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Numerical Computations

Assignment #5

Q1: let the amount of advertising be denoted as A, and the amount of sales by S.

Since the sales are dependent on how much ~~advertising~~ the product is advertised, then a linear relation can be made in the form of: $S = a_0 + a_1 A$

By using the data from the table, we get that:

$\sum S_i = 42570$, $\sum A_i = 3170$, $\sum A_i^2 = 2064900$

$\sum S_i A_i = 28034500$, $\bar{S} = 8514$, $\bar{A} = 634$

$$\therefore a_1 = \frac{n \times \sum S_i A_i - \sum S_i \sum A_i}{n \times \sum A_i^2 - (\sum A_i)^2} = 18.9608$$

$$\therefore a_0 = \bar{S} - a_1 \bar{A} = -3507.1472$$

$\therefore$ The linear relation is: $S = -3507.1472 + 18.9608A$

$\therefore$ The amount of sales if there were no advertising:

$S(0) = -3507.1472$

$\therefore$ The amount of sales if advertising has reached $A = 1000$: $S(1000) = 15453.6528$

Q2: a)

The Polynomial of degree 2 is: $y = a_0 X^2 + a_1 X + a_2$

$$\begin{bmatrix} n & \sum x_i & \sum x_i^2 \\ \sum x_i & \sum x_i^2 & \sum x_i^3 \\ \sum x_i^2 & \sum x_i^3 & \sum x_i^4 \end{bmatrix} \begin{bmatrix} a_2 \\ a_1 \\ a_0 \end{bmatrix} = \begin{bmatrix} \sum y_i \\ \sum x_i y_i \\ \sum x_i^2 y_i \end{bmatrix}$$

By using the values from the table, we get that:

$$\begin{bmatrix} 8 & 66 & 630 \\ 66 & 630 & 6732 \\ 630 & 6732 & 77826 \end{bmatrix} \begin{bmatrix} a_2 \\ a_1 \\ a_0 \end{bmatrix} = \begin{bmatrix} 99.5 \\ 790.5 \\ 7483.5 \end{bmatrix}$$

We get the results: $a_2 = 3.0116$, $a_1 = 2.2762$

$a_0 = -0.1251$

$\therefore$ The Polynomial that best suits this data is:

$$y = -0.1251X^2 + 2.2762X + 3.0116$$

Q2: b)

The Polynomial of degree 3 is: $y = a_0 X^3 + a_1 X^2 + a_2 X + a_3$

$$\begin{bmatrix} n & \sum x_i & \sum x_i^2 & \sum x_i^3 \\ \sum x_i & \sum x_i^2 & \sum x_i^3 & \sum x_i^4 \\ \sum x_i^2 & \sum x_i^3 & \sum x_i^4 & \sum x_i^5 \\ \sum x_i^3 & \sum x_i^4 & \sum x_i^5 & \sum x_i^6 \end{bmatrix} \begin{bmatrix} a_3 \\ a_2 \\ a_1 \\ a_0 \end{bmatrix} = \begin{bmatrix} \sum y_i \\ \sum x_i y_i \\ \sum x_i^2 y_i \\ \sum x_i^3 y_i \end{bmatrix}$$

By using the values from the table, we get:

$$\begin{bmatrix} 8 & 66 & 630 & 6732 \\ 66 & 630 & 6732 & 77826 \\ 630 & 6732 & 77826 & 948156 \\ 6732 & 77826 & 948156 & 11961690 \end{bmatrix} \begin{bmatrix} a_3 \\ a_2 \\ a_1 \\ a_0 \end{bmatrix} = \begin{bmatrix} 95.5 \\ 790.5 \\ 7483.5 \\ 79192.5 \end{bmatrix}$$

We get the results: $a_3 = -40.318$, $a_2 = 19.706$

$a_1 = -2.249$, $a_0 = 0.079$

We get the following Polynomial:

$$y = 0.079 X^3 - 2.249 X^2 + 19.706 X - 40.318$$

Q3: a)

$y = ae^{bx} \Rightarrow \ln(y) = \ln(ae^{bx}) \Rightarrow \ln(y) = \ln(a) + bx$

let $y' = \ln(y)$ and $a_0 = \ln(a)$, then we get the linear equation:

$y' = a_0 + bx$

x	4	5	8	10	13
$\ln(y)$	2.303	2.565	3.784	4.357	5.03

We can get that: $\sum x_i = 40$, $\sum x_i^2 = 374$

$\sum y'_i = 18.039$, $\sum x_i y'_i = 161.269$

$\bar{x} = 8$, $\bar{y'} = 3.6078$

$\therefore b = \dfrac{n\sum x_i y'_i - \sum y'_i \sum x_i}{n\sum x_i^2 - (\sum x_i)^2} = 0.31402$

$\therefore a_0 = \bar{y'} - b\bar{x} = 1.09564 \Rightarrow a_0 = \ln(a) \Rightarrow a = e^{a_0} = 2.991096$

$\therefore$ By substituting a and b in $y = ae^{bx}$, we get:

$y = 2.991096e^{0.31402x}$

$\therefore$ estimating y at $x = 18$, we get $y(18) = 852.352$

Q3: b)

$y = ax^b \Rightarrow \ln(y) = \ln(ax^b) \Rightarrow \ln(y) = \ln(a) + b\ln(x)$

let $y' = \ln(y)$, $a_0 = \ln(a)$, and $x' = \ln(x)$

$\therefore$ We get this linear equation: $y' = a_0 + bx'$

$\ln(x)$	1.386	1.609	2.079	2.303	2.565
$\ln(y)$	2.303	2.565	3.784	4.357	5.03

We can extract this information from the table above:

$\sum x_i' = 9.942$, $\sum y_i' = 18.039$, $\sum x_i'^2 = 20.715$

$\sum x_i' y_i' = 38.1221$, $\overline{x'} = 1.9884$, $\overline{y'} = 3.6078$

$$\therefore b = \frac{n\sum x_i' y_i' - \sum x_i' \sum y_i'}{n\sum x_i'^2 - (\sum x_i')^2} = 2.38116$$

$\therefore a_0 = \overline{y'} - b\overline{x'} = -1.126899 \Rightarrow a_0 = \ln(a) \Rightarrow a = e^{a_0} = 0.32404$

$\therefore$ by substituting a and b in $y = ax^b$, we get:

$$y = 0.32404\, x^{2.38116}$$

$\therefore$ estimating y when $x = 18$ would ~~yeild~~ yield:

$$y(18) = 315.93905$$

Q4:

$$L_0(x) = \frac{x-x_1}{x_0-x_1} \cdot \frac{x-x_2}{x_0-x_2} \cdot \frac{x-x_3}{x_0-x_3} = \frac{x-0}{-1} \cdot \frac{x-3}{-4} \cdot \frac{x-5}{-6}$$

$$= -\frac{1}{24}(x^3 - 8x^2 + 15x)$$

$$L_1(x) = \frac{x-x_0}{x_1-x_0} \cdot \frac{x-x_2}{x_1-x_2} \cdot \frac{x-x_3}{x_1-x_3} = \frac{x+1}{1} \cdot \frac{x-3}{-3} \cdot \frac{x-5}{-5}$$

$$= \frac{1}{15}(x^3 - 7x^2 + 7x + 15)$$

$$L_2(x) = \frac{x-x_0}{x_2-x_0} \cdot \frac{x-x_1}{x_2-x_1} \cdot \frac{x-x_3}{x_2-x_3} = \frac{x+1}{4} \cdot \frac{x}{3} \cdot \frac{x-5}{-2}$$

$$= -\frac{1}{24}(x^3 - 4x^2 - 5x)$$

$$L_3(x) = \frac{x-x_0}{x_3-x_0} \cdot \frac{x-x_1}{x_3-x_1} \cdot \frac{x-x_2}{x_3-x_2} = \frac{x+1}{6} \cdot \frac{x-0}{5} \cdot \frac{x-3}{2}$$

$$= \frac{1}{60}(x^3 - 2x^2 - 3x)$$

$$f(x) = \sum_{i=0}^{3} L_i(x) f(x_i) = -\frac{1}{3}x^3 + \frac{8}{3}x^2 - 5x + \frac{2}{3}x^3 - \frac{14}{3}x^2 + \frac{14}{3}x$$

$$+10 - \frac{2}{3}x^3 + \frac{8}{3}x^2 + \frac{10}{3}x + \frac{1}{3}x^3 - \frac{2}{3}x^2 - x$$

$$= 2x + 10 \Rightarrow f(x) = 2x + 10$$

$\therefore$ Estimating y at $x = 2$, $y(2) = 14$

Q5: Since there are 4 Points, then it must be Cubic interPolation.

By substituting the Points in $f(x)$, We get the 4 Points:

$(0, 0), (1, 0.595), (2, 0.407), (3, 0.045)$

$\therefore f(x)$ will take the form:

$$f(x) = b_0 + b_1(x - x_0) + b_2(x - x_0)(x - x_1) + b_3(x - x_0)(x - x_1)(x - x_2)$$

$$b_0 = f[x_0] = f(x_0) = 0, \quad b_1 = f[x_1, x_0] = \frac{f(x_1) - f(x_0)}{x_1 - x_0} = 0.595$$

$$b_2 = f[x_2, x_1, x_0] = \frac{f[x_2, x_1] - f[x_1, x_0]}{x_2 - x_0} = \frac{\frac{f(x_2) - f(x_1)}{x_2 - x_1} - f[x_1, x_0]}{x_2 - x_0}$$

$$= -0.3915$$

$$b_3 = f[x_3, x_2, x_1, x_0] = \frac{f[x_3, x_2, x_1] - f[x_2, x_1, x_0]}{x_3 - x_0}$$

$$= \frac{\dfrac{\frac{f(x_3) - f(x_2)}{x_3 - x_2} - \frac{f(x_2) - f(x_1)}{x_2 - x_1}}{x_3 - x_1} - f[x_2, x_1, x_0]}{x_3 - x_0} = 0.1015$$

$\therefore$ The Polynomial will be:

$$\cancel{f(x) = 0.595(x - 0) - 0.3915(x - 0)(x - 0.595) + 0.1015(x - 0)(x - 0.595)(x - 0.407)}$$

$$\cancel{= 0.1015x^3 - 0.4932x^2 + 0.8525x}$$

$$\bar{f}(x) = 0.1015x^3 - 0.696x^2 + 1.1895x$$

$$\bar{f}(x = 1.5) = 0.5608$$

$$\bar{f}(x = 2.5) = 0.2097$$

$$f(x = 1.5) = 0.5533$$

$$f(x = 2.5) = 0.2223$$

PRE of 1.5:

$$\frac{|\bar{f}(1.5) - f(1.5)|}{\bar{f}(1.5)} \times 100 = 1.3374\,\%$$

PRE of 2.5:

$$\frac{|\bar{f}(2.5) - f(2.5)|}{\bar{f}(2.5)} \times 100 = 6.0086\%$$