Relational Algebra

Motivation

We learnt how to use relations to model data

How can we retrieve (interesting) data?

We need a query language

- declarative (to allow for abstraction)
- optimisable (➔ less expressive than a programming language)
- relations as input and output

E.F. Codd (1970): Relational Algebra

Characteristics of an Algebra

Expressions
- are constructed with operators from atomic operands (constants, variables, ….)
- can be evaluated
- expressions can be equivalent
  - …if they return the same result for all values of the variables

Equivalence gives rise to identities between (schemas of) expressions

The value of an expression is independent of its context
- e.g., 5 + 3 has the same value, no matter whether it occurs as

$$10 - (5 + 3) \qquad \text{or} \qquad 4 \cdot (5 + 3)$$

Atomic expressions:

numbers and variables

Operators: $+, -, \cdot, :$

Identitities:

$x + y = y + x$

$x \cdot (y + z) = x \cdot y + x \cdot z$

… and so on

Consequence: subexpressions can be replaced by equivalent expressions without changing the meaning of the entire expression

Relational Algebra: Principles

Atoms are relations

Operators are defined for arbitrary instances of a relation

Two results have to be defined for each operator

- result schema (depending on the schemas of the argument relations)
- result instance (depending on the instances of the arguments)

Set theoretic operators

- union “$\cup$”, intersection “$\cap$”, difference “\”

Renaming operator ρ

Removal operators

- projection π, selection σ

Combination operators

- Cartesian product “$\times$”, joins “$\bowtie$”

Relational Algebra

Relational Algebra is a procedural data manipulation language (**DML**).

It specifies operations on relations to define new relations:

Unary Relational Operations: Select, Project

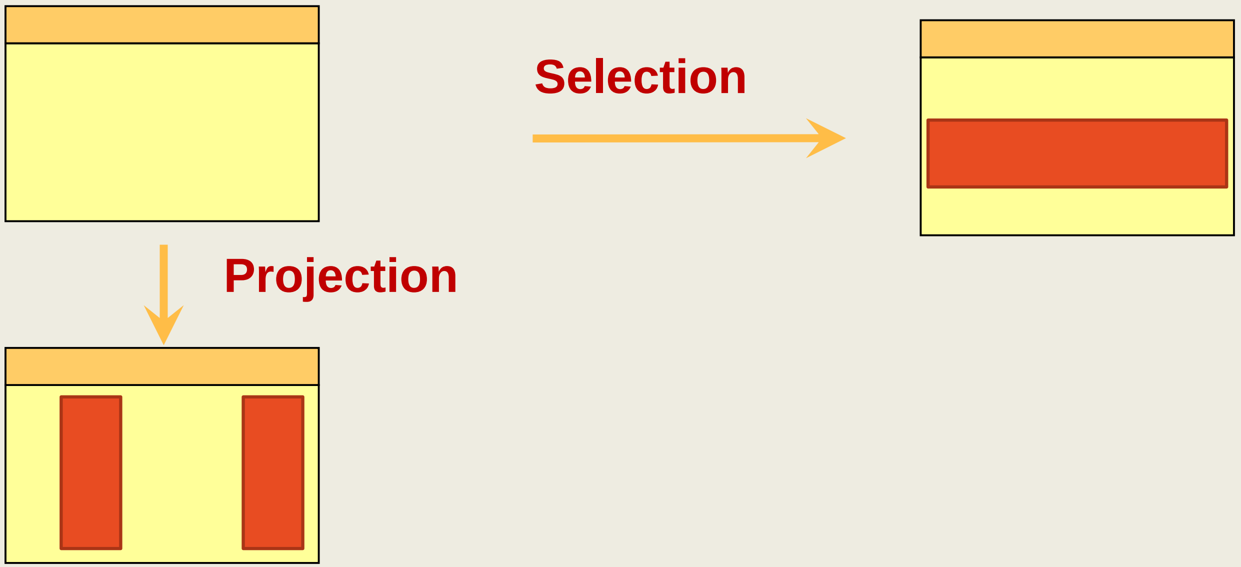
Operations from Set Theory: Union, Intersection, Difference, Cartesian Product

Binary Relational Operations: Join, Divide.

