Name: Ghazi Najeeb Al-Abbar

ID: 2181148914

Numerical Computations

Assignment #4

Question #1:

$6X_1 + 2X_2 = 12.5 \Rightarrow X_1 = \frac{12.5 - 2X_2}{6}$

$2X_1 - 5X_2 = 41 \Rightarrow X_2 = \frac{41 - 2X_1}{-5}$

itr #0: $X_1 = 0,\ X_2 = 0$

itr #1: $X_1 = \frac{12.5 - 2(0)}{6} = 2.08333,\ X_2 = \frac{41 - 2(2.08333)}{-5} = -7.36667$

itr #2: $X_1 = \frac{12.5 - 2(-7.36667)}{6} = 4.53889,\ X_2 = \frac{41 - 2(4.53889)}{-5} = -6.38444$

itr #3: $X_1 = \frac{12.5 - 2(-6.38444)}{6} = 4.21148,\ X_2 = \frac{41 - 2(4.21148)}{-5} = -6.51541$

itr #4: $X_1 = \frac{12.5 - 2(-6.51541)}{6} = 4.25514,\ X_2 = \frac{41 - 2(4.25514)}{-5} = -6.49794$

itr #5: $X_1 = \frac{12.5 - 2(-6.49794)}{6} = 4.24931,\ X_2 = \frac{41 - 2(4.24931)}{-5}$

$= -6.50028$

Question # 2:

$$\left[\begin{array}{cc|c} 6 & 2 & 12.5 \\ 2 & -5 & 41 \end{array}\right] \xrightarrow{R_1 = \frac{1}{6}R_1} \left[\begin{array}{cc|c} 1 & \frac{1}{3} & \frac{12.5}{6} \\ 2 & -5 & 41 \end{array}\right]$$

$$\xrightarrow{R_2 = -2R1 + R_2} \left[\begin{array}{cc|c} 1 & \frac{1}{3} & \frac{12.5}{6} \\ 0 & \frac{-17}{3} & \frac{110.5}{3} \end{array}\right]$$

$\therefore X_1 + \frac{1}{3}X_2 = \frac{12.5}{6}$, $\frac{-17}{3}X_2 = \frac{110.5}{3}$

$X_2 = \frac{110.5}{3} \times \frac{-3}{17} \Rightarrow X_2 = -6.5$

$X_1 + \frac{1}{3}(-6.5) = \frac{12.5}{6} \Rightarrow X_1 = \frac{12.5}{6} - \frac{1}{3}(6.5) \Rightarrow X_1 = 4.25$

$\therefore X_2 = -6.5$, $X_1 = 4.25$

Question #3:

Step one : ($A = LU$)

$$\begin{bmatrix} 6 & 2 \\ 2 & -5 \end{bmatrix} \xrightarrow{R_2 = -\frac{1}{3}R_1 + R_2} \begin{bmatrix} 6 & 2 \\ 0 & -\frac{17}{3} \end{bmatrix} = U, \quad L = \begin{bmatrix} 1 & 0 \\ \frac{1}{3} & 1 \end{bmatrix}$$

Step two : ($B = LY$)

$$\begin{bmatrix} 1 & 0 \\ \frac{1}{3} & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 12.5 \\ 41 \end{bmatrix} \Rightarrow \quad y_1 = 12.5, \quad \frac{1}{3}y_1 + y_2 = 41$$

$$\Rightarrow \frac{1}{3}(12.5) + y_2 = 41 \Rightarrow y_2 = 41 - \frac{1}{3}(12.5)$$

$$\Rightarrow y_2 = 36.83$$

$$\therefore y_1 = 12.5, \quad y_2 = 36.83$$

Step three : ($Y = UX$)

$$\begin{bmatrix} 6 & 2 \\ 0 & -\frac{17}{3} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} 12.5 \\ 36.83 \end{bmatrix} \Rightarrow \quad 6X_1 + 2X_2 = 12.5, \quad -\frac{17}{3}X_2 = 36.83$$

$$\Rightarrow X_2 = 36.83 \times \frac{-3}{17} = -6.499$$

$$\Rightarrow 6X_1 + 2(-6.499) = 12.5$$

$$\Rightarrow 6X_1 = 12.5 - 2(-6.499) \Rightarrow X_1 = \frac{12.5 - 2(-6.499)}{6}$$

$$\Rightarrow X_1 = 4.2497$$

$$\therefore X_1 = 4.2497, \quad X_2 = -6.499$$

Question #4:

$$\begin{bmatrix} 5 & 3 & -1 \\ -2 & 7 & 2 \\ 3 & 2 & 9 \end{bmatrix}$$

Rows one and two were exchanged since the previous form was not diagonally dominant.

