1. Definition of a Set: In mathematics, a set is a well-defined collection of distinct objects, considered as an entity on its own. The objects within a set are called its elements or members. The concept of a set is fundamental to various branches of mathematics and is denoted using curly braces **{}**. For example, if we have a set of numbers, it could be written as **{1, 2, 3, 4, 5}**.

Example 1: Set of Natural Numbers Let's consider the set of natural numbers less than 10. We represent it as:

A = {1, 2, 3, 4, 5, 6, 7, 8, 9}

Here, A is the set, and its elements are the numbers from 1 to 9 inclusive.

Example 2: Set of Even Numbers Now, let's look at a set containing even numbers:

B = {2, 4, 6, 8, 10, 12}

In this example, B is the set, and its elements are even numbers less than or equal to 12.

Example 3: Set of Vowels Sets can also contain non-numeric elements. Let's consider a set of vowels: C = {a, e, i, o, u }

Here, C is the set, and its elements are the five vowels in the English alphabet.

Example 4: Set of Colors Sets can have elements of various types. For instance, consider a set of colors:

D = {"red", "blue", "green", "yellow"}

In this case, D is the set, and its elements are color names.

Example 5: Set of Prime Numbers Let's look at a set containing prime numbers less than 20:

E = {2, 3, 5, 7, 11, 13, 17, 19}

In this example, E is the set, and its elements are the prime numbers up to 19.

Example 6: Set of Countries Sets can also include more complex elements, such as country names:

F = {"USA", "Canada", "India", "Australia", "Germany"}

Here, F is the set, and its elements are the names of different countries.

Kindly note that in a set, each element is unique, and the order of elements doesn't matter. For example, the set **{1, 2, 3}** is the same as the set **{3, 2, 1}**. If an element appears more than once, it is still considered only once in the set. For instance, the set of even numbers less than 10 can be written as **{2, 4, 6, 8, 2}** or any other combination, but it is still considered as **{2, 4, 6, 8}** since duplicate elements are merged into one.

1.1 Sets Representation:

Sets can be represented in two forms:

a) Roster or tabular form: This form lists all the elements of the set within braces { } and separates them by commas.

For example, if A is the set of all odd numbers less than 10, then in roster form, it can be expressed as $A = \{1, 3, 5, 7, 9\}$.

b) Set Builder form: In this representation, we list the properties fulfilled by all the elements of the set. It is noted as {x: x satisfies properties P} and read as 'the set of all x such that each x has properties P.'

For example, if $B = \{2, 4, 8, 16, 32\}$, then the set builder representation will be: $B = \{x: x = 2n, \text{ where } n \in N \text{ and } 1 \leq n \leq 5\}$

1.2 Venn Diagram:

A Venn diagram is a graphical representation of the relationships between different sets or groups of items. It uses circles or ellipses to depict these sets, with each circle representing a unique group, and the overlapping areas showing the intersections between the groups. John Venn, a renowned logician, introduced the concept of diagrams in 1918, which led to them being named after him.

Venn diagrams offer various advantages:

1. They facilitate the classification of data within the same category but with different sub-categories.
2. Venn diagrams make it simpler to compare different data sets and understand the relationships between them.
3. They aid in grouping information and identifying similarities and differences among the data.
4. Venn diagrams are useful for comprehending and determining various unknown parameters

Example of Venn diagram: Venn diagrams are of great utility when tackling set-related and other problems. They serve as valuable tools for visually representing data. Let's gain a deeper understanding of Venn diagrams through an illustrative example.

Example: Consider set A, which represents even numbers up to 10, and set B, which represents natural numbers less than 5. We can visualize their intersection using a Venn diagram.

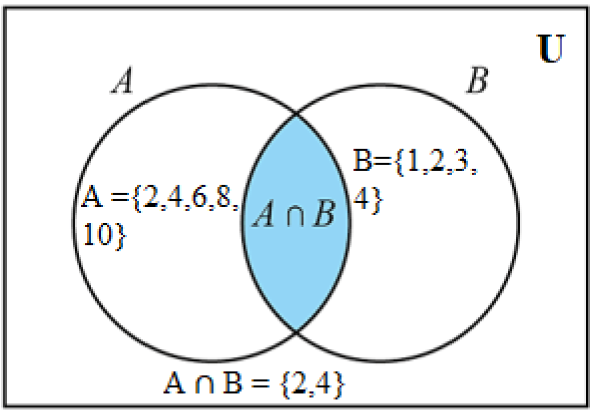


Figure: A ∩ B

1.2 Notation in set:

In the following section we will learn about various notations in set theory presented in tabular format:

Notation	**Explanation**	**Example**
Braces { }	Used to enclose the elements of a set.	Set A: {1, 2, 3}
Vertical Bar |	Used in set-builder notation to define a set's elements.	Set B: {x | x is a positive integer}
Comma ,	Separates elements within a set.	Set C: {a, b, c}
Ellipsis ...	Represents a sequence of elements.	Set D: {1, 2, 3, ..., 10}
Union ∪	Represents the union of sets.	Set E: A ∪ B
Intersection ∩	Represents the intersection of sets.	Set F: A ∩ B

Notation	Explanation	Example
Subset ⊆	Represents a subset relationship.	Set G: A ⊆ B
Proper Subset ⊂	Represents a proper subset relationship.	Set H: A ⊂ B
Complement '	Represents the complement of a set.	Set I: A' or ¬A
Empty Set ∅	Represents the set with no elements.	Empty Set: ∅

Understanding these notations is essential for working with sets and solving problems related to set theory and mathematics. They provide a standardized way to represent sets, set operations, and set relationships, making it easier to communicate and work with sets in various mathematical contexts.

