Tutorial 5 - Markov Chains and Hidden Markov Models

COMP9418 – Advanced Topics in Statistical Machine Learning

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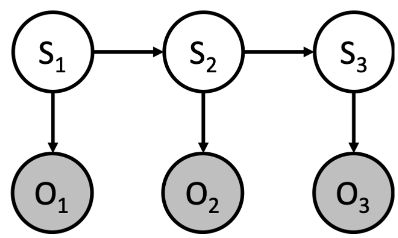
Lecture: Markov Chains and Hidden Markov Models

Topic: Questions from lecture topics

Last revision: Monday 12$^{\text{th}}$ October, 2020 at 05:57

Question 1

Answer the questions considering the Hidden Markov Model (HMM) shown in the following figure:



a. If each state has k different values, each observation has m different values, and the chain is length d, how many parameters are necessary to define this HMM?

b. Show the factorization for the joint distribution $P(S_1, S_2, S_3, O_1, O_2, O_3)$.

c. What conditional independences hold in this HMM?

Question 2

Lisa is given a fair coin C_1 and asked to flip it eight times in a row. Lisa also has a biased coin C_2 with probability 0.8 of landing heads. All we know is that Lisa flipped the fair coin initially, but we believe that she intends to switch to the biased coin and that she tends to be 10% successful in performing the switch. Suppose that we observe the outcome of the eight coin flips and want to find out whether Lisa managed to perform a coin switch and when.

a. Describe a Bayesian network and a corresponding query that solves this problem.

b. Write the probability tables necessary to model this problem.

c. Use the source code developed in the practical part of this tutorial to find the solution to this problem, assuming that the flips came out as follows:

   i. tails, tails, tails, heads, heads, heads, heads, heads.
   ii. tails, tails, heads, heads, heads, heads, heads, heads.

Question 3

Consider a cow that may be infected with a disease that can possibly be detected by performing a milk test. The test is performed on five consecutive days, leading to five outcomes. We want to determine the state of the cow's infection over these days, given the test outcomes. The prior probability of infection on day one is 1/10,000; the test false-positive rate is 5/1,000, and; its false-negative rate is 1/1,000. Moreover, the state of infection at a given day depends only on its state on the previous day. In particular, the probability of a new infection on a given day is 2/10,000, while the probability that an infection would persist to the next day is 7/10.

a. Describe a Bayesian network and a corresponding query that solves this problem.

b. Write the probability tables necessary to model this problem.

c. Use the source code developed in the practical part of this tutorial to find the most likely state of the cow's infection over the five days given the following test outcomes:

    1. positive, positive, negative, positive, positive
    2. positive, negative, negative, positive, positive
    3. positive, negative, negative, negative, positive

Question 4

For each transition matrix below, is the corresponding Markov chain irreducible? Is it aperiodic? What is its stationary distribution? The matrix $T(i, j)$ represents the transition probabilities from a state i in time $t-1$ to a state j in time t.

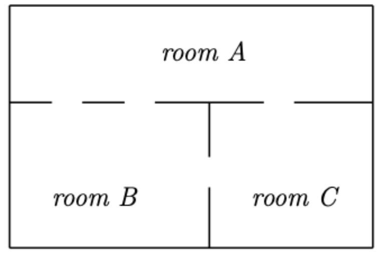
a. $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1/2 & 0 & 1/2 & 0 \\ 0 & 1/2 & 0 & 1/2 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

b. $\begin{bmatrix} 1/2 & 1/2 & 0 & 0 \\ 1/2 & 0 & 1/2 & 0 \\ 0 & 1/2 & 0 & 1/2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

c. $\begin{bmatrix} 1/2 & 1/2 & 0 & 0 \\ 1/2 & 1/2 & 0 & 0 \\ 0 & 0 & 1/2 & 1/2 \\ 0 & 0 & 1/2 & 1/2 \end{bmatrix}$

Question 5

A trained mouse lives in the house shown. A bell rings at regular intervals, and the mouse is trained to change rooms each time it rings. When it changes rooms, it is equally likely to pass through any of the doors in the room it is in. Approximately what fraction of its life will it spend in each room?


room A
room B
room C