Lambda Calculus Constructions

Greyson Knippel

CPSC 326-01 — Gonzaga University, compiled with LaTeX

Professor Johnson

January 23, 2026

to start off:

$$\text{SUCC } X = X + 1$$

$$\text{PRED } X = X - 1$$

$$(= X\,Y) \Rightarrow \text{T/F}$$

construction of ADD

using recursion: R, ADD can be defined as:

$$\text{R ADD } X\,Y = R\,\text{ADD}\,X\,Y$$
$$= (\text{if})\,(= Y\,0)\,(\text{then})\,X\,(\text{else})\,(\text{ADD}\,(\text{SUCC}\,X)\,(\text{PRED}\,Y))$$

construction of MULT

MULT can also be defined by using recursion R: but this time its simply just repeated addition.

$$\text{R MULT } X\,Y = R\,\text{MULT}\,X\,Y$$
$$= (\text{if})\,(= Y\,0)\,(\text{then})\,0\,(\text{else})\,(\text{ADD}\,X\,(\text{MULT}\,X\,(\text{PRED}\,Y)))$$

example — ADD 2 3

$$\text{R ADD } 2\,3 = (\text{if}) \, (= 3\,0) \, 2 \, (\text{ADD } (\text{SUCC } 2) \, (\text{PRED } 3))$$
$$= (\text{if}) \, (= 2\,0) \, 3 \, (\text{ADD } (\text{SUCC } 3) \, (\text{PRED } 2))$$
$$= (\text{if}) \, (= 1\,0) \, 4 \, (\text{ADD } (\text{SUCC } 4) \, (\text{PRED } 1))$$
$$= (\text{if}) \, (= 0\,0) \, 5 \, (\text{ADD } (\text{SUCC } 5) \, (\text{PRED } 0))$$

- basically $(= y\ 0)$ is your base case,
  then use recursion with SUCC AND PRED.

$$= 5$$

example — MULT 2 4

$$\text{MULT } 2\,4 = (\text{if}) \, (= 4\,0) \, (\text{then}) \, 0 \, (\text{else}) \, (\text{ADD } 2 \, (\text{MULT } 2 \, (\text{PRED } 4)))$$
$$= (\text{if}) \, (= 3\,0) \, (\text{then}) \, 0 \, (\text{else}) \, (\text{ADD } 2 \, (\text{MULT } 2 \, (\text{PRED } 3)))$$
$$= (\text{if}) \, (= 2\,0) \, (\text{then}) \, 0 \, (\text{else}) \, (\text{ADD } 2 \, (\text{MULT } 2 \, (\text{PRED } 2)))$$
$$= (\text{if}) \, (= 1\,0) \, (\text{then}) \, 0 \, (\text{else}) \, (\text{ADD } 2 \, (\text{MULT } 2 \, (\text{PRED } 1)))$$
$$= (\text{if}) \, (= 0\,0) \, (\text{then}) \, 0 \, (\text{else}) \, (\text{ADD } 2 \, (\text{MULT } 2 \, (\text{PRED } 0)))$$

and adding all the recursions together with the ADD block...
- basically we are just using ADD's recursion,
  doing repeated addition with MULT's recursion.

$$= 8$$