1.3 Types of Set: In the following section we will learn about various types of sets with examples.

1. Finite and Infinite Sets:

- **Finite Sets:** A set with a specific, definite number of elements is called a finite set. Example: Set A = {1, 2, 3} is a finite set as it has three elements.
- **Infinite Sets**: A set with an uncountable number of elements is called an infinite set. Example: The set of natural numbers N = {1, 2, 3, ...} is an infinite set as it continues indefinitely.

2. Empty Set (Null Set):

- The empty set, denoted by ∅ or {}, has no elements. Example: The set B = {} is an empty set.
- An empty set, also known as the null set, is a unique set that contains no elements. In set theory, it is denoted by the symbol ∅ or {}.
- **Characteristics of the Empty Set:**
- **It has no elements**: The defining characteristic of an empty set is that it contains nothing, and therefore, it is empty.

- **It is a subset of any set**: Since the empty set contains no elements, it is considered a subset of any other set. This property is known as the Empty Set Property.
- **Example**: Let's consider a few examples to understand the concept of the empty set:
- **Example 1**: Set A = {x | x is a positive integer greater than 10 and less than 5}
- **Explanation**: In this set builder notation, we are defining set A as a set of positive integers that satisfy the condition of being greater than 10 and less than 5. However, there are no integers that satisfy this condition, as there are no numbers greater than 10 and less than 5 at the same time. Therefore, the set A does not have any elements, and it is an empty set.
- **Example 2**: Set B = {x | x is a prime number and x is greater than 20} Explanation: In this set builder notation, we are defining set B as a set of prime numbers that are greater than 20. However, there are no prime numbers greater than 20. The closest prime number to 20 is 23, but it is not greater than 20. Therefore, the set B does not have any elements, and it is an empty set.
- **Example 3**: Set C = { } Explanation: In this example, the set C is explicitly defined as an empty set with no elements. This is denoted by the curly braces with nothing inside: { }.
- In short, an empty set is a set with no elements, and it is denoted by ∅ or {}. It is a unique set with distinct properties, such as being a subset of any other set. The empty set is a fundamental concept in set theory and is used in various mathematical contexts.

3. Singleton Set:

- A singleton set is a set that contains only one element. Example: Set C = {5} is a singleton set.

4. Equal Sets:

- Two sets are equal if they have exactly the same elements. Example: Set D = {1, 2, 3} and Set E = {3, 2, 1} are equal sets.

5. Subset and Superset:

- **Subset**: A set A is a subset of set B (denoted as A ⊆ B) if all the elements of A are also elements of B. Example: If Set F = {1, 2} and Set G = {1, 2, 3}, then F is a subset of G (F ⊆ G).
- **Proper Subset**: A set A is a proper subset of set B (denoted as A ⊂ B) if A is a subset of B, but A is not equal to B. Example: If Set H = {1, 2} and Set I = {1, 2, 3}, then H is a proper subset of I (H ⊂ I).
- **Superset**: If A is a subset of B, then B is called a superset of A (denoted as B ⊇ A). Example: If Set J = {1, 2, 3} and Set K = {1, 2}, then J is a superset of K (J ⊇ K).

6. Universal Set:

- In set theory, a universal set, also known as the universe, is a set that contains all the elements under consideration for a given discussion or problem. It is denoted by the symbol U.
- **Characteristics of the Universal Set:**
  - **All-inclusive**: The universal set includes all the elements that are relevant to a particular context or problem.
  - **Parent Set**: In many cases, all other sets under consideration are subsets of the universal set.
  - **Example**: Let's consider a simple example to understand the concept of the universal set:
  - **Example 1**: Consider the universal set U as the set of all letters in the English alphabet.

    U = {a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z}

    Now, we can have subsets of this universal set that represent specific categories. For instance:

1. Set A: The set of vowels in the English alphabet

   A = {a, e, i, o, u}

2. Set B: The set of consonants in the English alphabet.

   B = {b, c, d, f, g, h, j, k, l, m, n, p, q, r, s, t, v, w, x, y, z}

3. Set C: The set of letters in the first half of the English alphabet.

   C = {a, b, c, d, e, f, g, h, i, j, k, l, m}

4. Set D: The set of letters in the second half of the English alphabet.

   D = {n, o, p, q, r, s, t, u, v, w, x, y, z}

In this example, the universal set U contains all the letters in the English alphabet, and the other sets (A, B, C, and D) are subsets of this universal set. The elements in each subset are taken from the universal set based on specific criteria.

Remember, the universal set is context-specific and can change depending on the problem or topic being discussed. It helps us define and categorize elements in relation to a broader context or universe of discourse.

