Graphs
(8.1, 8.2)

Suppose we want to color a map of the world so that

(a) Each country is colored with one color.

(b) If two countries share a border, they have different colors.

How can we produce such a coloring that uses only five colors?

This problem can be modeled with a graph.

A graph consists of the following data:

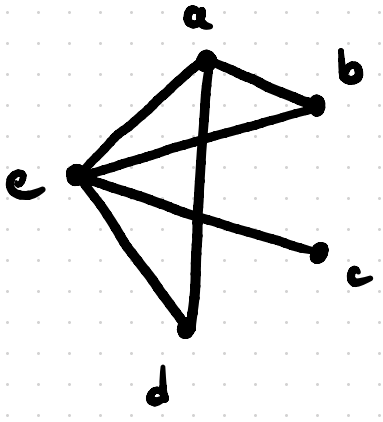
(1) A set of vertices. 

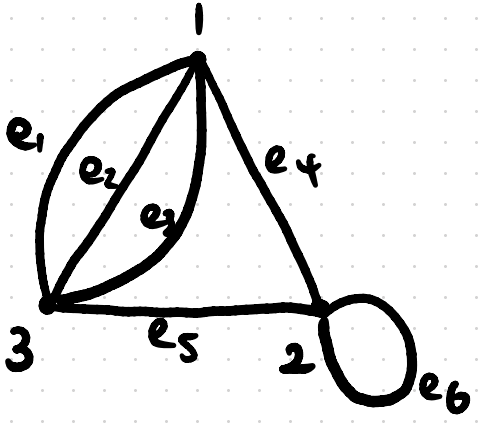
(2) A set of edges. Each edge connects two vertices, called the endpoints. (The endpoints can be the same, in which the edge is called a loop.) 

(3) (optional) A direction for each edge, pointing from one endpoint to the other. A graph with directed edges is called a directed graph or digraph.

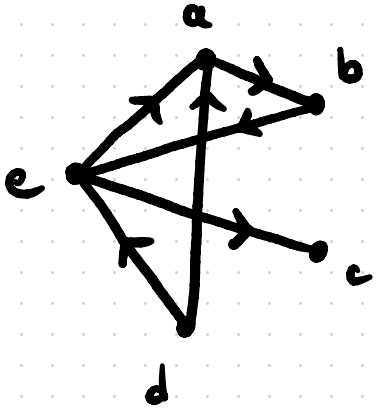
NOTE: We can communicate graphs through pictures, but picture itself is not a graph.

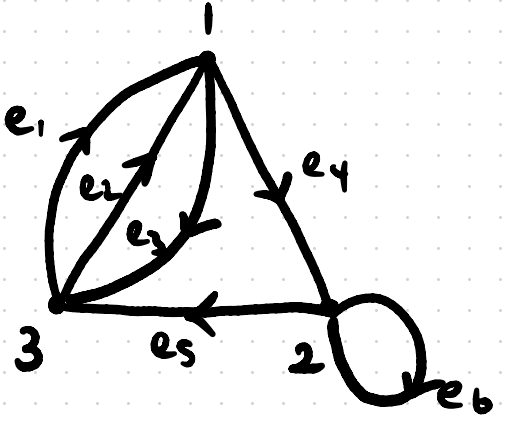
Examples of undirected graphs:





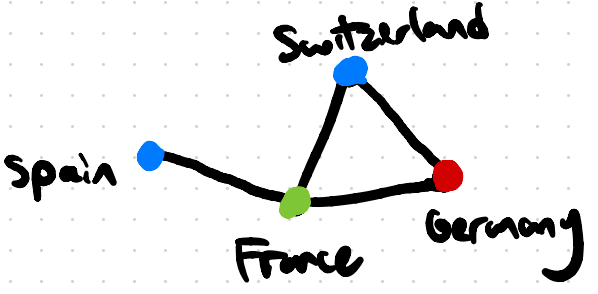
Examples of directed graphs:





Modeling the map problem as a graph:

Create a graph where the vertices are countries, with an edge between two vertices if they share a border.



We want to color the vertices with 5 colors so that no two vertices of the same color share an edge. This is called a graph coloring.