1. Which of the following is true about the market portfolio in CAPM?

    a. The market portfolio in CAPM defines "the domestic market".
    b. CAPM defines the market portfolio as the stock market.
    c. The true market portfolio holds all the world's assets – stocks, bonds, real estate, commodities as well as labor income – weighted in proportion to their market value.
    d. Holding the market portfolio is not optimal.

    Answer:

    The correct answer is c.

    The true market portfolio in CAPM consists of all the world's tradable and non-tradable assets, including stocks, bonds, commodities, real estate and all other risky assets. This of course presents a practical problem in measuring the market portfolio. This is why using the stock market as a proxy for the market portfolio has long been the common practice as it is accessible. However, we can build better market indices by looking beyond the domestic stock market benchmark.

2. Which of the following in not true about the Capital Market Line?

    a. The capital allocation line that goes through the risk-free asset and the market portfolio is called the Capital Market Line.
    b. The Capital Market Line maximizes the Sharpe ratio.
    c. The Capital Market Line is tangent to the efficient frontier.
    d. The Capital Market Line minimizes the Sharpe ratio.

    Answer:

    The correct answer is d.

    The Capital Market Line does not minimize the Sharpe ratio – in fact, it provides the portfolio combinations with the maximum Sharpe ratio.

3. Which of the following is not true about passive investing?

    a. Passive investing is buying a well-diversified portfolio to represent a broad-based market index without attempting to search out mispriced securities.
    b. Holding the market portfolio is efficient, so passive investing is efficient as well. No, that is not the correct answer. It is true that holding the market portfolio is efficient, so passive investing is efficient as well.
    c. Passive investing cannot be efficient because it involves passive investors.
    d. The amount of diversifiable risk in an index fund is near zero.

    Answer:

    The correct answer is c.

Passive investing is not about the behavior of investors.

4. Your investment portfolio consists of $150,000 invested in only one stock – Pocemon. Suppose the risk free rate is 3%. The Pocemon stock has an expected return of 12%, and a volatility of 40%, and the market portfolio has an expected return of 10% and volatility of 18%. Under the CAPM assumptions, what is the volatility of a better alternative portfolio that has the same expected return as Pocemon?

   a. σ = 15.52%
   b. σ = 23.14%
   c. σ = 30.25%
   d. σ = 35.13%

   Answer:

   The correct answer is b.

   Recall that under CAPM assumptions, the Capital Market line is the efficient frontier – that is the combinations of the market portfolio and the risk-free asset that maximize the Sharpe ratio. Holding Pocemon by itself has idiosyncratic risk, but one can construct a better alternative portfolio on the efficient frontier that has the same expected return, but lower volatility.

   The Capital Market Line is given by the following expression:

$$E(r) = \frac{(E(r_m) - r_f)}{\sigma_m} \cdot \sigma + r_f \Rightarrow E(r) = \frac{(10\% - 3\%)}{18\%} \cdot \sigma + 3\% \Rightarrow$$

$$E(r) = \frac{7}{18} \cdot \sigma + 3\%$$

   An efficient portfolio with the same expected return as Pocemon (but with lower volatility) will have the following volatility:

$$E(r) = \frac{7}{18} \cdot \sigma + 3\% \Rightarrow 12\% = \frac{7}{18} \cdot \sigma + 3\% \Rightarrow \boldsymbol{\sigma = 23.14\%}$$

5. Your investment portfolio consists of $150,000 invested in only one stock – Pocemon. Suppose the risk free rate is 3%. The Pocemon stock has an expected return of 12%, and a volatility of 40%, and the market portfolio has an expected return of 10% and volatility of 18%. Now what if you had the stomach for the kind of volatility Pocemon has. Under the CAPM assumptions, what would be the expected return you should then earn?

   a. E(r) = 13.65%
   b. E(r) = 15.30%
   c. E(r) = 18.56%
   d. E(r) = 20.38%

Answer:

The correct answer is c.

Recall that under CAPM assumptions, the Capital Market line is the efficient frontier – that is the combinations of the market portfolio and the risk-free asset that maximize the Sharpe ratio. Holding Pocemon by itself has idiosyncratic risk, but one can construct a better alternative portfolio on the efficient frontier that has the same expected return, but lower volatility.