7. Disjoint Sets:

- Two sets are disjoint if they have no common elements. Example: Set L = {1, 2, 3} and Set M = {4, 5, 6} are disjoint sets as they have no elements in common.

8. Power Set:

- The power set of a set A, denoted as P(A), is the set that contains all possible subsets of A, including the empty set and A itself. In other words, the power set of a set is the collection of all the subsets that can be formed by selecting zero or more elements from the original set A.
- **Example:** Let's consider a set A = {1, 2}. The power set P(A) will be: {{∅}, {1}, {2}, {1, 2}}. It contains all the subsets of A, which are the empty set, the set containing only {1}, the set containing only {2}, and the set containing both {1, 2}.
- What is the power set of the set {∅}?
  - **Answer**: The power set of the empty set contains only one subset, which is the empty set itself. Therefore, P (∅) = {∅}.
  - The power set of the set {∅} contains two subsets, which are the empty set (∅) and the set {∅} itself. Hence, P ({∅}) = {∅, {∅}}.
- **Properties of Power Set:**

  A. **Cardinality:** If the set A has 'n' elements, then the cardinality of its power set P(A) is 2^n. In other words, if A has 'n' elements, the power set will have 2^n subsets.

For Example: If A = {1, 2, 3}, the power set P(A) will have $2^3 = 8$ subsets.

Power Set P(A) = { {∅}, {1}, {2}, {3}, {1, 2}, {1, 3}, {2, 3}, {1, 2, 3} }

B. **Empty Set and Universal Set:** The power set always includes the empty set (∅) and the original set itself.

Example: For any set A, P(A) will include both {} and A.

C. **Subset Relation:** The power set contains all possible subsets of the original set, so any subset of A is an element of P(A).

Example: If A = {a, b, c}, and B = {a, b}, then B is an element of P(A).

D. **Size Comparison:** The power set is larger than the original set. If A has 'n' elements, P(A) has 2^n elements.

Example: If A = {1, 2}, then A has 2 elements, and P(A) has 2^2 = 4 elements.

E. **Uniqueness:** Each subset in the power set is unique; no two subsets are identical.

Example: In the power set P({1, 2}), {1} and {2} are distinct subsets, even though they have the same elements.

F. **Order of Elements:** The elements within each subset in the power set are listed in the same order as in the original set.

Example: For the set A = {x, y}, the power set P(A) will have subsets like {x, y} and {y, x}.

The power set is a fundamental concept in set theory and has applications in various areas of mathematics, computer science, and discrete mathematics. It allows us to explore all the possible combinations of elements in a set and plays a crucial role in solving problems related to sets and combinatorics.

9. Equal Cardinality Sets or Equivalent Set:

- Two sets have equal cardinality if they contain the same number of elements. Example: Set P = {a, b, c} and Set Q = {1, 2, 3} have equal cardinality as both sets contain three elements.

10. Complement of a Set:

- The complement of a set A with respect to the universal set U is the set of all elements in U that are not in A, denoted as A'. Example: If the universal set U = {1, 2, 3, 4, 5} and Set R = {2, 4}, then the complement of R is R' = {1, 3, 5}.

1.3 Cardinality of Set or cardinal number of set

The cardinality of a set is the number of elements it contains. It gives us the measure of the set's size or how many distinct elements are in the set.

Let's look at three examples to understand cardinality:

Example 1: Set A = {1, 2, 3, 4, 5} Cardinality of Set A = 5

Explanation: Set A has five elements, so its cardinality is 5.

Example 2: Set B = {apple, banana, orange} Cardinality of Set B = 3

Explanation: Set B has three elements (apple, banana, and orange), so its cardinality is 3.

Example 3: Set C = {x | x is a positive even number less than 10} Cardinality of Set C = 4
Explanation: Set C contains four positive even numbers less than 10, which are {2, 4, 6, 8}, so its cardinality is 4.

In short, the cardinality of a set represents the count of unique elements in the set, regardless of their individual values. It provides a simple way to quantify the size of a set.

1.4 Set Operation:

Set operations are fundamental operations used in set theory to manipulate and analyze sets. Let's explore the common set operations with examples:

Consider two sets: Set A = {1, 2, 3} and Set B = {3, 4, 5}.

1. Union (∪): The union of two sets A and B, denoted as A ∪ B, is the set containing all the elements that are in either A or B (or in both).

Example: A ∪ B = {1, 2, 3, 4, 5} (It includes all elements from both sets, without repetitions).

2. Intersection (∩): The intersection of two sets A and B, denoted as A ∩ B, is the set containing all the elements that are common to both A and B.

Example: A ∩ B = {3} (The element 3 is the only element present in both sets A and B).

3. Difference (-): The difference between two sets A and B, denoted as A - B or A \ B, is the set containing all the elements that are in A but not in B.

Example: A - B = {1, 2} (The elements 1 and 2 are in Set A but not in Set B).

- $A - B = \{x \mid x \in A \text{ and } x \notin B\}$

4. Complement ('): The complement of a set A with respect to a universal set U, denoted as A', is the set containing all the elements in U that are not in A.