$$5X_1 + 3X_2 - X_3 = 4.25 \Rightarrow X_1 = \frac{4.25 - 3X_2 + X_3}{5}$$

$$-2X_1 + 7X_2 + 2X_3 = -6.5 \Rightarrow X_2 = \frac{-6.5 + 2X_1 - 2X_3}{7}$$

$$3X_1 + 2X_2 + 9X_3 = 71.75 \Rightarrow X_3 = \frac{71.75 - 3X_1 - 2X_2}{9}$$

itr #0: $X_1 = 0, \quad X_2 = 0, \quad X_3 = 0$

itr #1:

$$X_1 = \frac{4.25 - 3(0) + 0}{5} = 0.85, \quad X_2 = \frac{-6.5 + 2(0.85) - 2(0)}{7} = -0.68571$$

$$X_3 = \frac{71.75 - 3(0.85) - 2(-0.68571)}{7} = 7.53651$$

itr #2:

$$X_1 = \frac{4.25 - 3(-0.68571) + (7.53651)}{5} = 2.76873$$

$$X_2 = \frac{-6.5 + 2(2.76873) - 2(7.53651)}{7} = -2.29079$$

$$X_3 = \frac{71.75 - 3(2.76873) - 2(-2.29079)}{9} = 7.55838$$

Question #4 Continuation:

itr #3:

$$X_1 = \frac{4.25 - 3(-2.29079) + (7.55838)}{5} = 3.73615$$

$$X_2 = \frac{-6.5 + 2(3.73615) - 2(7.55838)}{7} = -2.02064$$

$$X_3 = \frac{71.75 - 3(3.73615) - 2(-2.02064)}{9} = 7.17587$$

itr #4:

$$X_1 = \frac{41.25 - 3(-2.02064) + (7.17587)}{5} = 3.49756$$

$$X_2 = \frac{-6.5 + 2(3.49756) - 2(7.17587)}{7} = -1.97952$$

$$X_3 = \frac{71.75 - 3(3.49756) - 2(-1.97952)}{9} = 7.24626$$

itr #5:

$$X_1 = \frac{4.25 - 3(-1.97952) + (7.24626)}{5} = 3.48696$$

$$X_2 = \frac{-6.5 + 2(3.48696) - 2(7.24626)}{7} = -2.00266$$

$$X_3 = \frac{71.75 - 3(3.48696) - 2(7.24626)}{9} = 7.25494$$

Question #5:

$$\left[\begin{array}{ccc|c} -2 & 7 & 2 & -6.5 \\ 5 & 3 & -1 & 41.25 \\ 3 & 2 & 9 & 71.75 \end{array}\right] \xrightarrow{R_1 = -\frac{1}{2}R_1} \left[\begin{array}{ccc|c} 1 & -7/2 & -1 & 6.5/2 \\ 5 & 3 & -1 & 41.25 \\ 3 & 2 & 9 & 71.75 \end{array}\right]$$

$$\xrightarrow{R_2 = -5R_1 + R_2} \left[\begin{array}{ccc|c} 1 & -7/2 & -1 & 6.5/2 \\ 0 & 20.5 & 4 & -12 \\ 3 & 2 & 9 & 71.75 \end{array}\right] \xrightarrow{R_3 = -3R_1 + R_3} \left[\begin{array}{ccc|c} 1 & -7/2 & -1 & 6.5/2 \\ 0 & 20.5 & 4 & -12 \\ 0 & 12.5 & 12 & 62 \end{array}\right]$$

$$\xrightarrow{R_2 = \frac{1}{20.5}R_2} \left[\begin{array}{ccc|c} 1 & -7/2 & -1 & 6.5/2 \\ 0 & 1 & 8/41 & -24/41 \\ 0 & 12.5 & 12 & 62 \end{array}\right] \xrightarrow{R_3 = -12.5R_2 + R_3} \left[\begin{array}{ccc|c} 1 & -7/2 & -1 & 6.5/2 \\ 0 & 1 & 8/41 & -24/41 \\ 0 & 0 & \frac{392}{41} & \frac{2842}{41} \end{array}\right]$$

$$\xrightarrow{R_3 = \frac{41}{392}R_3} \left[\begin{array}{ccc|c} 1 & -7/2 & -1 & 6.5/2 \\ 0 & 1 & 8/41 & -24/41 \\ 0 & 0 & 1 & 7.25 \end{array}\right] \xrightarrow{R_2 = \frac{-8}{41}R_3 + R_2} \left[\begin{array}{ccc|c} 1 & -7/2 & -1 & 6.5/2 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 7.25 \end{array}\right]$$

$$\xrightarrow{R_1 = R_3 + R_1} \left[\begin{array}{ccc|c} 1 & -7/2 & 0 & 10.5 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 7.25 \end{array}\right] \xrightarrow{R_1 = \frac{7}{2}R_2 + R_1} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3.5 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 7.25 \end{array}\right]$$

