In [1]:
```
import numpy as np
import matplotlib.pyplot as plt
np.random.seed(2025)  # For reproducibility
from sklearn.linear_model import LogisticRegression
import time
```

Problem 1

Dataset Generation

Write a function to **generate a training set** of size m

- randomly generate a weight vector $w \in \mathbb{R}^{10}$, normalize length
- generate a training set $\{(x_i, y_i)\}$ of size m
  - x_i: random vector in $\mathbb{R}^{10}$ from $\mathbf{N}(0, I)$
  - y_i: $\{0, +1\}$ with $P[y = +1] = \sigma(w \cdot x_i)$ and $P[y = 0] = 1 - \sigma(w \cdot x_i)$

In [2]:
```
def sigmoid(z):
    return 1 / (1 + np.exp(-z))

def generate_data(m):
    w_star = np.random.randn(10)
    w_star /= np.linalg.norm(w_star)
    X = np.zeros((m, 10))
    Y = np.zeros(m)
    for i in range(m):
        X[i] = np.random.randn(10)
        prob = sigmoid(w_star @ X[i])
        Y[i] = 1 if np.random.rand() < prob else 0
    return w_star, X, Y

#Debugging purpose
# w, X, Y = generate_data(1000)
# print("True weights:", w)
# print("First 5 samples of X:\n", X[:5])
# print("First 5 labels of Y:", Y[:5])
```

Algorithm 1: logistic regression

The goal is to learn w. Algorithm 1 is logistic regression (you may use the built-in method LogisticRegression for this. Use max_iter=1000).

In [3]:
```
def logistic_regression(X, Y):
    clf = LogisticRegression(max_iter=1000)
    clf.fit(X, Y)
    w_hat = clf.coef_.flatten()
    return w_hat

# Debugging logistic regression function
# w_hat = logistic_regression(X, Y)
# print("Learned weights using logistic regression:", w_hat)
```

Algorithm 2: gradient descent with square loss

Define square loss as

$$L_i(w^{(t)}) = \frac{1}{2}\Big(\sigma(w^{(t)} \cdot x) - y_i\Big)^2$$

Algorithm 2 is gradient descent with respect to square loss (code this up yourself -- run for 1000 iterations, use step size eta = 0.01).

In [4]:
```
def gradient(w, x, y):
    yhat = sigmoid(np.dot(w, x))
    return (yhat - y) * yhat * (1 - yhat) * x
```

```
def gd_sl(X, Y, n_iter=1000, eta=0.01):
    w = np.zeros(X.shape[1])
    for t in range(n_iter):
        grad = np.zeros_like(w)
        for i in range(X.shape[0]):
            grad += gradient(w, X[i], Y[i])
        grad /= X.shape[0]
        w -= eta * grad
    return w


# Debugging gradient function
# test_w = np.random.randn(10)
# test_x = np.random.randn(10)
# test_y = 1
# grad = gradient(test_w, test_x, test_y)
# print("Gradient test:")
# print("w:", test_w)
# print("x:", test_x)
# print("y:", test_y)
# print("Computed gradient:", grad)

# # Debugging gd_sl function
# print("\nTesting gd_sl function:")
# w_hat = gd_sl(X, Y, n_iter=10, eta=0.01)
# print("Initial weights:", np.zeros(X.shape[1]))
# print("Learned weights after 10 iterations:", w_hat)
```

Algorithm 3: stochastic gradient descent with square loss

Similar to gradient descent, except we use the gradient at a single random training point every iteration.

In [5]:
```
def sgd(X, Y, n_iter=1000, eta=0.01):
    w = np.zeros(X.shape[1])
    for t in range(n_iter):
        i = np.random.randint(X.shape[0])  # Randomly select a training example
        grad = gradient(w, X[i], Y[i])  # Compute gradient
        w -= eta * grad  # Update weights
    return w

# Debugging SGD
# w_hat_sgd = sgd(X, Y, n_iter=10, eta=0.01)
# print("Initial weights:", np.zeros(X.shape[1]))
# print("Learned weights after 10 iterations:", w_hat_sgd)
```

Evaluation

Measure error $\|w - \hat{w}\|_2$ for each method at different sample size. For any fixed value of m, choose many different w's and average the values $\|w - \hat{w}\|_2$ for Algorithms 1, 2 and 3. Plot the results for for each algorithm as you make m large (use $m = 50, 100, 150, 200, 250$). Also record, for each algorithm, the time taken to run the overall experiment.

