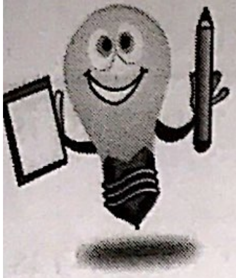
Numbers

Concept 1: Classification of Numbers

Class	Symbol	Description
Natural number	**N**	All countable numbers. N = {1, 2, 3, 4, ...}
Whole number	**W**	Zero added to the set of natural numbers becomes the set of whole numbers. W = {0, 1, 2, 3, 4, …}
Integer	**Z**	The integers are natural numbers (1, 2, 3, ...), their negatives (−1, −2, −3, ...) and 0.
Rational number	**Q**	Any number that can be expressed in the form of p/q, where p and q are integers and $q \neq 0$, is called a rational number. Q denotes the set of rational numbers. Example: 4, 4/5, -5/8, 1/3, 2.74, etc.
Irrational number	**Q'**	Any number that cannot be expressed in the form of p/q, where p and q are integers and $q \neq 0$, is called an irrational number. They are generally the non-terminating and non-recurring decimal fractions. Example: $\sqrt{3}$, π, e, etc.
Real number	**R**	Set of numbers that include both rational and irrational numbers is called real numbers. $R = Q \cup Q'$
	i	Square root of a negative real number denoted by the symbol i, where $i^2 = -1$. Ex: 5i, 3i, $\sqrt{3}i$
Complex number	**C**	A complex number consists of two parts: a real number and an imaginary number, and is expressed in the form a + bi (i is the notation for imaginary part of the number). Ex: 7 + 2i

iii. A number that is: Real, Rational and Non integer



Concept 2: Divisibility Tests

Divisibility Test	Condition	Example
2	The **last digit** is even (0,2,4,6,8)	25**6** is divisible by 2 25**7** is not divisible by 2
3	The **sum** of the digits is divisible by 3	471(4 + 7 + 1 = **12**, 12/3 = 4) is divisible by 3 322 (3 + 2 + 2 = **7**, 7 is not divisible by 3) is not divisible by 3
4	The **last two** digits are divisible by 4	25**16** (16/4 = 4) is divisible by 4 24**14** is not divisible by 4
5	The **last digit** is 0 or 5	46**5** is divisible by 5 71**6** is not divisible by 5
6	The number is divisible by both **2 and 3**	78 (is **even** and 7 + 8 = **15**, 15/3 = 5) is divisible by 6 80 (is **even** but 8 + 0 = 8, 8 is not divisible by 3) is not divisible by 6
7	Double the last digit and subtract it from the rest of the number and the answer should either be **0 or a number divisible by 7**	343 (twice 3 is 6, 34 – 6 = **28** and 28/7 = 4) is divisible by 7 147 (twice 7 is 14, 14 – 14 = **0**) is 192 (twice 2 is 4, 19 – 4 = **15** ≠ 0 nor a multiple of 7) is not divisible by 7
8	The **last three** digits are divisible by 8	102**192** (192/8 = 24) is divisible by 8 153**190** (190 is not divisible by 8) is not divisible by 8

9	The **sum** of the digits should be divisible by 9	6561 ($6 + 5 + 6 + 1 =$ **18** and $18/9 = 2$) is divisible by 9 611 ($6 + 1 + 1 = 8$ which is not divisible by 9) is not divisible by 9
10	The number **ends with 0**	15**0** is divisible by 10 14**1** is not divisible by 10
11	The difference between the sum of digits in odd places and the sum of digits in even places should be either **0 or a number divisible by 11**	12364 ($(1 + 3 + 4) - (2 + 6) =$ **0**) is divisible by 11 98736 ($(9 + 7 + 6) - (8 + 3) =$ **11**) is 4234 ($(4 + 3) - (2+4) =$ **1**, $\neq 0$ or 11) is not divisible by 11
12	The number is divisible by both **3 and 4**	144 ($1 + 4 + 4 =$ **9**, $9/3 = 3$ also last two digits **44** is divisible by 4) is divisible by 12 164 ($1 + 6 + 4 =$ **11**, 11 is not divisible by 3 but last two digits **64** is divisible by 4) is not divisible by 12
13	Multiply the last digit by 4 and add it to the rest of the number and the answer should be a **multiple of 13**	2379 ($237 + (9 \times 4) = 273$, $27 + (3 \times 4) =$ **39**, $39/13 = 3$) is divisible by 13 599 ($59 + (9 \times 4) = 95$, $9 + (5 \times 4) =$ **29**, not divisible by 13) is not divisible by 13

Did you know?

Square numbers can be generated by adding consecutive odd numbers.