$\therefore X_1 = 3.5, \quad X_2 = -2, \quad X_3 = 7.25$

Question #6:

$$A = \begin{bmatrix} -2 & 7 & 2 \\ 5 & 3 & -1 \\ 3 & 2 & 9 \end{bmatrix} \Rightarrow \det(A) = -2\begin{vmatrix} 3 & -1 \\ 2 & 9 \end{vmatrix} - 7\begin{vmatrix} 5 & -1 \\ 3 & 9 \end{vmatrix} + 2\begin{vmatrix} 5 & 3 \\ 3 & 2 \end{vmatrix}$$

$$= -2(3\cdot 9 + 2) - 7(5\cdot 9 + 3) + 2(10 - 9)$$

$$= -58 - 336 + 2 = -392$$

Finding Cofactors:

$$\Rightarrow \begin{bmatrix} \begin{vmatrix} 3 & -1 \\ 2 & 9 \end{vmatrix} & -\begin{vmatrix} 5 & -1 \\ 3 & 9 \end{vmatrix} & \begin{vmatrix} 5 & 3 \\ 3 & 2 \end{vmatrix} \\ -\begin{vmatrix} 7 & 2 \\ 2 & 9 \end{vmatrix} & \begin{vmatrix} -2 & 2 \\ 3 & 9 \end{vmatrix} & -\begin{vmatrix} -2 & 7 \\ 3 & 2 \end{vmatrix} \\ \begin{vmatrix} 7 & 2 \\ 3 & -1 \end{vmatrix} & -\begin{vmatrix} -2 & 2 \\ 5 & -1 \end{vmatrix} & \begin{vmatrix} -2 & 7 \\ 5 & 3 \end{vmatrix} \end{bmatrix}$$

$$= \begin{bmatrix} 29 & -418 & 1 \\ -59 & -24 & -25 \\ -13 & 8 & -41 \end{bmatrix} \Rightarrow \text{Finding Transpose: } \begin{bmatrix} 29 & -59 & -13 \\ -48 & -24 & 8 \\ 1 & -25 & -41 \end{bmatrix}$$

$$\Rightarrow \therefore A^{-1} = \frac{1}{\det(A)} A^T = -\frac{1}{392} \begin{bmatrix} 29 & -59 & -13 \\ -48 & -24 & 8 \\ 1 & -25 & -41 \end{bmatrix}$$

$$= \begin{bmatrix} +\frac{29}{392} & \frac{59}{392} & \frac{13}{392} \\ \frac{48}{392} & \frac{24}{392} & \frac{-8}{392} \\ \frac{-1}{392} & \frac{25}{392} & \frac{41}{392} \end{bmatrix}$$

Question #6 Continuation:

$$AX = B \Rightarrow A^{-1}AX = A^{-1}B \Rightarrow IX = A^{-1}B$$

$$\therefore A^{-1}B = \begin{bmatrix} -\frac{29}{392} & \frac{59}{392} & \frac{13}{392} \\ \frac{48}{392} & \frac{24}{392} & \frac{-8}{392} \\ \frac{-1}{392} & \frac{25}{392} & \frac{41}{392} \end{bmatrix} \begin{bmatrix} -6.5 \\ 41.25 \\ 71.75 \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{29}{392} \times -6.5 + \frac{59}{392} \times 41.25 + \frac{13}{392} \times 71.75 \\ \frac{48}{392} \times -6.5 + \frac{24}{392} \times 41.25 - \frac{8}{392} \times 71.75 \\ \frac{-1}{392} \times -6.5 + \frac{25}{392} \times 41.25 + \frac{41}{392} \times 71.75 \end{bmatrix}$$

$$= \begin{bmatrix} 3.5 \\ -2 \\ 7.25 \end{bmatrix}$$

$$\therefore X_1 = 3.5, \quad X_2 = -2, \quad X_3 = 7.25$$