Projection and Selection

Two "orthogonal" operators

- Selection:
  - horizontal decomposition
- Projection:
  - vertical decomposition



SELECT

- The SELECT operation is used to choose a *subset* of the tuples (rows) from a relation that satisfies a **selection condition**, denoted by:

$$\sigma_{<selection\ condition>}(R)$$

- Result:

  - Schema: the schema of R

  - Instance: the set of all $t \in R$ that satisfy select condition

- Intuition: Filters out all tuples that do not satisfy select condition

Selection Conditions

Elementary conditions:

`<attr> op <val>` or `<attr> op <attr>` or `<expr> op <expr>`

where op is “=”, “<”, “≤”, (on numbers and strings)
“LIKE” (for string comparisons),…

Example:

- age ≤ 24
- phone LIKE ‘0039%’
- salary + commission ≥ 24 000

Combined conditions (using Boolean connectives):

`C1 and C2` or `C1 or C2` or `not C`

STUDENT:

Person#	Name
1	Dr C.C.Chen
3	Ms K.Juliff
4	Ms J.Gledill
5	Ms B.K.Lee

RESEARCHER:

Person#	Name
1	Dr C.C.Chen
2	Dr R.G.Wilkinson

COURSE:

Department	Degree
Psychology	Ph.D.
Comp.Sci.	Ph.D.
Comp.Sci.	M.Sc.
Psychology	M.Sc.

ENROLMENT:

Enrolment#	Supervisee	Supervisor	Department	Degree
1	1	2	Psychology	Ph.D.
2	3	1	Comp.Sci.	Ph.D.
3	4	1	Comp.Sci.	M.Sc.
4	5	1	Comp.Sci.	M.Sc.

ENROLMENT:

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4	5	1	Comp.Sci.	M.Sc.

Q: Select the enrolment records for the students whose supervisor is Person 1

$$\sigma_{(Supervisor=1)}(ENROLMENT)$$

Enrolment#	Supervisee	Supervisor	Department	Degree
2	3	1	Comp.Sci	Ph.D.
3	4	1	Comp.Sci	M.Sc.
4	5	1	Comp.Sci	M.Sc.

ENROLMENT:

Enrolment#	Supervisee	Supervisor	Department	Degree
1	1	2	Psychology	Ph.D.
2	3	1	Comp.Sci.	Ph.D.
3	4	1	Comp.Sci.	M.Sc.
4	5	1	Comp.Sci.	M.Sc.

Q: Select the enrolment records for Person 1's non-Ph.D. students

$$\sigma_{(Supervisor=1\ AND\ Degree \neq 'Ph.D.')}(ENROLMENT)$$

$$\sigma_{(Supervisor=1\ AND\ NOT\ Degree='Ph.D.')}(ENROLMENT)$$

Enrolment#	Supervisee	Supervisor	Department	Degree
3	4	1	Comp.Sci	M.Sc.
4	5	1	Comp.Sci	M.Sc.

Properties of SELECT

- Commutative:

$$\sigma_{<cond1>}(\sigma_{<cond2>}(R)) = \sigma_{<cond2>}(\sigma_{<cond1>}(R))$$

- Consecutive selects can be combined:

$$\sigma_{<cond1>}(\sigma_{<cond2>}(R)) = \sigma_{<cond1>\ AND\ <cond2>}(R)$$

PROJECT

- The PROJECT operation is used to project a subset of the attributes (column) of a relation, denoted by:

- General form: $\pi_{<attribute\ list>}(R)$

- Result:
  - schema: attribute list $(A_1,\ldots,A_k)$
  - instance: the set of all subtuples $t[A_1,\ldots,A_k]$ where $t \in R$

- The PROJECT operation *removes any duplicate tuples*, so the result of the PORJECT operation is a set of distinct tuples, and this is known as **duplicate elimination**.

