COMP9444 Neural Networks and Deep Learning
Term 3, 2020

Solutions to Exercises 1: Perceptrons

This page was last updated: 09/21/2020 14:02:32

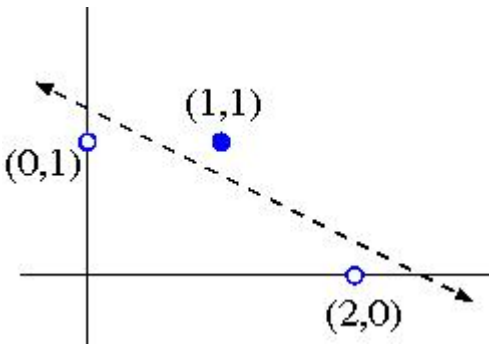
---

1. **Perceptron Learning**

   a. Construct by hand a Perceptron which correctly classifies the following data; use your knowledge of plane geometry to choose appropriate values for the weights w_0, w_1 and w_2.

Training Example	x_1	x_2	Class
a.	0	1	−1
b.	2	0	−1
c.	1	1	+1

The first step is to plot the data on a 2−D graph, and draw a line which separates the positive from the negative data points:



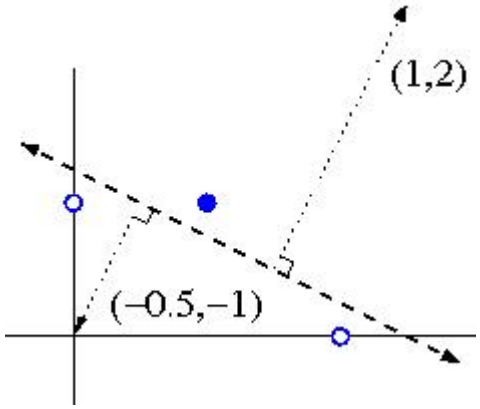
This line has slope −1/2 and x_2−intersect 5/4, so its equation is:

$x_2 = 5/4 - x_1/2$,
i.e. $2x_1 + 4x_2 - 5 = 0$.

Taking account of which side is positive, this corresponds to these weights:

$w_0 = -5$
$w_1 = 2$
$w_2 = 4$

Alternatively, we can derive weights w_1=1 and w_2=2 by drawing a vector normal to the separating line, in the direction pointing towards the positive data points:



The bias weight w_0 can then be found by computing the dot product of the normal vector with a perpendicular vector from the separating line to the origin. In this case $w_0 = 1(-0.5) + 2(-1) = -2.5$

(Note: these weights differ from the previous ones by a normalizing constant, which is fine for a Perceptron)

b. Demonstrate the Perceptron Learning Algorithm on the above data, using a learning rate of 1.0 and initial weight values of

$w_0 = -1.5$
$w_1 = 0$
$w_2 = 2$

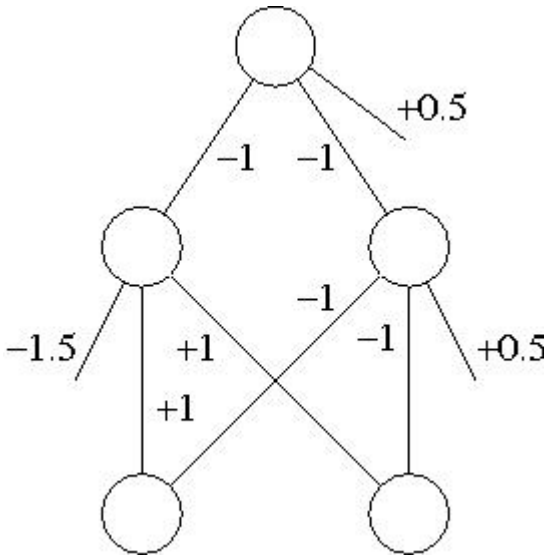
In your answer, you should clearly indicate the new weight values at the end of each training step.

Iteration	w_0	w_1	w_2	Training Example	x_1	x_2	Class	$s=w_0+w_1x_1+w_2x_2$	Action
1	–1.5	0	2	a.	0	1	–	+0.5	Subtract
2	–2.5	0	1	b.	2	0	–	–2.5	None
3	–2.5	0	1	c.	1	1	+	–1.5	Add
4	–1.5	1	2	a.	0	1	–	+0.5	Subtract
5	–2.5	1	1	b.	2	0	–	–0.5	None
6	–2.5	1	1	c.	1	1	+	–0.5	Add
7	–1.5	2	2	a.	0	1	–	+0.5	Subtract
8	–2.5	2	1	b.	2	0	–	+1.5	Subtract
9	–3.5	0	1	c.	1	1	+	–2.5	Add
10	–2.5	1	2	a.	0	1	–	–0.5	None
11	–2.5	1	2	b.	2	0	–	–0.5	None
12	–2.5	1	2	c.	1	1	+	+0.5	None

2. XOR Network

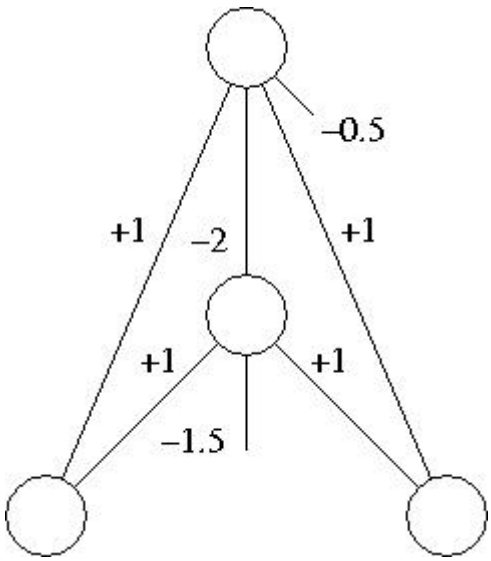
Construct by hand a Neural Network (or Multi-Layer Perceptron) that computes the XOR function of two inputs. Make sure the connections, weights and biases of your network are clearly visible.

