$$z := \vec{x} \cdot \vec{w}$$

$$L(\vec{z}, y_t) = -y_t \log(\sigma(z)) - (1 - y_t)\log(1 - \sigma(z))$$

$$\delta_0 := \frac{\partial L}{\partial \vec{z}} = -y_t \frac{1}{\sigma(z)} \sigma'(z) - (1 - y_t) \frac{1}{1 - \sigma(z)} \cdot -\sigma'(z)$$

$$= -y_t \frac{1}{\cancel{\sigma(z)}} \cancel{\sigma(z)} (1 - \sigma(z)) - (1 - y_t) \frac{1}{\cancel{1 - \sigma(z)}} \cdot -\sigma(z) \cancel{(1 - \sigma(z))}$$

$$\uparrow$$

$$\sigma'(z) = \sigma(z)(1 - \sigma(z))$$

$$= -y_t (1 - \sigma(z)) - (1 - y_t) \cdot (-\sigma(z))$$

$$= -y_t (1 - \sigma(z)) + (1 - y_t)\sigma(z)$$

$$= -y_t + \cancel{y_t \sigma(z)} + \sigma(z) - \cancel{y_t \sigma(z)}$$

$$= \sigma(z) - y_t$$

$$= \hat{y} - y_t$$

This happens to be the same value as in linear regression (though the losses are different!).