

1. Consider the following three investment options:

| Investment | Expected Return, $E(r)$ | Standard deviation (σ) |
|------------|-------------------------|---------------------------------|
| A | 20% | 30% |
| B | 13% | 15% |
| C | 23% | 45% |

Which investment would you choose if your preferences are represented by mean-variance utility and your risk aversion coefficient is equal to 3?

Answer:

The correct answer is investment B.

We compute the utility of each investment using mean-variance utility function:

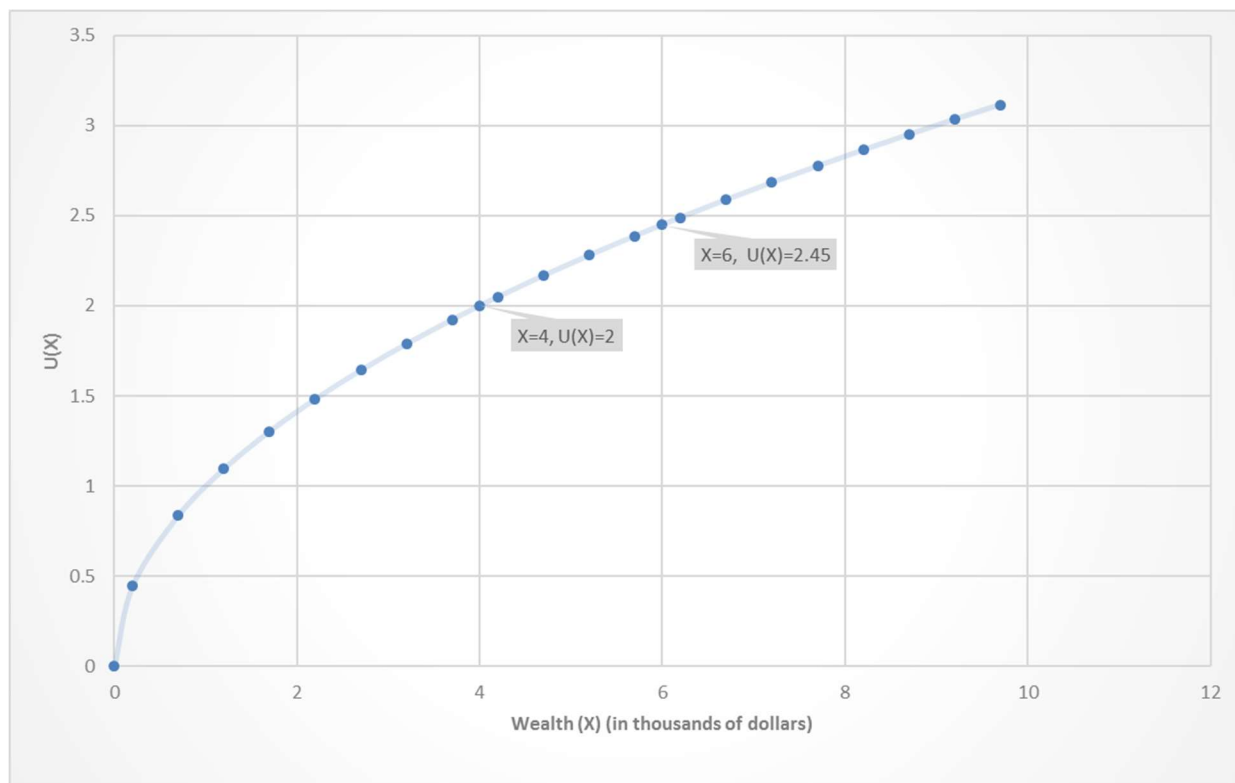
For investment A: $U_A = 0.20 - \frac{1}{2} \cdot 3 \cdot 0.30^2 = 0.07$

For investment B: $U_B = 0.13 - \frac{1}{2} \cdot 3 \cdot 0.15^2 = 0.10$

For investment C: $U_C = 0.23 - \frac{1}{2} \cdot 3 \cdot 0.45^2 = -0.07$

We would select investment B, since it provides us with the highest level of utility.

2. Suppose you have the following utility function. You have an investment opportunity that will give you a future wealth of either \$4,000 or \$6,000 with equal probability. **Which of the following statements is correct?**



- For a certain wealth of \$5,000, the utility you will get will be equal to 2.23.
- For a certain wealth of \$5,000, the utility you will get will be lower than 2.23.
- For a certain wealth of \$5,000, the utility you will get will be higher than 2.23.
- For a certain wealth of \$5,000, the utility you will get will be higher than 2.50.