In [6]:
```
sample_sizes = [50, 100, 150, 200, 250]

def evaluate(model, num_trials=10):
    errors = []
    for m in sample_sizes:
        trial_errors = []
        for _ in range(num_trials):
            w_star, X, Y = generate_data(m)
            w_hat = model(X, Y)
            error = np.linalg.norm(w_hat - w_star)
            trial_errors.append(error)
        errors.append(np.mean(trial_errors))
    return errors

# Track time for logistic regression
start_time = time.time()
lr_errors = evaluate(logistic_regression)
lr_time = time.time() - start_time

# Track time for gradient descent with square loss
start_time = time.time()
```

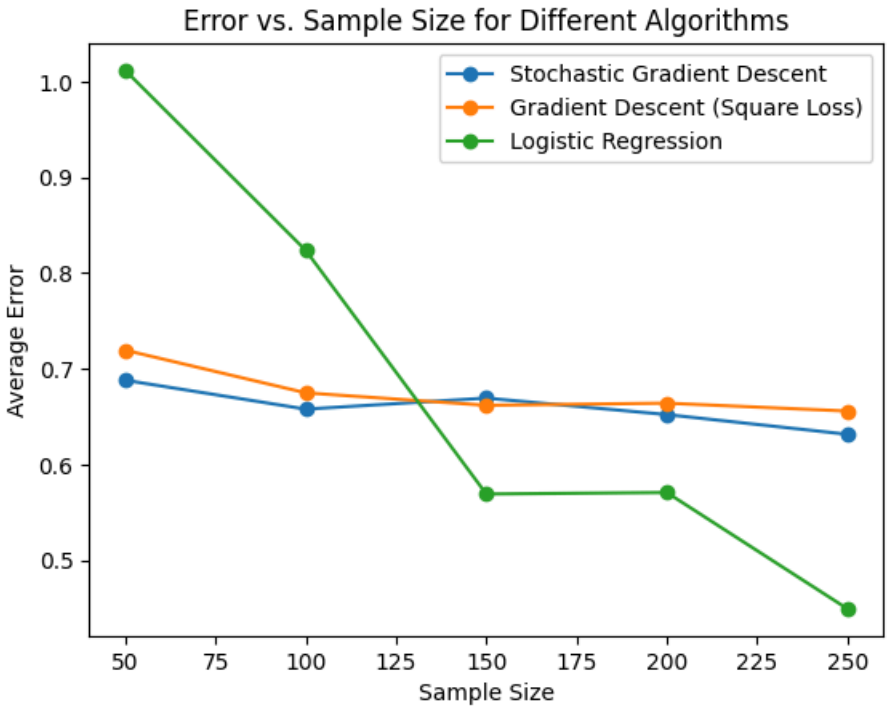
```
gd_errors = evaluate(gd_sl)
gd_time = time.time() - start_time

# Track time for stochastic gradient descent
start_time = time.time()
sgd_errors = evaluate(sgd)
sgd_time = time.time() - start_time

# Print the times for each algorithm
print(f"Logistic Regression Time: {lr_time:.2f} seconds")
print(f"Gradient Descent with Square Loss Time: {gd_time:.2f} seconds")
print(f"Stochastic Gradient Descent Time: {sgd_time:.2f} seconds")
```

```
Logistic Regression Time: 0.08 seconds
Gradient Descent with Square Loss Time: 12.66 seconds
Stochastic Gradient Descent Time: 0.17 seconds
```

In [7]:
```
plt.plot(sample_sizes, sgd_errors, label='Stochastic Gradient Descent', marker='o')
plt.plot(sample_sizes, gd_errors, label='Gradient Descent (Square Loss)', marker='o')
plt.plot(sample_sizes, lr_errors, label='Logistic Regression', marker='o')
plt.xlabel('Sample Size')
plt.ylabel('Average Error')
plt.title('Error vs. Sample Size for Different Algorithms')
plt.legend()
plt.show()
```



Problem 2

In [8]:
```
from sklearn import datasets
from sklearn.tree import DecisionTreeClassifier
from sklearn.ensemble import AdaBoostClassifier
from sklearn.model_selection import cross_val_score
```

In [9]:
```
cancer = datasets.load_breast_cancer()
```

For each depth in $1, \ldots, 5$, instantiate an AdaBoost classifier with the base learner set to be a decision tree of that depth (set `n_estimators=10` and `learning_rate=1`), and then record the 10-fold cross-validated error on the entire breast cancer data set. Plot the resulting curve of accuracy against base classifier depth. Use 101 as your random state for both the base learner as well as the AdaBoost classifier every time.

In [10]:
```
depths = range(1, 6)
errors = []
for d in depths:
    base = DecisionTreeClassifier(max_depth=d, random_state=101)
    clf = AdaBoostClassifier(estimator=base, n_estimators=10, learning_rate=1, random_state=101)
    scores = cross_val_score(clf, cancer.data, cancer.target, cv=10)
```

```
    error = np.mean(scores)
    errors.append(error)


plt.plot(depths, errors, marker='o')
plt.xlabel('Base Classifier Depth')
plt.ylabel('Accuracy')
plt.title('AdaBoost Accuracy vs. Base Classifier Depth')
plt.show()
```