Assignment 1 Due: 24.3.2022 (21.8.1443 AH)

Q1) Write a function that returns the real roots of the quadratic equation:

$$a\,x^2 + b\,x + c = 0 \qquad (a \neq 0)$$

It will always return two values:

- Either the two real roots (could be equal),
- Or: None, None (Recall that "None" is a data type in Python)

See this demo Program about "None" (Red color indicates keywords):

```python
def test(z):
    return z+1, None
x,y = test(7)
print (x,y)
if y == None:
    print("It is None")
```

Output:

```
8 None
It is None
```

Mathematical Background:

How to solve the quadratic equation: $a\,x^2 + b\,x + c = 0 \quad (a \neq 0)$

1- Compute the Discriminant = $b^2 - 4ac$

2- Check the Discriminant:

If it is Negative → No Real Solutions (Python: return None, None)

If it is Not Negative → Two Solutions:

(They are equal if Discriminant is Zero))

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Q2) Write a program that reads a 2x2 matrix (4 real values) and computes the real eigenvalues of the matrix (if any).

If no real eigenvalues are found, just give a message.

Mathematical Background:

The eigenvalues of a matrix A are all values λ such that there exists a vector X that satisfies:

$A\,X = \lambda\,X$

We will now see how to find the eigenvalues of a 2x2 matrix:

Let the matrix A = $\begin{bmatrix} t & h \\ k & u \end{bmatrix}$, and the vector X=$\begin{bmatrix} x \\ y \end{bmatrix}$,

Substitute in the definition:

$$\begin{bmatrix} t & h \\ k & u \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \lambda \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} t & h \\ k & u \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} - \lambda \begin{bmatrix} x \\ y \end{bmatrix} = \lambda \begin{bmatrix} x \\ y \end{bmatrix} - \lambda \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} t-\lambda & h \\ k & u-\lambda \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

This equation has a solution if the determinant of the matrix equals Zero, Recall that the determinant of $\begin{bmatrix} x & y \\ z & w \end{bmatrix} = xw - yz.$

So we have:

$(t - \lambda)(u - \lambda) - hk = 0$

$\lambda^2 - (t+u)\,\lambda + (tu - hk) = 0$

So Simply: call your quadratic equation solver function with:

a=1 (which is clearly $\neq$ 0), b = – (t+u) and c = (tu-hk)

Example:

Let A = $\begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}$ then the eigenvalues of A can be found by solving the quadratic equation:

x^2 -7x + + 10 = 0

Solution: The eigenvalues are: 2.000 and 5.000

Sample Run: (Note: read the input as a list and print results rounded to 3 decimal places)

Enter 4 elements of a 2x2 Matrix: 4 2 1 3

The eigenvalues are 2.000 and 5.000