Name: Ghazi Najeeb Al-Abbar

ID: 2181148914

Theory of Computation

Assignment #1

Q1:

Theorem: $let\ n \in \mathcal{N},\ if\ n^2\ is\ even,\ then\ n\ is\ even$

$Let\ \sqrt{8}\ be\ rational,\ then\ there\ exists\ two\ coprime\ numbers\ a, b \in \mathcal{N}$
$S.T\ \sqrt{8} = \frac{a}{b}$

$Then: \frac{a}{b} = \sqrt{8} \Rightarrow \frac{a}{b} = 2\sqrt{2} \Rightarrow \frac{a}{2b} = \sqrt{2} \Rightarrow Let\ c = 2b,\ then:$

$\frac{a}{c} = \sqrt{2} \Rightarrow \frac{a^2}{c^2} = 2 \Rightarrow a^2 = 2c^2\ --\ (1)$

$Since\ a^2 = 2c^2,\ then\ a^2\ must\ be\ even.\ Which\ also\ means\ a\ is\ even\ (Theorem)$
$since\ a\ is\ even,\ then\ it\ must\ have\ a\ form\ such\ as\ a = 2k,\ k \in \mathcal{N}$

$If\ a's\ new\ value\ is\ plugged\ in\ equation\ (1),\ then:$

$a^2 = 2c^2 \Rightarrow (2k)^2 = 2c^2 \Rightarrow 4k^2 = 2c^2 \Rightarrow 2k^2 = c^2\ --\ (2)$

$From\ equation\ (2),\ c^2\ is\ even,\ which\ means\ c\ is\ even\ (Theorem)$

$Since\ a\ and\ c\ are\ even,\ then\ they\ are\ not\ coprime.$
$Which\ means\ that\ \sqrt{8}\ is\ not\ rational.$ *CONTRADICTION*

$\therefore \sqrt{8}\ is\ irrational\ using\ proof\ by\ contradiction.$ ■

Q2:

Let the statement $P(n)$ *be* $\sum_{k=0}^{n} x^k = \frac{x^{n+1}-1}{x-1}$

Basis step: $P(1)$:

$$LHS = \sum_{k=0}^{1} x^k = x^0 + x^1 = 1 + x$$

$$RHS = \frac{x^{1+1}-1}{x-1} = \frac{x^2-1}{x-1} = \frac{(x-1)(x+1)}{x-1} = x + 1$$

Since the LHS = the RHS, then P(1) is true

Inductive step:

Assume that $P(n)$ *is true* $\forall\, n \in \mathcal{N}$, *then* $P(n+1)$ *should also be true*

and yields $P(n+1)$: $\sum_{k=0}^{n+1} x^k = \frac{x^{n+2}-1}{x-1}$

For $P(n+1)$:

$$\sum_{k=0}^{n+1} x^k = 1 + x + x^2 + ... + x^n + x^{n+1}$$

$$= (1 + x + x^2 + ... + x^n) + x^{n+1}$$

$$= \frac{x^{n+1}-1}{x-1} + x^{n+1} \text{ (Substitution)}$$

$$= \frac{x^{n+1}-1}{x-1} + \frac{x^{n+2}-x^{n+1}}{x-1} = \frac{x^{n+1}-1+x^{n+2}-x^{n+1}}{x-1} = \frac{x^{n+2}-1}{x-1}$$

$\therefore$ $P(n)$ *is true* $\forall\, n \in \mathcal{N}$ *by using mathematical induction* $\blacksquare$

Q3:

a) All words of size ≤ 4 are: {0, 000, 011}

b) $S \rightarrow A, \quad A \rightarrow 0B, \quad B \rightarrow 00B \mid C, \quad C \rightarrow 11C \mid \lambda$

Q4:

The grammar G on language L = { $a^{2k} b^{2m+1} c^{n} \mid k \geq 1, \; m, n \geq 0\}$ is:

$S \rightarrow A, \quad A \rightarrow aaA \mid aaB, \quad B \rightarrow bC \mid D, \quad C \rightarrow bbC \mid D \mid \lambda, \quad D \rightarrow cD \mid \lambda$

The derivation of the string "aabbbbbcc" using grammar G is:

$S \Rightarrow A \Rightarrow aaB \Rightarrow aabC \Rightarrow aabbC \Rightarrow aabbbC \Rightarrow aabbbbC$

$aabbbbbC \Rightarrow aabbbbbD \Rightarrow aabbbbbcD \Rightarrow aabbbbbccD$

$\Rightarrow aabbbbbcc$

Q5:

The grammar G on Language L = { $vwv\!: \; v, w \in \{a, b\}^{*}, \; |v| = 2$ } is:

$S \rightarrow aA \mid bB, \quad A \rightarrow aD \mid bD, \quad B \rightarrow aD \mid bD, \quad D \rightarrow aD \mid bD \mid E$

$E \rightarrow whatever\ substring\ that\ was\ chosen\ at\ the\ start.\ (Terminal\ symbols\ S, A, B)$

the derivation of the string "ababbaab" using grammar G is:

$S \Rightarrow aA \Rightarrow abD \Rightarrow abaD \Rightarrow ababD \Rightarrow ababbD \Rightarrow ababbaD \Rightarrow ababbaE$

$\Rightarrow ababbaab$