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Theory of Computation

Assignment #5

Disclaimer:

- When a transition is of the form "{x,y},{x,y},R", it means if the input is x then replace it with x and go right, and if it is y, then replace it with y and go right. They take their respective positions. This is done because of the lack of space when drawing the machines.

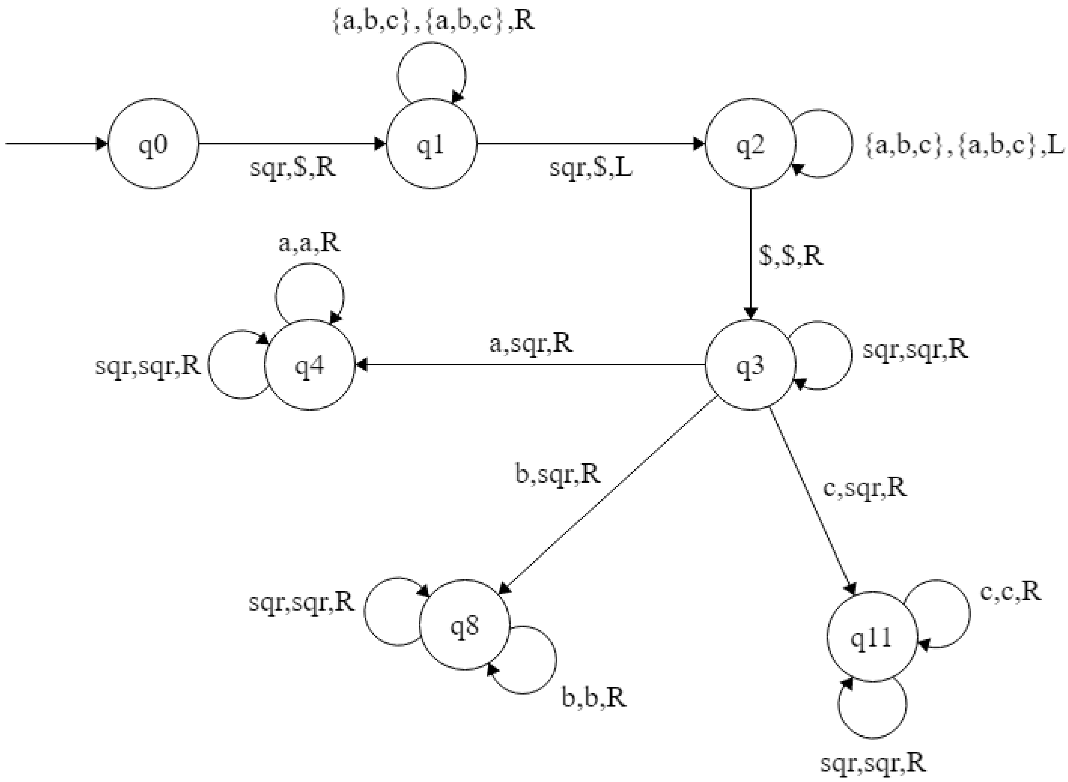
- In the transition functions written, some symbols are stacked on each other. I did this because of the lack of space on paper. They mean the same thing on the first bullet point. I hope you try to understand and not assume it as false as quickly as possible.

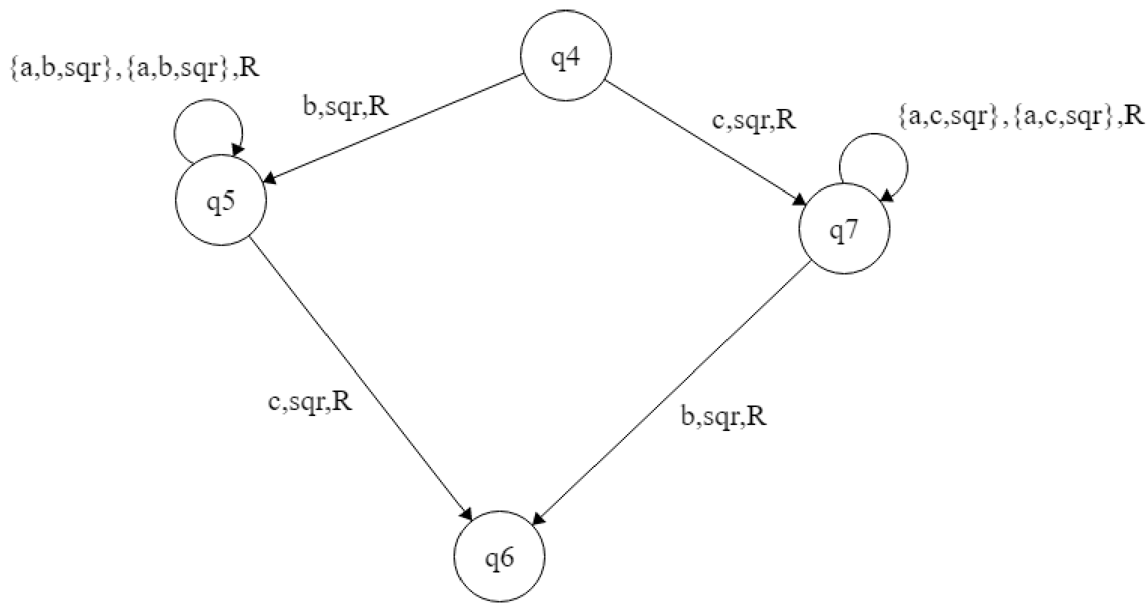
- The first question has a very long answer, so an example was given.