Example: Universal Set U = {1, 2, 3, 4, 5}. A' = {4, 5} (The elements 4 and 5 are in U but not in A).

5. Symmetric Difference (∆):

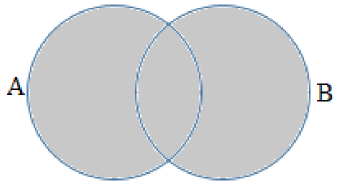
- The symmetric difference of two sets A and B, denoted as A ∆ B, is the set containing all the elements that are in either A or B but not in both.
- $A \Delta B = (A - B) \cup (B - A)$
- Example: A ∆ B = {1, 2, 4, 5} (It includes elements from both sets except the common element 3).

6. Cartesian Product (×): The Cartesian product of two sets A and B, denoted as A × B, is the set containing all possible ordered pairs (a, b) where a is in A and b is in B.

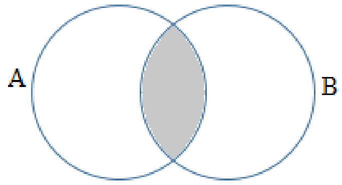
Example: A × B = {(1, 3), (1, 4), (1, 5), (2, 3), (2, 4), (2, 5), (3, 3), (3, 4), (3, 5)}.

These set operations allow us to analyze and manipulate sets to solve various mathematical problems and applications. Understanding these operations is essential for working with sets and set theory. Note that the Cartesian products A × B and B × A are not equal unless A = ∅ or B = ∅ (so that A × B = ∅) or A = B

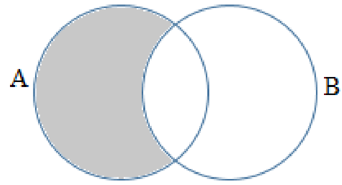
Venn diagram for Sets Operations



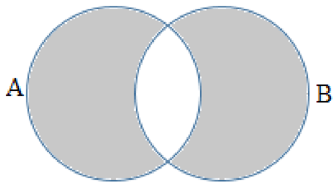
$A \cup B$



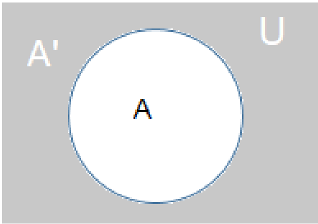
$A \cap B$

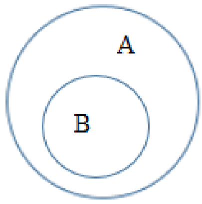


$A - B$

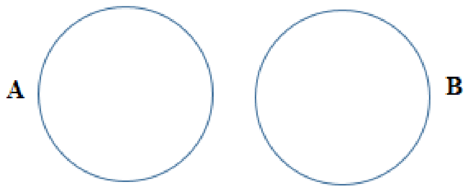


$A \Delta B$





$B \subseteq A$ / $A \supseteq B$



A Disjoint B

1.5 Properties of sets:

Set identities are mathematical equations or statements that hold true for any sets involved, regardless of their specific elements. These identities are analogous to algebraic identities in regular mathematics and are used to simplify and manipulate expressions involving sets. Understanding set identities is essential for working with sets efficiently. Here are some important set identities:

Serial No.	Property	Union	Intersection
1.	Commutative	$A \cup B = B \cup A$	$A \cap B = B \cap A$
2.	Associative	$(A \cup B) \cup C = A \cup (B \cup C)$	$(A \cap B) \cap C = A \cap (B \cap C)$
3.	Distributive	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$	$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Serial No.	Property	Union	Intersection
4.	Identity	A ∪ ∅ = A	A ∩ U = A
5.	Complement	A ∪ A' = U	A ∩ A' = ∅
6.	Absorption	A ∪ (A ∩ B) = A	A ∩ (A ∪ B) = A
7.	Set Difference	A - B = A ∩ B'	A - A = ∅
8.	De Morgan's Laws	(A ∪ B)' = A' ∩ B'	(A ∩ B)' = A' ∪ B'
9.	Identity Element	A ∪ ∅ = A	A ∩ U = A
10.	Empty Set	A ∪ ∅ = A	A ∩ ∅ = ∅
11.	Domination Law	A ∪ U = U	A ∩ ∅ = ∅
12.	Idempotent Laws	A ∪ A = A	A ∩ A = A
13.	Double Complementation Law	(A')' = A	(A')' = A

1.6 Numerical Examples

Question 1. If A= {1, 2, 3, 4}, B = {3, 4, 5, 6}, C = {5, 6, 7, 8}, D = {7, 8, 9, 10}, find A ∪ B, A ∪ B ∪ C, B ∪ C ∪ D and explain with its Venn diagram.

Solution:

To find the union of sets, you simply combine all the elements from the sets without repeating any element. Here are the unions asked for:

1. A ∪ B: A = {1, 2, 3, 4} B = {3, 4, 5, 6}

The union of A and B, denoted as A ∪ B, is {1, 2, 3, 4, 5, 6}.