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$$E(r) = \frac{7}{18} \cdot \sigma + 3\%$$

An efficient portfolio with the same volatility as Pocemon will have the following expected return:

$$E(r) = \frac{7}{18} \cdot \sigma + 3\% \Rightarrow E(r) = \frac{7}{18} \cdot 40\% + 3\% \Rightarrow \mathbf{E(r) = 18.56\%}$$

6. Suppose you have invested $30,000 in the following four stocks:

Security	Amount invested	Beta
Stock A	$5,000	0.75
Stock B	$10,000	1.10
Stock C	$8,000	1.36
Stock D	$7,000	1.88

The risk free rate is 2%, and the expected return on the market portfolio is 8%. Based on the CAPM, what is the beta of the portfolio?

a. $\beta_P = 0.95$
b. $\beta_P = 1.19$
c. $\beta_P = 1.29$
d. $\beta_P = 1.62$

Answer:

The correct answer is c.

Recall that the beta of the portfolio is simply the weighted average of the individual betas of the securities in the portfolio. The beta of the portfolio is calculated as follows:

$$\beta_p = \sum_{i=1}^{N} w_i \cdot \beta_i$$

where w_i is the weight of stock i and β_i is the beta of stock i with i=1,…,N

The weights of the stocks are:

$w_A = \frac{5{,}000}{30{,}000} = 0.167,$

$w_B = \frac{10{,}000}{30{,}000} = 0.333,$

$w_C = \frac{8{,}000}{30{,}000} = 0.267,$

$w_D = \frac{7{,}000}{30{,}000} = 0.233$

Hence,

$$\beta_p = 0.167 \cdot 0.75 + 0.333 \cdot 1.10 + 0.267 \cdot 1.36 + 0.233 \cdot 1.88 = \mathbf{1.29}$$

7. Based on the data provided in question 6 and based on the CAPM, what is the expected return of the portfolio?

   a. $E(r_P) = 9.74\%$
   b. $E(r_P) = 10.55\%$
   c. $E(r_P) = 13.00\%$
   d. $E(r_P) = 15.23\%$

Answer:

The correct answer is a.

The expected return-beta relationship is the following:

$$E(r_i) = r_f + \beta_i \cdot E(r_m - r_f)$$

where,
$E(r_i)$ is the expected return of asset i
$E(r_m)$ is the expected return of the market portfolio
r_f is the risk-free rate
β_i is the beta of asset i

The expected return of the portfolio is given by the following relationship:

$$E(r_P) = r_f + \beta_P \cdot E(r_m - r_f)$$

The beta of the portfolio is calculated as follows:

$$\beta_p = \sum_{i=1}^{N} w_i \cdot \beta_i$$

where w_i is the weight of stock i and β_i is the beta of stock i with i=1,…,N

The weights of the stocks are:

$$w_A = \frac{5{,}000}{30{,}000} = 0.167,\ w_B = \frac{10{,}000}{30{,}000} = 0.333,\ w_C = \frac{8{,}000}{30{,}000} = 0.267,\ w_D = \frac{7{,}000}{30{,}000} = 0.233$$

Hence,

$$\beta_p = 0.167 \cdot 0.75 + 0.333 \cdot 1.10 + 0.267 \cdot 1.36 + 0.233 \cdot 1.88 = \mathbf{1.29}$$

So,

$$E(r_P) = 2\% + 1.29 \cdot (8\% - 2\%) = \mathbf{9.74}\%$$

8. There are two stocks in the market, stock A and stock B. The price of stock A today is \$50. The price of stock A next year will be \$40 if the economy is in a recession, \$55 if the economy is normal, and \$60 if the economy is expanding. The probabilities of recession, normal times, and expansion are 0.1, 0.8, and 0.1 respectively. Stock A pays no dividend. Assume CAPM is true. The volatility of the return on the market portfolio (σ_M) is 10%. The expected return of stock B is 7% and the volatility of the return of stock B is 12%. Furthermore, the correlation between the return on stock A and the market portfolio return is 0.8, the correlation between the return on stock B and the market portfolio return is 0.5, and the correlation between the returns of the two stocks is 0.6. What is the beta of stock A?

   a. $\beta_A = 0.66$
   b. $\beta_A = 0.78$
   c. $\beta_A = 0.95$
   d. $\beta_A = 1.30$

Answer:

The correct answer is b.