> There are a number of ways to express XOR as a combination of simpler functions that are linearly separable. For example, using NOR as an abbreviation for "NOT OR", (x_1 XOR x_2) can be written as (x_1 AND x_2) NOR (x_1 NOR x_2). This decomposition allows us to compute XOR with a network like this:

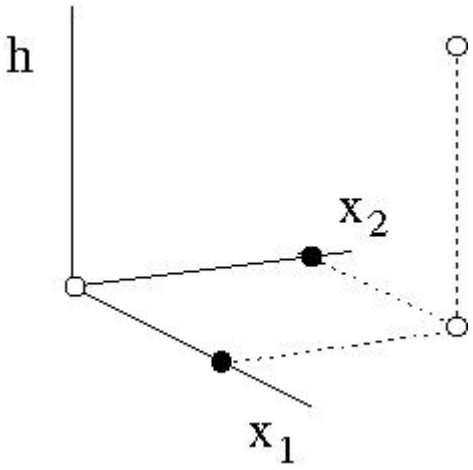


Challenge: Can you construct a Neural Network to compute XOR which has only one hidden unit, but also includes shortcut connections from the two inputs directly to the (one) output.
Hint: start with a network that computes the inclusive OR, and then try to think of how it could be modified.

> Exclusive OR (XOR) is very similar to normal (inclusive) OR, except for the case where both inputs are True, i.e. where (x_1 AND x_2) is True. We therefore introduce a single hidden unit which computes (x_1 AND x_2). This hidden unit is connected to the output with a negative weight, thus forcing that case to be classified negatively.



The addition of this hidden "feature" creates a 3–dimensional space in which the points can be linearly separated by a plane. The weights for the output unit (+1,+1,–2) specify a vector perpendicular to the separating plane, and its distance from the origin is determined by the output bias divided by the length of this vector.



3. **Computing any Logical Function with a 2–layer Network**

Assuming False=0 and True=1, explain how each of the following could be constructed:

a. Perceptron to compute the OR function of *m* inputs

Set the bias weight to –½, all other weights to 1.

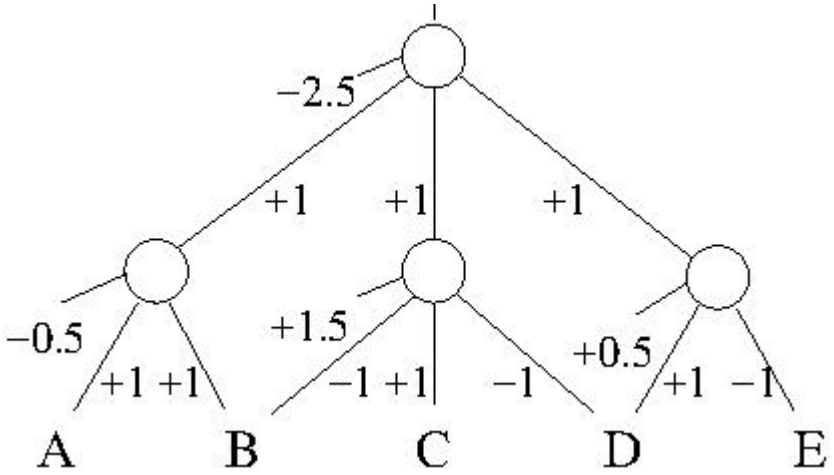
It makes sense for the input–to–output weights to be 1, because any of the inputs being True makes it more likely for the output to be True. In fact, the ONLY way the output can be False is if ALL the inputs are False. By setting the bias to –½, we insure that the linear combination is slightly negative when all of the inputs are False, but becomes positive when any of the inputs is True.

b. Perceptron to compute the AND function of *n* inputs

Set the bias weight to (½ – *n*), all other weights to 1.

The ONLY way the conjunction can be True is if ALL the inputs are True. By setting the bias to (½ – *n*), we insure that the linear combination is slightly positive when all of the inputs are True, but becomes negative when any of the inputs is False.

c. 2–layer Neural Network to compute the function (A ∨ B) ∧ (¬ B ∨ C ∨ ¬ D) ∧ (D ∨ ¬ E)



Each hidden node should compute one disjunctive term in the expression. The input–to–hidden weights are –1 for items that are negated, +1 for the others. The output node then computes the conjunction of all the hidden nodes, as in part 2.

d. 2–Layer Neural Network to compute any (given) logical expression, assuming it is written in Conjunctive Normal Form.

   As in the example above, each hidden node should compute one disjunctive term in the expression; the output node then computes the conjunction of all these hidden nodes. The input–to–hidden weights should be –1 for items that are negated, +1 for the others. The bias for each hidden node should be $(k - \frac{1}{2})$ where k is the number of items that are negated in the disjunctive term corresponding to that node.
