1. Consider the following distribution of returns:

Probability	R_A	R_B	R_C
30%	-20%	-5%	5%
40%	5%	10%	3%
30%	40%	15%	2%

What is the standard deviation of security A? Round off your answer to two digits after the decimal point (such as 5.55).

Answer:

The answer is that $\sigma_A = \mathbf{23.37}\%$.

The standard deviation of the return is defined as the square root of variance, which is the expected value of the squared deviations from the expected return. The variance is calculated as:

$$\sigma_x^2 = \sum\{[R_{xi} - E(R_x)]^2 \cdot P_i\}$$

The expected return is calculated as the probability-weighted average of the returns. $P(R_i)$ is the probability of each scenario and R_i the return in each scenario, where scenarios are labeled by i. So we write the expected return as:

$$E(R) = \sum R_i \cdot P(R_i)$$

$$E(R_A) = -20\% \cdot 0.30 + 5\% \cdot 0.40 + 40\% \cdot 0.30 = 8\%$$

The variance for A is calculated as:

$$\sigma_A^2 = \sum\{[R_{Ai} - E(R_A)]^2 \cdot P_i\}$$

$$\sigma_A^2 = (-20-8)^2 \cdot 0.30 + (5-8)^2 \cdot 0.40 + (40-8)^2 \cdot 0.30 = 546$$

Hence the standard deviation for A is equal to $\sigma_A = \sqrt{546} = \mathbf{23.37}\%$

2. Consider the following distribution of returns:

Probability	R_A	R_B	R_C
30%	-20%	-5%	5%
40%	5%	10%	3%
30%	40%	15%	2%

What is the standard deviation of security B? Round off your answer to two digits after the decimal point (such as 5.55).

Answer:

The answer is that $\sigma_B = \mathbf{8.12}\%$.

The standard deviation of the return is defined as the square root of variance, which is the expected value of the squared deviations from the expected return. The variance is calculated as:

$$\sigma_x^2 = \sum \{[R_{xi} - E(R_x)]^2 \cdot P_i\}$$

The expected return is calculated as the probability-weighted average of the returns. $P(R_i)$ is the probability of each scenario and R_i the return in each scenario, where scenarios are labeled by i. So we write the expected return as:

$$E(R) = \sum R_i \cdot P(R_i)$$

$$E(R_B) = -5\% \cdot 0.30 + 10\% \cdot 0.40 + 15\% \cdot 0.30 = 7\%$$

The variance for B is calculated as:

$$\sigma_B^2 = \sum \{[R_{Bi} - E(R_B)]^2 \cdot P_i\}$$

$$\sigma_B^2 = (-5 - 7)^2 \cdot 0.30 + (10 - 7)^2 \cdot 0.40 + (15 - 7)^2 \cdot 0.30 = 66$$

Hence the standard deviation for B is equal to $\sigma_B = \sqrt{66} = \mathbf{8.12}\%$

3. Consider the following distribution of returns:

Probability	R_A	R_B	R_C
30%	-20%	-5%	5%
40%	5%	10%	3%
30%	40%	15%	2%

What is the standard deviation of security C? Round off your answer to two digits after the decimal point (such as 5.55).

Answer:

The answer is that $\sigma_C = \mathbf{1.19}\%$.

The standard deviation of the return is defined as the square root of variance, which is the expected value of the squared deviations from the expected return. The variance is calculated as:

$$\sigma_x^2 = \sum \{[R_{xi} - E(R_x)]^2 \cdot P_i\}$$

The expected return is calculated as the probability-weighted average of the returns. $P(R_i)$ is the probability of each scenario and R_i the return in each scenario, where scenarios are labeled by i. So we write the expected return as:

$$E(R) = \sum R_i \cdot P(R_i)$$

$$E(R_C) = 5\% \cdot 0.30 + 3\% \cdot 0.40 + 2\% \cdot 0.30 = 3.3\ \%$$

The variance for C is calculated as:

$$\sigma_C^2 = \sum \{[R_{Ci} - E(R_C)]^2 \cdot P_i\}$$

$$\sigma_C^2 = (5 - 3.3)^2 \cdot 0.30 + (3 - 3.3)^2 \cdot 0.40 + (2 - 3.3)^2 \cdot 0.30 = 1.41$$

Hence the standard deviation for C is equal to $\sigma_C = \sqrt{1.41} = \mathbf{1.19}\%$

4. Consider the following distribution of returns:

Probability	R_A	R_B	R_C
30%	-20%	-5%	5%
40%	5%	10%	3%
30%	40%	15%	2%

What is the expected return of a portfolio with 40% in A, 20% in B and 40% in C? Round off your final answer to two digits after the decimal point. State your answer as a percentage rate (such as 5.55).

Answer:

The answer is that $E(r_P)$ = **5.92**%

Recall that the expected portfolio return is the weighted average of the expected returns on individual securities.

