# Lab Assignment – 5

1.

a) Write a python function to convert a given $n \times n$ symmetric matrix A into a symmetric tri-diagonal matrix T using Householder's method.

b) For the tridiagonal matrix T computed in part (a), find all the intervals of eigenvalues of T which are also the eigenvalues of A.

c) Find all the eigenvalues of A by using bisection method for the intervals calculated in part (b). You may use in-built python function "bisect" from "scipy.optimize" or any other function of your choice for the bisection method.

Test your implementation for the following two matrices:

i) $$\begin{bmatrix} 9 & 13 & 3 & 6 \\ 13 & 11 & 7 & 6 \\ 3 & 7 & 4 & 7 \\ 6 & 6 & 7 & 10 \end{bmatrix}$$

ii) A randomly generated $n \times n$ symmetric matrix where n is input by the user.

Compare eigenvalues calculated by you with the standard python function as well as the QR method. For QR method, use the function that you have already implemented in the assignment-4.

** A real symmetric tridiagonal matrix has real eigenvalues, and all the eigenvalues are distinct (simple) if all off-diagonal elements are nonzero. Ignore the case where you did not get the nonzero off-diagonal entries. **

2. For the given image do the following:

a) Reconstruct the image using SVD

b) Reconstruct the image using PCA

Compute the reconstructed image for different top k singular values (in case of SVD) and eigenvalues (in case of PCA). For different values of k plot the reconstructed image by SVD and PCA side by side. Also calculate the Frobenius norm of the difference matrix between the reconstructed and actual image for both SVD and PCA. Title of the reconstructed plot should be the value of k and the corresponding Frobenius norm.

Note: Direct use of SVD and PCA functions are not allowed. You may use inbuilt functions to find the eigenvalues and eigenvectors.