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### Assignment #1

#### Problem 1.1:

Expression	Dominating Term(s)	Big O notation
$5 + 0.001n^3 + 0.025n$	$0.001n^3$	$O(n^3)$
$10^{1000} + 2^{1000} + 4^{200} + 1$	$10^{1000} + 2^{1000} + 4^{200} + 1$	$O(1)$
$5n + n^{1.5} + 3n \log n$	$n^{1.5}$	$O(n^{1.5})$
$\sqrt[3]{n^9} + 10^{10^{10^{10}}}$	$\sqrt[3]{n^9} = n^3$	$O(n^3)$
$n! + 2^n + n \log n$	$n!$	$O(n!)$
$n! + n^n + n \log n$	$n^n$	$O(n^n)$
$2^{3^n} + 3^{2^n} + 5^n$	$3^{2^n} = 9^n$	$O(9^n)$
$\sqrt{n} + \log n$	$\sqrt{n}$	$O(\sqrt{n})$
$0.003 \log n + \log(\log n)$	$0.003 \log n$	$O(\log n)$
$\log_2 n + \log_3 n + \log_5 n$	$\log_2 n$	$O(\log_2 n)$
$\sum_{i=1}^{10} i * n^i$	$10n^{10}$	$O(n^{10})$
$\prod_{i=1}^4 n^i$	$n^{10}$	$O(n^{10})$

$\sum_{i=2}^{10} 30 \log(i)$	$\sum_{i=2}^{10} 30 \log(i)$	$O(1)$
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### Problem 1.2:

$O(1)$ ,  $O(1)$ ,  $O(\log n)$ ,  $O(\log_2 n)$ ,  $O(\sqrt{n})$ ,  $O(n^{1.5})$ ,  $O(n^3)$ ,  $O(n^{10})$ ,  $O(n^{10})$ ,  $O(9^n)$ ,  $O(n!)$ ,  $O(n^n)$

### Problem 2:

#### Function A:

```
1 void A(){
2     cout<<"Hello World"<<endl;
3 }
```

t = 1

$T(n) = 1 \Rightarrow$  Big O notation:  $O(1)$

#### Function B:

```
1 void B(int n){
2     int i = 0;
3     int sum = 0;
4     while (i < n){
5         sum += i;
6         i += 1;
7     }
8 }
```

t = 1  
t = 1  
t = n + 1  
t = n  
t = n

$T(n) = 1 + 1 + n + 1 + n + n = 3 + 3n \Rightarrow$  Big O notation:  $O(n)$

#### Function C:

```
1 void C(int n, int m){
2     int i = 0;
3     int sum = 0;
4     while (i < n){
5         int j = 0;
6         while (j < m){
7             sum += j;
8             j += 1;
9         }
10        i += 1;
11    }
12 }
```

t = 1  
t = 1  
t = n + 1  
t = n  
t = n\*(m+1)  
t = nm  
t = nm  
t = nm

$T(n,m) = 1 + 1 + n + 1 + n + nm + n + nm + nm + n = 3 + 3m + 3nm \Rightarrow$  Big O notation:  $O(nm)$

### Function D:

```

1 void D(int n, int m, int l){
2     int i = 0; → t = 1
3     int sum = 0; → t = 1
4     while (i < n){ → t = n + 1
5         int j = 0; → t = n
6         while (j < m){ → t = n*(m+1)
7             int k = 0; → t = nm
8             while( k < l ){ → t = nm*(l+1)
9                 sum += k; → t = nml
10                k += 1 → t = nml
11            }
12            j += 1; → t = nm
13        }
14        i += 1; → t = n
15    }
16 }

```

$$T(n,m,l) = 1 + 1 + n + 1 + n + nm + n + nm + nml + nm + nml + nml + nm + n$$

$$= 3 + 4n + 4nm + 3nml \Rightarrow \text{Big O notation: } O(nml)$$

### Function E:

```

1 int E(){
2     int n; → t = 1
3     cout<<"Enter a positive number:"; → t = 1
4     cin>>n; → t = 1
5     int i = 0; → t = 1
6     int sum = 0; → t = 1
7     while( i < n ){ → t = n + 1
8         sum += i; → t = n
9         i += 1; → t = n
10    }
11    return sum; → t = 1
12 }

```

$$T(n) = 1 + 1 + 1 + 1 + 1 + n + 1 + n + n + 1 = 7 + 3n \Rightarrow \text{Big O notation: } O(n)$$

### Function F:

```

1 void F(int n){
2     int sum = 0; → t = 1
3     for(int i = 0; i < n; i++){ → t = 1
4         for(int j = i; j < n; j++){ → t = n + 1
5             sum += 1; → t = (n-1)n/2
6         }
7     }
8 }

```

Annotations for Function F:

- Line 3:  $i = 0; t = 1$
- Line 3:  $i < n; t = n + 1$
- Line 3:  $i++; t = n$
- Line 4:  $j = i; t = n$
- Line 4:  $j < n; t = (n-1)n/2 + n$
- Line 4:  $j++; t = (n-1)n/2$

$$T(n) = 1 + 1 + n + 1 + n + \frac{1}{2}n^2 - \frac{1}{2}n + n + \frac{1}{2}n^2 - \frac{1}{2}n + \frac{1}{2}n^2 - \frac{1}{2}n = 3 + \frac{1}{2}n + \frac{3}{2}n^2$$

$$\Rightarrow \text{Big O notation: } O(n^2)$$

### Function G:

```

1 void G(int n){
2     int sum = 0; → t = 1
3     for(int i = 1; i < n; i = i*2){
4         sum += 1; → t = log(n)
5     }
6 }

