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Theory of Computations

Assignment #4

Question #1:

Let $a^n b^{3n} \in L$, where $n \geq 0$. Let S(n) be a string such that $S(n) = a^n b^{3n}$

Choosing n = 5, then: S(5) = $a^5 b^{15}$ = aaaaabbbbbbbbbbbbbbb

By choosing $x = a,\ y = aaaa,\ z = bbbbbbbbbbbbbbb$, then we have $S(5) = xyz$

Now suppose that $xy^i z$ is regular, then $xy^{i+1} z$ must be regular.

So if xyz is regular, then $xy^2 z$ must also be regular.

$\therefore\ xy^2 z = aaaaaaaaabbbbbbbbbbbbbbb = a^9 b^{15}$

Since 15 < 3 * 9, then there is a contradiction.

$\therefore$ L is not a regular language.

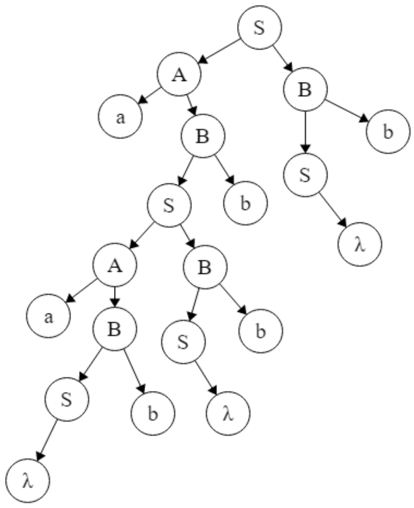
Question #2:

I. Rightmost derivation:

$S \Rightarrow AB \Rightarrow ASb \Rightarrow Ab \Rightarrow aBb \Rightarrow aSbb \Rightarrow aABbb \Rightarrow aASbbb \Rightarrow aAbbb$

$\Rightarrow aaBbbb \Rightarrow aaSbbbb \Rightarrow aabbbb$

Parse tree:

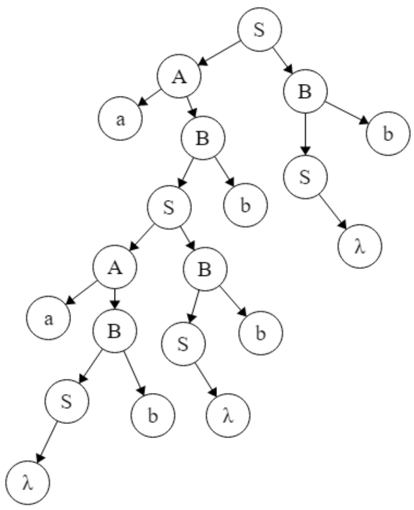


II. Leftmost derivation:

$S \Rightarrow AB \Rightarrow aBB \Rightarrow aSbB \Rightarrow aABbB \Rightarrow aaBBbB \Rightarrow aaSbBbB \Rightarrow aabBbB$

$\Rightarrow aabSbbB \Rightarrow aabbbSb \Rightarrow aabbbb$

Parse tree:



Question #3:

a) $S \Rightarrow aaB \Rightarrow aaAa \Rightarrow aabBba \Rightarrow aabbBbba \Rightarrow aabbAabba \Rightarrow aabbabba$

b) The string is not accepted since the grammar forces us to begin with "aa"

Question #4:

a)

$S \rightarrow aA$

$A \rightarrow aBC$

$B \rightarrow b \mid aBb$

b)

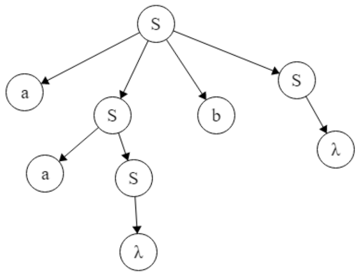
$S \rightarrow aA$

$A \rightarrow bbb \mid aAB$

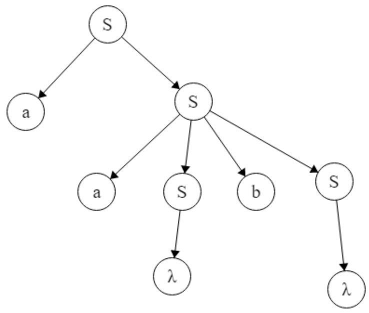
$B \rightarrow bb$

Question #5:

a)



One way to draw a parse tree for the string "aab"



Another way to draw a parse tree for the string "aab"

b)

$S \Rightarrow aSbS \Rightarrow aaSbS \Rightarrow aabS \Rightarrow aab$

$S \Rightarrow aS \Rightarrow aaSbS \Rightarrow aabS \Rightarrow aab$

Since there were two seperate derivations and parse trees for the same string, then the language is ambiguous.

