Lab Assignment – 2
Numerical Solutions of a System of Linear Equations

1. Write a program that computes the condition number of any given matrix with respect to the following norms:

   - 1-norm
   - Frobenius (Euclidean) norm
   - Infinity norm
   - Spectral norm

   Do not use any built functions such as sum or average of the matrices, however use of inverse function is allowed. Also, find the condition number of the following matrices:

   (a) Rotation matrix

   (b)
   $$A = \begin{bmatrix} 1 & 3 & 4 \\ 4 & 5 & 6 \\ -15 & 6 & 9 \end{bmatrix}$$

   (c)
   $$B = \begin{bmatrix} 2 & 1 & 0 & 0 & 0 \\ 3 & 3 & 12 & 0 & 0 \\ 0 & 4 & -33 & 21 & 0 \\ 0 & 0 & 12 & 0 & 23 \\ 5 & 0 & 0 & 14 & 67 \end{bmatrix}$$

2. Write a program to generate the Hilbert matrix of the following type:

   $$\begin{bmatrix} 1 & 1/2 & 1/3 & \cdots \\ 1/2 & 1/3 & 1/4 & \cdots \\ 1/3 & 1/4 & 1/5 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

   It is a $n \times n$ square matrix, where n is the input by the user. Find the condition number of the generated matrix and then solve the system $Ax = b$ for x, where b is given as:

   $$b_i = \sum_{j=1}^{n} A_{ij}$$

   Solve it by using LU decomposition method and Gauss elimination. Compare the values of x computed using both methods.

3. Generate a 60×60 matrix A with 1's on the main diagonal and in the last column, with -1's below the main diagonal, and with 0's everywhere else. Compute the condition number of the matrix A. Set a random vector x of length 60. Compute

$$b = A * x.$$

Solve the linear system $Ax' = b$ using different methods taught in the class. Compute the 2-norm of the difference between the computed vector x' and the true solution x generated previously. The relative difference is given by:

$$\frac{||x' - x||}{||x||}.$$

Compare the relative errors of all the methods.