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Numerical Computation

Assignment #3

Question 1:

Initial guess: $x_0 = 2$

$f(x) = x^5 - 20, \quad f'(x) = 5x^4$

itr = 1:

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 2 - \frac{12}{80} = 1.85$$

itr = 2:

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 1.85 - \frac{1.6699}{58.5675} = 1.8215$$

itr = 3:

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 1.8215 - \frac{0.0515}{55.041} = 1.8121$$

itr = 4:

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)} = 1.8121 - \frac{(-0.4606)}{53.9136} = 1.8206$$

Question 2:

Let $f(x) = 6(x - 3)^4$

Suppose that $f(x)$ can be solved using the bisection method, then there exists an interval

I = [a,b] such that $f(a) \cdot f(b) < 0$

Then either $f(a) < 0$ or $f(b) < 0$

For $f(a)$:

$f(a) = 6(a - 3)^4$, Since $(a - 3)^4$ is raised to the 4th power, then $f(a)$ can't be negative.

$\therefore f(a) \geq 0$ — (1)

For $f(b)$:

$f(b) = 6(b - 3)^4$, Since $(b - 3)^4$ is raised to the 4th power, then $f(b)$ can't be negative.

$\therefore f(b) \geq 0$ — (2)

$\therefore$ from (1) and (2), $f(a) \cdot f(b) \geq 0$, which is a contradiction.

$\therefore$ $f(x)$ cannot be solved using the bisection method ▮

Question 3:

The point of intersection between $h(x)$ and $g(x)$ is when $h(x) = g(x)$

Let $f(x) = h(x) - g(x) = e^{x} - x^{2}$

Applying the Newton-Raphson method:

Initial Guess: $x_0 = -1$, $f(x) = e^{x} - x^{2}$, $f'(x) = e^{x} - 2x$

itr = 1:

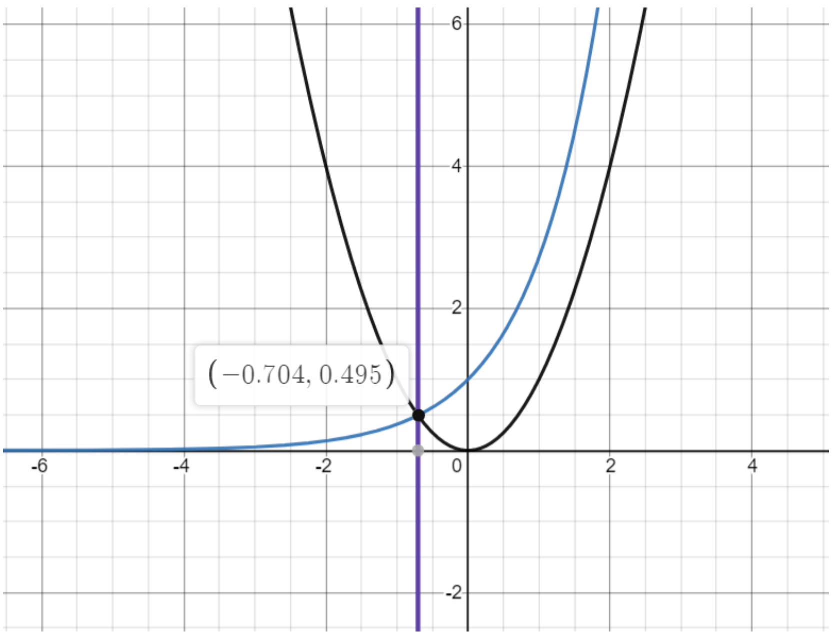
$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = -1 - \frac{-0.6321}{2.3679} = -0.7331$$

itr = 2:

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = -0.7331 - \frac{-0.057}{1.9466} = -0.7038$$

itr = 3:

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = -0.7038 - \frac{-6.3258\cdot 10^{-4}}{1.9023} = -0.7035$$



Question 4:

a) $f(x) = \frac{x^4 - x - 10}{x^3 + 7}$, $f(0) = \frac{-10}{7}$, $f(2) = \frac{4}{15}$

$f(0) \cdot f(2) = \frac{-8}{21} < 0$

itr = 1:

Mid point = $\frac{2+0}{2} = 1$, $f(1) = \frac{-5}{4} < 0$

New Bracket: [1, 2]

itr = 2:

Mid point = $\frac{1+2}{2} = \frac{3}{2}$, $f(\frac{3}{2}) = \frac{-103}{166} < 0$

New Bracket: [$\frac{3}{2}$, 2]

itr = 3:

Mid point = $\frac{\frac{3}{2}+2}{2} = \frac{7}{4}$, $f(\frac{7}{4}) = \frac{-607}{3164} < 0$

New Bracket: [$\frac{7}{4}$, 2]

itr = 4:

Mid point = $\frac{\frac{7}{4}+2}{2} = \frac{15}{8}$, $f(\frac{15}{8}) = \frac{1985}{55672} > 0$

New Bracket: [$\frac{7}{4}$, $\frac{15}{8}$]

b) $\frac{2-0}{2^n} \leq 10^{-3} \Rightarrow \frac{2}{2^n} \leq 10^{-3} \Rightarrow 2^{1-n} \leq 10^{-3} \Rightarrow (1 - n) \cdot ln(2) \leq ln(10^{-3})$

$\Rightarrow 1 - n \leq \frac{ln(10^{-3})}{ln(2)} \Rightarrow - n \Rightarrow \frac{ln(10^{-3})}{ln(2)} - 1 \Rightarrow n \leq 1 - \frac{ln(10^{-3})}{ln(2)}$

$n \leq 10.9658 \approx 11$

Question 5:

a) $x_0 = 2, \qquad x - sin(x) - 0.5 = 0 \Rightarrow x = sin(x) + 0.5$

itr = 1:

$x_1 = sin(x_0) + 0.5 = sin(2) + 0.5 = 1.4093$

itr = 2:

$x_2 = sin(x_1) + 0.5 = sin(0.5439) + 0.5 = 1.4869$

itr = 3:

$x_3 = sin(x_2) + 0.5 = sin(0.5093) + 0.5 = 1.4965$

itr = 4:

$x_4 = sin(x_3) + 0.5 = sin(0.5089) + 0.5 = 1.497$

b)

Main Equation: $x_{i+1} = x_i - \frac{f(x_i)\cdot(x_{i-1} - x_i)}{f(x_{i-1}) - f(x_i)}$, $x_0 = 1$, $x_1 = 2$

$f(x) = x - sin(x) - 0.5$

itr = 1:

$$x_2 = x_1 - \frac{f(x_1)\cdot(x_0 - x_1)}{f(x_0) - f(x_1)} = 1.1761$$

itr = 2:

$$x_3 = x_2 - \frac{f(x_2)\cdot(x_1 - x_2)}{f(x_1) - f(x_2)} = 1.4190$$

itr = 3:

$$x_4 = x_3 - \frac{f(x_3)\cdot(x_2 - x_3)}{f(x_2) - f(x_3)} = 1.5141$$

itr = 4:

$$x_5 = x_4 - \frac{f(x_4)\cdot(x_3 - x_4)}{f(x_3) - f(x_4)} = 1.4966$$