MEAN-VARIANCE OPTIMIZATION: OPTIMAL RISKY PORTFOLIO WITH TWO RISKY ASSETS

WHAT WILL YOU LEARN?

- ► What is the optimal portfolio of risky assets?
 - \blacktriangleright What is the best risky asset portfolio?
 - ► Mean-variance efficient (MVE) portfolio
- ► What is the optimal asset allocation?
 - ► How should we allocate our wealth between this optimal risky portfolio and the risk-free asset?
 - ► Capital allocation line (CAL)
 - ►Sharpe ratio

RECAP THE ANSWER TO THE SECOND QUESTION

- ► The CAL describes the menu of possible risk/return trade-offs between a risk-free and risky asset.
- ▶Its slope equals the Sharpe ratio.
- ▶The optimal allocation with a risky and risk-free asset for any investor is the tangency point of that investor's indifference curves with the CAL.

NOW TURN TO THE FIRST QUESTION

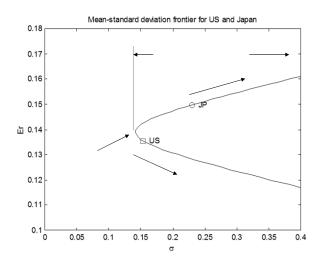
- ▶What is the optimal portfolio of risky assets?
 - ▶What is the best risky asset portfolio?
 - ► Mean-variance efficient (MVE) portfolio

WHAT IF WE HAVE TWO RISKY ASSETS?

- ▶With two risky assets, different CALs are available.
- ▶What is the composition of the best risky portfolio?

PORTFOLIO OF TWO RISKY ASSETS Mean-standard deviation frontier for US and Japan 0.18 0.17 0.16 0.15 0.13 0.12 0.11 0.10 0.





TWO RISKY ASSETS AND ONE RISK-FREE ASSET

- ► The "tangency portfolio" provides the "steepest" CAL.
- ►It maximizes the Sharpe ratio for all possible risky portfolios.
- ► This portfolio is known as the mean-variance efficient portfolio or MVE.

MEAN-VARIANCE EFFICIENT PORTFOLIO

- ► Mean-variance efficient (MVE) portfolio determines the optimal risky portfolio.
- ► Everyone will hold the same risky portfolio the MVE and mix it with the risk-free asset according to his or her preferences, namely risk aversion.
- ► CAL combining the risk-free asset and the MVE becomes the set of efficient portfolios.

SUMMARY

- ▶Step 1: Create the mean variance (minimum standard-deviation) frontier. (For two risky assets, this is all the possible combinations of the two assets.)
- ► Step 2: Find the portfolio that maximizes Sharpe ratio. This is the mean variance efficient (MVE) portfolio.
- ▶ Step 3: Construct the efficient frontier: the line starting from the risk-free asset, intersecting the frontier at the MVE.
- ► Step 4: Mix the MVE portfolio (fund) with the risk-free asset according to your preferences.

MEAN-VARIANCE OPTIMIZATION: OPTIMAL RISKY PORTFOLIO WITH MULTIPLE RISKY SECURITIES

OPTIMAL PORTFOLIO CHOICE PROBLEM

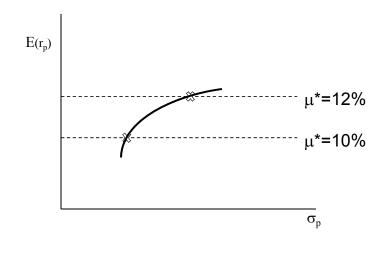
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- ► Step 3: Construct the efficient frontier: the line starting from the risk-free asset, intersecting the frontier at the MVE.
- ► Step 4: Mix the MVE portfolio (fund) with the risk-free asset according to your preferences.

WHAT WILL YOU LEARN?

- ► What is the optimal portfolio of risky assets you have multiple securities?
 - ► How do you construct the mean variance frontier when you have multiple securities?

MULTIPLE RISKY SECURITIES





MEAN VARIANCE FRONTIER WITH MULTIPLE RISKY SECURITIES

► Well, how many target levels do we have to try out to obtain the mean-variance frontier?

HOW DO WE CONSTRUCT THE MEAN VARIANCE FRONTIER?

- ▶ Find two portfolios on the frontier...
- ▶ If we can find two portfolios on the frontier that we can take all possible combinations of those to trace the mean variance frontier.
- ▶What two portfolios could you use???

HOW DO WE CONSTRUCT THE MEAN VARIANCE FRONTIER?

HOW DO WE CONSTRUCT THE MEAN VARIANCE FRONTIER

- ▶ Take all combinations of these two portfolios
 - ► Let w be the weight in the MVE and (1-w) in the minimum variance portfolio for various weights and draw the frontier.
- ► Use the formulas to compute the means and variances to trace the risk-return combinations of the two portfolios
- ► To actually find two frontier portfolios, Excel or some other optimizer can be used.

SUMMARY

- ► Mean variance frontier with multiple risky securities
- ► Two portfolios on the frontier generate the whole frontier
- ► Given a risk-free asset, the set of efficient portfolios has one risky portfolio that delivers the steepest CAL the MVE.
- ►The optimal asset allocation involves mixing the risk free asset with the MVE according to your preferences