A Note on Optimal Capital Allocation

This note accompanies the 'OptimalCapitalAllocationExample.xls' that illustrates the numerical example given in the lecture. Please first review the video lecture titled 'Optimal Capital Allocation Example: US Equities and Treasuries" along with the spreadsheet provided for you.

The first tab of the spreadsheet contains the data given to you.

	RETURN	DEVIATION
US Equities	0.1190	0.1915
Risk-free rate	0.0100	

The second tab in the spreadsheet derives the Capital Allocation Line by varying the weights of the U.S. equity market (column 1) and the risk-free asset (column 2) in the portfolio. This Capital Allocation Line plots all the possible portfolio combinations that can be constructed from a risky portfolio (US Equities) and a risk-free asset (US Treasuries). Notice that columns 3, 4 and 5 compute the expected return, the variance and the standard deviation for each different portfolio. The slope of the Capital Allocation Line is given by the Sharpe ratio given in column (6). Of course, the Sharpe ratio is the same for all portfolios that are on the Capital Allocation Line. Notice also that weights of the US equities greater than 1.0 correspond to levered positions where the investor is borrowing at the risk-free rate and investing in the risky asset.

(1)	(2)	(3)	(4)	(5)	(6)
weight_US	**weight_rf**	**E[r]**	**Variance**	**Standard Deviation**	**Sharpe Ratio**
0	1	0.010	0.0000	0.0000	-
0.1	0.9	0.021	0.0004	0.0192	0.5692
0.2	0.8	0.032	0.0015	0.0383	0.5692
0.3	0.7	0.043	0.0033	0.0575	0.5692
0.4	0.6	0.054	0.0059	0.0766	0.5692
0.5	0.5	0.065	0.0092	0.0958	0.5692
0.6	0.4	0.075	0.0132	0.1149	0.5692
0.7	0.3	0.086	0.0180	0.1341	0.5692
0.8	0.2	0.097	0.0235	0.1532	0.5692
0.9	0.1	0.108	0.0297	0.1724	0.5692
1	0	0.119	0.0367	0.1915	0.5692
1.1	-0.1	0.130	0.0444	0.2107	0.5692
1.2	-0.2	0.141	0.0528	0.2298	0.5692
1.3	-0.3	0.152	0.0620	0.2490	0.5692
1.4	-0.4	0.163	0.0719	0.2681	0.5692
1.5	-0.5	0.174	0.0825	0.2873	0.5692
1.6	-0.6	0.184	0.0939	0.3064	0.5692

1.7	-0.7	0.195	0.1060	0.3256	0.5692

The third tab in the spreadsheet is a graphical representation of the Capital Allocation Line given the data.

The fourth tab in the spreadsheet adds to the spreadsheet the solution to the optimal capital allocation problem for different risk aversion coefficients. Column 1 is the risk aversion coefficient. Column 2 solves for the optimal solution for the weight in the risky asset as given in the lecture. For example, columns 2 and 3 indicate that, based on the given assumptions of expected return and volatility for the US equities and Treasuries, the optimal capital allocation for an investor with a risk aversion coefficient of 3 is to invest almost 100% of her wealth in the risky asset, that is, the US equities in this example. Finally, columns 4 and 5 then compute the expected portfolio return and standard deviation for each optimal allocation for different levels of risk aversion.

(1)	(2)	(3)	(4)	(5)
A	w* (US)	1-w*	E[r]	Standard Deviation
1	2.972	-1.972	45.30%	56.92%
2	1.486	-0.486	18.20%	28.46%
3	0.991	0.009	11.80%	18.97%
4	0.743	0.257	9.10%	14.23%
5	0.594	0.406	7.48%	11.38%
6	0.495	0.505	6.40%	9.49%
7	0.425	0.575	5.63%	8.13%
8	0.372	0.628	5.05%	7.11%
9	0.330	0.670	4.60%	6.32%
10	0.297	0.703	4.24%	5.69%

Finally, the last tab in the spreadsheet depicts where the optimal allocations for different investors of different risk aversion lie on the CAL.