We therefore need to first find the expected return for each security:

$$E(r_A) = 0.3 \times (-20\%) + 0.4 \times 5\% + 0.3 \times 40\% = 8\%$$

$$E(r_B) = 0.3 \times (-5\%) + 0.4 \times 10\% + 0.3 \times 15\% = 7\%$$

$$E(r_C) = 0.3 \times (5\%) + 0.4 \times 3\% + 0.3 \times 2\% = 3.3\%$$

The weights of A, B, and C are respectively 40%, 20%, and 40%.

$$E(r_P) = 0.4 \times 8\% + 0.2 \times 7\% + 0.4 \times 3.3\% = 5.92\%$$

5. Consider the following distribution of returns:

Probability	R_A	R_B	R_C
30%	-20%	-5%	5%
40%	5%	10%	3%
30%	40%	15%	2%

Which of the following pairwise covariance measures are correct? (Multiple answers are accepted)

a. $\sigma_{AB} = 174$
b. $\sigma_{AC} = -26.4$
c. $\sigma_{BC} = 26.4$
d. $\sigma_{BC} = -9.60$

Answer:

The correct answers are a, b and d.

Covariance between the returns of two assets is computed as the expected value of the product of the deviations from the mean:

$$\sigma_{AB} = \Sigma\ \{[R_{Ai} - E(R_A)] \times [R_{Bi} - E(R_B)] \times p_i\}$$

You already computed the expected return on A and B in a previous quiz.

$$E(R_A) = 0.3 \times -20\% + 0.4 \times 5\% + 0.3 \times 40\% = 8\%$$
$$E(R_B) = 0.3 \times -5\% + 0.4 \times 10\% + 0.3 \times 15\% = 7\%$$
$$E(R_C) = 0.3 \times 5\% + 0.4 \times 3\% + 0.3 \times 2\% = 3.3\%$$

Hence the covariance between A and B is calculated as:

σ_{AB} = (-20 - 8) x (-5 - 7) x 0.30 + (5-8) x (10-7) x 0.40 + (40-8) x (15-7) x 0.30 =174

Hence the covariance between A and C is calculated as:

σ_{AC} = (-20 - 8) x (5 - 3.3) x 0.30 + (5-8) x (3-3.3) x 0.40 + (40-8) x (2-3.3) x 0.30 = -26.4

Hence the covariance between B and C is calculated as:
σ_{BC} = (-5 - 7) x (5 - 3.3) x 0.30 + (10-7) x (3-3.3) x 0.40 + (15-7) x (2-3.3) x 0.30 = -9.6

6. Consider the following distribution of returns:

Probability	R_A	R_B	R_C
30%	-20%	-5%	5%
40%	5%	10%	3%
30%	40%	15%	2%

Which of the following pairwise correlation coefficients are correct? (Hint: Your answers to the previous questions may be useful.)

a. $\rho_{AC} = -0.95$
b. $\rho_{AC} = 0.95$
c. $\rho_{CB} = -0.99$
d. $\rho_{CB} = 0.99$
e. $\rho_{AB} = -0.92$
f. $\rho_{AB} = 0.92$

Answer:

The correct answers are a, c and f.

Recall that the correlation coefficient is scaled covariance. That is, the correlation coefficient between A and C is calculated as:

$$\rho_{AC} = \sigma_{AC}/(\sigma_A \times \sigma_C)$$

We therefore need the covariance between A and C and the individual standard deviations for A and C. In previous questions, you computed these to be:

$\sigma_A = 23.37\%$
$\sigma_C = 1.19\%$
$\sigma_{AC} = -26.4$
The correlation coefficient between A and C is therefore computed as:

$$\rho_{AC} = (-26.40)/(23.37\text{x}1.19) = -0.95$$

The correlation coefficient between C and B is calculated as

$$\rho_{CB} = \sigma_{CB}/(\sigma_C \times \sigma_B)$$

We therefore need the covariance between C and B and the standard deviations for C and B. You have already computed these measures:

$\sigma_B = 8.12\%$
$\sigma_C = 1.19\%$
$\sigma_{BC} = -9.60$

The correlation coefficient between C and B is therefore computed as:

$$\rho_{CB} = (-9.60)/(8.12x1.19) = -0.99$$

The correlation coefficient between A and B is calculated as

$$\rho_{AB} = \sigma_{AB}/(\sigma_A \times \sigma_B)$$

We therefore need the covariance between A and B and the standard deviations for A and B. You already computed these measures:
$\sigma_A = 23.37\%$
$\sigma_B = 8.12\%$
$\sigma_{AB} = 174$
The correlation coefficient between A and B is therefore computed as:

$$\rho_{AB} = 174/(23.37 \text{ x } 8.12) = 0.92$$

7. Consider the following distribution of returns:

Probability	R_A	R_B	R_C
30%	-20%	-5%	5%
40%	5%	10%	3%
30%	40%	15%	2%

Find the expected return of a portfolio that is equally weighted between securities A and C. Round off your final answer to two digits after the decimal point. State your answer as a percentage rate (such as 5.55) (Hint: Your answers to previous questions may be useful.)