Question #6:

a)

$S \rightarrow ASBb \mid SBb \mid aA \mid a$

$A \rightarrow aAS \mid aS \mid a$

$B \rightarrow SbS \mid A \mid bb \mid CD$

$C \rightarrow AC \mid C \mid CC$

$D \rightarrow DA \mid D \mid aA$

b)

$S \rightarrow ASBb \mid SBb \mid aA \mid a$

$A \rightarrow aAS \mid aS \mid a$

$B \rightarrow SbS \mid a \mid bb \mid CD$

$C \rightarrow AC \mid C \mid CC$

$D \rightarrow DA \mid a \mid b$

c)

Part I: Find all Non-Terminal symbols that derive the elements of Σ and put them in a set. The process is done repeatedly until the last two sets are equal.

$S1 = \{S, A, B, D\}$, $S2 = \{S, A, B, D\}$

Since S1 and S2 are equal, then part I is complete.

The new grammar:

$S \rightarrow ASBb \mid SBb \mid aA \mid a$

$A \rightarrow aAS \mid aS \mid a$

$B \rightarrow SbS \mid a \mid bb$

$D \rightarrow DA \mid a \mid b$

Part II: Have the start symbol in a set, and append all the non-Terminal symbols that are derived from it until a dead-end is reached.

$S1 = \{S\},\ S2 = \{S, A, B\},\ S3 = \{S, A, B\}$

Since S2 and S3 are equal, then part II is complete.

The new grammar:

$S \rightarrow ASBb \mid SBb \mid aA \mid a$

$A \rightarrow aAS \mid aS \mid a$

$B \rightarrow SbS \mid a \mid bb$

d)

$S \rightarrow S_1B_2 \mid SB_2 \mid A_1A \mid a$

$A \rightarrow S_1S \mid A_1S \mid a$

$B \rightarrow SS_2 \mid B_1B_1 \mid a$

$B_1 \rightarrow b$

$B_2 \rightarrow B\ B_1$

$A_1 \rightarrow a$

$S_1 \rightarrow AS$

$S_2 \rightarrow B_1S$

Question #7:

Let $a^n b^n c^{n+1} \in L$, where $n \geq 0$. Let $S(n) = a^n b^n c^{n+1}$

By choosing n = 4, then $S(4) = aaaabbbbccccc$

By choosing $u = aa,\ v = aa,\ x = bbbbc,\ y = c,\ z = ccc$

Then $uvxyz = S(4)$

Since $uv^i xy^i z$ is context free, then $uv^{i+1} xy^{i+1} z$ must also be context free.

$\therefore$ By applying the property on $uvxyz$, $uv^2 xy^2 z$ must also be context free

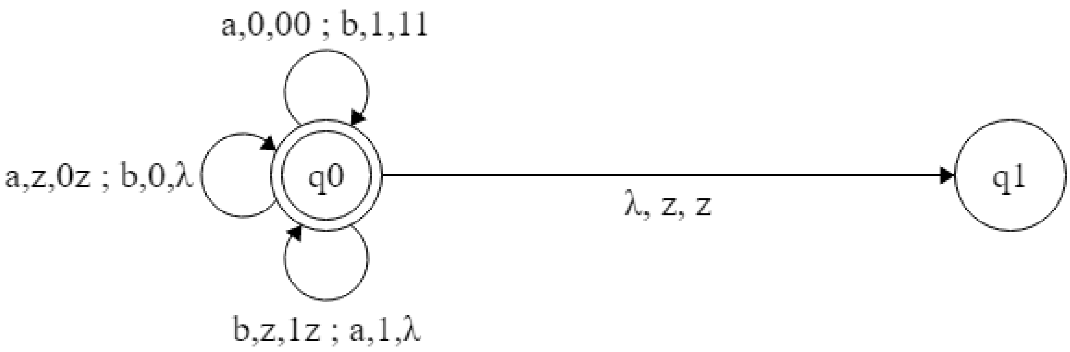
$\therefore uv^2 xy^2 z = aaaaaabbbbcccccc = a^6 b^4 c^6$

Since the language L was defined as $L = \{a^n b^n c^m \mid n \geq 0,\ m > n\}$

|a| is greater than |b| and |a| is equal to |c|, which contradicts the definition.

