Tutorial 9 - MAP and MPE inference

COMP9418 – Advanced Topics in Statistical Machine Learning

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Lecture: MAP and MPE inference

Topic: Questions from lecture topics

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Question 1

Given the following Bayesian network

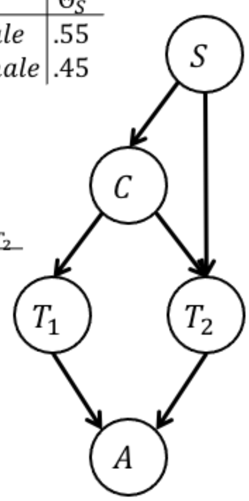
S	C	$\Theta_{S\|C}$
male	*yes*	.05
male	*no*	.95
female	*yes*	.01
female	*no*	.99

C	T_1	$\Theta_{T_1\|C}$
yes	*ve*	.80
yes	$\overline{ve}$	.20
no	*ve*	.20
no	$\overline{ve}$	.80

S	Θ_S
male	.55
female	.45

S	C	T_2	$\Theta_{T_2\|C,S}$
male	*yes*	*ve*	.80
male	*yes*	$\overline{ve}$	.20
male	*no*	*ve*	.20
male	*no*	$\overline{ve}$	.80
female	*yes*	*ve*	.95
female	*yes*	$\overline{ve}$	.05
female	*no*	*ve*	.05
female	*no*	$\overline{ve}$	.95

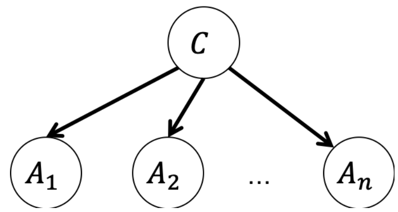
T_1	T_2	A	$\Theta_{A\|T_1,T_2}$
ve	*ve*	*yes*	1
ve	*ve*	*no*	0
ve	$\overline{ve}$	*yes*	0
ve	$\overline{ve}$	*no*	1
$\overline{ve}$	*ve*	*yes*	0
$\overline{ve}$	*ve*	*no*	1
$\overline{ve}$	$\overline{ve}$	*yes*	1
$\overline{ve}$	$\overline{ve}$	*no*	0



a. Compute the MPE probability and corresponding MPE instantiation given evidence $A = no$.
b. What is the most likely outcome of the two tests T_1 and T_2 for a female on whom the tests came differently?

Question 2

Consider the naive Bayes stucture



What is the complexity of computing the MPE probability for this network using algorithm MPE VE? What is the complexity of computing the MAP probability for MAP variables $A_1, \ldots, A_n$ using algorithm MAP VE? How does the complexity of these computations change when we have evidence on variable C?

Question 3

Can we compute a MAP instantiation $MAP(\mathbf{M}, e)$ by computing the MAP instantiations $\mathbf{m}_x = MAP(\mathbf{M}, \mathbf{e}x)$ for every value of x of some variable $X \notin \mathbf{M}$ and then returning $argmax_{\mathbf{m}_x} P(\mathbf{m}_x, \mathbf{e})$? What if $X \in \mathbf{M}$? Justify your response.

Question 4

Consider the classical jointree algorithm as defined by the following equations for message M_{ij} and factor f_i

$$M_{ij} = \sum_{\mathbf{C}_i \setminus \mathbf{S}_{ij}} \Phi_i \prod_{k \neq j} M_{ki}$$

$$f_i(\mathbf{C}_i) = \Phi_i \prod_k M_{ki}$$

Suppose we replace all summations by maximizations when computing messages,

$$M_{ij} = \max_{\mathbf{C}_i \setminus \mathbf{S}_{ij}} \Phi_i \prod_{k \neq j} M_{ki}$$

What are the semantics of the factor $f_i(\mathbf{C}_i)$ in this case? In particular, can we use it to recover answers to MPE queries?

Question 5

Consider the classical jointree algorithm as defined by the following equations for message M_{ij} and factor f_i

$$M_{ij} = \sum_{\mathbf{C}_i \setminus \mathbf{S}_{ij}} \Phi_i \prod_{k \neq j} M_{ki}$$

$$f_i(\mathbf{C}_i) = \Phi_i \prod_k M_{ki}$$

Suppose we replace all summations over MAP variables $\mathbf{M}$ by maximizations when computing messages, as follows:

$$M_{ij} = \max_{(\mathbf{C}_i \setminus \mathbf{S}_{ij}) \cap \mathbf{M}} \sum_{(\mathbf{C}_i \setminus \mathbf{S}_{ij}) \setminus \mathbf{M}} \Phi_i \prod_{k \neq j} M_{ki}$$

What are the semantics of the factor $f_i(\mathbf{C}_i)$ in this case? In particular, can we use it to recover answers to MAP queries?