A note on finding the Mean Variance Efficient (MVE) Portfolio:
The case with two risky assets and one risk-free asset

This note accompanies the Excel spreadsheet CAL_US_JAPAN_RF_ASSET.xlsx that illustrates the example discussed in the lecture.

Suppose there are two risky assets, the US equity market and the Japanese equity market, in addition to risk-free US Treasuries.

The first tab in the spreadsheet includes the data and the analysis. Let's first start with the data and focus on just the data given on the US, Japan, and the risk-free asset.

	RETURN	DEVIATION
US	0.1355	0.1535
UK	0.1589	0.2430
France	0.1519	0.2324
Germany	0.1435	0.2038
Japan	0.1497	0.2298

Risk-free rate	0.0500

	US	UK	France	Germany	Japan
US	1.0000	0.5003	0.4398	0.3681	0.2663
UK	0.5003	1.0000	0.5420	0.4265	0.3581
France	0.4398	0.5420	1.0000	0.6032	0.3923
Germany	0.3681	0.4265	0.6032	1.0000	0.3663
Japan	0.2663	0.3581	0.3923	0.3663	1.0000

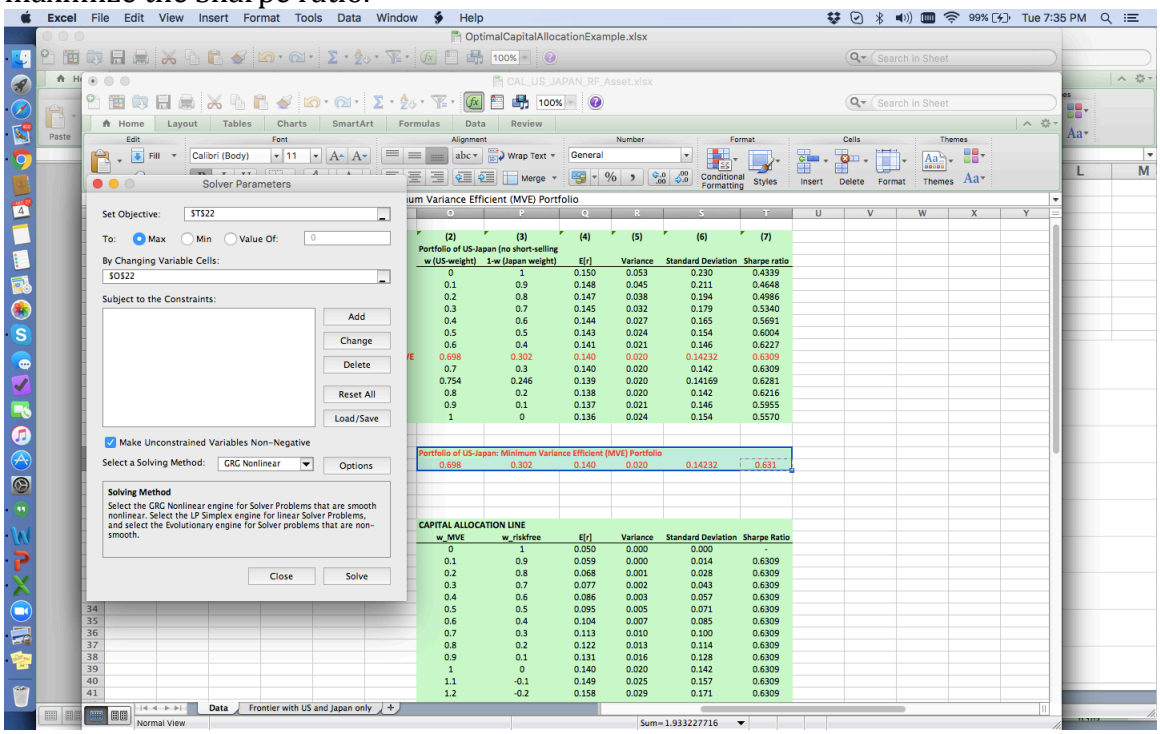
We can plot the mean-variance frontier that can be constructed from US and Japan, as we have seen before:

(1)	(2)	(3)	(4)	(5)	(6)	(7)
	Portfolio of US-Japan (no short-selling					
	w (US-weight)	1-w (Japan weight)	E[r]	Variance	Standard Deviation	Sharp ratio
	0	1	0.150	0.053	0.230	0.433
	0.1	0.9	0.148	0.045	0.211	0.464
	0.2	0.8	0.147	0.038	0.194	0.498
	0.3	0.7	0.145	0.032	0.179	0.534
	0.4	0.6	0.144	0.027	0.165	0.569
	0.5	0.5	0.143	0.024	0.154	0.600

	0.6	0.4	0.141	0.021	0.146	0.622
MVE	0.698	0.302	0.140	0.020	0.14232	0.630
	0.7	0.3	0.140	0.020	0.142	0.630
	0.754	0.246	0.139	0.020	0.14169	0.628
	0.8	0.2	0.138	0.020	0.142	0.621
	0.9	0.1	0.137	0.021	0.146	0.595
	1	0	0.136	0.024	0.154	0.557

Columns 2 and 3 are the portfolio weights for US and Japan, respectively. Columns 4 through 6 show the expected return, variance and volatility for various portfolio combinations. You can of course choose to plot more combinations if you wanted by varying the weights in a more fine fashion.

Finally, column 6 gives the Sharpe ratio. What we are looking for is the mean variance efficient (MVE) portfolio with the maximum Sharpe ratio. How can you find this? Well, we can again use the EXCEL Solver to find the portfolio weights that will maximize the Sharpe ratio.



Once we have found the weights for the MVE, we can draw the Capital Allocation Line by constructing portfolios by combining the MVE portfolio with the risk free asset:

(1)	(2)	(3)	(4)	(5)	(6)
CAPITAL ALLOCATION LINE					
w_MVE	**w_riskfree**	**E[r]**	**Variance**	**Standard Deviation**	**Sharpe Ratio**
0	1	0.050	0.000	0.000	-
0.1	0.9	0.059	0.000	0.014	0.6309
0.2	0.8	0.068	0.001	0.028	0.6309
0.3	0.7	0.077	0.002	0.043	0.6309
0.4	0.6	0.086	0.003	0.057	0.6309
0.5	0.5	0.095	0.005	0.071	0.6309
0.6	0.4	0.104	0.007	0.085	0.6309
0.7	0.3	0.113	0.010	0.100	0.6309
0.8	0.2	0.122	0.013	0.114	0.6309
0.9	0.1	0.131	0.016	0.128	0.6309
1	0	0.140	0.020	0.142	0.6309
1.1	-0.1	0.149	0.025	0.157	0.6309
1.2	-0.2	0.158	0.029	0.171	0.6309
1.3	-0.3	0.167	0.034	0.185	0.6309
1.4	-0.4	0.176	0.040	0.199	0.6309
1.5	-0.5	0.185	0.046	0.213	0.6309
1.6	-0.6	0.194	0.052	0.228	0.6309
1.7	-0.7	0.203	0.059	0.242	0.6309

Column 1 shows the weight of the optimal risky portfolio in the capital allocation decision; column 2 shows the weight of the risk-free asset. Columns 3 through 5 show the expected return, variance and volatility of the different portfolios along the Capital Allocation Line. Notice that regardless what the allocation is they all have the maximum Sharpe ratio because the optimal risky portfolio is the same regardless. Of course, where along the CAL an investor would choose will depend on her risk aversion. But we already know the solution to that problem. The weight in the risky portfolio in the optimal capital allocation problem is given by:

$$w = \frac{E(r_p - r_f)}{A\sigma_p^2}$$

The second tab depicts the mean-variance frontier, the MVE and the Capital Allocation Line constructed based on these data.