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Assignment - 02

Q.1 (a.) Explain forward and backward difference.

Ans ⇒ Forward Difference ⇒ The differences $y_1 - y_0$, $y_2 - y_1$, $y_3 - y_2$, ---, $y_n - y_{n-1}$ when denoted by dy_0, dy_1, dy_2 -- dy_{n-1} are respectively, called the first forward differences. Thus the first forward difference are:

$$\Delta y_r = y_{r+1} - y_r$$

Backward Differences ⇒ The differences $y_1 - y_0$, $y_2 - y_1$ ---- $y_n - y_{n-1}$ when denoted by dy_1, dy_2 -- dy_n, respectively, are called first backward difference, are called first backward difference. Thus the first backward difference are:

$$\nabla y_r = y_r - y_{r-1}$$

b. Apply Bessel's formula to obtain y_{25} given.

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$y_{20} = 2854, y_{24} = 3162, y_{28} = 3544, y_{32} = 3992$

Solve ⇒

x	y_u	Δy_u	$\Delta^2 y_u$	$\Delta^3 y_u$
20	2854			
	~~2854~~	308		
24	3162		74	
		382		-8
28	3544		66	
		448		
32	3992			

formula

$$y_u = y_0 + u\Delta y_0 + \frac{u(u-1)}{2}\left(\frac{\Delta^2 y_{-1} + \Delta^2 y_0}{2}\right) + \frac{u(u-\frac{1}{2})(u-1)}{3!}$$

$$\times \Delta^3 y_{-1} + \frac{(u+1)u(u-1)(u-2)}{4!}\left(\frac{\Delta^4 y_{-2} + \Delta^4 y_{-1}}{2}\right) + \ldots$$

here $u = \frac{x - x_0}{h} = \frac{25-24}{4} = \frac{1}{4}$

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$$y_{25} = 3160 + (0.25)(382) + \frac{(0.25)(0.25-1)}{2!}\left(\frac{74-66}{2}\right) + \frac{(0.25)(0.25-\frac{1}{2})(0.25-1)}{3!}(-8)$$

$$y_{25} = 3250.87 \quad \text{ans}$$

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c. Using Newton's divide difference formula evaluate f(8) and f(15) given

X:	4	5	6	7	11	13
Y:	48	100	294	900	1210	2028

Solve: Given Arguments: 4, 5, 6, 7, 11, 13.
Co-ordinating arguments with formula we'll get

$$f(x) = f(4) + (x-4)\Delta f(4) + (x-4)(x-5)\Delta^2_{5} f(4) + (x-4)(x-5)(x-6)\Delta^3_{5,6} f(4) + (x-4)(x-5)(x-6)(x-7)\Delta^4_{5,6,7} f(4) + (x-4)(x-5)(x-6)(x-7)(x-11)\Delta^5_{5..13} f(4) \quad \text{— (A)}$$

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X	f(x)	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$	$\Delta^5 (f(x))$
4	48	$\frac{100-48}{5-4} = 52$				
5	100	$\frac{294-100}{6-5} = 194$	$\frac{194-52}{6-4} = 71$			
6	294	$\frac{900-294}{7-6} = 606$	$\frac{606-194}{7-5} = 206$	$\frac{206-71}{7-4} = 45$		
7	900	$\frac{1210-900}{11-7} = 77.5$	$\frac{77.5-606}{11-6} = -105.7$	$\frac{-105.7-206}{11-5} = -51.95$	$\frac{-51.95-45}{11-4} = -13.85$	$\frac{9.36+13.85}{13-4} = 2.51$
11	1210	$\frac{2028-1210}{13-11} = 409$	$\frac{409-77.5}{13-7} = 55.25$	$\frac{55.25+105.7}{13-6} = 22.99$	$\frac{22.99+51.95}{}$ 9.3675	
13	2028					

Now put the value in eqn (A).

$$f(x) = 48 + (x-4)(52) + (x-4)(x-5)(71) + (x-4)(x-5)(x-6)(45) + (x-4)(x-5)(x-6)(x-7)(-13.85) + (x-4)(x-5)(x-6)(x-7)(x-11)(2.51)$$

To find f(8) and f(15

$f(8) = 48+(8-4)52+(8-4)(8-5)71+(8-4)(8-5)$
$(8-6)45+(8-4)(8-5)(8-6)(8-7)(-13.85)$
$+(8-4)(8-5)(8-6)(8-7)(8-11)(2.51)$

$f(8) = 1675.6$. Ans

$f(15) = 48+(15-4)52+(15-4)(15-5)71+$
$(15-4)(15-5)(15-6)45+(15-4)(15-5)$
$(15-6)(15-7)(-13.85)+(15-4)(15-5)(15-6)(15-7)$
$(15-11)\ 2.51$

$f(15) = 22488$ Ans

Q.2 (a) Write Taylor's series method.

Ans ⇒ The Taylor's series of a real or complex valued function f(x) that is infinitely differentiable at a real or complex no. a is the power series

$$= f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3$$

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(b) Using R-K method of fourth order, solve $\frac{dy}{dx} = y^2 - x^2y^2 + x^2$ with $y(0) = 1$

at $x = 0.2, 0.4$.

