B.E. (Civil Engineering) Third Semester (C.B.S.)

Applied Mathematics - III

P. Pages : 3 **NIR/KW/18/3292**

Time : Three Hours Max. Marks : 80

Notes :
1. All questions carry marks as indicated.
2. Solve Question 1 OR Questions No. 2.
3. Solve Question 3 OR Questions No. 4.
4. Solve Question 5 OR Questions No. 6.
5. Solve Question 7 OR Questions No. 8.
6. Solve Question 9 OR Questions No. 10.
7. Solve Question 11 OR Questions No. 12.
8. Assume suitable data whenever necessary.
9. Use of non programmable calculator is permitted.

1. Sketch the function $f(x) = |\sin x|, -\pi < x < \pi$. Hence obtain Fourier series for $f(x)$. **7**

OR

2. Obtain Half range sine series for $f(x) = \pi x - x^2$ in the interval $(0, \pi)$. **7**

3. a) Solve $y^2 p - xyq = x(z - 2y)$ **6**

b) solve $\frac{\partial^2 z}{\partial x^2} + 3\frac{\partial^2 z}{\partial x \partial y} + 2\frac{\partial^2 z}{\partial y^2} = 24xy$ **6**

c) Solve using method of separation of variables **6**
$4\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 3u$, given that $u = 3e^{-y} - e^{-5y}$ when $x = 0$.

OR

4. a) An insulated rod of length ℓ has its ends A and B maintained at 0ºC and 100ºC respectively until steady conditions are reached. If B is then suddenly reduced to 0ºC and maintained at 0ºC, find the temperature at a distance x from A at any time t. **8**

b) Solve $x^2(y - z)p + y^2(z - x)q = z^2(x - y)$ **5**

c) Solve $\left(D^3 - 3DD'^2 + 2D'^3\right)z = (x + 2y)^{1/2}$ **5**

5. Find the extremals of the functional $\int_1^2 \frac{\sqrt{1 + (dy/dx)^2}}{x} dx$, given y (1) = 0, y (2) = 1. **7**

OR

6. Find the curve joining points (0,0) and (1,0) for which the integral. **7**

$\int_0^1 (y'')^2\, dx$ is minimum, if $y(0)=0=y(1),\ y'(0)=a,\ y'(1)=b$

7. a) Find whether given set of vectors are linearly dependent. If so find the relationship between them: **6**

$X_1=[1,2,-1,3],\ X_2=[2,-1,3,2],\ X_3=[-1,8,-9,5]$.

b) Verify Caylay – Hamilton's theorem and hence find A^{-1}, where $A=\begin{bmatrix}1&2&4\\2&1&2\\4&2&1\end{bmatrix}$ **6**

c) Find modal matrix for $A=\begin{bmatrix}3&1&4\\0&2&6\\0&0&5\end{bmatrix}$ **6**

OR

8. a) If $A=\begin{bmatrix}3&2\\2&3\end{bmatrix}$, verify $\log_e e^A = A$ by using Sylvester's theorem. **6**

b) Solve $\frac{d^2y}{dt^2}-3\frac{dy}{dt}-10y=0$ given $y(0)=3,\ y'(0)=15$ by matrix method. **6**

c) Reduce the given quadratic form to canonical form by orthogonal Transformation **6**

$3x^2+5y^2+3z^2-2yz+2zx-2xy$.

9. a) Find by Newton – Raphson method, the red root of 3x – cos x – 1 = 0. **6**

b) Solve $4x+11y-z=33,\ 6x+3y+12z=36,\ 8x-3y+2z=20$ **6**

by Guass – seidal method.

c) Solve $\frac{\partial y}{\partial x}=-xy^2,\ y=z,$ when $x=0$ to find y (0.2) by Modified Euler's method. **6**

(Take h = 0.1)

OR

10. a) Using Regula – Falsi method, find the root of **6**

$x\log_{10} x-1.2=0$

Correct to three decimal places.

b) Solve $3x+2y+7z=4: 2x+3y+z=5$ **6**

$3x+4y+z=7$ by Crout's method.

c) Solve $\frac{\partial y}{\partial x}=x-2y$ by Runge – Kutta method given $y(0)=1,$ to find y (0.2). **6**

11. a) A manufacturer produces two types of models M_1 and M_2. Each M_1 model requires 4 hrs of grinding and 2 hrs of polishing where as each M_2 model requires 2 hrs of grinding and 5 hrs of polishing. The manufacturer has 2 grinders and 3 polishers. Each grinder works for 40 hrs a week and each polisher works for 60 hrs a week. Profit as on M_1 model is Rs. 3 and on M_2 model is Rs. 4 Whatever is produced in a week is sold in market. How should the manufacturer allocate his production capacity to the two types of models so that he may make the maximum profit in a week. Formulate the LPP. **6**

b) Solve the given LPP by graphical method. **6**

Max. $z = 3x_1 + 5x_2$

Subject to

$$x_1 + 2x_2 \le 200$$
$$x_1 + x_2 \le 150$$
$$x_1 \le 60$$
$$x_1, x_2 \ge 0$$

OR

12. Using simplex Method, Solve the LPP. **12**

Maximize $z = 10x_1 + x_2 + 2x_3$

Subject to

$$x_1 + x_2 - 2x_3 \le 10$$
$$4x_1 + x_2 + x_3 \le 20$$
$$x_1, x_2, x_3 \ge 0$$

****

B.E. (Civil Engineering) Third Semester (C.B.S.)

