ODE HW 1

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problem 6

Show that the function

$$y(t) = \frac{4}{1 + Ce^{-4t}}, \quad C = 1, 2, \ldots, 5$$

is a general solution of the differential equation

$$y' = y(4 - y).$$

solution

rewrite so we dont have to divide.

$$y = 4(1 + Ce^{-4t})^{-1}.$$

differentiate using chain rule and power rule.

$$y' = 4(-1)(1 + Ce^{-4t})^{-2} \cdot \frac{d}{dt}(1 + Ce^{-4t}).$$

since

$$\frac{d}{dt}(1 + Ce^{-4t}) = -4Ce^{-4t},$$

we obtain

$$y' = \frac{16Ce^{-4t}}{(1 + Ce^{-4t})^2}.$$

now compute $y(4 - y)$:

$$4 - y = 4 - \frac{4}{1 + Ce^{-4t}} = \frac{4Ce^{-4t}}{1 + Ce^{-4t}}.$$

thus,

$$y(4 - y) = \frac{4}{1 + Ce^{-4t}} \cdot \frac{4Ce^{-4t}}{1 + Ce^{-4t}} = \frac{16Ce^{-4t}}{(1 + Ce^{-4t})^2}.$$

since $y' = y(4 - y)$, the function satisfies the differential equation. therefore, $y(t) = \frac{4}{1+Ce^{-4t}}$ is a general solution.

MATLAB plot

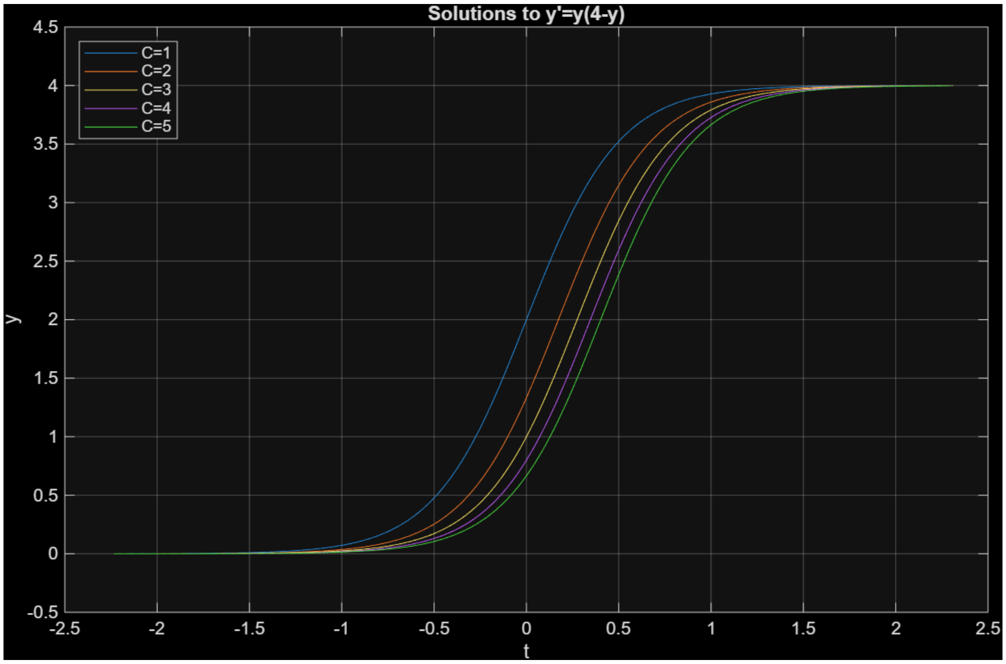

Figure 1: Solutions of the differential equation $y' = y(4-y)$ for $C = 1, 2, \ldots, 5$.

problem 7

A general solution may fail to produce all solutions of a differential equation. In Problem 6, show that y = 0 is a solution of the differential equation, but no value of C in the given general solution will produce this solution.

solution

Take

$$y(t) = 0.$$

Then

$$y'(t) = 0,$$

and substituting into the differential equation gives

$$y(4-y) = 0 \cdot (4-0) = 0.$$

Since $y' = y(4-y)$, the constant function $y = 0$ satisfies the differential equation.

The constant function $y = 0$ satisfies the differential equation but cannot be obtained from the general solution because no value of C makes $\frac{4}{1+Ce^{-4t}} = 0$.

problem 11