Optimization and Decision Analysis

Operations Research, Constrained Optimization, Linear Programming, Sensitivity Analysis

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Nonlinear Programming

Objective function minimize/maximize

Optimize $z = f(x_1, x_2, \ldots x_n)$

Subject to $g_1(x_1, x_2, \ldots x_n) \leq or \geq or = b_1$

$g_2(x_1, x_2, \ldots x_n) \leq or \geq or = b_2$

$g_3(x_1, x_2, \ldots x_n) \leq or \geq or = b_3$

Constraints (Inequality and equality)

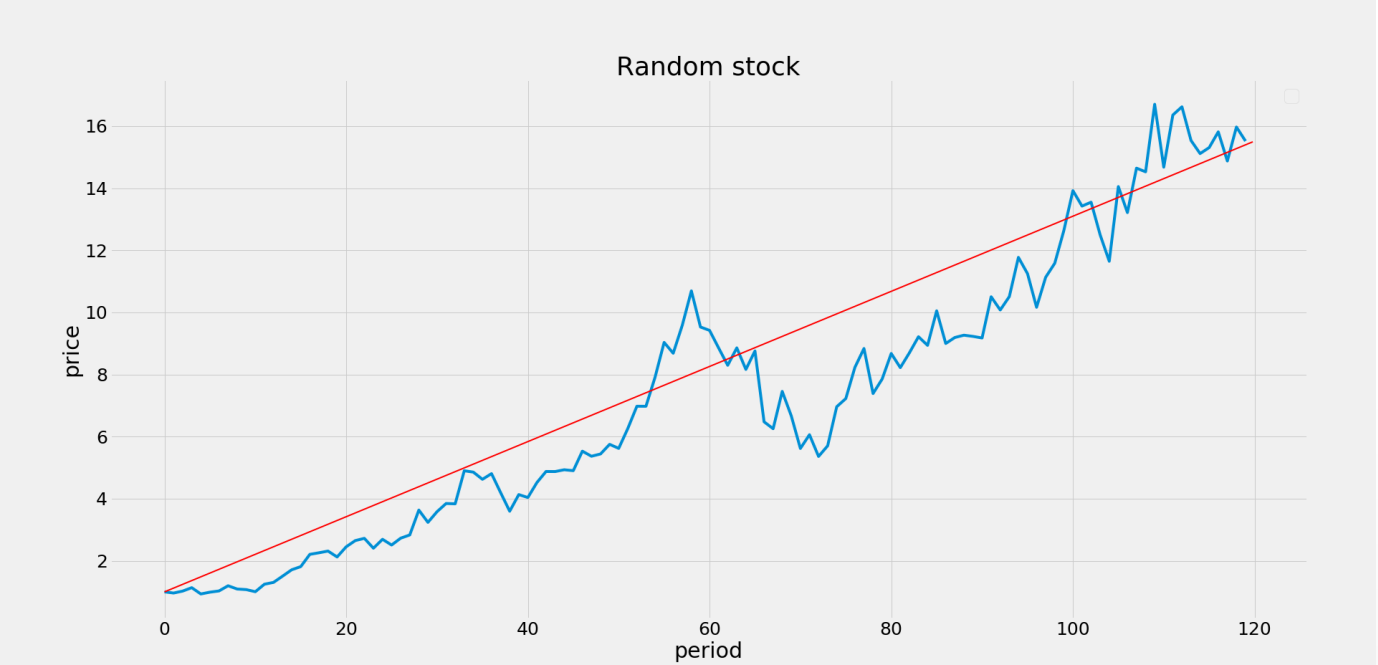
$Nonlinear function$

The optimization problem is generally specified using matrices and vectors.