Applied Mathematics - III

P. Pages : 3 **NJR/KS/18/4347**

Time : Three Hours *0108* Max. Marks : 80

Notes :
1. All questions carry marks as indicated.
2. Solve Question 1 OR Questions No. 2.
3. Solve Question 3 OR Questions No. 4.
4. Solve Question 5 OR Questions No. 6.
5. Solve Question 7 OR Questions No. 8.
6. Solve Question 9 OR Questions No. 10.
7. Solve Question 11 OR Questions No. 12.
8. Assume suitable data whenever necessary.
9. Use of non programmable calculator is permitted.
10. Use of simple graph paper is permitted.

1. Sketch the function $f(x) = |\sin x|, -\pi < x < \pi,$ hence obtain Fourier series for f(x). **7**

OR

2. Obtain half range cosine series for $f(x) = 2x - 1$, in interval 0<x<1. Hence show that **7**

$$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + --- + \infty .$$

3. a) Solve $\left(x^2 - y^2 - z^2\right)p + 2xyq = 2xz$. **6**

b) Solve $\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = 2\cos y - x\sin y$ **6**

c) Solve using method of separation of variables $4\frac{\partial U}{\partial x} + \frac{\partial U}{\partial y} = 3U,$ given that $U = 3e^{-y} - e^{-5y}$ when x=0. **6**

OR

4. a) A tightly stretched string with fixed end points x = 0, x = l is initially at rest in its equilibrium position. If it is set vibrating by giving each point a velocity $\left(\frac{\partial y}{\partial t}\right)_{t=0} = v_0 \sin^3\left(\frac{\pi x}{\ell}\right)$. Find the displacement at any point from one end at any time t. **8**

b) Solve $xq = yp + xe^{x^2 + y^2}$ **5**

c) Solve $\left(D^2 - 3DD' + 2D'^2\right)z = e^{2x+3y} + \sin(x - 2y)$. **5**

5. Find the curve passing through the points (x_1, y_1) and (x_2, y_2) which when rotated about X-axis gives minimum surface area. **7**

OR

6. Find the extremals of the isoperimetric problem $v[y(x)] = \int_0^1 \left[\left(y^1\right)^2 + x^2 \right] dx$ given that $\int_0^1 y^2 dx = 2,\ y(0) = 0,\ y(1) = 0.$ **7**

7. a) Find whether the following set of vectors are linearly dependent or otherwise, find the relation between them **6**
$X_1 = (1, 2, 4),\ X_2 = (2, -1, 3),\ X_3 = (0, 1, 2),\ X_4 = (-3, 7, 2)$.

b) Find the modal matrix B and verify $B^{-1}AB$ a diagonal matrix, if $A = \begin{bmatrix} 3 & 4 \\ 4 & -3 \end{bmatrix}$. **6**

c) Verify Cayley – Hamilton theorem for matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and hence find A^{-1}. **6**

OR

8. a) Use Sylvester's theorem to show that $e^A = e^x \begin{bmatrix} \cosh x & \sinh x \\ \sinh x & \cosh x \end{bmatrix}$, where $A = \begin{bmatrix} x & x \\ x & x \end{bmatrix}$. **6**

b) Solve $\frac{d^2y}{dx^2} + 4y = 0$, given $y = 8,\ \frac{dy}{dx} = 0$ when x=0. **6**

c) Reduce the quadratic form $3x^2 + 5y^2 + 3z^2 - 2yz + 2zx - 2xy$ to canonical form. **6**

9. a) Find the real root of the equation $3x - \cos x = 1$ by method of felse position. **6**

b) Apply Gauss – Seidel method to solve the equations **6**

$$x + 7y - 3z = -22$$
$$5x - 2y + 3z = 18$$
$$2x - y + 6z = 22$$

c) Use Runge – Kutta method to find approximate value of y for x = 0.2, when $\frac{dy}{dx} = \frac{x^2 + y^2}{10}$, given $y(0) = 1,\ h = 0.2.$ **6**

OR

10. a) Find the real root of the equation $x\log_{10}^{x} - 1.2 = 0$ by Newton-Raphson method. **6**

b) Use Crout's method to solve the equations **6**

$$3x + 2y + 7z = 4$$
$$2x + 3y + z = 5$$
$$3x + 4y + z = 7$$

c) Use modified Euler's method to solve the equation $\frac{dy}{dx} = x + y$ for x=0.1 given that y(0)=1 and h=0.05. **6**

