311 – Numerical Computations
Lab 12: Splines & Linear Programming

A) Splines: This program demonstrates Splines in Python:

```
from scipy.interpolate import interp1d
from scipy.interpolate import CubicSpline
import numpy as np

X = np.array([3,9,15,21])
Y = np.array([2, 10, 13, 15])

r1 = interp1d(X, Y ,kind = 'linear')

r2 = interp1d(X, Y, kind = 'quadratic')

r3 = interp1d(X, Y, kind = 'cubic')
r4 = CubicSpline(X, Y)        #another way to do
                              #cubic splines
print(r1(6),r1(9), r1(14))

print(r2(6),r2(9), r2(14))

print(r3(6),r3(9), r3(14))

print(r4(6),r4(9), r4(14))
```

Output:

```
6.0  10.0  12.5
6.708333333333334  10.0  12.63425925925926
6.875000000000002  10.000000000000002  12.677469135802468
6.875  10.0  12.67746913580247
```

B) Linear Programming

General Form of LP:

Maximize (or Minimize): $c_1x_1 + c_2x_2 + \ldots + c_n x_n$

Due to conditions:

$$c_{11}x_1 + c_{12}x_2 + \ldots + c_{1n} x_n \quad (\leq, \geq) \quad b_1$$

$$c_{21}x_1 + c_{22}x_2 + \ldots + c_{2n} x_n \quad (\leq, \geq) \quad b_2$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$c_{m1}x_1 + c_{m2}x_2 + \ldots + c_{mn} x_n \quad (\leq, \geq) \quad b_m$$

Possibly with bounds on some or all x_i:

For example: $x_1 \leq 9$, $x_2 \geq 7$, $1 \leq x_3 \leq 14$

Example:

Minimize: $-x_1 + 4x_2$

Such that: $-3x_1 + x_2 \leq 6$

$-x_1 - 2x_2 \geq -4$

$x_2 \geq -3$

Python:

1- **Convert the problem to minimization (if it is not already so). (How??)**
2- **Convert all inequality signs to <= (How??)**

Code (of our example):

```
from scipy.optimize import linprog
c = [-1, 4]                    #objective function
A = [[-3, 1], [1, 2]]          # constraints coefficient matrix
b = [6, 4]                     # constraints vector (RHS of inequalities)
x1_bounds = (None, None)       #bounds on x1
x2_bounds = (-3, None)         #bounds on x2

res = linprog(c, A_ub=A, b_ub=b, bounds=[x1_bounds, x2_bounds])
print(res)
```

```
message: 'Optimization terminated successfully.'
      nit: 6
    slack: array([3.89999997e+01, 8.46872314e-08])
   status: 0
  success: True
        x: array([ 9.99999989, -2.99999999])          # x1=10  x2=-3
```

Lab Task:

Use Python to solve the following Linear Programming Problem:

Maximize: $5x_1 + 12x_2 + 4x_3 + 8x_4$

Subject to: $2x_1 + x_2 + x_3 \leq 45$

$3x_2 + x_3 + x_4 \leq 18$

$-4x_1 - x_3 \geq -40$

$-x_1 - x_2 - x_4 \geq -11$

and bounds: $x_1 \geq 0$, $x_2 \geq 5$, $x_3 \geq 0$, $x_4 \leq 10$