Assignment 4

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1 P151, 3.1 Computer Problems: 3.

Write a Matlab function polyinterp.m that takes as input a set of (x,y) interpolating points and another x_0 , and outputs y_0 , the value of the interpolating polynomial at x_0 . The first line of the file should be function $y_0 = polyinterp(x, y, x_0)$, where x and y are input vectors of data points.

The Matlab code is as follows:

```
\begin{array}{l} \textbf{function} \  \  \, y0 = & polyinterp\,(x\,,y\,,x0\,) \\ k = & 0; \\ \textbf{while}\,(k < \textbf{length}\,(x) \&\& \textbf{length}\,(x) = = \textbf{length}\,(y)\,) \\ k = & k+1; \\ c = & newtdd\,(x\,,y\,,k)\,; \\ x1 = & -1:.01:4; \\ y1 = & nest\,(k-1,c\,,x1\,,x)\,; \\ y0 = & nest\,(k-1,c\,,x0\,,x)\,; \\ \textbf{plot}\,(x\,,y\,,\,'bo\,'\,,x1\,,y1\,,x0\,,y0\,,\,'r -\!\!\!-\!\!\!o\,') \\ \textbf{end} \end{array}
```

The input is:

$$x = 1$$
 2 3 $y = 3.0000$ 5.6000 4.0000

predictable point is $x_0 = 6$, The output is:

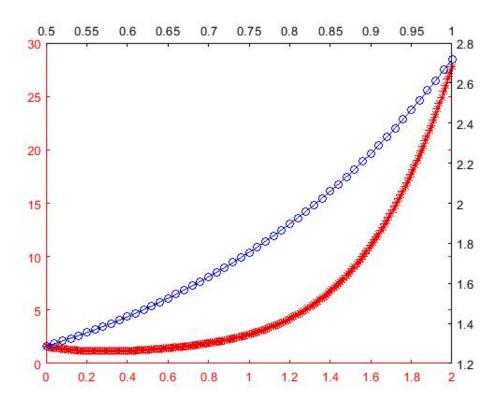
ans = -26.0000

2 P157, 3.2 Computer Problems: 1.

(a) Use the method of divided differences to find the degree 4 interpolating polynomial P4(x) for the data (0.6,1.433329), (0.7,1.632316), (0.8,1.896481), (0.9,2.247908), and (1.0,2.718282). (b) Calculate P4(0.82) and P4(0.98). (c) The preceding data come from the function $f(x) = e^{x^2}$. Use the interpolation error formula to find upper bounds for the error at x = 0.82 and x = 0.98, and compare the bounds with the actual error. (d) Plot the actual interpolation error $P(x)-e^{x^2}$ on the intervals [.5,1] and [0,2].

```
x0 = (0.6:0.1:1);
y0 = [1.433329 \ 1.632316 \ 1.896481 \ 2.247908]
   2.718282];
c=newtdd(x0,y0,5);
syms x y;
y=c(1)+c(2)*(x-x0(1))+c(3)*(x-x0(1))*(x-x0(2))
+c(3)*(x-x0(1))*(x-x0(2))*(x-x0(3))+c(4)*(x-x0(3))
   (1))*(x-x0(2))*(x-x0(4));
y=vpa(simplify(y),6);
disp(y)
x1 = [0.82, 0.98];
y1=zeros(1, length(x1));
r=zeros(1, length(x1));
tmp=ones(1, length(x1));
ff = diff(exp(x^5), 5);
t = [-1:0.01:1];
a=zeros(1, length(t));
```

```
for i = 1:2
     y1(i)=nest(4,c,x1(i),x0);%拟合值
     r(i) = abs(exp(x1(i)^2) - y1(i));
     for j=1:4
     tmp(i) = tmp(i) *(x1(i) - x0(j));
     end
     for k=1:length(t)
     a(k)=subs(ff,t(k));
     end
     tmp(i)=abs(tmp(i)/factorial(5))*max(k);
     end
     for i = 1:2
     fprintf('预测值%d:%d',i,y1(i))
     fprintf('误差上界%d:%d',i,tmp(i))
     end
     t1 = [0.5:0.01:1];
     t2 = [0:0.01:2];
     ans1 = nest(4, c, t1, x0);
     ans2=nest (4, c, t2, x0);
     hl1 = plot(t2, ans2, '-*r');
     ax1 = gca;
     set(ax1, 'XColor', 'r', 'YColor', 'r')
     ax2 = axes('Position', get(ax1, 'Position'),...
     'XAxisLocation', 'top',...
     'YAxisLocation', 'right',...
     'Color', 'none',...
     'XColor', 'k', 'YColor', 'k');
     hl2 = line(t1, ans1, 'Color', 'b', 'Parent', ax2);
     hl2. Marker='o';
The output is:
  P_4(x) = 6.93957 * x^3 - 11.6823 * x^2 + 8.36355 * x - 0.878137
       预测值 1:1.958910
                          误差上界 1:7.075200e-05
```