11. Formulate and solve by simplex method. **12**

The manager of a company, which supplies office furniture has asked to prepare a profit maximizing schedule for the manufacturing of desks, chairs and book-cases. The wooden materials have to be cut properly by machines. Each unit of desks, chairs and book-cases require 0.8, 0.4 and 0.5 machine hours respectively. The total machine hours available for cutting are 100. The company has 650 man-hours available for painting and polishing. Each unit of desks, chairs and book-cases require 5, 3 and 3 man-hours for painting and polishing respectively. The total capacity of a warehouse where there are to be stored is 1260 sq.ft. The floor space per unit of each product required by these products are 9, 6 and 9 sq. ft respectively. Each product is supplied at a profit of Rs. 30, Rs. 16 and Rs. 25 per unit respectively. What should be the number of units of each product to be manufactured so that the profit is maximum?

OR

12. a) Use Graphical method to solve the following L.P. problem. **6**

Minimize $z = 20x_1 + 10x_2$

Subject to
$$x_1 + 2x_2 \le 40$$
$$3x_1 + x_2 \ge 30$$
$$4x_1 + 3x_2 \ge 60$$
$$x_1, x_2 \ge 0.$$

b) The standard weight of a special purpose brick is 5 kg and it contains two basic ingredients B_1 and B_2. B_1 costs Rs. 5 per kg and B_2 costs Rs. 8 per kg. Strength considerations dictate that the brick contains not more than 4 kg of B_1 and a minimum of 2 kg of B_2. Since the demand for the product is likely to be related to the price of the brick. Formulate L.P.P. model to minimize the cost of the brick satisfying the above conditions. **6**

****

B.E. (Civil Engineering) Third Semester (C.B.S.)

Applied Mathematics - III

P. Pages : 3 **NRJ/KW/17/4347**

Time : Three Hours Max. Marks : 80

Notes :
1. All questions carry marks as indicated.
2. Solve Question 1 OR Questions No. 2.
3. Solve Question 3 OR Questions No. 4.
4. Solve Question 5 OR Questions No. 6.
5. Solve Question 7 OR Questions No. 8.
6. Solve Question 9 OR Questions No. 10.
7. Solve Question 11 OR Questions No. 12.
8. Assume suitable data whenever necessary.
9. Use of non programmable calculator is permitted.
10. Use of graph sheet is permitted.

1. Find Fourier series for $f(x) = x - x^2, \quad -\pi < x < \pi$. **7**

OR

2. Find half range cosine series for $f(x) = (x-1)^2, \; 0 < x < 1$. **7**

3. a) Solve $z(x+y)p + z(x-y)q = 2x^2 + y$. **6**

b) Solve $\left(D^2 + 2DD' - 8D'^2\right)z = e^{2x+y} + \sqrt{2x+3y}$. **6**

c) Solve $3\frac{\partial u}{\partial x} - 2\frac{\partial u}{\partial y} = 0$ given that $u(x,0) = 4e^{-x}$ by method of separation of variables. **6**

OR

4. a) A tightly stretched string with fixed end points x=0 and x=l, is initially at rest in its equilibrium position. If it is set vibrating by giving each point a velocity $\lambda x(\ell - x)$. Find the displacement of the string at any distance from one end at any time t, if $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$. **8**

b) Solve $z(p-q) = z^2 + (x+y)^2$. **5**

c) Solve $\left(D^2 - 3DD' + 2D'^2\right)z = e^{2x+3y} + \sin(x-2y)$. **5**

5. Find the extremal of the functional $\int_a^b \frac{\sqrt{1+(dy/dx)^2}}{x^2}\,dx$ given y(1)=0, y(2)=1. **7**

OR

6. Find the plane closed curve of fixed perimeter and maximum area. **7**

7. a) If $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ -4 & 4 & 3 \end{bmatrix}$, find the eigen values of $A^3 - 6A^2 + 3A - 2I$. Also find the spectral radius of the matrix represented by $A^3 - 6A^2 + 3A - 2I$. **6**

b) Diagonalize the matrix $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$. **6**

c) Using Sylvester's theorem show that $e^A = e^x \begin{bmatrix} \cosh x & \sinh x \\ \sinh x & \cosh x \end{bmatrix}$ where $A = \begin{bmatrix} x & x \\ x & x \end{bmatrix}$. **6**

OR

8. a) Verify Cayley Hamilton theorem and hence find A^{-1}, where $A = \begin{bmatrix} 1 & 2 & 4 \\ 2 & 1 & 2 \\ 4 & 2 & 1 \end{bmatrix}$. **6**

b) Solve by matrix method the equation $\frac{d^2y}{dt^2} - 5\frac{dy}{dt} + 6y = 0$ given y(0)=2, y'(0)=5. **6**

c) Reduce the quadratic form $5x^2 + 6y^2 + 7z^2 - 4xy + 4yz$ to canonical form by orthogonal transformation. **6**