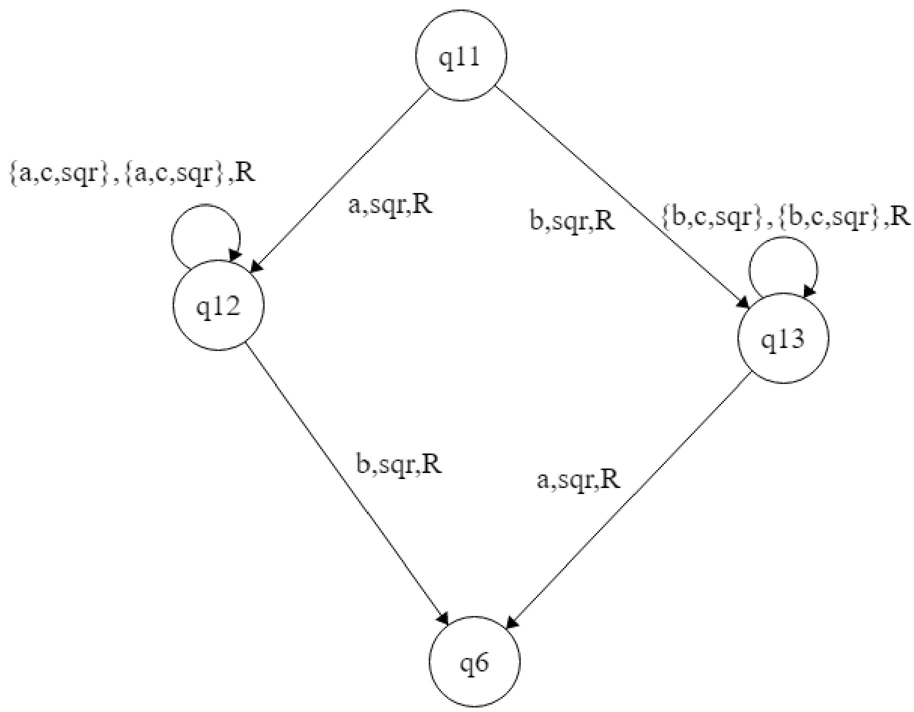
- Every diagram image that has the same two states is actually just a continuation. This was done because there was not enough space to draw. For example, the first diagram has q4, q8, and q11. Also, the 2nd, 3rd, and 4th have q4, q8, and q11, respectively. They are a continuation of the states.

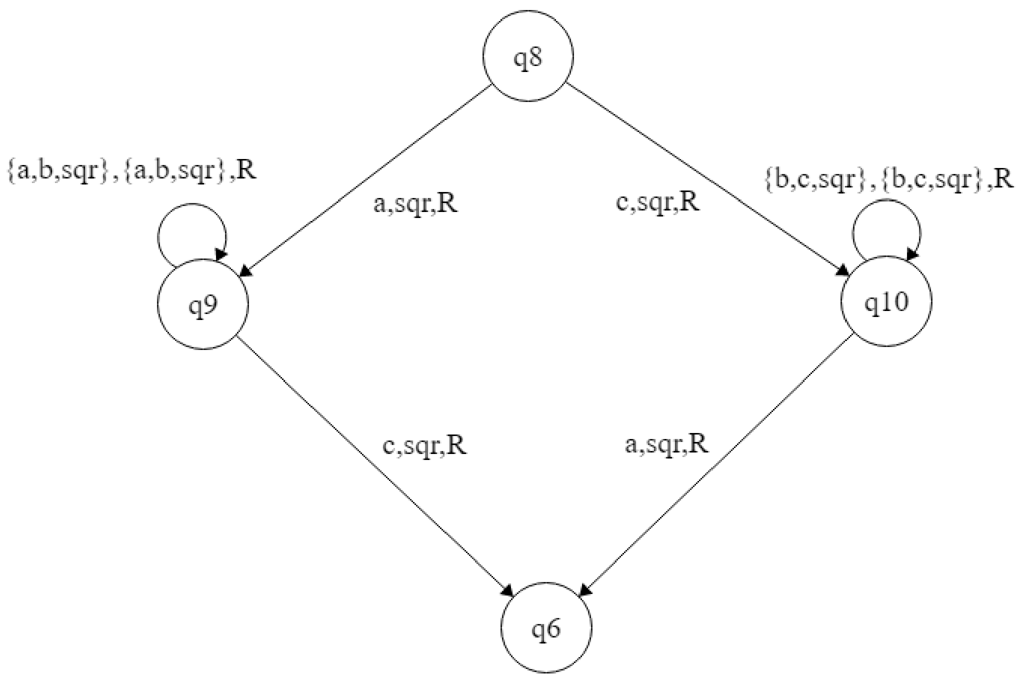
- In each diagram, there is a symbol named "sqr". This is nothing but the space symbol. There are no symbols that denote square or space in the applications that I use.

