

A note on measuring risk aversion and certainty equivalent

If you are a risk-averse investor, it means that you would rather prefer the sure outcome to a fair gamble. Risk aversion measures how much more the investor prefers the sure thing to a fair gamble. Technically speaking, risk aversion measures the degree of concavity in an investor's utility function.

Let's illustrate this by the graph down below:

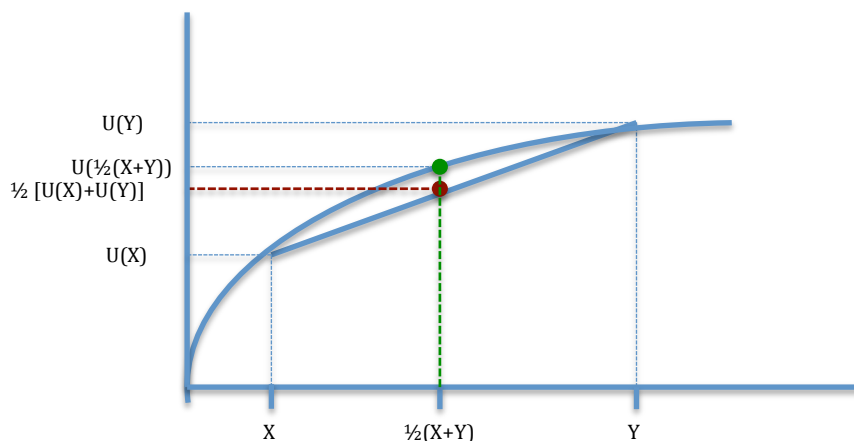
Suppose there are two equally likely outcomes: X (low) and Y high, giving you a utility level of $U(X)$ and $U(Y)$ respectively.

What is your expected utility? Well, it is either $U(X)$ with probability $\frac{1}{2}$ or $U(Y)$ with probability $\frac{1}{2}$. In other words, the expected utility is given by $\frac{1}{2}U(X) + \frac{1}{2}U(Y)$ shown by the red point in the graph below.

What if you were given the certain outcome of $\frac{1}{2}X + \frac{1}{2}Y$? The level of utility that corresponds to the certain outcome of $\frac{1}{2}X + \frac{1}{2}Y$ is given by the green point, shown in the graph below. In other words, utility is higher for the certain outcome than for the uncertain outcome:

$$U\left(\frac{1}{2}X + \frac{1}{2}Y\right) \geq \frac{1}{2}U(X) + \frac{1}{2}U(Y)$$

The greater the difference between the red point and the green point, the more risk averse is the investor. Therefore, the more concave the utility function, the more risk averse is the investor. Notice that a risk-averse investor would require a premium to be indifferent between these two.



Alternatively, if the investor is *risk-loving*, or *risk-seeking*, the utility function becomes convex. Finally, if the investor is risk-neutral, the utility function would be linear. In this case, $U\left(\frac{1}{2}X + \frac{1}{2}Y\right) = \frac{1}{2}U(X) + \frac{1}{2}U(Y)$ and a risk-neutral investor would require a zero risk premium.

Economists measure the certainty equivalent (CE) by asking investors what sure amount of wealth – or risk-free rate of return – investors would accept instead of a

risky outcome (or a lottery). Formally, we can write the certainty equivalent CE as the amount that satisfies $U(CE) = E[U(X)]$ for a risky lottery X.

For example, say you were given a lottery where you can win \$1000 with 50% probability or win \$500 with 50% probability. That is, you will get \$500 for sure, but you have the possibility of winning \$1000. If you were asked how much you would be willing to pay for this opportunity, your answer reveals your certainty equivalent. If you are willing to pay \$750, which is the fair value of the lottery, your certainty equivalent is equal to the fair value of the lottery. In other words, you are risk neutral and have a risk aversion coefficient of 0. Most people would be willing to pay between \$540 and \$707 to enter this lottery, which correspond to risk aversion coefficient between 1 and 10.