A note on mean variance frontier with multiple risky assets

This note accompanies the Excel spreadsheet MeanVarianceFrontier_Multiple RiskyAssets.xlsx that illustrates how to plot the mean variance frontier with multiple risky assets.

How do we find the mean variance frontier when there are a large N number of risky assets? Unlike the two risky assets case, now the number of combinations that one can create is countless. So how do we find the locus of portfolios that has the minimum variance for each level of expected return?

As I explained in the lecture, all we need to find is two portfolios that we know are on the frontier: the global minimum variance (GMV) portfolio – the portfolio with the minimum variance of all possible combinations – and the mean variance efficient (MVE) portfolio which we know is the optimal risky portfolio on the frontier with the maximum Sharpe ratio. Once we find these two portfolios, then we are back in the two risky assets world, and we can plot the mean variance frontier by varying the weights on these two portfolios to trace the frontier.

The accompanying spreadsheet MeanVarianceFrontier_MultipleRiskyAssets.xlsx illustrates the steps using the equity market data for G5 countries.

The first tab is the raw data.

	RETURN	DEVIATION
US	0.1355	0.1535
UK	0.1589	0.2430
France	0.1519	0.2324
Germany	0.1435	0.2038
Japan	0.1497	0.2298
Risk-free rate	0.0500	

	US	UK	France	Germany	Japan
US	1.0000	0.5003	0.4398	0.3681	0.2663
UK	0.5003	1.0000	0.5420	0.4265	0.3581
France	0.4398	0.5420	1.0000	0.6032	0.3923
Germany	0.3681	0.4265	0.6032	1.0000	0.3663
Japan	0.2663	0.3581	0.3923	0.3663	1.0000

The second tab shows the analysis. In this analysis, you will use the Excel Solver twice: One to find the Global Minimum Variance portfolio weights and one to find the Mean Variance Efficient Portfolio. All of this analysis is much simpler if you are comfortable using matrix formulation in Excel. In the absence of matrix formulation study how I use the bordered variance/covariance matrix and the portfolio weights to solve for the GMV and MVE portfolios.

Having found the portfolio weights for the GMV, I copied and pasted them over here:

Global Minimum Variance (GMV) Portfolio	
	Portfolio Wieghts
US	0.635
UK	-0.016
France	-0.015
Germany	0.216
Japan	0.181
Expected return	0.1392
Standard Deviation	0.1364
Sharpe Ratio	0.6539

Similarly, having solved for the MVE weights, I copied and pasted them over here:

Mean Variance Efficient Portfolio (MVE)	
	Portfolio Wieghts
US	0.507
UK	0.075
France	0.024
Germany	0.190
Japan	0.204
Expected return	0.1421
Standard Deviation	0.1386
Sharpe Ratio	0.6644

You also need to find the covariance (or the correlation) between these two portfolios:

Covariance	0.0186
Corrleation	0.9842

All that is left to do now is to construct the mean variance frontier by varying the weights on the GMV and MVE portfolios and computing the expected return and volatility for each combination.