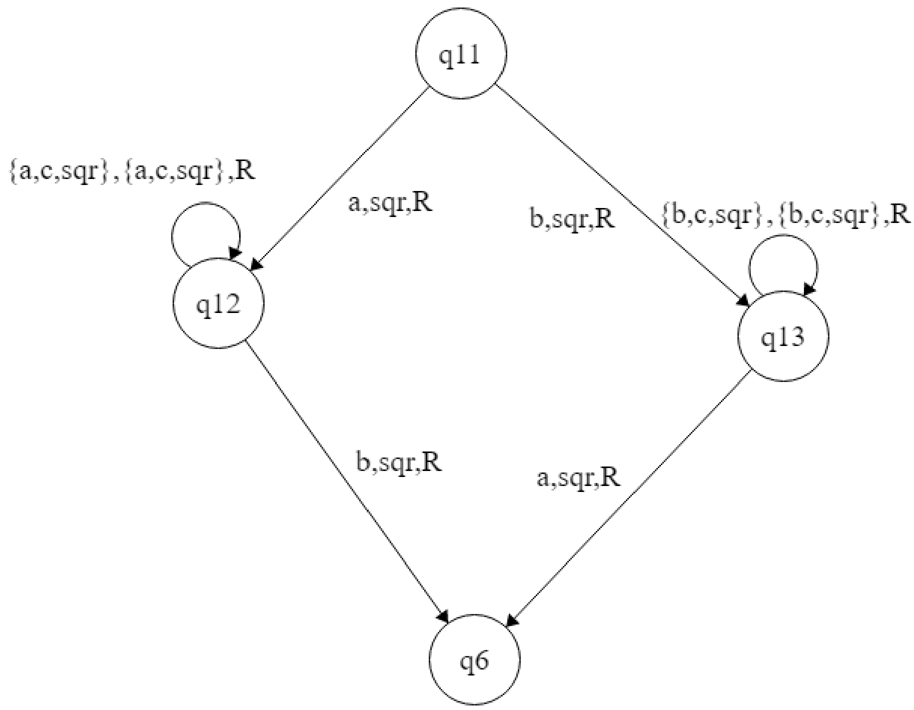
Question 1:
Diagram:

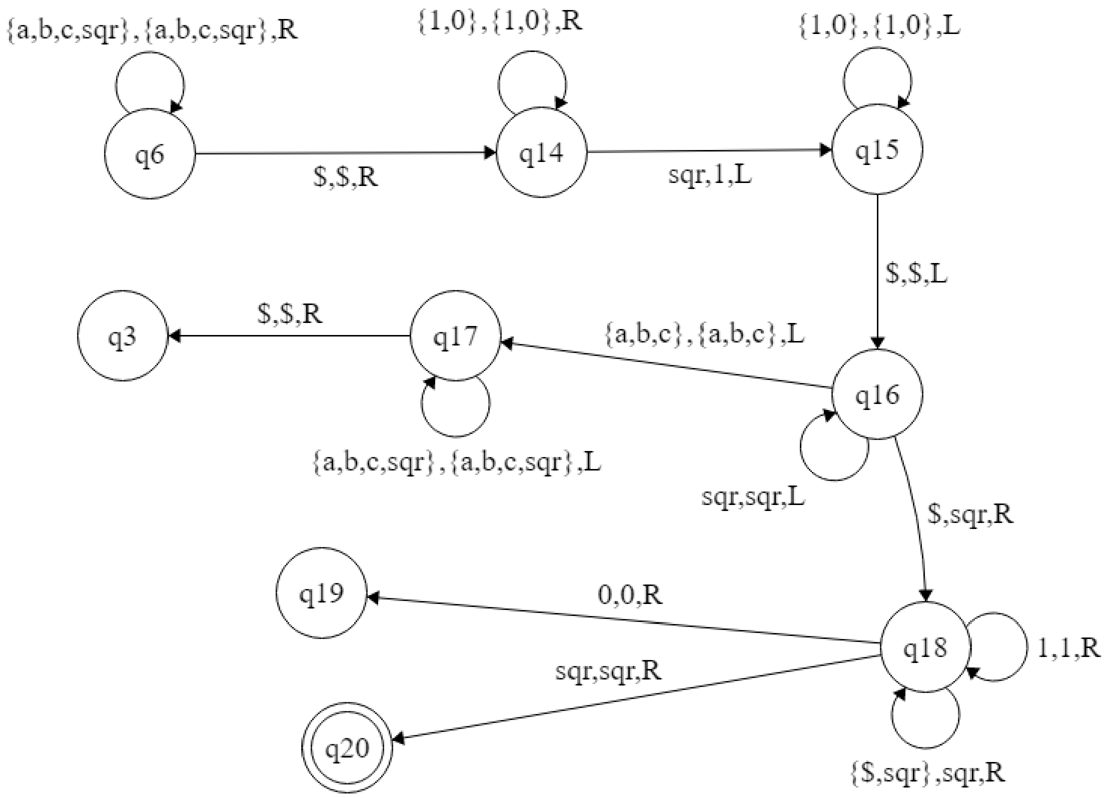


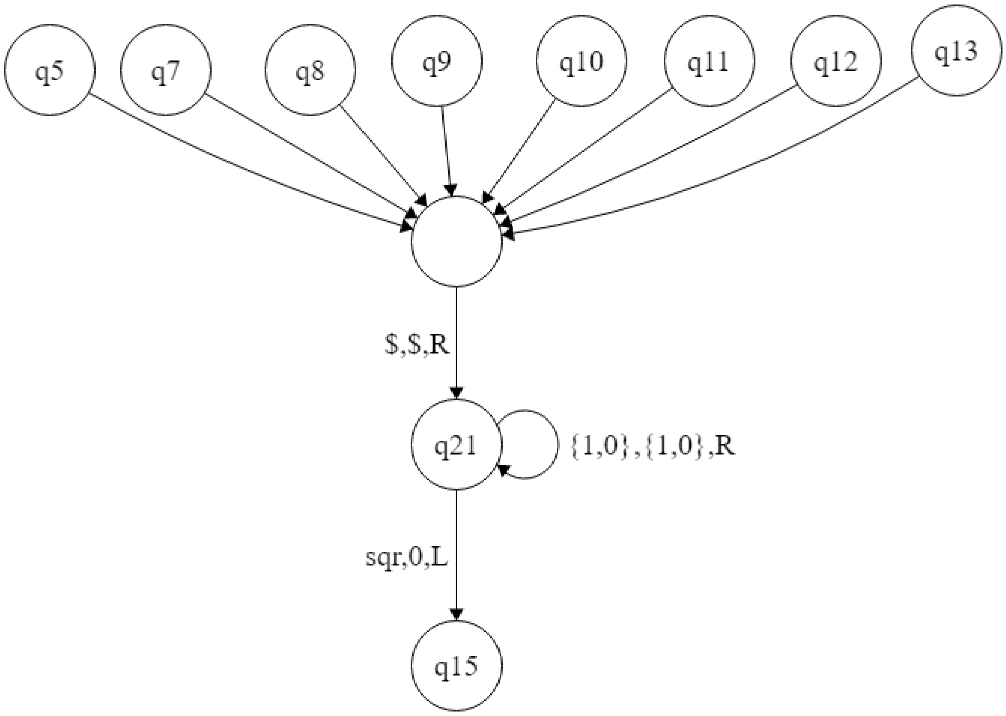



q11
{a,c,sqr},{a,c,sqr},R
a,sqr,R
b,sqr,R
{b,c,sqr},{b,c,sqr},R
q12
q13
b,sqr,R
a,sqr,R
q6


q8
{a,b,sqr},{a,b,sqr},R
a,sqr,R
c,sqr,R
{b,c,sqr},{b,c,sqr},R
q9
q10
c,sqr,R
a,sqr,R
q6


q11
a,sqr,R
b,sqr,R
{a,c,sqr},{a,c,sqr},R
{b,c,sqr},{b,c,sqr},R
q12
q13
b,sqr,R
a,sqr,R
q6