Answer:

The correct answer is c.

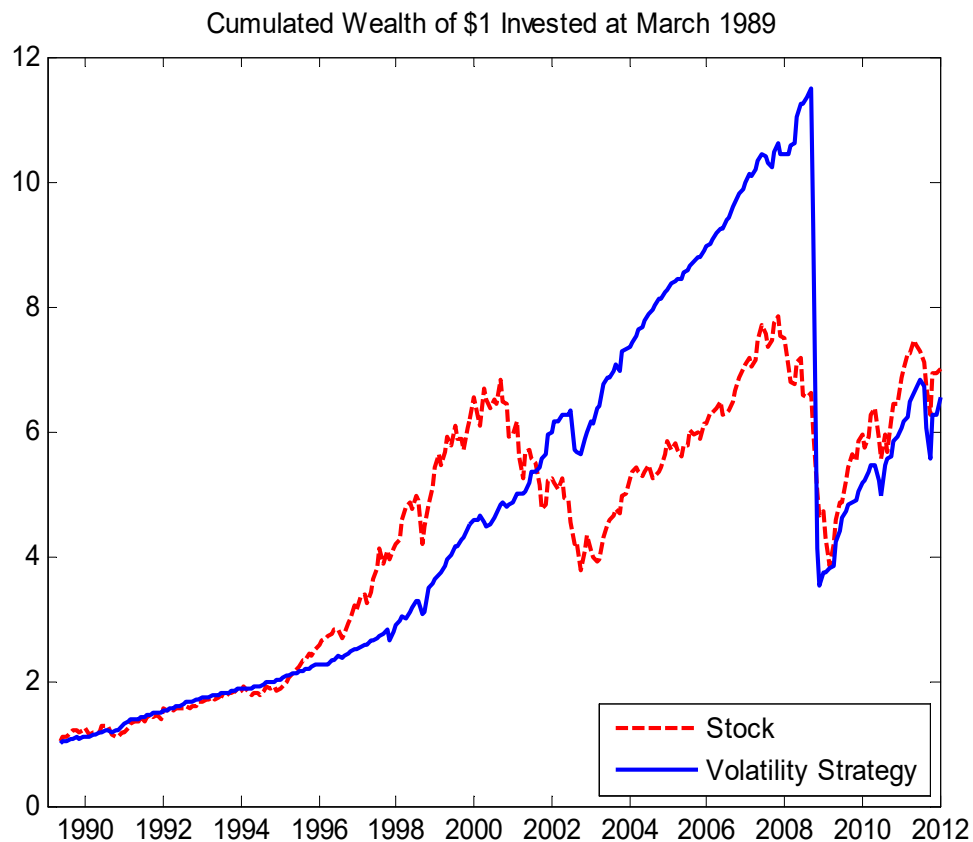
The expected utility from this investment opportunity will be equal to:

$$\frac{1}{2}U(4,000) + \frac{1}{2}U(6,000) = \frac{1}{2} \cdot 2 + \frac{1}{2} \cdot 2.45 = 2.23$$

A sure outcome that is equal to the expected outcome, which is equal to: $\frac{1}{2}4,000 + \frac{1}{2}6,000 = 5,000$, will have a higher utility than 2.23. So answers a and b are false. Moreover, based on the graph, the utility could not be as high as 2.50, so answer d is false.

3. **Compare S&P 500 and volatility strategies (Ang (2014)):** Below is a graph that compares the cumulative wealth obtained from investing \$1 in March 1989 from two

strategies: investing in the S&P 500 equity index and volatility strategy. Volatility strategy is an investment strategy that earns premiums during stables times (essentially equivalent to selling volatility insurance), but has large losses during volatile times such as the financial crisis of 2008-09.



| | Volatility strategy | S&P 500 |
|--------------------|---------------------|---------|
| Mean | 9.9% | 9.7% |
| Standard Deviation | 15.2% | 15.1% |
| Skewness | -8.3% | -0.6% |
| Kurtosis | 104.4 | 4.0 |

Which one of the following is a limitation of mean-variance preferences in comparing these two strategies?