$\therefore$ L is not context free

Question 8:



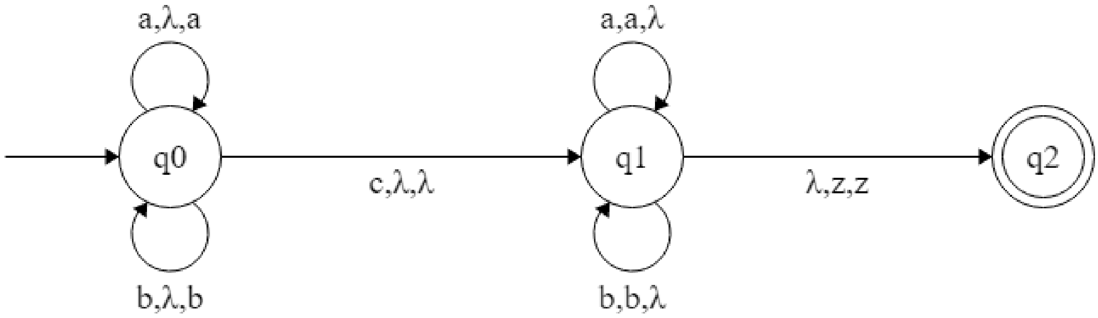
Initial State is q0

$\delta(q0, a, z) = (q0, 0z)$, $\delta(q0, b, z) = (q0, 1z)$, $\delta(q0, b, 0) = (q0, \lambda)$

$\delta(q0, a, 1) = (q0, \lambda)$, $\delta(q0, a, 0) = (q0, 00)$, $\delta(q0, b, 1) = (q0, 11)$

$\delta(q0, \lambda, z) = (q1, z)$

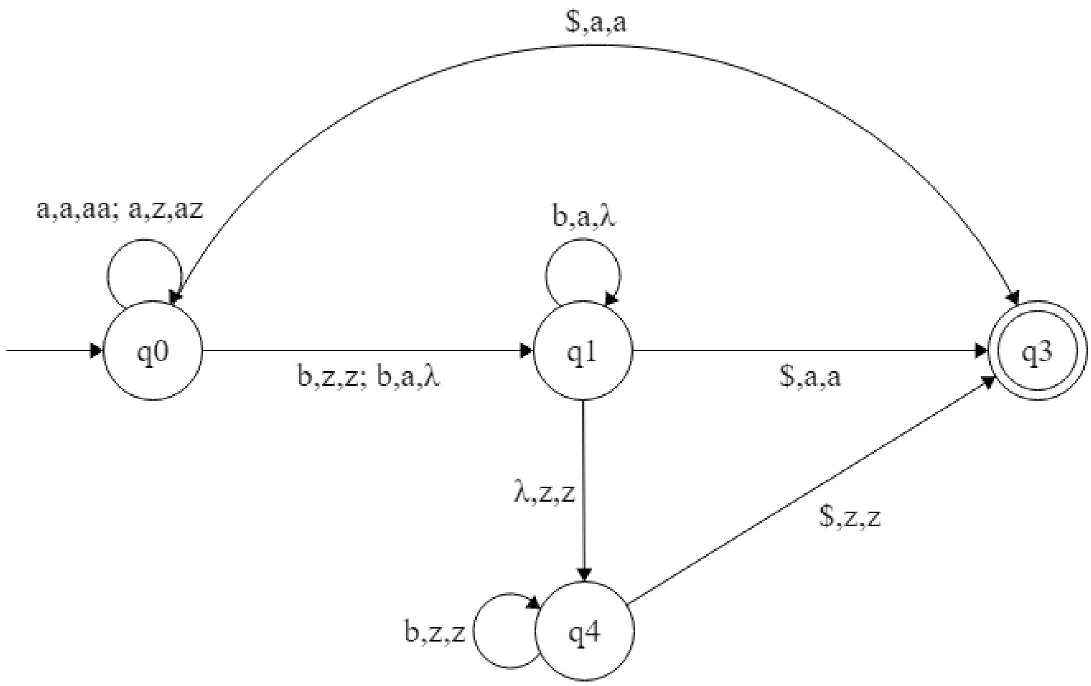
Question 9:



$\delta(q0, a, \lambda) = (q0, a)$, $\delta(q0, b, \lambda) = (q0, b)$, $\delta(q0, c, \lambda) = (q1, \lambda)$

$\delta(q1, a, a) = (q1, \lambda)$, $\delta(q1, b, b) = (q1, \lambda)$, $\delta(q1, \lambda, z) = (q2, z)$

Question 10:



$\delta(q0, a, a) = (q0, aa)$, $\delta(q0, a, z) = (q0, az)$, $\delta(q0, \$, a) = (q3, a)$

$\delta(q0, b, z) = (q1, z)$, $\delta(q0, b, a) = (q1, \lambda)$, $\delta(q1, b, a) = (q1, \lambda)$

$\delta(q1, \lambda, z) = (q4, z)$, $\delta(q1, \$, a) = (q3, a)$, $\delta(q4, b, z) = (q4, z)$

$\delta(q4, \$, z) = (q3, z)$