ENROLMENT:

Enrolment#	Supervisee	Supervisor	Department	Degree
1	1	2	Psychology	Ph.D.
2	3	1	Comp.Sci.	Ph.D.
3	4	1	Comp.Sci.	M.Sc.
4	5	1	Comp.Sci.	M.Sc.

Q: Find departments and degree requirements for the courses that students enroll.

$$\pi_{\{department,degree\}}(ENROLMENT)$$

Department	Degree
Psychology	Ph.D.
Comp.Sci	Ph.D.
Comp.Sci	M.Sc.

Properties of PROJECT

- if <list2> contains all the attributes in <list1> then

$$\pi_{<list1>}(\pi_{<list2>}(R)) = \pi_{<list1>}(R)$$

else

The operation is not well defined.

- commutes with selection:

$$\pi_X(\sigma_B(R)) = \sigma_B(\pi_X(R)) \text{ (?)}$$

Commutes follows if and only if the attribute names used in SELECT is a subset of the attribute list in PROJECT

Check the example below:

$$\pi_{\{degree\}}(\sigma_{(Department='Psychology')}(ENROLMENT)) =$$

Degree
Ph.D.

$$\sigma_{(Department='Psychology')}(\pi_{\{degree\}}(ENROLMENT)) =$$ Error as SELECT cannot find Department

UNION

- UNION is a relation that includes all tuples that are either in the left relation or in the right relation or in both relations, denoted by

$$R \cup S = \{t : t \in R \, or \, t \in S\}$$

- Note: Union requires R and S to be **union compatible**:
  that there is a 1-1 correspondence between their attributes,
  in which corresponding attributes are over the same domain

Example:

$R1 \leftarrow \sigma_{(Supervisor=2)} (ENROLMENT)$

$R2 \leftarrow \sigma_{(Name='M.Sc')} (ENROLMENT)$

$R1 \cup R2 =$

Enrolment#	Supervisee	Supervisor	Department	Name
1	1	2	Psych.	Ph.D.
3	4	1	Comp.Sci	M.Sc
4	5	1	Comp.Sci	M.Sc

Example: $STUDENT \cup RESEARCHER =$

Person#	Name
1	Dr C.C.Chen
3	Ms K.Juliff
4	Ms J.Gledhill
5	Ms B.K.Lee
2	Dr R.G.Wilkinson

INTERSECTION

- INTERSECTION is a relation that includes all tuples that are in both relations, denoted by

$$R \cap S = \{t : t \in R \, and \, t \in S\}$$

- Example:

$$R_1 \leftarrow \sigma_{(Supervisor=1)}(ENROLMENT)$$

$$R_2 \leftarrow \sigma_{(Degree='Ph.D.')}(ENROLMENT)$$

$$R_1 \cap R_2 =$$

Enrolment#	Supervisee	Supervisor	Department	Name
2	3	1	Comp.Sci.	Ph.D.

STUDENT:

Person#	Name
1	Dr C.C.Chen
3	Ms K.Juliff
4	Ms J.Gledill
5	Ms B.K.Lee

RESEARCHER:

Person#	Name
1	Dr C.C.Chen
2	Dr R.G.Wilkinson

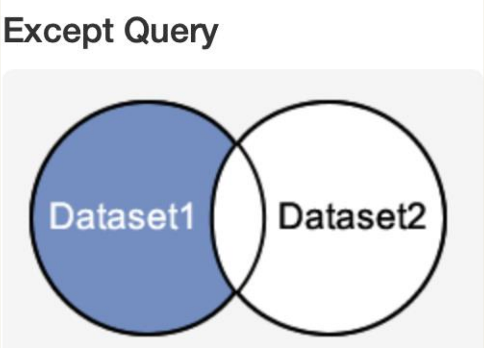
Example: STUDENT ∩ RESEARCHER =

Person#	Name
1	Dr C.C. Chen

DIFFERENCE

- SET DIFFERENCE is a relation that includes all tuples that are in the left relation but not in the right relation, denoted by

$$R - S = \{t : t \in R \, and \, t \notin S\}$$



- Example: STUDENT – RESEARCHRER =

Person#	Name
3	Ms K. Juliff
4	Ms J. Gledhill
5	Ms B.K. Lee

Renaming

- The renaming operator ρ changes the name of one or more attributes
- It changes the schema, but not the instance of a relation

