# Tutorial 8 - Factor Elimination and Jointrees

COMP9418 – Advanced Topics in Statistical Machine Learning

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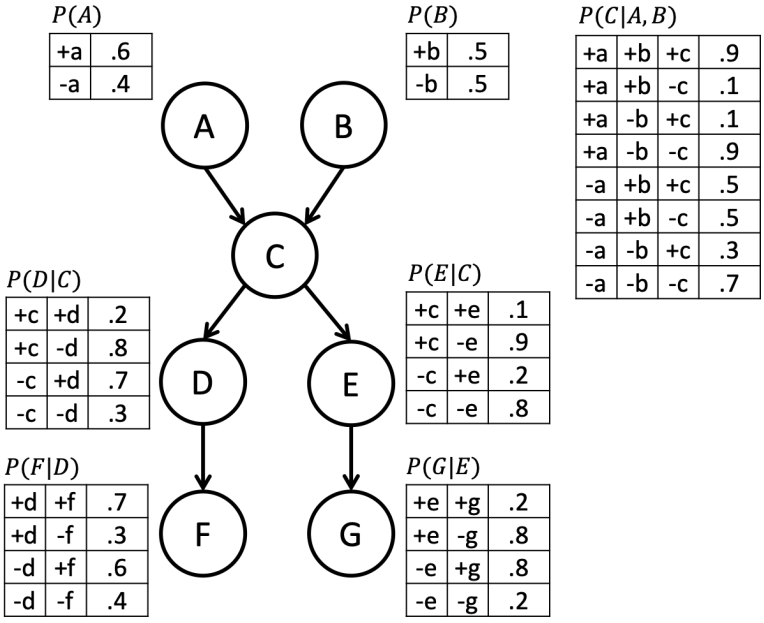
**Lecture:** Factor Elimination and Jointrees

**Topic:** Questions from lecture topics

**Last revision:** Monday 2nd November, 2020 at 00:04

## Question 1

Answer the following queries with respect to the Bayesian network below:




$P(A)$

| | |
|---|---|
| +a | .6 |
| -a | .4 |

$P(B)$

| | |
|---|---|
| +b | .5 |
| -b | .5 |

$P(C|A,B)$

| | | | |
|---|---|---|---|
| +a | +b | +c | .9 |
| +a | +b | -c | .1 |
| +a | -b | +c | .1 |
| +a | -b | -c | .9 |
| -a | +b | +c | .5 |
| -a | +b | -c | .5 |
| -a | -b | +c | .3 |
| -a | -b | -c | .7 |

$P(D|C)$

| | | |
|---|---|---|
| +c | +d | .2 |
| +c | -d | .8 |
| -c | +d | .7 |
| -c | -d | .3 |

$P(E|C)$

| | | |
|---|---|---|
| +c | +e | .1 |
| +c | -e | .9 |
| -c | +e | .2 |
| -c | -e | .8 |

$P(F|D)$

| | | |
|---|---|---|
| +d | +f | .7 |
| +d | -f | .3 |
| -d | +f | .6 |
| -d | -f | .4 |

$P(G|E)$

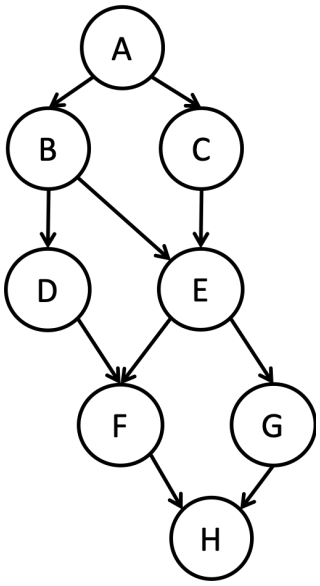
| | | |
|---|---|---|
| +e | +g | .2 |
| +e | -g | .8 |
| -e | +g | .8 |
| -e | -g | .2 |

1. $P(B, C)$
2. $P(C, D = true)$
3. $P(A|D = true, E = true)$

You may prune the network before attempting each computation and use any inference method you find most appropriate.

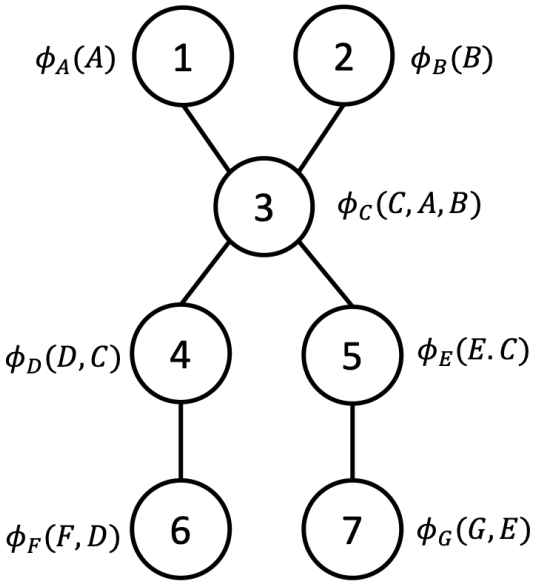
## Question 2

Consider the Bayesian network in the figure below. Construct an elimination tree for the Bayesian network CPTs that has the smallest width possible and assigns at most one CPT to each tree node. Compute the separators, clusters and width of the elimination tree. It may be useful to know that this network has the following jointree $ABC - BCE - BDE - DEF - EFG - FGH$.




## Question 3

Consider the Bayesian network of Question 1 and the corresponding elimination tree:




Suppose that the evidence indicator for each variable is assigned to the node corresponding to that variable (e.g. $\lambda_C$ is assigned to node 3). Suppose we are answering the following queries according to the given order:

$P(G = true)$, $P(G, F = true)$, $P(F, A = true, G = true)$ using the following algorithm:

**Data:** $N$: Bayesian network
**Data:** $(T, \Phi)$: elimination tree for the CPTs of network $N$
**Data:** $\mathbf{e}$: evidence
**Result:** the joint marginal $P(\mathbf{C}, \mathbf{e})$ for each node $i$ in elimination tree

```
begin
    for each variable E in evidence e do
        i ← node in the tree T such that E ∈ C_i;
        λ_E ← evidence indicator for variable E {λ_E(e) = 1 if e ∼ e and λ_E(e) = 0 otherwise };
        φ_i ← φ_i λ_E {entering evidence at node i };
    end
    Choose a root node r in the tree T;
    Pull/collect messages toward root r;
    Push/distribute messages away from root r;
end
return φ_i ∏_k M_ki for each node i in the tree T {joint marginal P(C_i, e)}
```

1. Compute the separators, clusters, and width of the given elimination tree.
2. What messages are computed while answering each query according to the previous sequence? State the origin, destination, and value of each message. Use node 7 as the root to answer the first two queries, and node 6 as the root to answer the last query. For each query, compute only the messages directed toward the corresponding root.
3. What messages are invalidated due to new evidence as we attempt each new query?
4. What is the answer to each of the previous queries?