<u>Answer:</u>

The answer is $E(r_P) = 5.65\%$.

Recall the expected return on A and C:

$$E(R_A) = 0.3 \times -20\% + 0.4 \times 5\% + 0.3 \times 40\% = 8\%$$

$$E(R_C) = 0.3 \times (5\%) + 0.4 \times 3\% + 0.3 \times 2\% = 3.3\%$$

The expected return of a portfolio is calculated as the weighted average of the individual expected returns:

$$E(r_P) = w_A \times E(R_A) + w_C \times E(R_C) = 0.5 \times 8\% + 0.5 \times 3.3\% = 5.65\%$$

8. Consider the following distribution of returns:

Probability	R_A	R_B	R_C
30%	-20%	-5%	5%
40%	5%	10%	3%
30%	40%	15%	2%

Compute the volatility of a portfolio that is equally invested in A and C. Round off your final answer to two digits after the decimal point (such as 5.55) (Hint: Your answers to previous questions may be useful.)

Answer:

The answer is $\sigma_P = 11.12\%$.

Portfolio volatility is measured by the standard deviation of the portfolio return. The variance of a two-asset portfolio is given by:

$$\sigma_P^2 = w_1^2 \times \sigma_1^2 + w_2^2 \times \sigma_2^2 + 2 \times w_1 \times w_2 \times \sigma_{12}$$

$$\sigma_P^2 = w_1^2 \times \sigma_1^2 + w_2^2 \times \sigma_2^2 + 2 \times w_1 \times w_2 \times \sigma_1 \times \sigma_2 \times \rho_{12}$$

Previously we computed:

$\sigma_A = 23.37\%$
$\sigma_C = 1.19\%$
$\sigma_{AC} = -26.4$

Hence:

$$\sigma_P^2 = (0.5)^2 \times (23.37\%)^2 + (0.5)^2 \times (1.19\%)^2 + 2 \times (0.5) \times (0.5) \times (-26.4) = 123.64 \Rightarrow \sigma_P = 11.12\%$$

9. Consider the following distribution of returns:

Probability	R_A	R_B	R_C
30%	-20%	-5%	5%
40%	5%	10%	3%
30%	40%	15%	2%

Find the expected return of a portfolio with 60% in A and 40% in B. Round off your final answer to two digits after the decimal point. State your answer as a percentage rate (such as 5.55) (Hint: Your answers to previous questions may be useful.)

Answer:

The answer is $E(r_P) = 7.6\%$.

Recall the expected return on A and B:

$$E(R_A) = 0.3 \times -20\% + 0.4 \times 5\% + 0.3 \times 40\% = 8\%$$

$$E(R_B) = 0.3 \times -5\% + 0.4 \times 10\% + 0.3 \times 15\% = 7\%$$

The expected return of a portfolio is calculated as the weighted average of the individual expected returns:

$$E(r_P) = w_A \times E(R_A) + w_B \times E(R_B) = 0.6 \times 8\% + 0.4 \times 7\% = 7.6\%$$

10. Consider the following distribution of returns:

Probability	R_A	R_B	R_C
30%	-20%	-5%	5%
40%	5%	10%	3%
30%	40%	15%	2%

Compute the volatility of a portfolio that is 60% invested in A and 40% invested in B. Round off your final answer to two digits after the decimal point. State your answer as a percentage rate (such as 5.55) (Hint: Your answers to previous questions may be useful.)

<u>Answer:</u>

The answer is $\sigma_P = 17.05$.

Portfolio volatility is measured by the standard deviation of the portfolio return. The variance of a two-asset portfolio is given by:

$$\sigma_P^2 = w_1^2 \times \sigma_1^2 + w_2^2 \times \sigma_2^2 + 2 \times w_1 \times w_2 \times \sigma_{12}$$

$$\sigma_P^2 = w_1^2 \times \sigma_1^2 + w_2^2 \times \sigma_2^2 + 2 \times w_1 \times w_2 \times \sigma_1 \times \sigma_2 \times \rho_{12}$$

Previously we computed:

$\sigma_A = 23.37\%$
$\sigma_B = 8.12\%$
$\sigma_{AB} = 174$

Hence:

$$\sigma_P^2 = (0.6)^2 \times (23.37\%)^2 + (0.4)^2 \times (8.12\%)^2 + 2 \times (0.6) \times (0.4) \times (174) = 290.64 \Rightarrow \sigma_P = 17.05\%$$