9. a) Find the root of the equation $3x - \cos x - 1 = 0$ correct to third decimal place by regula falsi method. **6**

b) Solve by using Crout's method. **6**

$$4x + y - z = 13$$
$$3x + 5y + 2z = 21$$
$$2x + y + 6z = 14$$

c) Solve by Runge – Kutta fourth order method $\frac{dy}{dx} = \frac{y-x}{y+x}$, y(0)=1 by taking h=0.2. **6**

OR

10. a) Using Euler's modified method, solve the equation $\frac{dy}{dx} + xy^2 = 0$, $y(0) = 2$, find $y(0.2)$ taking h=0.1. **6**

b) Find the root of $x \log_{10} x - 2 = 0$ by Newton – Raphson method correct upto three decimal places. **6**

c) Solve by Gauss – Seidel method the following system of equations. **6**

$$x + 7y - 3z = -22$$

$$x - 2y + 3z = 18$$

$$2x - y + 6z = 22$$

11. a) A company produces three products P_1, P_2 and P_3 from two raw materials A and B, and labour L. One unit of product P_1 requires one unit of A, 3 units of B and 2 units of L. One unit of product P_2 requires 2 units of A and B each, and 3 units of L, while one unit of P_3 needs 2 units of A, 6 units of B and 4 unit of L. The company has a daily availability of 8 units of A, 12 units of B and 12 units of L. It is further known that the unit contribution margin for the products is Rs. 3, 2 and 5 respectively for P_1, P_2 and P_3. Formulate this problem as a linear programming problem. **6**

b) Solve graphically **6**

maximize $z = 6x_1 + 14x_2$

subject to $5x_1 + 4x_2 \geq 60$

$3x_1 + 7x_2 \leq 84$

$x_1 + 2x_2 \geq 18 \quad \& \quad x_1, x_2 \geq 0$

OR

12. Use simplex method to solve the following l.p.p. **12**

maximize $z = 5x + 3y$

sub. to $x + y \leq 2$

$5x + 2y = 10$

$3x + 8y \geq 12$

$x, y \geq 0$

****

B.E.Third Semester (Civil Engineering) (C.B.S.)

Mathematics - III

P. Pages : 3 **NKT/KS/17/7207**

Time : Three Hours Max. Marks : 80

Notes :
1. All questions carry marks as indicated.
2. Solve Question 1 OR Questions No. 2.
3. Solve Question 3 OR Questions No. 4.
4. Solve Question 5 OR Questions No. 6.
5. Solve Question 7 OR Questions No. 8.
6. Solve Question 9 OR Questions No. 10.
7. Solve Question 11 OR Questions No. 12.
8. Use of non programmable calculator is permitted.

1. Sketch the function **7**

$$f(x) = \begin{cases} \pi + x, & -\pi < x \le 0 \\ \pi - x, & 0 \le x < \pi \end{cases}$$

and find fourier series for f(x). Hence show that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + ----+\infty = \frac{\pi^2}{8}$

OR

2. Obtain half range cosine series for $f(x) = 2x - 1,\ 0 < x < 1.$ **7**

3. a) Solve $xq = yp + xe^{(x^2+y^2)}$ **6**

b) Solve $(D^2 - 3DD' + 2D'^2)\ Z = e^{2x+3y} + \sin(x - 2y)$ **6**

c) Solve by method of separation of variables the equation $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$, given that $u(x,0) = 6e^{-3x}$. **6**

OR

4. a) A stretched string with fixed ends at $x = o, x = \ell$ is initially in a position given by $y(x, o) = a\sin\left(\frac{\pi x}{\ell}\right)$. If it is released from the rest, show that the displacement of any point at a distance x from one end at any time t is given by $y(x,t) = a\sin\left(\frac{\pi x}{\ell}\right)\cos\left(\frac{\pi x}{\ell}\right)$ **8**

b) Solve $x^2(y - z)p + y^2(z - x)q = z^2(x - y)$ **5**

c) Solve $(D^2 + DD' - 6D'^2)z = y\cos x$. **5**

5. Prove that the sphere is the solid of revolution which, for given surface area has maximum volume. **7**

OR

6. Find the extrenals of the functionals $\int_{x_0}^{x_1} \left[y^2 - (y')^2 - 2y\sin x\right] dx$. **7**

7. a) Show that the vectors $x_1 = [1, 0, 2, 1]$, $x_2 = [3, 1, 2, 1]$, $x_3 = [4, 6, 2, -4]$, $x_4 = [-6, 0, -3, -4]$ are linearly dependent. Find the relation. **6**

b) Find the model matrix of $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ **6**

c) Verify Cayley-Hamilton theorem and express $A^5 - 4A^4 - 7A^3 + 11A^2 - A - 10I$ as a linear polynomial of A, if $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$. **6**