Solve ⇒ Given

$$f(x, y) = y^2 - x^2y^2 + x^2$$

(a)

To find $y(0.2)$

Here $x_0 = 0$, $y_0 = 1$, $h = 0.2$

$$K_1 = h \times f(x_0, y_0) = 0.2 \times f(0, 1)$$
$$= 0.2 \times [1^2 - 0^2 1^2 + 0^2]$$
$$= 0.2$$

$$K_1 = 0.2$$

$$K_2 = h \times f(x_0 + h/2, y_0 + K_1/2)$$
$$= 0.2 \times f(0 + 0.2/2, 1 + 0.2/2)$$
$$= 0.2 \times f(0.1, 1.1)$$
$$= 0.2 \times [(1.1)^2 - (0.1)^2(1.1)^2 + (0.1)^2]$$

$$K_2 = 0.241$$

$$K_3 = h \times f(x_0 + h/2, y_0 + K_2/2)$$
$$= 0.2 \times f(0 + 0.2/2, 1 + 0.24/2)$$
$$= 0.2 \times f(0.1, 1.12)$$

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$$= 0.2\times\left[(1.12)^2 - (0.1)^2 (1.12)^2 + (0.1)^2\right]$$

$$K_3 = 0.250$$

$$K_4 = h\times f(x_0+h, y_0+K_3)$$
$$= 0.2\times f(0+0.2, 1+0.250)$$
$$= 0.2\times f(0.2, 1.25)$$
$$= 0.2\times \left[(1.25)^2 - (1.25)^2(0.2)^2 + (0.2)^2\right]$$
$$K_4 = 0.3081$$

We know

$$K = \frac{1}{6}\left[K_1 + 2K_2 + 2K_3 + K_4\right]$$

$$= \frac{1}{6}\left[0.2 + 0.241 + 0.250 + 0.3081\right]$$

$$K = 0.26$$

Now

$$y(0.2) = y_0 + K$$
$$y(0.2) = 1 + 0.26$$
$$y(0.2) = 1.26 \quad \text{Ans}$$

(b) To find $y(0.4)$

Here

$$x_1 = 0.2,\ y_1 = 1.26,\ h = 0.2$$

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$K_1 = hf(x_1, y_1)$

$K_1 = 0.2 \times f(0.2, 1.26)$

$K_1 = 0.2 \times [(1.26)^2 - (0.2)^2 (1.26)^2 + (0.2)^2]$

$K_1 = 0.312$

$K_2 = hf(x_1 + h/2, y_1 + K_1/2)$

$K_2 = 0.2 \times f(0.2 + 0.2/2, 1.26 + 0.312/2)$

$K_2 = 0.2 \times f(0.3, 0.19)$

$K_2 = 0.2 \times [(0.19)^2 - (0.3)^2 (0.19)^2 + (0.3)^2]$

$K_2 = 0.024$

$K_3 = h \times f(x_1 + h/2, y_1 + K_2/2)$

$K_3 = 0.2 \times f(0.2 + 0.2/2, 1.26 + 0.024/2)$

$K_3 = 0.2 \times f(0.3, 0.015)$

$K_3 = 0.2 \times [(0.015)^2 - (0.3)^2 (0.015)^2 + (0.015)^2]$

$K_3 = 0.018$

$K_4 = h \times f(x_1 + h, y_1 + K_3)$

$K_4 = 0.2 \times f(0.2 + 0.2, 1.26 + 0.018)$

$K_4 = 0.2 \times f(0.4, 1.271)$

$K_4 = 0.2 \times [(1.27)^2 - (0.4)^2 (1.27)^2 + (0.4)^2]$

$K_4 = 0.3037$

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$$K = \frac{1}{6}\left[K_1 + 2K_2 + 2K_3 + K_4\right]$$

$$K = \frac{1}{6}\left[0.312 + 0.024 + 0.018 + 0.3037\right]$$

$$K = 0.1166$$

Now,

$y(0.4) = y_1 + K$

$y(0.4) = 1.26 + 0.1166$

$y(0.4) = 1.38$ // Ans

C. Using Eular's method, find an approximate value of y corresponding to $x = 1$, given that dy/dx $x+y$ and $y = 1$ when $x = 0$.

Solve⇒ Let $n = 10$

So $h = \frac{y-x}{n} = \frac{1-0}{10} = 0.1$

x	y	$\frac{dy}{dx} = x+y$	old $y + h\left(\frac{dy}{dx}\right) =$ new y
0	1	1	$1 + 0.1(1) = 1.1$
0.1	1.1	1.2	$1.1 + 0.1(1.2) = 1.22$
0.2	1.22	1.42	$1.22 + 0.1(1.42) = 1.36$
0.3	1.36	1.66	$1.36 + 0.1)(1.66) = 1.53$

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x	y	$\frac{dy}{dx} = x+y$	$old\,y + h\left(\frac{dy}{dx}\right) = new\,y$
0.4	1.53	1.93	$1.53+0.1(1.93)=1.72$
0.5	1.72	2.22	$1.72+0.1(2.22)=1.94$
0.6	1.94	2.54	$1.94+0.1(2.54)=2.19$
0.7	2.19	2.18	$2.19+0.1(2.18)=2.48$
0.8	2.48	3.29	$2.48+0.1(3.29)=2.81$
0.9	2.81	3.71	$2.81+0.1(3.71)=3.18$
1	3.18	—	—

Hence the value of y when $x = 1$ is

$= 3.18$ Ans

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