# 01204211 Discrete Mathematics Lecture 15: Binomial Coefficients (2)

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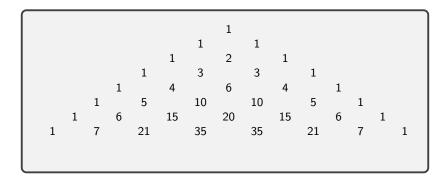
### The binomial coefficients<sup>1</sup>

There is a reason why the term  $\binom{n}{k}$  is called the binomial coefficients. In this lecture, we will discuss

▶ identities on binomial coefficients.

<sup>&</sup>lt;sup>1</sup>This lecture mostly follows Chapter 3 of [LPV].

# Identities in the Triangle



#### Odd and even subsets

Let's try to prove this identity with the Pascal's triangle

$$\binom{n}{0} - \binom{n}{1} + \binom{n}{2} + \dots + (-1)^n \binom{n}{n} = 0.$$

## A more formal proof

$$\binom{n}{0} - \binom{n}{1} + \binom{n}{2} + \dots + (-1)^n \binom{n}{n} = 0.$$

### The next experiment

Let's try to compute the sum of squares of numbers in each row.

$$1^{2} = 1$$

$$1^{2} + 1^{2} = 2$$

$$1^{2} + 2^{2} + 1^{2} = 6$$

$$1^{2} + 3^{2} + 3^{2} + 1^{2} = 20$$

$$1^{2} + 4^{2} + 6^{2} + 4^{2} + 1^{2} = 70$$

#### Theorem:

$$\binom{n}{0}^2 + \binom{n}{1}^2 + \binom{n}{2}^2 + \dots + \binom{n}{n}^2 = \binom{2n}{n}.$$

# Another identity

This suggests

$$\binom{n}{0} + \binom{n+1}{1} + \binom{n+2}{2} + \dots + \binom{n+k}{k} = \binom{n+k+1}{k}.$$

#### Theorem:

$$\binom{n}{0} + \binom{n+1}{1} + \binom{n+2}{2} + \dots + \binom{n+k}{k} = \binom{n+k+1}{k}.$$