8. a) Diagonalise the matrix by orthogonal transformation. $A = \begin{bmatrix} 3 & 1 & 1 \\ 1 & 3 & -1 \\ 1 & -1 & 3 \end{bmatrix}$ **6**

b) Use Sylvester's theorem to verify $\log_e e^A = A$ where $A = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix}$. **6**

c) Solve $\frac{d^2y}{dt^2} - 4\frac{dy}{dt} + 3y = 0$ given $y(0) = 2,\ y'(0) = 2$ by matrix method. **6**

9. a) Use Regula-false method to find the root of the equation $\tan x + \tanh x = 1$ correct to third decimal place. **5**

b) Solve by Gauss - Seidel method **6**

$$2x + 10y + z = 13$$
$$2x + 2y + 10z = 14$$
$$10x + y + z = 12$$

c) Solve by using modified Euler's method the equation $\frac{dy}{dx} = \log(x + y)$, given y(0) = 2 for x = 0.2 take h = 0.1. **7**

OR

10. a) Find by Newton-Raphson method the root of the equation $\sin x - \frac{x+1}{x-1} = 0$ near to $x = -0.4$. **5**

b) Solve by Runge-Kutta fourth order method $\frac{dy}{dx} = \frac{y-x}{y+x}$, $y(0) = 1$, find $y(0 \cdot 2)$ by taking $h = 0.2$. **6**

c) Solve the system of equations by Crout's method **7**

$$5x + 2y + z = 12$$
$$x + 4y + 2z = 15$$
$$x + 2y + 5z = 20$$

11. Solve by simplex method the L.P.P. **12**

Minimize : $z = 3x_1 - 7x_2 + 5x_3$

Subject to : $5x_1 - x_2 + 4x_3 \le 15$

$-3x_1 + 4x_2 \le 8$

$4x_1 + 3x_2 - 8x_3 \le 31$

OR

12. a) A farmer wants to make sure that his herd get the minimum daily requirement of three basic nutrients A, B, C. Daily requirements are 15 units of A, 20 units of B, 30 units of C. One gram of product P has 2 units of A, 1 unit of B and 1 unit of C. One gram of product Q has 1 unit of A, 1 unit of B & 3 units of C. The cost of P is Rs 12/gram and cost of Q is Rs. 18/gram. Formulate the L. P. P. to minimize the cost. **6**

b) Solve the L. P. P. by graphical method. **6**

Maximize : $z = 6x_1 + 4x_2$

Subject to : $2x_1 + 3x_2 \le 30$

$3x_1 + x_2 \le 24$

$x_1 + x_2 \ge 3$

$x_1, x_2 \ge 0$

****

B.E.(Civil Engineering) Semester Third (C.B.S.)

Mathematics - III

P. Pages : 3 **KNT/KW/16/7207**

Time : Three Hours Max. Marks : 80

Notes :
1. All questions carry marks as indicated.
2. Solve Question 1 OR Questions No. 2.
3. Solve Question 3 OR Questions No. 4.
4. Solve Question 5 OR Questions No. 6.
5. Solve Question 7 OR Questions No. 8.
6. Solve Question 9 OR Questions No. 10.
7. Solve Question 11 OR Questions No. 12.
8. Due credit will be given to neatness and adequate dimensions.
9. Assume suitable data whenever necessary.
10. Use of non programmable calculator is permitted.

1. Find the Fourier series for the function. **7**

$$f(x) = \begin{cases} \pi + 2x, & -\pi \le x \le 0 \\ \pi - 2x, & 0 \le x \le \pi \end{cases}$$

Hence deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + ... + \infty = \frac{\pi^2}{8}$

OR

2. Find the half range sine series for the function. **7**

$$f(x) = \begin{cases} x, & 0 \le x \le 2 \\ 4 - x, & 2 \le x \le 4 \end{cases}$$

3. a) Solve $(x^2 - y^2 - z^2)p + 2xy\,q = 2xz$ **6**

b) Solve $\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x \partial y} - 8\frac{\partial^2 z}{\partial y^2} = e^{2x+y} + \sqrt{2x+3y}$. **6**

c) Solve $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + U$ given that $U(x,0) = 6e^{-3x}$ by the method of separation of variables. **6**

OR

4. a) A tightly stretched string with fixed end points $x = 0, x = \ell$ is initially at rest in its equilibrium position. It is set vibrating by giving each point a velocity $\lambda x(\ell - x)$, find the displacement of the string at any distance from one end at any time 't'. **8**

b) Solve $\frac{\partial^3 z}{\partial x^3} - 3\frac{\partial^3 z}{\partial x \partial y} - 2\frac{\partial^3 z}{\partial y^3} = \cos(x + 2y)$. **5**

c) Solve : $x(y - z)p + y(z - x)q = z(x - y)$ **5**

5. Find the curve passing through the points (x_1, y_1) and (x_2, y_2) which when rotated about x - axis gives minimum surface area. **7**

OR

6. Find the extremals of the isoperimetric problem **7**

$$I = \int_0^{\pi} \left[(y^1)^2 - y^2\right] dx$$

given that $\int_0^{\pi} y dx = 1,\ y(0) = 0,\ y(\pi) = 1$.