Case Study: Portfolio Allocation of Stocks

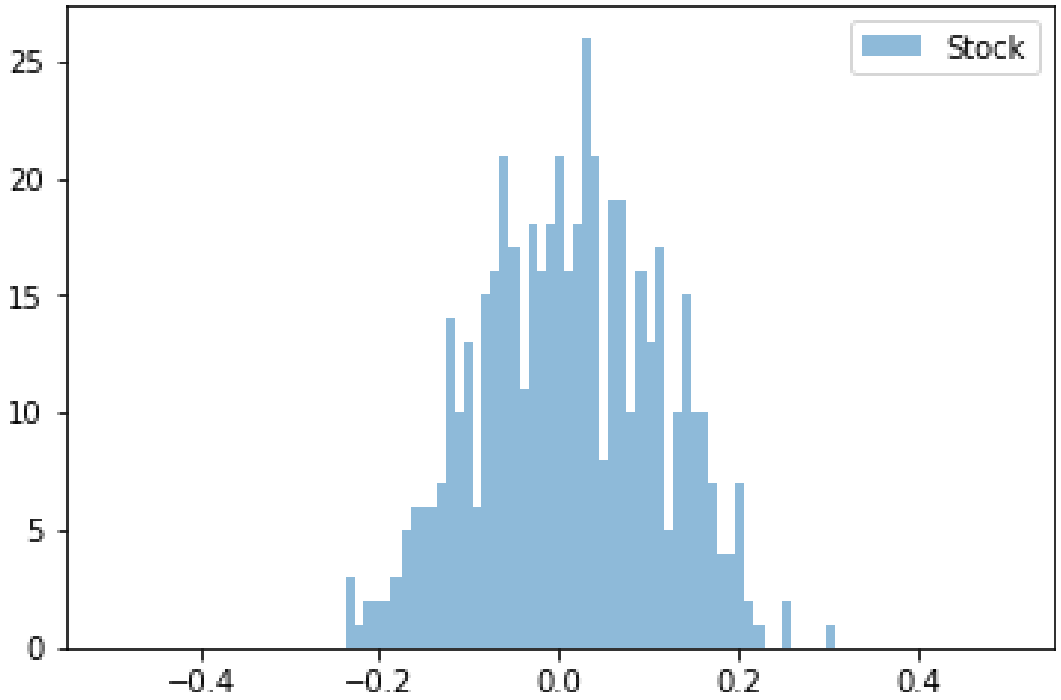
How does one quantify Risk?

