Game theory
(cont.)

Rival networks: During the 8-9 PM timeslot, two networks are competing for an audience of 100 million viewers. Each network can air either a comedy, drama, or reality show. The number of viewers network 1 will have for each pair of choices is shown. Assume all other viewers will watch network 2.

N1 \ N2	C	D	R
C	35, 65	15, 85	60, 40
D	45, 55	58, 42	50, 50
R	45, 55	58, 42	70, 30

Payoff matrix for N1

Payoff matrix for N2

constant sum game

Network 1's optimal strategy:

Assume N1 uses mixed strategy (x_C, x_D, x_R).

N2 will respond with the strategy that minimizes N1's expected payoff.

N1 \ N2	C	D	R
C	35	15	60
D	45	58	50
R	45	58	70

N1's expected payoff

N2 plays C: $35x_C + 45x_D + 45x_R$

N2 plays D: $15x_C + 58x_D + 58x_R$

N2 plays R: $60x_C + 50x_D + 70x_R$

Thus, NI's expected payoff is

$$\min\left(35x_C + 45x_D + 45x_R,\; 15x_C + 58x_D + 58x_R,\; 60x_C + 50x_D + 70x_R\right) = z$$

NI wants to maximize z.

Max z

s.t. $z \le 35x_C + 45x_D + 45x_R$

$z \le 15x_C + 58x_D + 58x_R$

$z \le 60x_C + 50x_D + 70x_R$

$0 \le x_C, x_D, x_R \le 1$

$x_C + x_D + x_R = 1$

Optimal solution:

$$z = 45 \qquad x_C = 0 \qquad x_D = 1 \qquad x_R = 0$$

A strategy where one of the probabilities is 1 is a pure strategy.

Another optimal solution:

$$z = 45 \qquad x_C = 0 \qquad x_D = 0 \qquad x_R = 1$$

Actually, any strategy of the form $(0, \alpha, 1-\alpha)$ will be optimal.

for arbitrary α

Network 2's optimal strategy:

N1 \ N2	C	D	R
C	35	15	60
D	45	58	50
R	45	58	70

Assume N2 plays the mixed strategy (y_C, y_D, y_R).

N1 will choose the strategy which maximize N1's expected payoff.

	N1's expected payoff
N1 plays C	$35y_C + 15y_D + 60y_R$
N1 plays D	$45y_C + 58y_D + 50y_R$
N1 plays R	$45y_C + 58y_D + 70y_R$

Thus, N1's expected payoff is

$$\max\left(35y_C+15y_D+60y_R,\; 45y_C+58y_D+50y_R,\; 45y_C+58y_D+70y_R\right) = z$$

N2 wants to choose y_C, y_D, y_R to minimize this.

$\min z$

s.t. $z \geq 35y_C+15y_D+60y_R$ $\quad 0 \leq y_C, y_D, y_R \leq 1$

$z \geq 45y_C+58y_D+50y_R$ $\quad y_C+y_D+y_R = 1$

$z \geq 45y_C+58y_D+70y_R$

Optimal solution: still N1's expected payoff

$$z = 45 \quad y_C = 1 \quad y_D = 0 \quad y_R = 0$$

An equilibrium is N1 airs drama, N2 airs comedy, N1 gets 45 million viewers and N2 gets 55 million viewers.

(Another equilibrium is N1 airs reality, N2 airs comedy, same viewership.)

N1 \ N2	C	D	R
C	35	15	60
D	45	58	50
R	45	58	70

N1's optimal strategy

No matter what N2 does, if N1 plays R, the outcome for N1 will be at least as good as if they played C or D.

This means R dominates C and D.

A strategy x **dominates** a strategy x' if no matter what the opponent does, x does at least as well as x', and in some situation better.

If x' is dominated by x, then we can eliminate x' from the table when solving for the optimal strategies.

(Note: If there are multiple optimal strategies, this might eliminate some of them.)

N1 \ N2	C	D	R
C	35	15	60
D	45	58	50
R	45	58	70

N2's optimal strategy

C dominates R, since no matter what N1 does, N1's payoff will be lower if N2 plays C as opposed to R.

Since C dominates R, we can eliminate column R from the table.

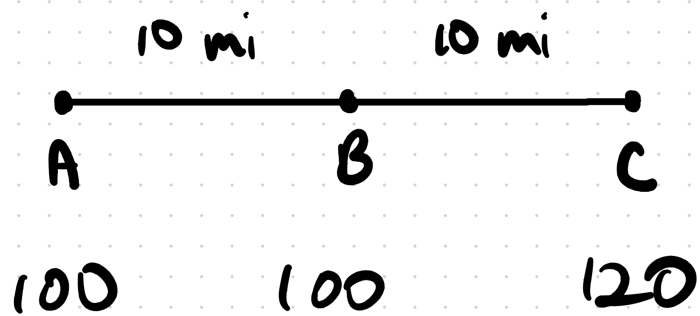
N1 \ N2	C	D	~~R~~
~~C~~	~~35~~	~~15~~	~~60~~
~~D~~	~~45~~	~~58~~	~~50~~
R	$\boxed{45}$	58	~~70~~

Eliminate rows C and D, and column R.

In the smaller game, N1's optimal strategy is purely play R.

N2's optimal strategy is purely play C.

Two competing firms are deciding whether to locate a new store at point A, B, or C. 100 customers live at A, 100 live at B, and 120 at C. Each customer will shop at the nearest store. If a customer is equidistant from two stores, they will shop at either with probability $\frac{1}{2}$. Where will each firm locate their stores?



constant sum game

P1 \ P2	~~A~~	B	~~C~~
~~A~~	~~160~~	~~100~~	~~150~~
B	~~220~~	160	~~200~~
~~C~~	~~170~~	~~120~~	~~160~~

P1's payoff matrix

For P1, B dominates A and B

Same for P2

Optimal strategy for both players is build at B.
Each will get 160 customers.