7. a) Examine the following system of vectors for linearly dependent. Find the relation between them. $X_1 = (1,1,-1,1),\ X_2 = (1,-1,2,-1)\ X_3 = (3,1,0,1)$ **6**

b) Find eigen values, eigen vectors and model matrix for the matrix. **6**

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

c) Given $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$, evaluate $A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$ by using Cayley - Hamilton theorem. **6**

OR

8. a) Use Sylvester's theorem to show that **6**
$3\tan[A] = (\tan 3)[A]$, where $A = \begin{bmatrix} -1 & 4 \\ 2 & 1 \end{bmatrix}$.

b) Solve $\frac{d^2y}{dt^2} - 3\frac{dy}{dt} - 10y = 0$, given $y(0) = 3,\ y'(0) = 15$ by matrix method. **6**

c) Reduce the quadratic form **6**
$6x^2 + 3y^2 + 3z^2 - 4xy + 4zx - 2yz$
to the canonical form.

9. a) Find the real root of the equation $x e^x = \cos x$ by Regula - Falsi method. **6**

b) Apply Gauss - Seidal method to solve the equations **6**
$2x - 3y + 20z = 25,\ 20x + y - 2z = 17$ and $3x + 20y - z = -18$.

c) Using modified Euler method, solve the differential equation **6**

$\frac{dy}{dx} = x + \sqrt{y}$ for $0 \le x \le 0.4$

given y = 1 when x = 0 and h = 0.2

OR

10. a) Use Runge - Kutta method to find approximate value of y for x = 0.2 when $\frac{dy}{dx} = x^2 + y^2$, given y (0) = 1, h = 0.1. **6**

b) Find a real root of the equation $x^3 - 2x - 5 = 0$ by Newton - Raphson method. **6**

c) Use Crout's method to solve the equations. **6**

$$4x + y - z = 13$$
$$3x + 5y + 2z = 21$$
$$2x + y + 6z = 14$$

11. A company manufactures three products A, B and C. Each product has to undergo operations on three types of machines m_1, m_2, m_3 before these are ready for sale. The time that each product requires on each machine are given in the following table. The table also show the net profit per unit on the sale of the three products. Formulate the mathematical model for this problem to maximize the total net profit of the company per day and obtain its solution by Simplex method. **12**

Machine ↓ Product →	Time required per unit (in minutes) A	B	C	Total time available per day (in minutes)
m_1	1	2	1	480
m_2	2	1	0	540
m_3	1	0	3	510
Profit Per Unit (Rs.)	4	3	5	

OR

12. a) Solve the following L.P.P. using Graphical method. **6**

Minimize $Z = 5x_1 + 8x_2$

Subject to $x_1 \le 4,\ x_2 \ge 2,\ x_1 + x_2 = 5$ and $x_1, x_2 \ge 0$

b) The standard weight of a special purpose brick is 5kg and it contains two basic ingredients B_1 and B_2. B_1 costs Rs. 5 per kg and B_2 costs Rs. 8 per kg. Strength considerations dictate that the brick contains not more than 4kg of B_1 and a minimum of 2kg of B_2. Since the demand for the product is likely to be related to the price of the brick. Formulate L.P.P. model to minimize the cost of the brick satisfying the above conditions. **6**

****

B.E. (Civil Engineering) Third Semester (C.B.S.)

Mathematics - III

P. Pages : 3 **TKN/KS/16/7295**

Time : Three Hours Max. Marks : 80

Notes :
1. All questions carry marks as indicated.
2. Solve Question 1 OR Questions No. 2.
3. Solve Question 3 OR Questions No. 4.
4. Solve Question 5 OR Questions No. 6.
5. Solve Question 7 OR Questions No. 8.
6. Solve Question 9 OR Questions No. 10.
7. Solve Question 11 OR Questions No. 12.
8. Use of non programmable calculator is permitted.

1. Obtain Fourier Series for **7**

$f(x) = |\sin x|, -\pi < x < \pi$

Hence show that

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots\ldots\infty$$

OR

2. Obtain half range sine series for $f(x) = \pi x - x^2$ in the interval (o, π). **7**

3. a) Solve : $(x^2 - yx)p + (y^2 - zx)q = z^2 - xy$ **6**

b) Solve : $\frac{\partial^3 z}{\partial x^3} - 2\frac{\partial^2 z}{\partial x^2 \partial y} = 2e^{2x} + 3x^2 y$. **6**

c) Solve using the method of separation of variables. **6**

$4\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 3u$, given that

$u = 3e^{-y} - e^{-5y}$, when x = 0.