2. A ∪ B ∪ C: A = {1, 2, 3, 4} B = {3, 4, 5, 6} C = {5, 6, 7, 8}

The union of A, B, and C, denoted as A ∪ B ∪ C, is {1, 2, 3, 4, 5, 6, 7, 8}.

3. B ∪ C ∪ D: B = {3, 4, 5, 6} C = {5, 6, 7, 8} D = {7, 8, 9, 10}

The union of B, C, and D, denoted as B ∪ C ∪ D, is {3, 4, 5, 6, 7, 8, 9, 10}.

So, the results are:

1. A ∪ B = {1, 2, 3, 4, 5, 6}
2. A ∪ B ∪ C = {1, 2, 3, 4, 5, 6, 7, 8}
3. B ∪ C ∪ D = {3, 4, 5, 6, 7, 8, 9, 10}

Question 2: In class of 200 students who appeared certain examinations 35 students failed in CET, 40 in NEET and 40 in JEE, 20 failed in CET and NEET, 17 failed in NEET and JEE, 15 in CET and JEE and 5 failed in all three examination find how many students.

1. Did not fail in any examination

2. Failed in NEET or JEE entrance

Solution:

1. **Did not fail in any examination.**

- Start with the total number of students: There are 200 students in the class.
- Calculate the number of students who failed in at least one examination:
    1. Add the number of students who failed in CET, NEET, and JEE: 35 + 40 + 40 = 115
    2. However, we counted some students twice because they failed in more than one exam. So, we need to subtract those cases:
        - Subtract the number of students who failed in both CET and NEET: 20
        - Subtract the number of students who failed in both NEET and JEE: 17
        - Subtract the number of students who failed in both CET and JEE: 15
    3. But, in the process, we subtracted the students who failed in all three exams twice, so we need to add them back:

        - Add the number of students who failed in all three exams: 5

    4. Putting it all together: 115 - 20 - 17 - 15 + 5 = 68 students failed in at least one examination.

- Calculate the number of students who did not fail in any examination:

    1. Simply subtract the number of students who failed in at least one exam from the total number of students:

    2. 200 - 68 = 132 students did not fail in any examination.

So, the final answer is that 132 students did not fail in any examination.

2. **Failed in NEET or JEE**

To find the number of students who failed in NEET or JEE entrance, we can use the principle of inclusion-exclusion.

Let's denote the following:

- F(CET): Number of students who failed in CET
- F(NEET): Number of students who failed in NEET
- F(JEE): Number of students who failed in JEE

Given:

- F(CET) = 35
- F(NEET) = 40
- F(JEE) = 40
- F(CET ∩ NEET) = 20 (failed in both CET and NEET)
- F(NEET ∩ JEE) = 17 (failed in both NEET and JEE)
- F(CET ∩ JEE) = 15 (failed in both CET and JEE)
- F(CET ∩ NEET ∩ JEE) = 5 (failed in all three examinations)

We need to find the total number of students who failed in NEET or JEE (F(NEET ∪ JEE)).

We can use the formula for the union of two sets: F(NEET ∪ JEE) = F(NEET) + F(JEE) - F(NEET ∩ JEE)

F(NEET ∪ JEE) = 40 + 40 - 17 = 63

So, 63 students failed in either NEET or JEE entrance exams

Question 3: Let A={1,2,3,4}, B={4,5,6}, C={5,6} verify that:

i) **A × (B ∩ C) = (A × B)** ∩ (**A × C**)

ii) **A × (B ∪ C) = (A × B)** ∪ (**A × C**)

Solution:

i) To verify the given equation i.e., A × (B ∩ C) = (A × B) ∩ (A × C), we need to show that both sets contain the same elements.

Let's go step-by-step:

Step 1: Find A × (B ∩ C)

A = {1, 2, 3, 4}

B = {4, 5, 6}

C = {5, 6}

First, find B ∩ C, which is the intersection of sets B and C:

B ∩ C = {5, 6}

Now, we need to take the Cartesian product of A with (B ∩ C). This means we combine each element from set A with each element from the intersection of sets B and C:

A × (B ∩ **C)** = {(1, 5), (1, 6), (2, 5), (2, 6), (3, 5), (3, 6), (4, 5), (4, 6)}

Step 2: Find (A × B) ∩ (A × C)

Now, find the Cartesian products A × B and A × C:

A × B = {(1, 4), (1, 5), (1, 6), (2, 4), (2, 5), (2, 6), (3, 4), (3, 5), (3, 6), (4, 4), (4, 5), (4, 6)}

A × C = {(1, 5), (1, 6), (2, 5), (2, 6), (3, 5), (3, 6), (4, 5), (4, 6)}

Now, find the intersection of sets A × B and A × C:

(A × B) ∩ (A × C) = {(1, 5), (1, 6), (2, 5), (2, 6), (3, 5), (3, 6), (4, 5), (4, 6)}

Step 3: Compare the two sets

Now, let's compare the sets **A × (B ∩ C)** and **(A × B)** ∩ **(A × C):**

A × (B ∩ C) = {(1, 5), (1, 6), (2, 5), (2, 6), (3, 5), (3, 6), (4, 5), (4, 6)}

(A × B) ∩ (A × C) = {(1, 5), (1, 6), (2, 5), (2, 6), (3, 5), (3, 6), (4, 5), (4, 6)}

Since both sets contain the exact same elements, we can conclude that A × (B ∩ C) = (A × B) ∩ (A × C). Thus, the equation is verified.

ii) . A × (B ∪ C) = (A × B) ∪ (A × C)

To verify the given equation, i.e., A × (B ∪ C) = (A × B) ∪ (A × C), we need to show that both sets contain the same elements.