- a. It does not account for tail events
- b. It does not account for risk aversion
- c. It takes into account only expected returns of each strategy
- d. It takes into account only the volatility of each strategy

Answer:

The correct answer is a.

An investor with mean-variance preferences cares only about means and variances, so she would not take into account the skewness and the kurtosis of the respective distributions. So she does not account for tail events.

4. Consider the following data:

| Investment | Expected Return, $E(r)$ | Standard deviation (σ) |
|------------|-------------------------|---------------------------------|
| A | 20% | 30% |
| B | 13% | 15% |
| C | 23% | 45% |

Which investment would you choose if your preferences are represented by mean-variance utility function and you are risk-neutral?

Answer:

The correct answer is investment C.

We compute the utility of each investment, based on the utility formula. In the case of risk neutral investors ($A=0$), because they judge risky prospects solely by their expected rates of return.

$$\text{For investment A: } U_A = 0.20 - \frac{1}{2} \cdot 0 \cdot 0.30^2 = 0.20$$

$$\text{For investment B: } U_B = 0.13 - \frac{1}{2} \cdot 0 \cdot 0.15^2 = 0.13$$

$$\text{For investment C: } U_C = 0.23 - \frac{1}{2} \cdot 0 \cdot 0.45^2 = 0.23$$

We would select investment C, since it provides us with the highest level of utility.

5. Which of the following statements is correct?
- In the mean-variance space, the indifference curve of a less risk-averse investor will be steeper compared to that of a more risk-averse investor.
 - Indifference curves are graphical representations of choices or combinations that give individuals the same level of utility.
 - Moving northwest in mean-variance space, indifference curves are associated with lower levels of utility.
 - The risk aversion of an investor is irrelevant to indifference curves.

Answer:

The correct answer is b.

Indifference curves are combinations of choices that give individuals the same level of utility. In mean-variance space, an investor would be indifferent among portfolios on the same indifference curve because they represent combinations of return and risk that give the same level of utility.

6. Your financial advisor offers you a portfolio of equities with an expected return of 20% and standard deviation of 30%. Suppose at the time of this offer, U.S. Treasuries offer a safe return of 5%. Suppose your preferences can be described by mean-variance preferences, and you know your risk aversion parameter A to be equal to 2. Which would you prefer?

- a. Equities
- b. U.S. Treasuries

Answer:

The correct answer is a.

In choosing between two alternative investment options, an investor will maximize expected utility given her mean-variance preferences.

Let's compute the level of utility associated with each investment. Under mean-variance preferences, utility is given by:

$U = E(r) - \frac{1}{2} A \sigma^2$ where A represents the coefficient of risk aversion.

Given that $A = 2$,

Equities: $U = 20\% - \frac{1}{2} \times 2 \times (30\%)^2 = 11\%$

Treasuries: $U = 5\%$

An investor who is maximizing utility would choose equities over Treasuries.

7. Suppose you have two investment opportunities: A gives you a future wealth of either \$150,000 or \$50,000 with equal probability. B gives you a future wealth of 125,000 with probability 2/3 and 55,000 with probability of 1/3. Suppose, also, that your preferences can be described by a log utility function. In other words, $U(W) = \ln(W)$. If you are maximizing your expected utility, which investment would you choose?

- a. Investment A
- b. Investment B

Answer:

The correct answer is b.

The expected utility associated with investment A is given by:

$$E[U(W)] = \frac{1}{2} \times U(W_1) + \frac{1}{2} \times U(W_2) = \frac{1}{2} \times \ln(150,000) + \frac{1}{2} \times \ln(50,000) = 11.37$$

The expected utility associated with investment B is given by:

$$E[U(W)] = \frac{2}{3} \times U(W_1) + \frac{1}{3} \times U(W_2) = \frac{2}{3} \times \ln(125,000) + \frac{1}{3} \times \ln(55,000) = 11.46$$

So the correct answer is b.

8. Maximizing utility in mean-variance space graphically corresponds to
- a. Choosing the steepest indifference curve you can attain
 - b. Choosing the flattest indifference curve you can attain
 - c. Choosing the highest indifference curve moving northwest
 - d. Choosing the lowest indifference curve moving southeast

Answer:

The correct answer is c.

Maximizing utility in mean-variance space graphically corresponds to getting on the highest possible indifference curve you can attain moving northwest in the mean-variance space.