OR

4. a) A tightly stretched string with fixed end points x = o and $x = \ell$ is initially in a position given by $y = y_o \sin^3\left(\frac{\pi x}{\ell}\right)$. If it is released from rest from this position. Find the displacement y (x, t). **8**

b) Solve : $p+3q=5z+\tan(y-3x)$. **5**

c) Solve : $\frac{\partial^3 z}{\partial x^3}-7\frac{\partial^3 z}{\partial x\,\partial y^2}+6\frac{\partial^3 z}{\partial y^3}=\sin(x+2y)$ **5**

5. Find the extremals of the functional $\int_1^2 \frac{\sqrt{1+\left(\frac{dy}{dx}\right)^2}}{x}\,dx$ given y (1) = 0, y (2) = 1. **7**

OR

6. Find the plane closed curve of fixed perimeter and maximum area. **7**

7. a) Investigate the linear dependence of the vectors $X_1=(3,1,-4)$, $X_2=(2,2,-3)$, $X_3=(0,-4,1)$, $X_4=(-4,-4,6)$ and if possible find the relation between them. **6**

b) Find eigen values, eigen vectors and modal matrix for $A=\begin{bmatrix}8 & -6 & 2\\ -6 & 7 & -4\\ 2 & -4 & 3\end{bmatrix}$. **6**

c) Verify Cayley Hamilton theorem for matrix $A=\begin{bmatrix}2, & -1 & 1\\ -1 & 2 & -1\\ 1 & -1 & 2\end{bmatrix}$ and find A^{-1}. **6**

OR

8. a) Diagonalise the following matrix by orthogonal transformation. **6**

$$A=\begin{bmatrix}6 & -2 & 2\\ -2 & 3 & -1\\ 2 & -1 & 3\end{bmatrix}$$

b) Use Sylvester theorem to show that **6**

$$e^A=e^x\begin{bmatrix}\cosh x & \sinh x\\ \sinh x & \cosh x\end{bmatrix}$$

where $A=\begin{bmatrix}x & x\\ x & x\end{bmatrix}$

c) Solve $\frac{d^2y}{dx^2}+4y=0$ given y = 8, $\frac{dy}{dx}=0$ when x = 0 **6**

9. a) Using the method of false position, find the root of the equation $x\log_{10}^{x}-1.2=0$; correct upto three places of decimal. **5**

b) Apply Crout's method to solve the equations **6**

$$3x+2y+7z=4$$
$$2x+3y+z=5$$
$$3x+4y+z=7$$

c) Given $\frac{dy}{dx} = x + y$, $y(0) = 1$, find y upto five terms by Picard's method and hence find y when x = 0.1 and x = 0.2 **7**

OR

10. a) Solve by 4th order Runge – Kutta method $\frac{dy}{dx} = xy + y^2$ given $y(0) = 1$, $h = 0.1$ find $y(0.1)$ and $y(0.2)$. **7**

b) Solve by Gauss Seidal Method. **6**

$$x + 7y - 3z = -22$$
$$5x - 2y + 3z = 18$$
$$2x - y + 6z = 22$$

c) Find by Newton Raphson method the root of the equation $e^x - 4x = 0$ near to 2, correct to three decimal places. **5**

11. A firm produces 3 products. These products are processed on 3 different machines. The time required to manufacture 1 unit of each of the 3 products and the daily capacity of the 3 machines are given in the following table. **12**

Machine	Time per Unit			
	Product 1	Product 2	Product 3	Machine capacity
M1	2	3	2	440
M2	4	–	3	470
M3	2	5	–	430

It is required to determine the daily number of units to be manufactured for each product. The profit per unit for product 1, 2 and 3 are Rs. 4, Rs. 3 and Rs. 6 respectively. It is assumed that the amounts produced are consumed in the market. Formulate and solve by Simplex Method.

OR

12. a) A farmer want to make sure that his herd get the minimum daily requirement of three basic nutrient A, B, C. Daily requirement are 15 unit of A 20 unit of B and 30 unit of C one gram of product P has 2 unit of A, 1 unit of B and 1 unit of C. One gram of product Q has 1 unit of A, 1 unit of B and 3 unit of C. The cost of P is Rs. 12/gram and cost at Q is Rs. 18/gram formulate this problem as linear programming problem so that the cost is minimum. **6**

b) Solve the linear programming using graphical method : **6**

Maximize $Z = 40x_1 + 60x_2$

Subject to : $4x_1 + 9x_2 \le 2000$

$12x_1 + 5x_2 \le 5000$

$6x_1 + 10x_2 \le 900$

$x_1, x_2 \ge 0$

****

Third Semester B. E. (Civil Engg.) (C.B.S.) Examination

APPLIED MATHEMATICS – III

Paper–III

Time : Three Hours]   [Max. Marks : 80

N. B. : (1) All questions carry marks as indicated.
(2) Solve **SIX** questions as follows :
Q. No. 1 OR Q. No. 2.
Q. No. 3 OR Q. No. 4.
Q. No. 5 OR Q. No. 6.
Q. No. 7 OR Q. No. 8.
Q. No. 9 OR Q. No. 10.
Q. No. 11 OR Q. No. 12.
(3) Use of non programmable calculator is permitted.