Father-Child

Father	Child
Adam	Abel
Adam	Cain
Abraham	Isaac

$\rho_{\text{Parent} \leftarrow \text{Father}}$ (Father-Child)

Parent	Child
Adam	Abel
Adam	Cain
Abraham	Isaac

Exercise

RESEARCHER:

Person#	Name
1	Dr C.C.Chen
2	Dr R.G.Wilkinson

STUDENT:

Person#	Name
1	Dr C.C.Chen
3	Ms K.Juliff
4	Ms J.Gledill
5	Ms B.K.Lee

COURSE:

Department	Degree
Psychology	Ph.D.
Comp.Sci.	Ph.D.
Comp.Sci.	M.Sc.
Psychology	M.Sc.

ENROLMENT:

Enrolment#	Supervisee	Supervisor	Department	Degree
1	1	2	Psychology	Ph.D.
2	3	1	Comp.Sci.	Ph.D.
3	4	1	Comp.Sci.	M.Sc.
4	5	1	Comp.Sci.	M.Sc.

Exercise

Write relational algebra that retrieve:

1. The names of persons who are either a student or a researcher
2. The names of persons who are a student and a researcher
3. The names of persons who are a student but not a researcher
4. The IDs of persons who are supervisors in the Computer Science Department
5. The departments and degrees of Courses which are not enrolled by any student

Exercise Answer:

1. The names of persons who are either a student or a researcher

$\pi_{\{Name\}}(STUDENT \cup RESEARCHER)$

Name
Dr C.C.Chen
Dr R.G.Wilkinson
Ms K.Juliff
Ms J.Gledill
Ms B.K.Lee

Exercise Answer (continue):

2. The names of persons who are a student and a researcher

$\pi_{\{Name\}}(STUDENT \cap RESEARCHER)$

Name
Dr C.C.Chen

3. The names of persons who are a student but not a researcher

$\pi_{\{Name\}}(STUDENT - RESEARCHER)$

Name
Ms K.Juliff
Ms J.Gledill
Ms B.K.Lee

Exercise Answer (continue):

4. The IDs of persons who are supervisors in the Computer Science Department

$\pi_{\{Supervisor\}}(\sigma_{<Department=\text{'}Comp.Sci.\text{'}>}(ENROLMENT))$

Supervisor
1

5. The departments and degrees of Courses which are not enrolled by any student

$Course\ - \pi_{\{Department,\ Degree\}}\ (ENROLMENT)$

Department	Degree
Psychology	M.Sc.

CARTESIAN PRODUCT

$$R \times S = \{t_1 || t_2 : t_1 \in R \, and \, t_2 \in S\}$$

- Where $t_1 || t_2$ indicates the concatenation of tuples.
- Intuition: to put together every tuple in *R* with every tuple in *S*
- The number of tuples in *R X S* : |R| * |S|

Cartesian Product

- Example: ENROLMENT X RESEARCHRER =

E'ment#	S'ee	S'or	D'ment	Degree	Person#	Name
1	1	2	Psych.	Ph.D.	1	Dr C.C. Chen
1	1	2	Psych.	Ph.D.	2	Dr R.G.Wilkinson
2	3	1	Comp.Sci	Ph.D.	1	Dr C.C. Chen
2	3	1	Comp.Sci	Ph.D.	2	Dr R.G.Wilkinson
3	4	1	Comp.Sci	M.Sc.	1	Dr C.C. Chen
3	4	1	Comp.Sci	M.Sc.	2	Dr R.G.Wilkinson
4	5	1	Comp.Sci	M.Sc.	1	Dr C.C. Chen
4	5	1	Comp.Sci	M.Sc.	2	Dr R.G.Wilkinson

There are 4 tuples in ENROLMENT, 2 tuples in RESEARCHER. In the result, there are 8 tuples.