```

— int i = 1: t = 1  
 — i < n: t = log(n) + 1  
 — i = i\*2: t = log(n)

$$T(n) = 1 + 1 + \log(n) + 1 + \log(n) + \log(n) = 3 + 3\log(n) \Rightarrow \text{Big O notation: } O(\log(n))$$

### Function H:

```

1 void H(int n){
2     int sum = 0; → t = 1
3     for(int i = n; i > 0; i = i/2){
4         sum += 1; → t = log(n)
5     }
6 }

```

— int i = n: t = 1  
 — i > 0: t = log(n) + 1  
 — i = i/2: t = log(n)

$$T(n) = 1 + 1 + \log(n) + 1 + \log(n) + \log(n) = 3 + 3\log(n) \Rightarrow \text{Big O notation: } O(\log(n))$$

### Problem 3:

#### Function A1:

```

1 void S(int n){
2     for(int i = 1; i < n; i = i*2){
3         cout<<"i_U=U"<<i; → t = log(n)
4     }
5 }
6 void L(int n){
7     for(int i = 0; i < n; i = i + 2){
8         cout<<"L..i_U=U"<<i; → t = ((n + 2)/2)
9     }
10 }
11 void M(int n){
12     while(n > 0){ → t = log5 n + 1 (log base 5 of n)
13         cout<<"M..i_U=U"<<i; → t = log5 n
14         n = n/5; → t = log5 n
15     }
16 }
17 void A1(int n){
18     S(n);
19     L(n);
20     M(n);
21 }

```

— int i = 1: t = 1  
 — i < n: t = log(n) + 1  
 — i = i\*2: t = log(n)

— int i = 0: t = 1  
 — i < n: t = ((n + 2)/2) + 1  
 — i = i + 2: t = ((n + 2)/2)

$$S(n): T1(n) = 1 + \log(n) + 1 + \log(n) + \log(n) = 2 + 3\log(n)$$

$$L(n): T2(n) = 1 + \frac{1}{2}n + 1 + 1 + \frac{1}{2}n + 1 + \frac{1}{2}n + 1 = 5 + \frac{3}{2}n$$

$$M(n): T3(n) = \log_5 n + 1 + \log_5 n + \log_5 n = 1 + 3\log_5 n$$

$$\therefore T(n) = T1 + T2 + T(3) = 2 + 3\log(n) + 5 + \frac{3}{2}n + 1 + 3\log_5 n = 8 + 3\log(n) + \frac{3}{2}n + 3\log_5 n$$

$$\Rightarrow \text{Big O notation: } O(n)$$

### Function B1:

```
1 void B1(int n){
2     int sum = 0; → t = 1
3     for(int i = 1; i < n; i = i*2){ → int i = 0: t = 1
4         sum += 100; → t = 6           → i < n: t = 6
5         if( sum > 500 ){ → t = 6       → i++: t = 6
6             break; → t = 1
7         }
8     }
9 }
```

$T(n) = 1 + 1 + 6 + 6 + 6 + 6 + 1 = 27 \Rightarrow \text{Big O notation: } O(1)$

### Function C1:

```
1 int C1(int n){
2     if( n == 0) → t = 1 + n
3     return 0; → t = 1
4     return C1(n-1) + n; → t = n
5 }
```

$T(n) = 1 + n + 1 + n = 2 + 2n \Rightarrow \text{Big O notation: } O(n)$

### Function D1:

```
1 int D1(int n){
2     if(n == 0) → t = 1
3     return 0;
4     return 2*C1(n-1) + n; → t = 2 + 2n
5 }
```

$T(n) = 1 + 2 + 2n = 3 + 2n \Rightarrow \text{Big O notation: } O(n)$

### Function E1:

```
1 int E1(int n){
2     if(n == 0) → t = 1
3     return 0;
4     return C1(n-1) + C1(n-1) + n; → t = 2 + 2n + 2 + 2n + n
5 }
```

$T(n) = 1 + 2 + 2n + 2 + 2n + n = 5 + 5n \Rightarrow \text{Big O notation: } O(n)$

### Function F1:

```
1 void F1(int n){
2     if(n == 0) → t = 1 + n      | int i = 0: t = 1 + n
3     return 0; → t = 1
4     for(int i = 0; i < n; i++){ | i < n: t = n + 1 + n*(n-1)/2 + n
5         cout<<n<<" "<<i; t = n + n(n-1)/2 | i++: t = n + n*(n-1)/2
6     }
7     F1(n-1); → t = n
8 }
```

$$\begin{aligned} T(n) &= 1 + n + 1 + 1 + n + n + 1 + \frac{1}{2}n^2 - \frac{1}{2}n + n + n + \frac{1}{2}n^2 - \frac{1}{2}n + n + \frac{1}{2}n^2 - \frac{1}{2}n + n \\ &= 4 + \frac{11}{2}n + \frac{3}{2}n^2 \Rightarrow \text{Big O notation: } O(n^2) \end{aligned}$$

### Function G1:

```
1 void G1(int n){
2     if(n == 0) → t = 1 + log(n)
3     return 0; → t = 1
4     return G1(n/2) + n; → t = log(n)
5 }
```

$$T(n) = 1 + \log(n) + 1 + \log(n) = 2 + 2\log(n) \Rightarrow \text{Big O notation: } O(\log(n))$$