Let's go step-by-step:

Step 1: Find A × (B ∪ C)

A = {1, 2, 3, 4}

B = {4, 5, 6}

C = {5, 6}

First, find B ∪ C, which is the union of sets B and C:

B ∪ C = {4, 5, 6}

Now, we need to take the Cartesian product of A with (B ∪ C). This means we combine each element from set A with each element from the union of sets B and C:

A × (B ∪ C) = {(1, 4), (1, 5), (1, 6), (2, 4), (2, 5), (2, 6), (3, 4), (3, 5), (3, 6), (4, 4), (4, 5), (4, 6)}

Step 2: Find (A × B) ∪ (A × C)

Now, find the Cartesian products A × B and A × C:

A × B = {(1, 4), (1, 5), (1, 6), (2, 4), (2, 5), (2, 6), (3, 4), (3, 5), (3, 6), (4, 4), (4, 5), (4, 6)}

$A \times C = \{(1, 5), (1, 6), (2, 5), (2, 6), (3, 5), (3, 6), (4, 5), (4, 6)\}$

Now, find the union of sets $A \times B$ and $A \times C$:

$(A \times B) \cup (A \times C) = \{(1, 4), (1, 5), (1, 6), (2, 4), (2, 5), (2, 6), (3, 4), (3, 5), (3, 6), (4, 4), (4, 5), (4, 6)\}$.

Step 3: Compare the two sets

Now, let's compare the sets $A \times (B \cup C)$ and $(A \times B) \cup (A \times C)$:

$A \times (B \cup C) = \{(1, 4), (1, 5), (1, 6), (2, 4), (2, 5), (2, 6), (3, 4), (3, 5), (3, 6), (4, 4), (4, 5), (4, 6)\}$

$(A \times B) \cup (A \times C) = \{(1, 4), (1, 5), (1, 6), (2, 4), (2, 5), (2, 6), (3, 4), (3, 5), (3, 6), (4, 4), (4, 5), (4, 6)\}$

Since both sets contain the exact same elements, we can conclude that $A \times (B \cup C) = (A \times B) \cup (A \times C)$. Thus, the equation is verified.

Question 4: What is the Cartesian product $A \times B \times C$, where $A = \{0, 1\}$, $B = \{1, 2\}$, and $C = \{0, 1, 2\}$?

Solution:

Let’s find the Cartesian product $A \times B \times C$ step-by-step:

Step 1: First Find $A \times B$

$A = \{0, 1\}$ $B = \{1, 2\}$

To find $A \times B$, we need to consider all possible ordered pairs:

$A \times B = \{(0, 1), (0, 2), (1, 1), (1, 2)\}$

Step 2: Find $A \times B \times C$

Now, we need to take the Cartesian product of $A \times B$ with set C. This means we combine each ordered pair from $A \times B$ with each element from set C:

$A \times B \times C = \{ (0, 1, 0), (0, 1, 1), (0, 1, 2), (0, 2, 0), (0, 2, 1), (0, 2, 2), (1, 1, 0), (1, 1, 1), (1, 1, 2), (1, 2, 0), (1, 2, 1), (1, 2, 2) \}$

So, the Cartesian product A × B × C contains 12 ordered triples, where each triple is a combination of an element from set A, an element from set B, and an element from set C.

Question 5: Let A = {0, 2, 4, 6, 8}, B = {0, 1, 2, 3, 4}, and C = {0, 3, 6, 9}.

- **What are A ∪ B ∪ C and A ∩ B ∩ C?**

Solution:

Let's find A ∪ B ∪ C and A ∩ B ∩ C step-by-step:

Given: A = {0, 2, 4, 6, 8}, B = {0, 1, 2, 3, 4}, C = {0, 3, 6, 9}

Step 1: Find A ∪ B ∪ C (Union of sets A, B, and C)

A ∪ B ∪ C is the set that contains all the unique elements from sets A, B, and C.

To find A ∪ B ∪ C, combine all the elements from A, B, and C without repetitions:

A ∪ B ∪ C = {0, 1, 2, 3, 4, 6, 8, 9}

Step 2: Find A ∩ B ∩ C (Intersection of sets A, B, and C)

A ∩ B ∩ C is the set that contains all the elements that are common to sets A, B, and C.

To find A ∩ B ∩ C, identify the elements that are present in all three sets:

A ∩ B ∩ C = {0}

So, the results are:

A ∪ B ∪ C = {0, 1, 2, 3, 4, 6, 8, 9} and

A ∩ B ∩ C = {0}

Question 6: Let A = {1, 2, 3, 4, 5} and B = {0, 3, 6},

Find

a) A ∪ B. b) A ∩ B. c) A − B. d) B − A.

