UNIVERSITY OF IBADAN

B.Ed and B.Sc Degree Examinations

First Semester Examination 2020/2021 Session

Mathematics Department.

MAT 251: Numerical Methods and Computation

Carefully write legible and correct answers to **Three** questions

Thursday: July 13, 2021 2hrs

1. (a) Outline the types of approximation errors that are common in Mathematical calculations.

   (b) Suppose y_1 and y_2 are two numbers, Y_1 and Y_2 their respective approximations. If ϵ_1 and ϵ_2 are the resective approximate errors. Determine the

   (i) error in sum.

   (ii) error in difference.

   (iii) error in the product.

   (c) Determine in which of the intervals $[-4,-3]$, $[4,5]$, $[0,1]$, $[2,3]$ does the cubic equation $x^3 - 10x + 4 = 0$ has roots.

2. (a) Given that Δ is the forward difference operator, ∇ is the backward difference operator and δ is the central difference operator. By first defining each of the operators with respect to a function f_k, show derivation formulas for

   (i) $\Delta^4 f_k$.

   (ii) $\nabla^4 f_k$.

   (iii) $\delta^3 f_k$.

   (b) (i) From 2(a)(iii) above, approximate the derivative of $f(t) = t^2+2t$ at $t = -3$

   (ii) Write out suitable iteration formulas for the equation $f(x) = sinx + x - 1$

3. (a) By first defining the step up operator E, prove that

   (i) $\Delta = E - 1$

   (ii) $E^2 = (\Delta + 1)^2$

   (iii) $\delta = E^{\frac{1}{2}}\nabla = E^{-\frac{1}{2}}\Delta$

   (iv) $\delta^2 = \nabla\Delta$

   v) $DE = ED$

   (b) Use the Taylor's series to obtain a solution of $y' = -xy$ correct to four decimal places for values $x_0 = 0.0$, $x_1 = 0.1$, $x_2 = 0.2$, $x_3 = 0.3$, $x_4 = 0.4$, $x_5 = 0.5$, with initial conditions $y(0) = 1$. .

4. (a) Given the following system of equations :

   $2a + 3b + c = 9$

   $a + 2b + 3c = 6$

   $3a + b + 2c = 8$

   (i) Express the system of equations above in the form $AX = B$.

   (ii) Using 4(a)(i) above solve for X.

   (iii) Determine eigenvalues and eigenvectors for A in 4(a)(i) above (if any)

(b) (i) Distinguish between the Taylors' series and the Runge-Kutta methods for sloving ODE's (in just a statement).

(ii) Hence state formulas for

(a) Runge-Kutta method of order2

(b) Runge-Kutta method of order3

(c) Runge-Kutta method of order 4.

What necessitates the higher order methods?

$A\cancel{X} = \lambda X$

$(A - \lambda I)X = 0$

$X = A^{-1}\lambda X$

$\lambda/^{1}\Big(\begin{pmatrix} 4 & 6 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 4 \end{pmatrix}$

Best of luck

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