

## **MEAN-VARIANCE OPTIMIZATION: OPTIMAL ASSET ALLOCATION**

### **WHAT WILL YOU LEARN?**

- ▶ What is the optimal portfolio of risky assets?
  - ▶ What is the best risky asset portfolio?
  - ▶ Mean-variance efficient (MVE) portfolio
- ▶ What is the optimal asset allocation?
  - ▶ How should we allocate our wealth between this optimal risky portfolio and the risk-free asset?
  - ▶ Capital allocation line (CAL)
  - ▶ Sharpe ratio

## **LET'S START WITH THE SECOND QUESTION**

- ▶ What is the optimal asset allocation?
  - ▶ How should we allocate our wealth between this optimal risky portfolio and the risk-free asset?
  - ▶ Capital allocation line (CAL)
  - ▶ Sharpe ratio

## **PORTFOLIO OF ONE RISKY ASSET AND A RISK-FREE ASSET**

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## **CAPITAL ALLOCATION LINE**

## **OPTIMAL CAL PORTFOLIO**

## **OPTIMAL CAL PORTFOLIO**

## SUMMARY

- ▶ Capital allocation line (CAL) describes the set of portfolio – risk-return trade-offs – that can be constructed with the risky portfolio and a risk-free asset.
- ▶ The slope of the CAL is Sharpe ratio.
- ▶ The optimal asset allocation over one risky asset and one risk-free asset is the tangency point of the highest indifference curve with the CAL.

## **CAPITAL ALLOCATION LINE: A NUMERICAL EXAMPLE**

### **WHAT WILL YOU LEARN?**

- ▶ Let's plug some numbers in with some data.
  - ▶ How should we allocate our wealth between this optimal risky portfolio and the risk-free asset?
  - ▶ Capital allocation line (CAL)
  - ▶ Sharpe ratio

**CAPITAL ALLOCATION LINE**

**CAPITAL ALLOCATION LINE**

## OPTIMAL ALLOCATION DECISION – ANALYTICAL SOLUTION

► Maximize utility

## OPTIMAL ALLOCATION DECISION – NUMERICAL EXAMPLE

Risk aversion (A)	$W^*$	$(1-w^*)$	$E[r_p]$	$\sigma_p$
1.0	2.972	-1.972	33.4%	56.92%
2.0	1.486	-0.486	17.2%	28.46%
3.0	0.991	0.009	11.8%	18.97%
4.0	0.743	0.257	9.10%	14.23%
5.0	0.594	0.406	7.48%	11.38%
6.0	0.495	0.505	6.40%	9.49%
7.0	0.425	0.575	5.63%	8.13%



## **OPTIMAL PORTFOLIOS – LEVERAGE**

### **SUMMARY**

- Solve the optimal asset allocation problem for a given risky portfolio and a risk-free asset