Risk => Measure of Uncertainty



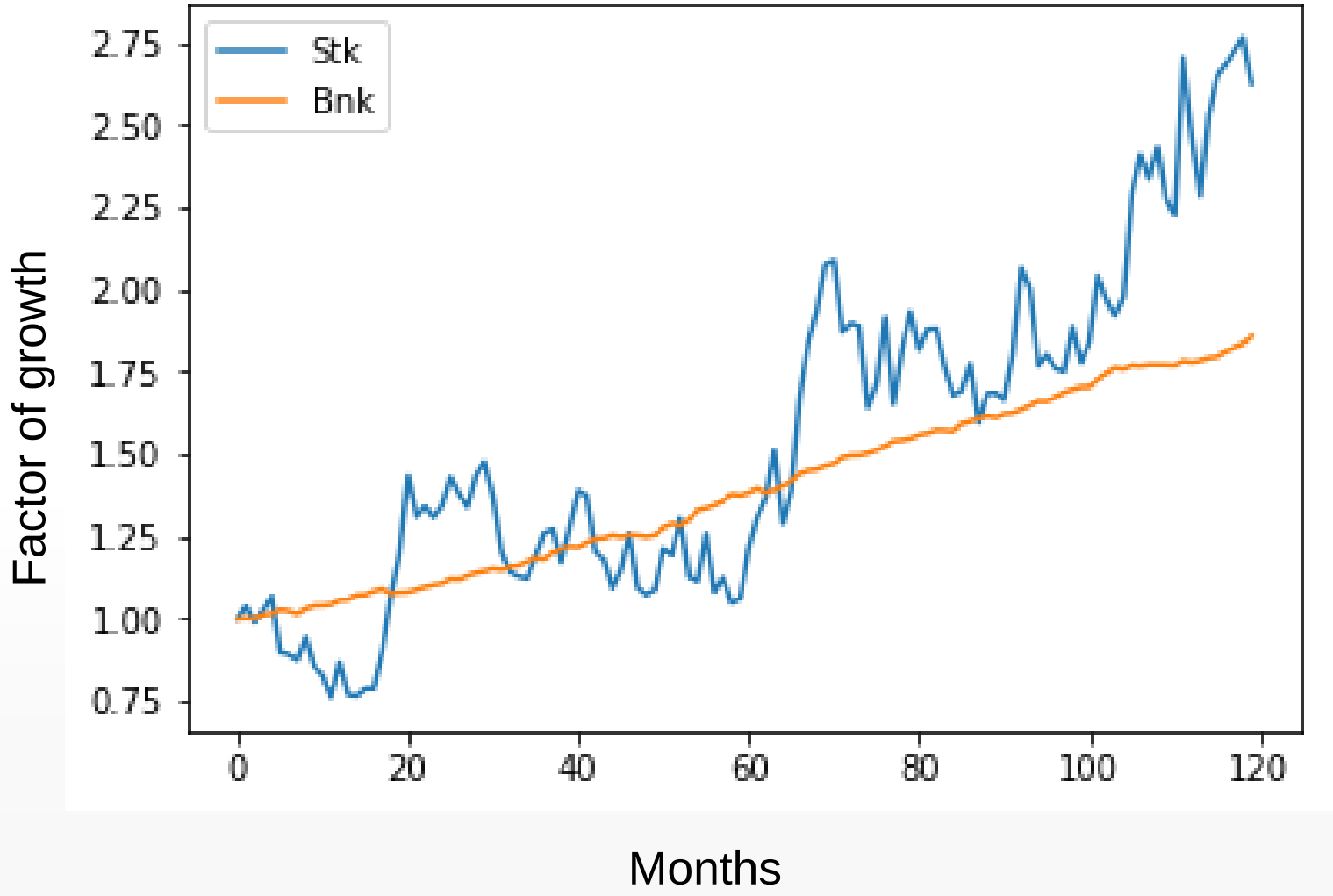
One Definition of Risk = Std(daily %price changes)

A Simple Example: Stock vs Bank Deposit



- Stock: mean annual ret=15%, std dev=0.35
- Bank: mean ret = 6% , std dev = 0.02
- How do you invest?

- 60% in Stocks & 40% in Bank
- Expected Return = 0.6* 15% + 0.4*5% = 11%
- Regret not investing fully in stock



$$\text{Sharpe Ratio} = \frac{\text{Total Returns}}{\text{Resultant Variance}}$$

Investment Goal = Maximize Sharpe Ratio

Stock 1: R_1, w_1, σ_1 Stock 2: R_2, w_2, σ_2

Total Return: $R = w_1 R_1 + w_2 R_2$

Total Standard Deviation: $\sigma = \sqrt{w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1 w_2 cov(\sigma_1, \sigma_2)}$

Objective Function: Sharpe Ratio: $\frac{R}{\sigma} = \frac{w_1 R + w_2 R_2}{\sqrt{w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1 w_2 cov(\sigma_1, \sigma_2)}}$

Goal: Maximize Sharpe Ratio

Constraints: $w_1, w_2 \geq 0$ $w_1 + w_2 = 1$

Stock 1: 15%, 35%, w_1 Stock 2: 6%, 2%, w_2

Total Return: $R = 0.15w_1 + 0.06w_2$

Total Standard Deviation: $\sigma = \sqrt{w_1^2 0.35^2 + w_2^2 0.02^2 + 2w_1 * w_2 cov(\sigma_1, \sigma_2)}$

Thank You