = \begin{bmatrix} \underline{v}_a^{sp/a} \\ -\frac{\mu}{r^3} \underline{r}_a^{sp} + \frac{3\mu J_2 R_p^2}{2r^5} \left(\left(\frac{5}{r^2} \left(\underline{r}_a^{sp^T} \underline{1}_3 \right)^2 - 1 \right) - 2 \left(\underline{r}_a^{sp^T} \underline{1}_3 \right) \underline{1}_3 \right) \end{bmatrix}$

e) Are we suppose to re-derive V_{sp} if we want an accurate stm →

↱ gravity potential perturbed

$$V_{sp} = \cancel{\phi_p} m_s + -\frac{\mu m_s}{r}$$

$$= -\frac{\mu m_s}{r^3} J_2 R_p^2 \left(\frac{3}{2}\left(\frac{z}{r}\right)^2 - \frac{1}{2}\right) - \frac{\mu m_s}{r}$$

$$z = \underline{r} \cdot \underline{\underline{1}}_3, \quad \frac{z}{r} = \frac{\underline{r}}{r} \cdot \underline{\underline{1}}_3$$

$$V_{sp} = \frac{-\mu m_s J_2 R_p^2}{r^3}\left(\frac{3}{2}\left(\frac{\underline{r}_a^{sp^T} \cdot \underline{\underline{1}}_3}{r}\right)^2 - \frac{1}{2}\right) - \frac{\mu m_s}{r}$$

$$E_{spca} = \frac{1}{2} m_s \underline{v}_a^{sp^T} \underline{v}_a^{sp} - \frac{\mu m_s}{r} - \frac{\mu m_s J_2 R_p^2}{r^3}\left(\frac{3}{2}\left(\frac{\underline{r}_a^{sp^T} \cdot \underline{\underline{1}}_3}{r}\right)^2 - \frac{1}{2}\right)$$

$$E_{spca} = \frac{1}{2} m_s \underline{v}_a^{sp^T} \underline{r}_a^{sp} - \frac{\mu m_s}{r} - \frac{\mu m_s J_2 R_p^2}{r^3}\left(\frac{3}{2r^2}\left(\underline{r}_a^{sp^T} \underline{\underline{1}}_3\right)^2 - \frac{1}{2}\right)$$