More useful is: specify the condition

$$R_1 \leftarrow ENROLMENT \times RESEARCHER$$

$$\sigma_{(Supervisor=Person\#)}(R_1) =$$

E’ment#	S’ee	S’or	D’ment	E’ment. Name	Person#	R’cher. Name
1	1	2	Psych.	Ph.D.	2	Dr R.G.Wilkinson
2	3	1	Comp.Sci.	Ph.D.	1	Dr C.C. Chen
3	4	1	Comp.Sci.	M.Sc.	1	Dr C.C. Chen
4	5	1	Comp.Sci.	M.Sc.	1	Dr C.C. Chen

Or even better: record equal attributes only once

$$R_1 \leftarrow ENROLMENT \times RESEARCHER$$

$$R_2 \leftarrow \sigma_{(Supervisor=Person\#)}(R_1)$$

$$\pi_{\{E'ment\#,S'ee,S'or,Name,D'ment,Degree\}}(R_2) =$$

E'ment#	S'ee	S'or	Name	D'ment	Degree
1	1	2	Dr R.G.Wilkinson	Psych.	Ph.D.
2	3	1	Dr C.C. Chen	Comp.Sci.	Ph.D.
3	4	1	Dr C.C. Chen	Comp.Sci.	M.Sc.
4	5	1	Dr C.C. Chen	Comp.Sci.	M.Sc.

The last of these is also known as natural join, the next to last is equi-join.

JOIN

- JOIN is used to combine related tuples from two relations into single "longer" tuples.
- **Theta-join**

$$R \bowtie_{<join\ condition>} S = \{t_1 || t_2 : t_1 \in R\ and\ t_2 \in S\ and\ <join\ condition>\}$$

- A general join condition is of the form:

<condition> **AND** <condition> **AND** … **AND** <condition>

- where each condition is of the form $A_i \theta B_j$, in which A_i is an attribute of R, B_j is an attribute of S, A_i and B_j have the same domain, and θ is a comparison operator. A JOIN operation with such a general join condition is called a **THETA JOIN**.

JOIN: Equi-join

EQUI-JOIN is a theta-join where the only comparison operator used is "=".

Example:

$$ENROLMENT \bowtie_{(Supervisor=Person\#)} RESEARCHER$$

JOIN: Natural join

NATURAL JOIN is an equi-join which requires that the two join attributes (or each pair of join attributes) have the same name in both relations.

Question: If two relations have no join attributes, how do you define the join result? Why?

$$R\,(A, B) \Join S\,(B, C) \Join T\,(C, D)$$

Notes:

1. In a natural join, there may be several pairs of

join attributes.

Example:

COURSE		
Department	Name	By
Comp.Sci	Ph.D.	Research
Comp.Sci.	M.Sc.	Research
Psychology	M.Sc.	Coursework

Calculate

$$ENROLMENT \bowtie_{(Department,Name),(Department,Name)} COURSE$$

2. If the pairs of joining attributes are exactly those that are identically named, we can write

$$ENROLMENT \bowtie COURSE$$

Exercise

RESEARCHER:

Person#	Name
1	Dr C.C.Chen
2	Dr R.G.Wilkinson

STUDENT:

Person#	Name
1	Dr C.C.Chen
3	Ms K.Juliff
4	Ms J.Gledill
5	Ms B.K.Lee

COURSE:

Department	Degree
Psychology	Ph.D.
Comp.Sci.	Ph.D.
Comp.Sci.	M.Sc.
Psychology	M.Sc.

ENROLMENT:

Enrolment#	Supervisee	Supervisor	Department	Degree
1	1	2	Psychology	Ph.D.
2	3	1	Comp.Sci.	Ph.D.
3	4	1	Comp.Sci.	M.Sc.
4	5	1	Comp.Sci.	M.Sc.