{a,b,c,sqr},{a,b,c,sqr},R
{1,0},{1,0},R
{1,0},{1,0},L
q6
$,$,R
q14
sqr,1,L
q15
$,$,L
q3
$,$,R
q17
{a,b,c},{a,b,c},L
q16
{a,b,c,sqr},{a,b,c,sqr},L
sqr,sqr,L
$,sqr,R
q19
0,0,R
q18
1,1,R
sqr,sqr,R
q20
{$,sqr},sqr,R


q5
q7
q8
q9
q10
q11
q12
q13
$,$,R
q21
{1,0},{1,0},R
sqr,0,L
q15

Transition functions:

$$\delta(q_0, \square) = (q_1, \$, R),\ \delta(q_1, \substack{a\\b\\c}) = (q_1, \substack{a\\b\\c}, R),\ \delta(q_1, \square) = (q_2, \$, L)$$

$$\delta(q_2, \substack{a\\b\\c}) = (q_2, \substack{a\\b\\c}, L),\ \delta(q_2, \$) = (q_3, \$, R),\ \delta(q_3, a) = (q_4, \square, R)$$

$$\delta(q_4, b) = (q_5, \square, R),\ \delta(q_5, c) = (q_6, \square, R),\ \delta(q_4, c) = (q_7, \square, R)$$

$$\delta(q_7, b) = (q_6, \square, R),\ \delta(q_4, a) = (q_4, a, R),\ \delta(q_5, \substack{a\\b\\\square}) = (q_5, \substack{a\\b\\\square}, R)$$

$$\delta(q_7, \substack{a\\c\\\square}) = (q_7, \substack{a\\c\\\square}, R),\ \delta(q_3, b) = (q_8, \square, R),\ \delta(q_8, a) = (q_9, \square, R)$$

$$\delta(q_9, c) = (q_6, \square, R),\ \delta(q_8, c) = (q_{10}, \square, R),\ \delta(q_{10}, a) = (q_6, \square, R)$$

$$\delta(q_8, b) = (q_8, b, R),\ \delta(q_9, \substack{a\\b\\\square}) = (q_9, \substack{a\\b\\\square}, R),\ \delta(q_{10}, \substack{b\\c\\\square}) = (q_{10}, \substack{b\\c\\\square}, R)$$

$$\delta(q_3, c) = \delta(q_{11}, \square, R),\ \delta(q_{11}, a) = (q_{12}, \square, R),\ \delta(q_{12}, b) = (q_6, \square, R)$$

$$\delta(q_{11}, b) = (q_{13}, \square, R),\ \delta(q_{13}, a) = (q_6, \square, R),\ \delta(q_{11}, c) = (q_{11}, c, R)$$

$$\delta(q_{12}, \substack{a\\c\\\square}) = (q_{12}, \substack{a\\c\\\square}, R),\ \delta(q_{13}, \substack{b\\c\\\square}) = (q_{13}, \substack{b\\c\\\square}, R),\ \delta(q_6, \begin{smallmatrix}a&b\\c&\square\end{smallmatrix}) = (q_6, \begin{smallmatrix}a&b\\c&\square\end{smallmatrix}, R)$$

$$\delta(q_6, \$) = (q_{14}, \$, R),\ \delta(q_{14}, \square) = (q_{15}, 1, L),\ \delta(q_{15}, \substack{1\\0}) = (q_{15}, \substack{1\\0}, L)$$

$$\delta(q_{15}, \$) = (q_{16}, \$, L),\ \delta(q_{16}, \square) = (q_{16}, \square, L),\ \delta(q_{16}, \substack{a\\b\\c}) = (q_{17}, \substack{a\\b\\c}, L)$$

$$\delta(q_{17}, \begin{smallmatrix}a&b\\c&\square\end{smallmatrix}) = (q_{17}, \begin{smallmatrix}a&b\\c&\square\end{smallmatrix}, L),\ \delta(q_{17}, \$) = (q_3, \$, R),\ \delta(q_{16}, \$) = (q_{18}, \square, R)$$

$$\delta(q_{18}, \substack{\square\\\$}) = (q_{18}, \square, R),\ \delta(q_{18}, 1) = (q_{22}, 1, R),\ \delta(q_{22}, 0) = (q_{19}, 0, R)$$

$$\delta(q_{22}, \square) = \delta(q_{20}, \square, R),\ \text{let } Z = \{q_5, q_7, q_9, q_{10}, q_{12}, q_{13}, q_8, q_{11}\}$$

