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Roll No. [redacted] **Exam Code : M-23**

Subject Code—59009

B. A. EXAMINATION

(Main/Reappear)

(Batch 2018 Onwards)

(Sixth Semester)

MATHEMATICS

BAMH-306

Real and Complex Analysis

(B.A. Mathematics)

Time : 3 *Hours* *Maximum Marks* : 24

Note : Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. 1 is compulsory. All questions carry equal marks.

(Compulsory Question)

1. (a) Define improper integral of second kind. **0.5**

   (b) What is period of Sinz function ? **0.5**

   (c) Give example of the connected space. **0.5**

   (d) Define open set. **0.5**

   (e) Give one example of continuous function. **0.5**

   (f) Define limit point a set. **0.5**

   (g) Boundary point a set in a metric space. **0.5**

   (h) Evaluate : **0.5**

$$\lim_{z\to 0}\frac{1-\cos z}{z^2}$$

Unit I

2. (a) Let (X, d) be a metric space and let $d^*(x, y) = \min\{1, d(x, y)\}$. Show that d^* is a metric space. **2.5**

   (b) In a metric space (X, d), every open sphere is an open set. **2.5**

3. (a) In a metric space, the intersection of a finite number of open sets is open. Prove. **2.5**

   (b) If A is a subset of a metric space (X, d), then closure of A is a closed set. Prove. **2.5**

Unit II

4. (a) A point 'a' is a limit point of a subset A of a metric space (X, d), iff there is a sequence $< a_n >$ of points of A, all distinct 'a' which converges to a. Prove. **2.5**

   (b) Every Cauchy sequence is bounded in a metric space. Prove. **2.5**

5. (a) Let (X, d) and (Y, d^*) be two metric spaces and f, g be two continuous functions of X into Y. Prove.

   Then the set $\{x \in X : f(x) = g(x)\}$ is a closed subset of X. Prove. **2.5**

(b) Every countably compact metric space has BWP. Prove. 2.5

Unit III

6. (a) Evaluate the improper integral : 2.5

$$\int_0^1 \frac{dx}{x^2}$$

(b) Discuss the convergence of integral : 2.5

$$\int_0^1 \frac{dx}{x^3(1+x^2)^5}$$

7. (a) Examine the convergence of integral : 2.5

$$\int_0^\infty \frac{x \tan^{-1}}{(1+x^4)^{1/3}} dx$$

(b) Show that : 2.5

$$\int_0^1 \frac{x^\alpha - 1}{\log x} dx = \log(1+\alpha)$$

Unit IV

8. (a) Prove that $i^i = e^{-(4n+1)\frac{\pi}{2}}$ and show that its values for a geometrical. 2.5

(b) Show that the function $f(z) = |z^2|$ is the continuous everywhere but nowhere differentiable expect at origin. 2.5

9. (a) Find the regular function whose imaginary part is $V = e^x(x \sin y + y \cos y)$. 2.5

(b) Prove that $f(z) = xy + iy$ is everywhere continuous but not analytic. 2.5