

You are trying to schedule training meetings for a large group of people, each with different availability times. How can you schedule the minimum number of meetings so that everyone can attend at least one?

Suppose you can only schedule four meetings. How can you do so while maximizing the number of people that can attend?

Solving integer programs

Linear programs can be solved efficiently, both in theory and practice. (It is in complexity class P .)

Integer and mixed integer programs are theoretically difficult to solve. (It is NP -complete.)

In practice, there are algorithms for solving IP's which are fast enough on real-world problems.

Branch-and-bound is a general paradigm for optimization where one repeatedly branches into subproblems, while eliminating branches that cannot contain the optimum.

To solve an IP with this method, we first solve it as a **linear program**. If we get an integer solution, we are done.

Otherwise, we use the solution to **branch** the original feasible region into two feasible regions. We repeat the process on each branch.

Each integer solution we obtain in the process gives a **bound** for the actual solution. If a branch cannot beat this solution, the entire branch can be eliminated.

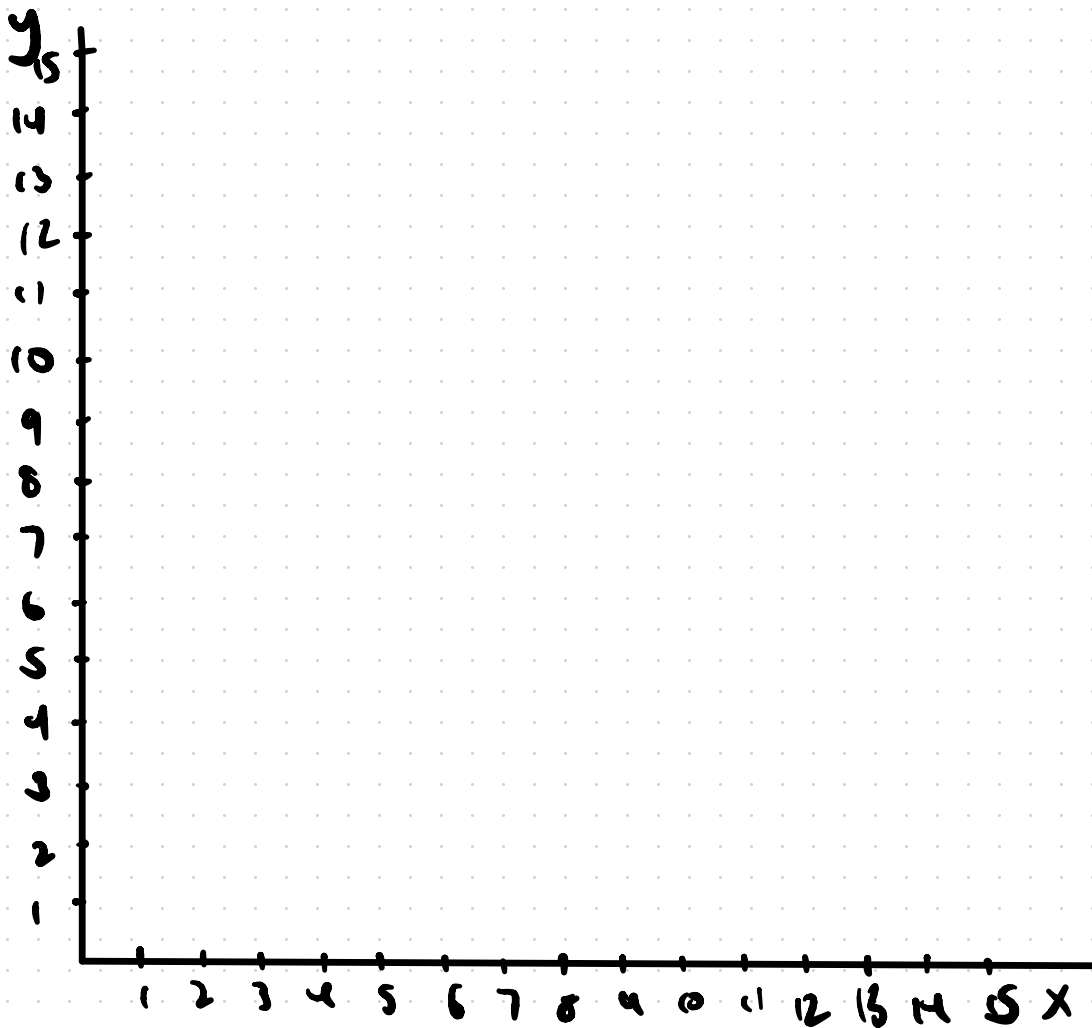
$$\text{Max } 3x + 4y$$

$$\text{s.t. } x + y \leq 10$$

$$2x + 5y \leq 30$$

$$x, y \geq 0$$

$$x, y \in \mathbb{Z}$$



In the worst-case scenario, the branch-and-bound method yields exponentially many branches. In practice, many branches may be eliminated early.

In practice, integer solutions are usually vertex solutions of branches. Therefore simplex method is generally preferred for solving IP's.