$$\delta(Z, \$) = (q_{21}, \$, R),\ \delta(q_{21}, \substack{1\\0}) = (q_{21}, \substack{1\\0}, R),\ \delta(q_{21}, \square) = (q_{15}, 0, L)$$

$$\delta(q_3, \square) = (q_3, \square, R),\ \delta(q_4, \square) = (q_4, \square, R)$$

$$\delta(q_{14}, \substack{1\\0}) = (q_{14}, \substack{1\\0}, R),\ \delta(q_8, \square) = (q_8, \square, R)$$

$$\delta(q_{11}, \square) = (q_{11}, \square, R)$$

Example:

$$q_0 abaccb\square \vdash \$q_1 abaccb\square \vdash \$a q_1 baccb$$

$$\vdash \$ab q_1 baccb \vdash \ldots \vdash \$abaccb q_1 \square \vdash \$abaccb q_2 \$$$

$$\vdash \$abacc q_2 b\$ \vdash \$abac q_2 cb\$ \vdash \ldots \vdash \$q_3 abaccb\$$$

$$\vdash \$\square q_4 baccb\$ \vdash \$\square\square q_5 accb\$ \vdash \$\square\square a q_5 ccb\$$$

$$\vdash \$\square\square a\square q_6 cb\$ \vdash \$\square\square a\square c q_6 b\$ \vdash \$\square\square a\square cb q_6 \$$$

$$\vdash \$\square\square a\square cb\$ q_{14} \square \vdash \$\square\square a\square cb\$ q_{15} 1 \vdash \$\square\square a\square cb q_{16} \$1$$

$$\vdash \$\square\square a\square c q_{17} b\$1 \vdash \$\square\square a\square q_{17} cb\$1 \vdash \ldots \vdash \$q_{17}\square\square a\square cb\$1$$

$$\vdash \$q_3\square\square a\square cb\$1 \vdash \$\square q_3\square a\square cb\$1 \vdash \$\square\square q_3 a\square cb\$1$$

$$\vdash \$\square\square\square q_4\square cb\$1 \vdash \$\square\square\square\square q_4 cb\$1 \vdash \$\square\square\square\square\square q_7 b\$1$$

$$\vdash \$\square\square\square\square\square\square q_6 \$1 \vdash \$\square\square\square\square\square\square\$ q_{14} 1 \vdash \$\square\square\square\square\square\square\$1 q_{14}\square$$

$$\vdash \$\square\square\square\square\square\square\$1 q_{15} 1 \vdash \$\square\square\square\square\square\square\$ q_{15} 11 \vdash \$\square\square\square\square\square\square q_{16} \$11$$

$$\vdash \$\square\square\square\square\square q_{16}\square\$11 \vdash \$\square\square\square\square q_{16}\square\square\$11 \vdash \ldots \vdash \$q_{16}\square\square\square\square\square\square\$11$$

$$\vdash \square q_{18}\square\square\square\square\square\square\square\$11 \vdash \square\square q_{18}\square\square\square\square\square\$11 \vdash \ldots \vdash \square\square\square\square\square\square\square\square q_{18}\$11$$

$$\vdash \square\square\square\square\square\square\square\square\square q_{18} 11 \vdash \square\square\square\square\square\square\square\square\square 1 q_{22} 1\square$$

$$\vdash \square\square\square\square\square\square\square\square\square 11 q_{22}\square \vdash \square\square\square\square\square\square\square\square\square 11 q_{20}\square$$

Steps:

1- At the start, put $ then go to the end of the string and add another $ so we can know when the string starts and ends.

2- Go back to the beginning of the string, and move right while replacing every first instance of a symbol with a space. For example, if 'a' is seen, replace it with a space and continue. While moving to the right, ignore all other 'a's and find a 'b' or a 'c'. suppose 'b' was found. It should be replaced with a space and continue to the right while ignoring all other 'a's and 'b's. If ‘c’ was found, then replace it with a space and continue to the right while ignoring all other inputs until you reach $.

3- Since $ is reached, find the nearest space symbol and replace it with 1. If $ has been reached and you have not found three separate instances of ‘a’, ‘b’, and ‘c’, then instead of 1, add 0.

4- Repeat parts 2 and 3 until the region between the two $ are all space symbols.

5- If part 4 is achieved, then go back to the first $.