Solution:

Let’s find the results step-by-step:

Given: A = {1, 2, 3, 4, 5}, B = {0, 3, 6}

a) A ∪ B (Union of sets A and B) the union of two sets A and B, denoted as A ∪ B, is the set containing all the elements that are in A or in B, or in both.

To find A ∪ B, combine all the elements from sets A and B without repetitions:

A ∪ B = {0, 1, 2, 3, 4, 5, 6}

b) A ∩ B (Intersection of sets A and B) the intersection of two sets A and B, denoted as A ∩ B, is the set containing all the elements that are common to both A and B.

To find A ∩ B, identify the elements that are present in both sets A and B:

A ∩ B = {3}

c) A − B (Set difference of A and B) the set difference of two sets A and B, denoted as A − B, is the set containing all the elements that are in A but not in B.

To find A − B, remove the elements of set B from set A:

A − B = {1, 2, 4, 5}

d) B − A (Set difference of B and A) The set difference of two sets B and A, denoted as B − A, is the set containing all the elements that are in B but not in A.

To find B − A, remove the elements of set A from set B:

B − A = {0, 6}

So, the results are:

a) A ∪ B = {0, 1, 2, 3, 4, 5, 6}

b) A ∩ B = {3}

c) A − B = {1, 2, 4, 5}

d) B − A = {0, 6}.

Question 7: Let A = {a, b, c, d, e} and B = {a, b, c, d, e, f, g, h}.

Find

a) A ∪ B. **b) A ∩ B.**

c) A − B. **d) B − A.**

Solution:

Let's perform the operations:

Given: A = {a, b, c, d, e} B = {a, b, c, d, e, f, g, h}

a) A ∪ B (Union of A and B): The union of A and B contains all the elements from both sets, without any duplicates.

A ∪ B = {a, b, c, d, e, f, g, h}

b) A ∩ B (Intersection of A and B): The intersection of A and B contains only the elements that are common to both sets.

A ∩ B = {a, b, c, d, e}

c) A − B (Set Difference between A and B): The set difference between A and B contains all the elements that are in A but not in B.

A − B = {} (Since all elements in A are also present in B, there are no elements left in the set difference.)

d) B − A (Set Difference between B and A): The set difference between B and A contains all the elements that are in B but not in A.

B − A = {f, g, h} (These are the elements present in B but not in A.)

So, the results are:

a) A ∪ B = {a, b, c, d, e, f, g, h}

b) A ∩ B = {a, b, c, d, e}

c) A − B = { }

d) B − A = {f, g, h}

Question 8. Find the sets A and B if A − B = {1, 5, 7, 8}, B − A = {2, 10}, and A ∩ B = {3, 6, 9}.

Solution:

To find sets A and B using the given information, we can break down the set operations step by step. Let's start:

Given: $A - B = \{1, 5, 7, 8\}$ (Set difference between A and B) $B - A = \{2, 10\}$ (Set difference between B and A) $A \cap B = \{3, 6, 9\}$ (Intersection of A and B)

Step 1: Find the elements that are in both A and B (Intersection): $A \cap B = \{3, 6, 9\}$

Step 2: Find the elements that are only in A but not in B (Set difference $A - B$): $A - B = \{1, 5, 7, 8\}$

Step 3: Find the elements that are only in B but not in A (Set difference $B - A$): $B - A = \{2, 10\}$

Now, we can use the information from Step 1, Step 2, and Step 3 to construct sets A and B:

Step 4: Combine elements from Step 1 and Step 2 to form set A: A = (Elements in $A \cap B$) $\cup$ (Elements in $A - B$) $A = \{3, 6, 9\} \cup \{1, 5, 7, 8\}$ $A = \{1, 3, 5, 6, 7, 8, 9\}$

Step 5: Combine elements from Step 1 and Step 3 to form set B: B = (Elements in $A \cap B$) $\cup$ (Elements in $B - A$) $B = \{3, 6, 9\} \cup \{2, 10\}$ $B = \{2, 3, 6, 9, 10\}$

So, the sets A and B are as follows: $A = \{1, 3, 5, 6, 7, 8, 9\}$ $B = \{2, 3, 6, 9, 10\}$.

Question 9. Find the symmetric difference of $\{1, 3, 5\}$ and $\{1, 2, 3\}$

Solution:

The symmetric difference of two sets A and B is the set of elements that are in either of the sets but not in their intersection. It is denoted by $A \triangle B$.