Exercise

Write relational algebra that retrieve:

1. The name of supervisor who supervises student with ID 3
2. The names of students who are studying MSc in computer science
3. The IDs of students who are supervised by Dr C.C.Chen
4. The ID of supervisor who supervises both MSc and PhD students

Exercise Answers:

1. The name of supervisor who supervises student with ID 3

$$\pi_{\{Name\}}(\sigma_{<Supervisee=3>}(ENROLMENT) \bowtie_{(Supervisor),(Person\#)} RESEARCHER)$$

Name
Dr C.C.Chen

2. The names of students who are studying MSc in computer science

$$\pi_{\{Name\}}(\sigma_{<Degree=\text{'M.Sc.'} \text{ and } Department=\text{'Comp.Sci.'}>}(ENROLMENT) \bowtie_{(Supervisee),(Person\#)} STUDENT)$$

Name
Ms J.Gledill
Ms B.K.Lee

Exercise Answers (continue):

3. The IDs of students who are supervised by Dr C.C.Chen

$\pi_{\{Supervisee\}}(ENROLMENT \bowtie_{(Supervisor),(Person\#)} \sigma_{<Name='Dr\ C.C.Chen'>}(RESEARCHER))$

Supervisee
3
4
5

4. The name of supervisor who supervises both MSc and PhD students

$\pi_{\{Name\}}(\sigma_{<Degree='M.Sc.'>}(ENROLMENT) \bowtie_{(Supervisor),(Person\#)} RESEARCHER) \cap \pi_{\{Name\}}(\sigma_{<Degree='Ph.D.'>}(ENROLMENT) \bowtie_{(Supervisor),(Person\#)} RESEARCHER)$

Name
Dr C.C.Chen

DIVIDE

The DIVISION operation is applied to two Relations

$$R(Z) \div S(X)$$

Where the attributes of S are a subset of the attributes of R.

Let Y be the set of attributes of R that are not attributes of S

R	
A	B
a_1	b_1
a_1	b_2
a_2	b_1
a_3	b_2
a_4	b_1
a_5	b_1
a_5	b_2

S
B
b_1
b_2

Example: $X \subseteq Z$

X = {B}, Z = {A, B}

and Y = Z - X = {A}

DIVIDE

DIVISION is a relation T(Y) that includes a tuple t if tuples t_R appear in R with $t_R[Y] = t$, and with $t_R[X] = t_S$ for every tuple t_S in S.

$$R \div S = \{t : t \times S \subseteq \mathrm{R}\}$$

R	
A	B
a_1	$\boxed{b_1}$
a_1	$\boxed{b_2}$
a_2	b_1
a_3	b_2
a_4	b_1
a_5	$\boxed{b_1}$
a_5	$\boxed{b_2}$

S
B
b_1
b_2

Example:

$X = \{B\}$, $Z = \{A, B\}$, $Y = \{A\}$

$t_R[X] = t_S = \{b_1, b_2\}$

In R, there are two satisfied t_R pairs:

$\{a_1b_1, a_1b_2\}$ and $\{a_5b_1, a_5b_2\}$

So $t = t_R[Y] = \{a_1, a_5\}$

T
A
a_1
a_5

DIVIDE

R	
A	B
a_1	b_1
a_1	b_2
a_2	b_1
a_3	b_2
a_4	b_1
a_5	b_1
a_5	b_2

S
B
b_1
b_2

$$R(Z) \div S(X) =$$

T
A
a_1
a_5

DIVIDE

Typical use: which courses are offered by all departments?

$$COURSE \div (\pi_{Department} COURSE)$$

Exercise

RESEARCHER:

Person#	Name
1	Dr C.C.Chen
2	Dr R.G.Wilkinson

STUDENT:

Person#	Name
1	Dr C.C.Chen
3	Ms K.Juliff
4	Ms J.Gledill
5	Ms B.K.Lee

COURSE:

Department	Degree
Psychology	Ph.D.
Comp.Sci.	Ph.D.
Comp.Sci.	M.Sc.
Psychology	M.Sc.