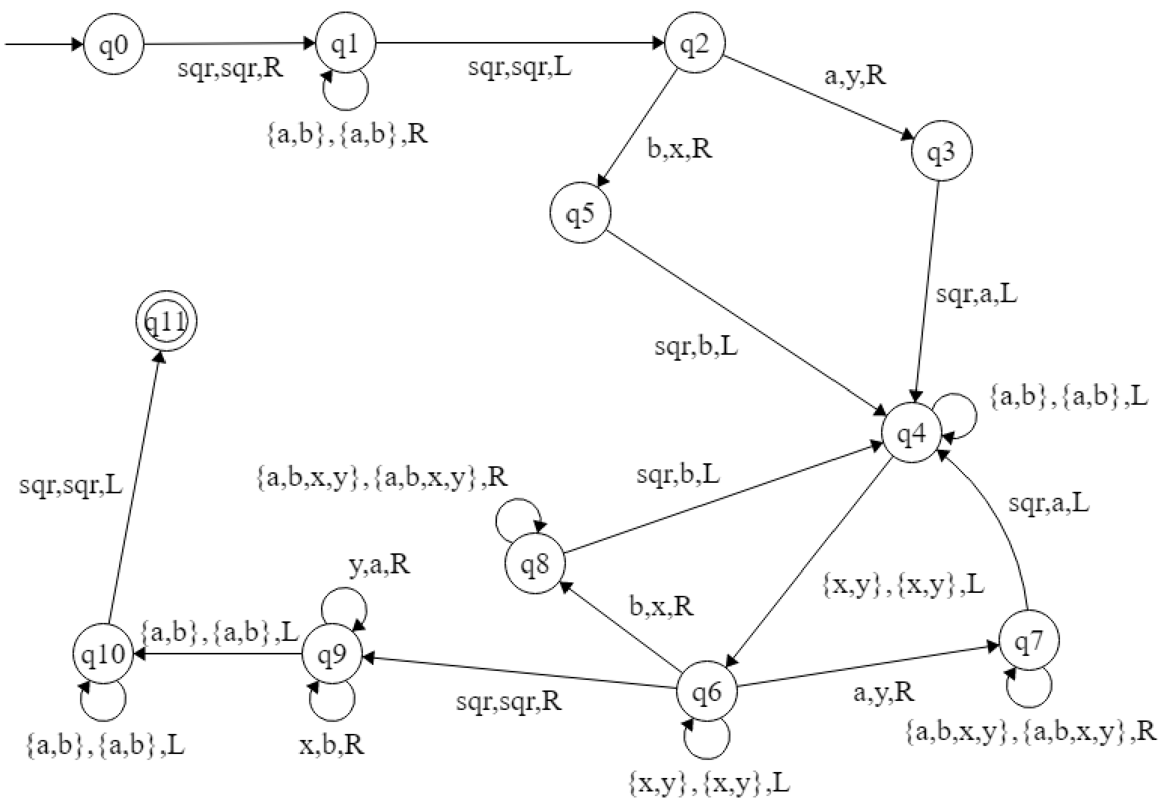
6- Move your way up until the second $, replacing the $ symbols with space symbols as you go.

7- if the 1's and 0's part has been reached, then go through each one. If a single 0 was found, then the number of 'a', 'b', and 'c' are not equal.

8- If they are all 1's, then the number of 'a', 'b', and 'c' are equal.

Question 2:

Diagram:



Transition functions:

$\delta(q_0, \square) = (q_1, \square, R)$, $\delta(q_1, \substack{a \\ b}) = (q_1, \substack{a \\ b}, R)$,

$\delta(q_1, \square) = (q_2, \square, L)$, $\delta(q_2, a) = (q_3, y, R)$, $\delta(q_3, \square) = (q_4, a, L)$

$\delta(q_2, b) = (q_5, x, R)$, $\delta(q_5, \square) = (q_4, b, L)$

$\delta(q_4, \substack{a \\ b}) = (q_4, \substack{a \\ b}, L)$, $\delta(q_4, \substack{x \\ y}) = (q_6, \substack{x \\ y}, L)$, $\delta(q_6, a) = (q_7, y, R)$

$\delta(q_7, \substack{x\ a \\ y\ b}) = (q_7, \substack{x\ a \\ y\ b}, R)$, $\delta(q_6, b) = (q_8, x, R)$

$\delta(q_8, \substack{x\ a \\ y\ b}) = (q_8, \substack{x\ a \\ y\ b}, R)$, $\delta(q_7, \square) = (q_4, a, L)$

$\delta(q_8, \square) = (q_4, b, L)$, $\delta(q_6, \square) = (q_9, \square, R)$

$\delta(q_9, x) = (q_9, b, R)$, $\delta(q_9, y) = (q_9, a, R)$

$\delta(q_9, \substack{a \\ b}) = (q_{10}, \substack{a \\ b}, L)$, $\delta(q_{10}, \substack{a \\ b}) = (q_{10}, \substack{a \\ b}, L)$

$\delta(q_6, \substack{x \\ y}) = (q_6, \substack{x \\ y}, L)$, $\delta(q_{10}, \square) = (q_{11}, \square, L)$

Steps:

1- Begin by going to the right until you reached the space at the end of the string.