First state of the economy (recession):

$$\%\ \text{Return} = \frac{\text{Capital gain} + \text{Dividend}}{\text{Purchase price}} * 100$$

$$R_1 = \frac{40 - 50 + 0}{50} * 100 = -20\%$$

Second state of the economy (normal):

$$\%\ \text{Return} = \frac{\text{Capital gain} + \text{Dividend}}{\text{Purchase price}} * 100$$

$$R_2 = \frac{55 - 50 + 0}{50} * 100 = 10\%$$

Third state of the economy (expansion):

$$\%\ \text{Return} = \frac{\text{Capital gain} + \text{Dividend}}{\text{Purchase price}} * 100$$

$$\mathrm{R_3} = \frac{60 - 50 + 0}{50} * 100 = 20\%$$

The expected return is calculated as the probability-weighted average of the returns. P(Ri) is the probability of each scenario and Ri the return in each scenario, where scenarios are labeled by i. So we write the expected return as:

$$E(R) = \sum R_i \cdot P(R_i)$$

$$\mathrm{E(R_A)} = -20\% \cdot 0.10 + 10\% \cdot 0.80 + 20\% \cdot 0.10 = \mathbf{8}\%$$

The standard deviation of the return is defined as the square root of variance, which is the expected value of the squared deviations from the expected return. The variance is calculated as:

$$\sigma_x^2 = \sum \{[\mathrm{R_{xi}} - \mathrm{E(R_x)}]^2 \cdot \mathrm{P_i}\}$$

The variance of stock A is calculated as:

$$\sigma_A^2 = \sum \{[\mathrm{R_{Ai}} - \mathrm{E(R_A)}]^2 \cdot \mathrm{P_i}\}$$

$$\sigma_A^2 = (-20 - 8)^2 \cdot 0.10 + (10 - 8)^2 \cdot 0.80 + (20 - 8)^2 \cdot 0.10 = 96$$

Hence the standard deviation for stock A is equal to $\sigma_\mathrm{A} = \sqrt{96} = \mathbf{9.80}\%$

The beta of the stock A is given by the following relationship:

$$\beta_\mathrm{A} = \frac{\mathrm{cov(r_A, r_m)}}{\sigma_\mathrm{m}^2} = \frac{\rho_\mathrm{Am} \cdot \sigma_\mathrm{A} \cdot \sigma_\mathrm{m}}{\sigma_\mathrm{m}^2} = \frac{\rho_\mathrm{Am} \cdot \sigma_\mathrm{A}}{\sigma_\mathrm{m}}$$

where,
ρ_{Am}= 0.80
$\sigma_\mathrm{A} = \sqrt{96} = 9.80\%$
$\sigma_m = 10\%$

Hence,

$$\beta_\mathrm{A} = \frac{0.80 \cdot 9.80\%}{10\%} = \mathbf{0.78}$$

9. There are two stocks in the market, stock A and stock B. The price of stock A today is $50. The price of stock A next year will be $40 if the economy is in a recession, $55 if the economy is normal, and $60 if the economy is expanding. The probabilities of

recession, normal times, and expansion are 0.1, 0.8, and 0.1 respectively. Stock A pays no dividend. Assume CAPM is true. The volatility of the return on the market portfolio (σ_M) is 10%. The expected return of stock B is 7% and the volatility of the return of stock B is 12%. Furthermore, the correlation between the return on stock A and the market portfolio return is 0.8, the correlation between the return on stock B and the market portfolio return is 0.54, and the correlation between the returns of the two stocks is 0.6. What is the beta of stock B?

a. $\beta_B = 0.65$
b. $\beta_B = 0.87$
c. $\beta_B = 0.96$
d. $\beta_B = 1.25$

Answer:

The correct answer is a.

The beta of the stock B is given by the following relationship:

$$\beta_B = \frac{\text{cov}(r_B, r_m)}{\sigma_m^2} = \frac{\rho_{Bm} \cdot \sigma_B \cdot \sigma_m}{\sigma_m^2} = \frac{\rho_{Bm} \cdot \sigma_B}{\sigma_m}$$

where,
$\rho_{Bm} = 0.54$
$\sigma_B = 12\%$
$\sigma_m = 10\%$

So,

$$\beta_B = \frac{0.54 \cdot 12\%}{10\%} = 0.65$$

10. You invest in a stock with beta equal to 0.80. The risk free rate is 2% and the expected return on the market portfolio is 8%. Based on the CAPM, what is the expected return of the stock?

a. E(r) = 6.80%
b. E(r) = 5.35%
c. E(r) = 10.25%
d. E(r) = 13.70%

Answer:

The correct answer is a.

