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Numerical Computations

Assignment #7

Question #1:

Step 1: Put our constraints and minimization function in a matrix:

$$\begin{bmatrix} 2 & 1 & 8 \\ 1 & 2 & 8 \\ 3 & 9 & 1 \end{bmatrix}$$

Step 2: Take the transpose of that matrix:

$$\begin{bmatrix} 2 & 1 & 3 \\ 1 & 2 & 9 \\ 8 & 8 & 1 \end{bmatrix}$$

Step 3: Turn the minimization proplem to a maximization proplem using the dat from the matrix:

Maximize: $\cancel{8x} \; 8x_1 + 8x_2 = c$

Constraints:

$$2x_1 + x_2 \leq 3$$
$$x_1 + 2x_2 \leq 9$$

Now, We introduce the slack variables:

$$-8X_1 - 8X_2 + C = 0$$

$$2X_1 + X_2 + S_1 = 3$$

$$X_1 + 2X_2 + S_2 = 9$$

Now, we add all the data in a matrix:

$$\begin{array}{cccccc} X_1 & X_2 & S_1 & S_2 & C & \\ \end{array}$$
$$\left[\begin{array}{ccccc:c} 2 & 1 & 1 & 0 & 0 & 3 \\ 1 & 2 & 0 & 1 & 0 & 9 \\ -8 & -8 & 0 & 0 & 1 & 0 \end{array}\right]$$

Now, we find the smallest number in the bottom row, and try to find a Pivot Point:

-8 was chosen, and the data shows that: $\frac{3}{1} < \frac{9}{2}$

So the Pivot Point is Row[1] coloumn[2]

Now we turn everything under it to zeros using gauss elimination:

$$\begin{bmatrix} 2 & 1 & 1 & 0 & 0 & 3 \\ 1 & 2 & 0 & 1 & 0 & 9 \\ -8 & -8 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\xrightarrow[R_3 = 8R_1 + R_3]{R_2 = -2R_1 + R_2} \begin{bmatrix} 2 & 1 & 1 & 0 & 0 & 3 \\ -3 & 0 & -2 & 1 & 0 & 3 \\ +8 & 0 & 8 & 0 & 1 & 24 \end{bmatrix}$$

Since the bottom row does not contain any negative numbers, then the maximization is done

$$2X_1 + X_2 + S_1 = 3$$

$$-3X_1 - 2S_1 + S_2 = 3$$

$$8X_1 + 8S_1 + C = 24$$

∴ For the minimization Problem:

$X_1 = S_1 = 8$, $X_2 = S_2 = 0$ $\Rightarrow$ ∴ $\boxed{X_1 = 8 \; , \; X_2 = 0}$

To test: $3(8) + 9(0) = 24$

uestion #2:

$_1(x) = a_1x^2 + b_1x + c_1$, $2 \leq x \leq 3$

$f_2(x) = a_2x^2 + b_2x + c_2$, $3 \leq x \leq 4$

$f_3(x) = a_3x^2 + b_3x + c_3$, $4 \leq x \leq 5$

nce we have 9 unknowns, then we need 9 equations

: By substituting the ~~...~~ x values for the equations:

1) $4a_1 + 2b_1 + c_1 = 5$, 2) $9a_1 + 3b_1 + c_1 = 8$

3) $9a_2 + 3b_2 + c_2 = 8$, 4) $16a_2 + 4b_2 + c_2 = 14$

5) $16a_3 + 4b_3 + c_3 = 14$, 6) $25a_3 + 5b_3 + c_3 = 20$

ow we get the equations when functions meet at one point:

$f_i'(x) = 2a_ix + b_i$

$\therefore$ at $x = 3$: $f_1'(3) = f_2'(3) \Rightarrow 6a_1 + b_1 = 6a_2 + b_2$

$\Rightarrow 6a_1 + b_1 - 6a_2 - b_2 = 0$

$\therefore$ at $x = 4$: $f_2'(4) = f_3'(4) \Rightarrow 8a_2 + b_2 = 8a_3 + b_3$

$\Rightarrow 8a_2 + b_2 - 8a_3 - b_3 = 0$

Now, we get one more equation from either the starting point or the end. It is chosen by which Δx is less.
Since $\Delta x = 1$ for all points, then the last equatio will be:

$$\boxed{a_1 = 0}$$

$\therefore$ if we plug all equations in a matrix, we get:

$$\left[\begin{array}{ccccccccc|c}
4 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 5 \\
9 & 3 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 8 \\
0 & 0 & 0 & 9 & 3 & 1 & 0 & 0 & 0 & 8 \\
0 & 0 & 0 & 16 & 4 & 1 & 0 & 0 & 0 & 14 \\
0 & 0 & 0 & 0 & 0 & 0 & 16 & 4 & 1 & 14 \\
0 & 0 & 0 & 0 & 0 & 0 & 25 & 5 & 1 & 20 \\
6 & 1 & 0 & -6 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 8 & 1 & 0 & -8 & -1 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right]$$

We get that:

$a_1 = 0$, $b_1 = 3$, $c_1 = -1$

$a_2 = 3$, $b_2 = -15$, $c_2 = 26$

$a_3 = -3$, $b_3 = 33$, $c_3 = -70$

∴ The equations are:

$$f_1(X) = 3X - 1 \quad , \quad 2 \leq X \leq 3$$

$$f_2(X) = 3X^2 - 15X + 26 \quad , \quad 3 \leq X \leq 4$$

$$f_3(X) = -3X^2 + 33X - 70 \quad , \quad 4 \leq X \leq 5$$