2- Go to the left (the last character of your input). If the last character is 'a', then replace it with 'y' then go to the nearest space on the right and replace it with 'a'. If the last character was a 'b', then replace it with 'x' and go to the nearest space on the right and replace it with 'b'.

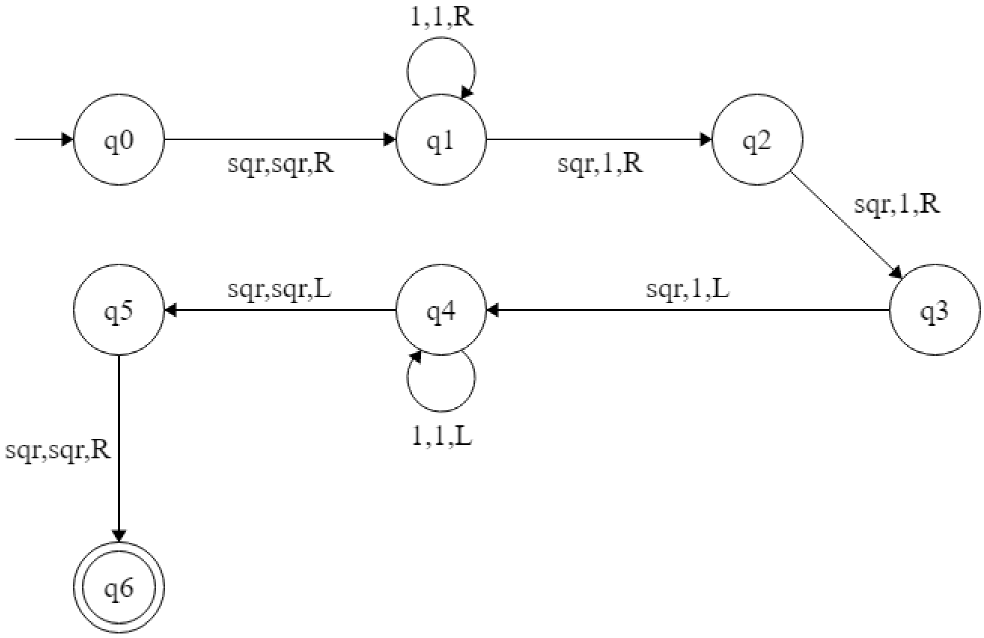
3- Go back and forth replacing every ‘a’ and ‘b’ from the original string with ‘y’ and ‘x’, respectively until the lefthand side is all x and y’s, and the rightmost side is the original string reversed.

4 - If part 3 is achieved, then you should be at the leftmost. Start by heading to the left and ignoring all {a, b} symbols. If any {x, y} symbols are reached, then replace every 'x' with 'b', and every 'y' with 'a'.

5- When you have reached the rightmost, then you are in the final state.

Question 3:

Diagram:



Transition functions:

$\delta(q_0, \square) = (q_1, \square, R)$, $\delta(q_1, 1) = (q_1, 1, R)$

$\delta(q_1, \square) = (q_2, 1, R)$, $\delta(q_2, \square) = (q_3, 1, R)$

$\delta(q_3, \square) = (q_4, 1, L)$, $\delta(q_4, 1) = (q_4, 1, L)$

$\delta(q_4, \square) = (q_5, \square, L)$, $\delta(q_5, \square) = (q_6, \square, R)$

Steps:

1- Go right until you reach the space at the end of the string.

2- Add a '1' symbol then go to the space on the right.

3- Apply step 2 two more times.

4- Now go to the rightmost of the string, then you are in the final state.

Question 4:

a) $S \rightarrow aS \mid aA$

$A \rightarrow bbA \mid bbB$

$B \rightarrow cccB \mid ccc$

b) $S \rightarrow AABB$

$A \rightarrow aAc, \;\; B \rightarrow bBd$

$cAb \rightarrow bAc, \;\; bB \rightarrow Bb$

$cBb \rightarrow bBc, \;\; cbA \rightarrow bcA$

$Bb \rightarrow bB, \;\; dB \rightarrow Bd$

$Ab \rightarrow bA, \;\; cB \rightarrow Bc$

$Bc \rightarrow c, \;\; bA \rightarrow b$