The expected return-beta relationship is the following:

$$E(r_i) = r_f + \beta_i \cdot E(r_m - r_f)$$

where,

$E(r_i)$ is the expected return of asset i
$E(r_m)$ is the expected return of the market portfolio
r_f is the risk-free rate
β_i is the beta of asset i

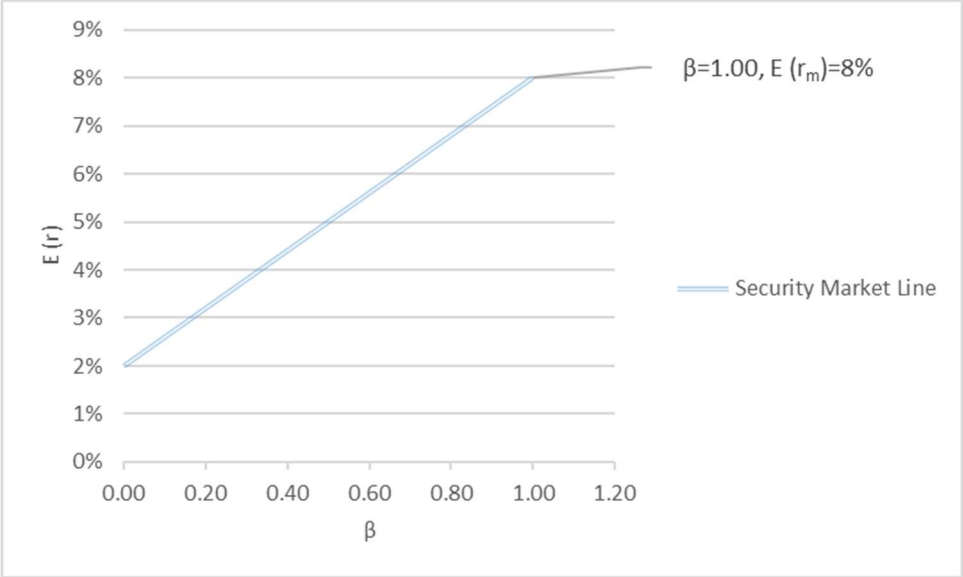
The expected return of the stock is given by the following relationship:

$$E(r) = r_f + \beta_P \cdot E(r_m - r_f)$$

So,

$$E(r) = 2\% + 0.80 \cdot (8\% - 2\%) = \mathbf{6.80}\%$$

11. Based on the following plot of the Security Market Line, which of the following statements is correct?



a. The risk-free rate is equal to 3%
b. The expected return of the market portfolio is equal to 8%.
c. The expected return of the market portfolio is equal to 2%.
d. The risk-free rate is equal to 8%

Answer:

The correct answer is b.

The market portfolio is placed where on x-axis beta is equal to 1 and on y-axis the expected return is equal to the expected return of the market portfolio. So the expected return of the market portfolio is equal to 8%. The risk-free asset as we observe on the plot is on the y-axis, since the beta of the risk-free asset is equal to 0. The risk-free rate is equal to 2%.

12. Suppose you find an asset that, based on its price today, has a lower expected return than what is suggested by the SML. If the Capital Asset Pricing Model holds, which of the following statements is/are correct? (more than one answer can be correct)

    a. If the asset has a lower expected return than the one suggested by the SML, that means that the price is too low.
    b. If the asset has a lower expected return than the one suggested by the SML, that means that the price is too high.
    c. The asset would plot below the SML.
    d. The asset would plot above the SML.

    Answer:

    The correct answers are b and c.

    If the asset has a lower expected return than the one suggested by the SML, that means that the price is too high. This means that none would want to buy the asset, so the price will drop. As the price of the asset goes down, the expected return goes up, until we reach an equilibrium. So initially the asset would be plotted below the SML, but as equilibrium is reached the asset should be plotted on the SML.