Modeling with linear programs

(3.4, 3.5, 3.6, 3.7, 3.8, 3.9
3.10, 3.11)

An orchard has two farms that grow wheat and corn. Because of differing soil conditions, there are differences in yields and costs of growing crops on the two farms, shown below. Each farm has 100 acres available. 11,000 bushels of wheat and 7000 bushels of corn must be grown. Determine a planting plan to minimize cost.

	Farm 1	Farm 2
Corn yield/acre (bushels)	500	650
Corn cost/acre ($)	100	120
Wheat yield/acre (bushels)	400	350
Wheat cost/acre ($)	90	80

What variables should we use?

C_1 = acres of corn planted on farm 1

C_2 " " " 2

W_1 = " wheat " 1

W_2 = " " " 2

Objective:

Min $100C_1 + 120C_2 + 90W_1 + 80W_2$

Constraints:

$c_1 + w_1 \leq 100$

$c_2 + w_2 \leq 100$

$500c_1 + 650c_2 \geq 7000$

$400w_1 + 350w_2 \geq 11000$

$c_1, c_2, w_1, w_2 \geq 0$

$$\text{Min} \quad 100c_1 + 120c_2 + 90w_1 + 80w_2$$

s.t.

$$c_1 + w_1 \leq 100$$

$$c_2 + w_2 \leq 100$$

$$500c_1 + 650c_2 \geq 7000$$

$$400w_1 + 350w_2 \geq 11000$$

$$c_1, c_2, w_1, w_2 \geq 0$$

During each 4-hour period, a building requires the number of on-duty security guards shown below. Each guard works two consecutive 4-hour shifts. Design a schedule that meets the requirements and minimizes the number of guards.

Period		Guards needed
0	0:00 - 4:00	8
1	4:00 - 8:00	7
2	8:00 - 12:00	6
3	12:00 - 16:00	6
4	16:00 - 20:00	5
5	20:00 - 0:00	4

Does not give us a nice plan

Let x_i be the number of guards on duty during period i.

$\text{Min } \frac{1}{2}(x_0 + \cdots + x_5)$

$x_0 \geq 8$ $\quad x_3 \geq 6$

$x_1 \geq 7$ $\quad x_4 \geq 5$

$x_2 \geq 6$ $\quad x_5 \geq 4$

Let y_i be the number of guards that start at the beginning of period i.

Min $y_0 + y_1 + \cdots + y_5$

s.t.

$y_0 + y_1 \geq 7$

$y_1 + y_2 \geq 6$

$y_2 + y_3 \geq 6$

$y_3 + y_4 \geq 5$

$y_4 + y_5 \geq 4$

$y_5 + y_0 \geq 8$

$y_0, y_1, \ldots, y_5 \geq 0$

$y_0, y_1, \ldots, y_5$ are integers

↓ ignore for now

A brief discussion of LP solvers

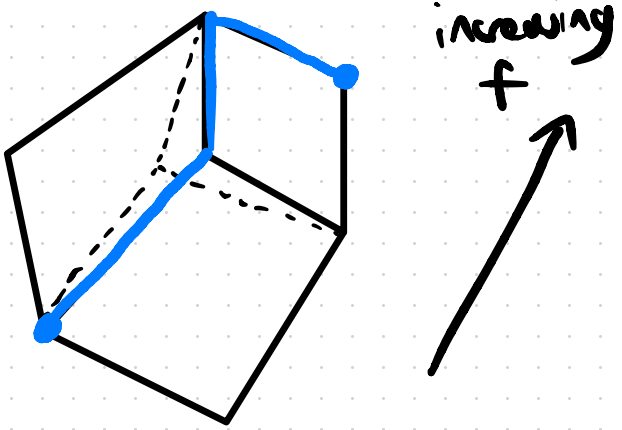
There are two main classes of algorithms used to solve LP's.

1) Simplex method

2) Interior-point methods

Simplex method: The basic idea is to travel along the boundary of the polyhedron until an optimal vertex is reached.

Max f



Feasible region

Pros

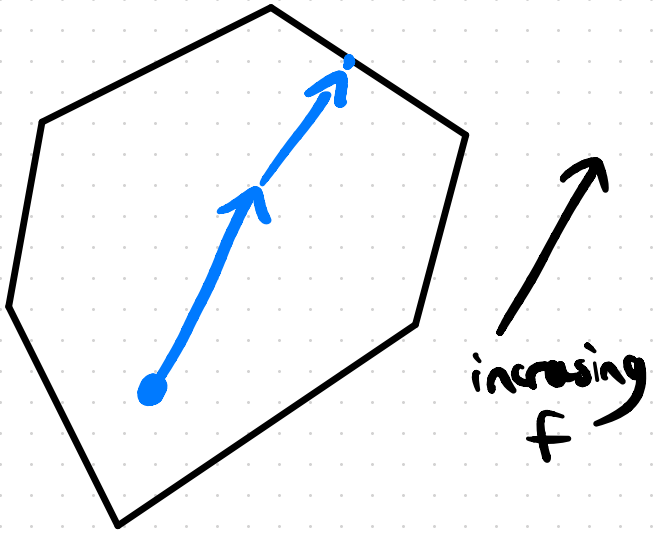
- Good performance on most LP's
- Easy to understand and adapt to situation
- Numerically robust
- Always returns a vertex optimal solution.

Cons

- Very bad performance on some LP's.
- Doesn't scale well to very large LP's.

Interior-point (barrier, second-order) methods: The idea is to traverse the interior of the polyhedron to reach an optimal solution.

Max f



Pros

- Polynomial-time worst case behavior
- Reliable in practice
- Scales well to large LP's.

Cons

- Doesn't usually return a vertex if there are multiple optimal solutions.
- Most algorithms are proprietary.

There are also first-order methods, which use gradient data to converge to a solution (similar to gradient descent). These algorithms can be effective for finding approximate solutions to LP's.

In practice, it is common to run both algorithms in parallel and return the answer of whichever finishes first.

In this class, the LP's will be very small and it doesn't matter what you use. I recommend GLOP (simplex method solver developed by Google) for this class.

Outside this class, I recommend a commercial solver (Gurobi, CPLEX, Xpress). You can get an academic license for free.