3 P166, 3.3 Computer Problems: 2.

Build a Matlab program to evaluate the cosine function correct to 10 decimal places using Chebyshev interpolation. Start by interpolating on a fundamental domain $[0,\pi/2]$, and extend your answer to inputs between 40^{-4} and 10^4 . You may want to use some of the Matlab code written in this chapter.

Code is below:

function $y=\cos 1(x)$

```
\begin{array}{l} n\!=\!10; \\ b\!=\!\mathbf{pi}/4\!+\!(\mathbf{pi}/4)\!*\!\mathbf{cos}\left((1\!:\!2\!:\!2\!*\!n\!-\!1)\!*\!\mathbf{pi}/(2\!*\!n)\right); \\ yb\!=\!\mathbf{cos}(b); \\ c\!=\!\operatorname{newtdd}(b,yb,n); \\ s\!=\!1; \\ x1\!=\!\operatorname{mod}(x,2\!*\!\mathbf{pi}); \\ \mathbf{if} \ 3\!*\!\mathbf{pi}/2\!>\!x1\!>\!\mathbf{pi}/2 \\ x1 = \mathbf{abs}(\mathbf{pi}\!-\!x1); \\ s = -1; \\ \mathbf{end} \\ \mathbf{if} \ x1 > 3\!*\!\mathbf{pi}/2 \\ x1 = 2\!*\!\mathbf{pi}\!-\!x1; \\ \mathbf{end} \\ y = s\!*\!\operatorname{nest}(n\!-\!1,c,x1,b); \end{array}
```

4 P178, 3.4 Computer Problems: 1.

Find the equations and plot the natural cubic spline that interpolates the data points (a) (0,3), (1,5), (2,4), (3,1) (b) (-1,3), (0,5), (3,1), (4,1), (5,1).

```
x0=[0 1 2 3];
y0=[3 5 4 1];
x1=[-1 0 3 4 5];
y1=[3 5 1 1 1];
splineplot(x0,y0,4)
figure;
splineplot(x1,y1,5)
```

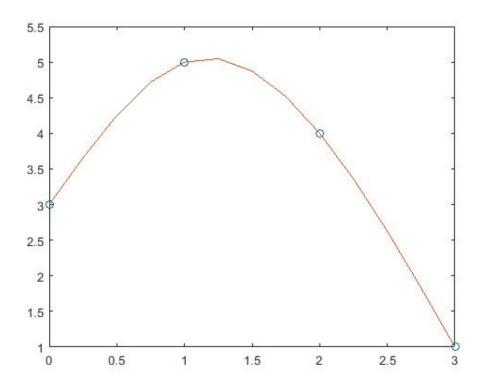


图 2: Curve 1.

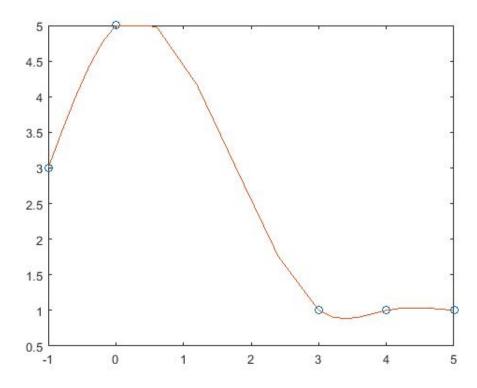


图 3: Curve 2.