$1^2 = 1 = 1$

$2^2 = 4 = 1 + 3$

$3^2 = 9 = 1 + 3 + 5$

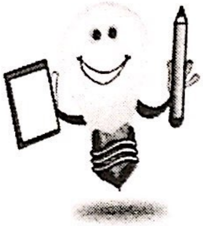
$4^2 = 16 = 1 + 3 + 5 + 7$

$5^2 = 25 = 1 + 3 + 5 + 7 + 9$

$6^2 = 36 = 1 + 3 + 5 + 7 + 9 + 11$

$7^2 = 49 = 1 + 3 + 5 + 7 + 9 + 11 + 13$ and so on…

Conversely, the sum of N odd integers starting with $1 = N^2$



Concept 3: Power Cycles & Remainders

The units digit of a number when raised to various powers exhibit cyclicity i.e., they follow a pattern and repeat themselves after a certain number of times.

For instance, 2^1 ends with 2; 2^2 ends with 4; 2^3 ends with 8; 2^4 ends with 6; 2^5 ends with 2 again.

The same sequence of the last digits shall be repeated for the subsequent powers. So if we want to find the last digit of 2^{45} divide 45 by 4. The remainder is 1. So the last digit of the number would be the same as last digit of 2^1. Here, we are dividing by 4 because, in the case of powers of 2, the pattern consists of 4 numbers. This concept can be extended to other numbers as follows:

Number	Repeating pattern (Power cycle)
0	0
1	1
2	2, 4, 8, 6
3	3, 9, 7, 1
4	4, 6
5	5
6	6
7	7, 9, 3, 1
8	8, 4, 2, 6
9	9, 1

Note: Similar to the last digits of powers of numbers (cyclicity), remainders of powers of numbers divided by the same divisor also form a certain pattern.

Concept 4: Factors & Multiples

Factors

Factors of a number are the numbers which will divide the given number without leaving any remainders.

Multiples

Multiples of a number are generated by multiplying other integers with the given number.

Highest Common Factor (HCF)

The highest common factor of two numbers is the largest number that will divide both the numbers without leaving a remainder. It is also called as the greatest common divisor or GCD.

Example: Consider the numbers 12 and 15.

Factors of 12: 1, 2, 3, 4, 6 and 12

Factors of 15: 1, 3, 5 and 15

Common factors: 1, 3

Highest common factor: 3

Least Common Multiple (LCM)

The least common multiple of two numbers is the lowest number that is a multiple of both the numbers.

Example: Consider the numbers 12 and 15.

Multiples of 12: 12, 24, 36, 48, 60, 72, 84, 96, 108, 120, 132 …

Multiples of 15: 15, 30, 45, 60, 75, 90, 105, 120, 135 …

Common multiples: 60, 120 …

Least common multiple: 60

Note: The same concepts of HCF and LCM can be extended for more than two numbers.

Finding HCF & LCM with prime factorization

Suppose we want to find the HCF and LCM of the numbers 60 and 72, we start by writing each number as a product of its prime factors.

$$60 = 2 \times 2 \times 3 \times 5$$
$$72 = 2 \times 2 \times 2 \times 3 \times 3$$

To make the next stage easier, we need to write these so that each new prime factor begins in the same place:

$$60 = 2 \times 2 \times 3 \times 5$$
$$72 = 2 \times 2 \times 2 \times 3 \times 3$$

All the "2"s are now above each other, as are the "3"s etc. This allows us to match up the prime factors.

The highest common factor is found by multiplying all the factors which appear in both lists.

$$\begin{array}{lcccccccccc} 60 = & 2 & \times & 2 & & & \times & 3 & \times & 5 & \\ 72 = & 2 & \times & 2 & \times & 2 & \times & 3 & \times & 3 & \\ \hline \text{HCF} = & 2 & \times & 2 & & & \times & 3 & & & = 12 \end{array}$$

The lowest common multiple is found by multiplying all the unique factors which appear in either list.

So the LCM of 60 and 72 is 2 x 2 x 2 x 3 x 3 x 5 which is 360.

> **Question:** Why have we never come across HCM or LCF in basic mathematics?

Concept 5: Applications of HCF & LCM

Type	Approach
Find the greatest number that will exactly divide a, b and c.	HCF (a, b, c)
Find the greatest number that will divide a, b and c leaving remainders of x, y and z respectively.	HCF (a-x, b-y, c-z)
Find the greatest number which when it divides a, b and c will leave the same remainder in each case.	HCF (a-b, b-c, c-a)
Find the least number which is exactly divisible by a, b and c.	LCM (a, b, c)
Find the least number which when divided by a, b and c leaves the same remainder 'r' in each case.	LCM (a, b, c) + r
Find the least number which when divided by a, b and c leaves the remainders x, y and z respectively.	Check if a-x = b-y = c-z = K. If this is the case, then LCM (a, b, c) – K