1. (a) Sketch the function

$$f(x) = \begin{cases} \pi + x \; ; & -\pi < x \leq 0 \\ \pi - x \; ; & 0 \leq x < \pi \end{cases}$$

and hence find Fourier series for f(x). Hence show that

$$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \text{-------}$$

7

OR

2. Obtain Fourier series for

$$f(x) = \begin{cases} \pi x \; ; & 0 \leq x \leq 1 \\ \pi\,(2 - x) \; ; & 1 \leq x \leq 2 \end{cases}$$ hence

Show that

$$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \text{-------}$$

7

3. (a) Solve $xq = yp + x\, e^{(x^2 + y^2)}$ 5

(b) Solve :

$$\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = 2\cos y - x\sin y$$ 7

(c) Solve using method of separation of variables,

$$3\frac{\partial u}{\partial x} - 2\frac{\partial u}{\partial y} = 0 \ , \ u(x, 0) = 4e^{-x}$$ 6

OR

4. (a) A Tightly stretched string with fixed end points $x = 0$, $x = \boldsymbol{l}$ is initially at rest in its equilibrium position. If it is set vibrating by giving each point a velocity $\lambda x(\boldsymbol{l} - x)$, find the displacement of the string at any distance from one end at any time t. 8

(b) Solve $(D^2 - 3DD' + 2D'^2)z = e^{2x+3y} + \sin(x - 2y)$. 5

(c) Solve $y^2 p - xyq = x(z - 2y)$. 5

5. Find the extremal of the functional

$$\int_{x_0}^{x_1} \{x^2(y')^2 + 2y^2 + 2xy\}\, dx$$ 7

OR

6. Find the plane closed curve of fixed perimeter and maximum area. 7

7. (a) Show by matrix, the vectors $X_1[2, 3, 1, -1]$, $X_2[2, 3, 1, -2]$, $X_3[4, 6, 2, -3]$ are linearly dependent. Find the relation between them. 5

(b) Reduce the matrix

$$A = \begin{bmatrix} 0 & 1 \\ 12 & -4 \end{bmatrix}$$

to the diagonal form. 6

(c) Verity Cayley – Hamilton theorem for given matrix A and hence find A^{-2}

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$$ 7

OR

8. (a) Use Sylvester's theorem to show that $3\tan A = (\tan 3)A$, where

$$A = \begin{bmatrix} -1 & 4 \\ 2 & 1 \end{bmatrix}$$ 6

(b) Solve the following differential equation by using matrix method

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0 \text{ , given } y(0) = 2,$$

$y'(0) = 5$ 6

(c) If $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -3 & 5 \\ 7 & 4 \\ 6 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$ and

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 2 & 5 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$$

Express x_1, x_2, x_3 in terms of z_1 and z_2. 6

9. (a) Find the root of equation $x \log_{10} x - 1.2 = 0$ by method of false position, correct upto four places of decimal. 6

(b) Apply Gauss – Seidal iteration method to solve the equations

$$2x - 3y + 20z = 25,$$
$$20x + y - 2z = 17,$$
$$3x + 20y - z = 18.$$ 6

(c) Use Runge–Kutta method to find approximate value of y for x = 0.2, when

$$\frac{dy}{dx} = x - 2y \text{ , given } y(0) = 1, \ h = 0.1 \qquad 6$$

OR

10. (a) Write Newton– Raphson formula for finding $\sqrt[3]{N}$, where N is a real number. Use it to find $\sqrt[3]{18}$ by assuming 2.5 as initial appr-oximation. 6

(b) Use Crout's method to solve the equations

$$5x + 2y + z = 12,$$
$$x + 4y + 2z = 15,$$
$$x + 2y + 5z = 20. \qquad 6$$

(c) By Milne's predictor–corrector method

$$\frac{dy}{dx} = \frac{1}{x+y} \ ; \ y(0) = 2, \ y(0.2) = 2.0933,$$

$y(0.4) = 2.1755$, $y(0.6) = 2.2493$, find $y(0.8)$. 6

11. The standard weight of a special purpose brick is 5 kg and it contains two basic ingredients B_1 and B_2. B_1 costs Rs 5/kg and B_2 costs Rs 8/kg. Strength considerations dictate that the brick contains not more than 4 kg of B_1 and a minimum of 2 kg of B_2. Since the demand for the product is likely to be related to the price of the brick, find graphically the minimum cost of the brick satisfying the above conditions. 12

OR

12. Solve the following L.P.P.

Maximize $Z = 12x_1 + 15x_2 + 14x_3$
subject to

$$-x_1 + x_2 \leq 0$$
$$-x_2 + 2x_3 \leq 0$$
$$x_1 + x_2 + x_3 \leq 100$$
$$x_1, x_2, x_3 \geq 0 \qquad 12$$