ENROLMENT:

Enrolment#	Supervisee	Supervisor	Department	Degree
1	1	2	Psychology	Ph.D.
2	3	1	Comp.Sci.	Ph.D.
3	4	1	Comp.Sci.	M.Sc.
4	5	1	Comp.Sci.	M.Sc.

Exercise:

Write relational algebra that retrieve:

1. The departments which offer all degrees

Exercise Answers:

1. The departments which offer all degrees

$$Course \div \pi_{\{Degree\}}(Course)$$

Department
Psychology
Comp.Sci.

Exercise

R:

A	B	C
a_1	b_1	c_1
a_1	b_1	c_2
a_1	b_1	c_3
a_1	b_2	c_2
a_2	b_1	c_1
a_2	b_2	c_2
a_3	b_1	c_1
a_3	b_2	c_1
a_3	b_2	c_2

S:

B	C
b_1	c_1
b_1	c_2

Exercise:

Write relational algebra that retrieve:

2. Find A of *R* that contains all *S*.

3. Find (A, B) of *R* that contains all C of *S*.

Exercise Answers:

2. $R \div S$

A
a_1

3. $R \div \pi_{\{c\}}(S)$

A	B
a_1	b_1
a_3	b_2

OPERATION	PURPOSE	NOTATION
SELECT	Selects all tuples that satisfy the selection condition from a relation R	$\sigma_{<selection\ condition>}(R)$
PROJECT	Produces a new relation with only some of the attributes of R, and removes duplicate tuples.	$\pi_{<attribute\ list>}(R)$
THETA-JOIN	Produces all combinations of tuples from R and S that satisfy the join condition.	$R \bowtie_{<join\ condition>} S$
EQUI-JOIN	Produces all the combinations of tuples from R and S that satisfy a join condition with only equality comparisons.	$R \bowtie_{<join\ condition>} S$
NATURAL-JOIN	Same as EQUIJOIN except that the join attributes of S are not included in the resulting relation; if the join attributes have the same names, they do not have to be specified at all.	$R \bowtie_{<join\ condition>} S$
UNION	Produces a relation that includes all the tuples in R or S or both R and S; R and S must be union compatible.	$R \cup S$
INTERAECTION	Produces a relation that includes all the tuples in both R and S; R and S must be union compatible.	$R \cap S$
DIFFERENCE	Produces a relation that includes all the tuples in R that are not in S; R and S must be union compatible.	R - S
CARTESIAN PRODUCT	Produces a relation that has the attributes of R and S and includes as tuples all possible combinations of tuples from R and S.	$R \times S$
DIVISION	Produces a relation T(X) that includes all tuples t[X] in R(Z) that appear in R in combination with every tuple from S(Y), where Z = X ∪ Y.	$R(Z) \div S(Y)$

Aggregation

Often, we want to retrieve aggregate values, like the "sum of salaries" of employees, or the "average age" of students.

This is achieved using aggregation functions, such as SUM, AVG, MIN, MAX, or COUNT.

Such functions are applied by the grouping and aggregation operator γ.

If $R =$

A	B
1	2
3	4
3	5
1	1

, then $\gamma_{SUM(A)}(R) =$

SUM(A)
8

and $\gamma_{AVG(B)}(R) =$

AVG(B)
3

Grouping and Aggregation

More often, we want to retrieve aggregate values for groups, like the "sum of employee salaries" per department, or the "average student age" per faculty.

As additional parameters, we give γ attributes that specify the criteria according to which the tuples of the argument are grouped.

E.g., the operator γA,SUM(B) (R)

- partitions the tuples of R in groups that agree on A,
- returns the sum of all B values for each group.

If R =

A	B
1	2
3	4
3	5
1	3

, then $\gamma_{A,SUM(B)}(R)$ =

A	SUM(B)
1	5
3	9

Learning Outcome

- Write relational algebra expressions for given queries