The formula to find the symmetric difference of two sets A and B is:

$A \triangle B = (A \cup B) - (A \cap B)$

Given sets: $A = \{1, 3, 5\}$ $B = \{1, 2, 3\}$

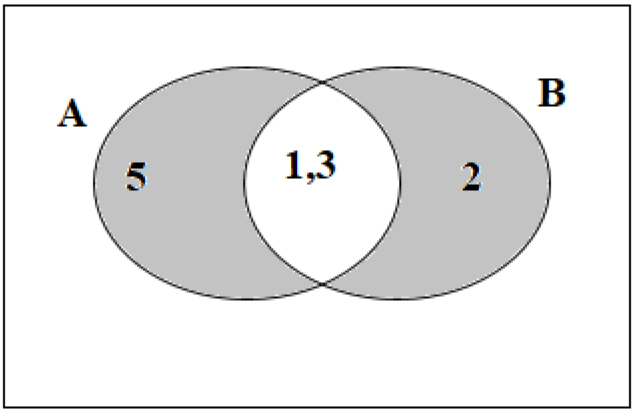
Step 1: Find the union of sets A and B ($A \cup B$): $A \cup B = \{1, 2, 3, 5\}$

Step 2: Find the intersection of sets A and B ($A \cap B$): $A \cap B = \{1, 3\}$

Step 3: Subtract the intersection from the union to get the symmetric difference (A △ B): A △ B = {2, 5}

So, the symmetric difference of {1, 3, 5} and {1, 2, 3} is {2, 5}

Alternatively you can also draw its Venn diagram as shown in the following:



A △ B = { 5, 2}

A △ B

Question 10: Let A = {0, 2, 4, 6, 8, 10}, B = {0, 1, 2, 3, 4, 5, 6}, and C = {4, 5, 6, 7, 8, 9, 10}. Find a) A ∩ B ∩ C. b) A ∪ B ∪ C. c) (A ∪ B) ∩ C. d) (A ∩ B) ∪ C.

Solution:

Let's find the sets as per the given operations:

Given sets: A = {0, 2, 4, 6, 8, 10} B = {0, 1, 2, 3, 4, 5, 6} C = {4, 5, 6, 7, 8, 9, 10}

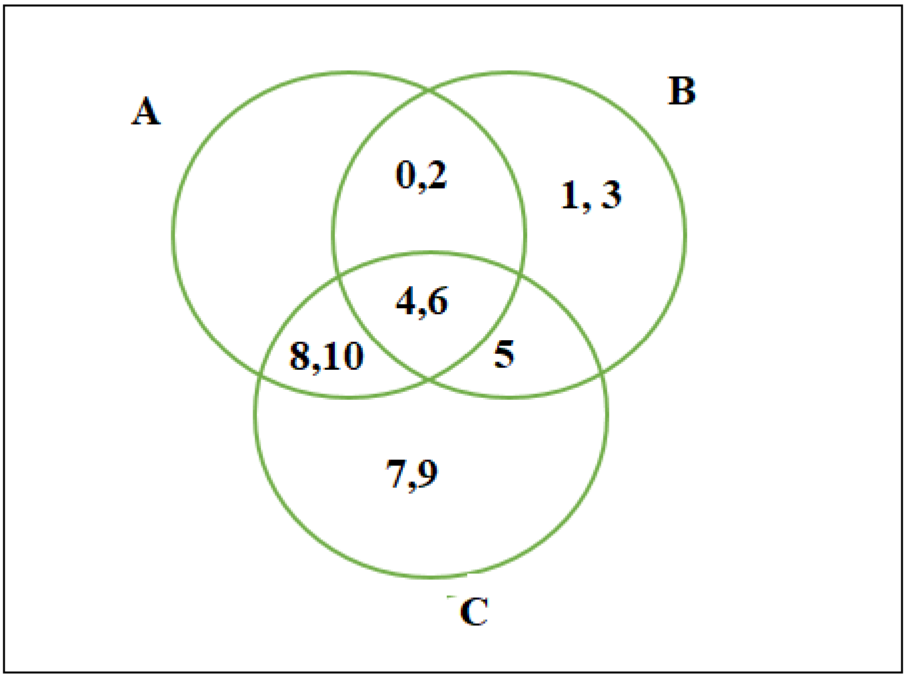
a) A ∩ B ∩ C (Intersection of sets A, B, and C): A ∩ B = {0, 2, 4, 6} (elements common to sets A and B) (A ∩ B) ∩ C = {4, 6} (elements common to sets A, B, and C)

b) A ∪ B ∪ C (Union of sets A, B, and C): A ∪ B = {0, 1, 2, 3, 4, 5, 6, 8, 10} (elements present in either set A or set B) (A ∪ B) ∪ C = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10} (elements present in either set A, set B, or set C)

c) (A ∪ B) ∩ C (Intersection of the union of sets A and B with set C): (A ∪ B) = {0, 1, 2, 3, 4, 5, 6, 8, 10} (A ∪ B) ∩ C = {4, 5, 6, 8, 10} (elements common to the union of sets A and B and set C)

d) (A ∩ B) ∪ C (Union of the intersection of sets A and B with set C): (A ∩ B) = {0, 2, 4, 6} (A ∩ B) ∪ C = {0, 2, 4, 5, 6, 7, 8, 9, 10} (elements present in either the intersection of sets A and B or set C)

Alternatively you can solve the above problem using Venn diagram which is demonstrated below.



Now just by observing the above Venn diagram you can find the following answer

a) $A \cap B \cap C = \{4, 6\}$

b) $A \cup B \cup C = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

c) $(A \cup B) \cap C = \{4, 5, 6, 8, 10\}$

d) $(A \cap B) \cup C = \{0, 2, 4, 5, 6, 7, 8, 9, 10\}$