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Numerical Computations

Assignment #6

Question #1:

$\Delta x = 2$ seconds

a) $\dfrac{f(x+\Delta x) - f(x)}{\Delta x} = \dfrac{f(0+2) - f(0)}{2} = \dfrac{f(2) - f(0)}{2}$

$= \dfrac{8.25 - 0}{2} = 4.125$ m/s

b) $\dfrac{f(x) - f(x-\Delta x)}{\Delta x} = \dfrac{f(8) - f(8-2)}{2} = \dfrac{f(8) - f(6)}{2}$

$= \dfrac{34.5 - 22.7}{2} = 5.9$ m/s

c) $\dfrac{f(x+\Delta x) - f(x-\Delta x)}{2\Delta x} = \dfrac{f(6+2) - f(6-2)}{2 \times 2}$

$= \dfrac{f(8) - f(4)}{4} = \dfrac{34.5 - 13.9}{4} = 5.15$ m/s

Question #2:

$f(x) = x^3 e^{-x}$, $f(0) = 0$, $f(1) = 0.3679$, $f(2) = 1.0827$

$f(3) = 1.3443$, $f(4) = 1.1722$, $f(5) = 0.8422$

$f(6) = 0.5354$

Trapezoidal rule:

$h = \frac{6-0}{6} = 1$, $\sum_{i=1}^{n-1} f(x_i) = 4.8093$

$$\int_a^b f(x)\,dx = \frac{h}{2}\left[f(a) + 2\sum_{i=1}^{n-1} f(x_i) + f(b)\right]$$

$$= \frac{1}{2}\left(0 + 2(4.8093) + 0.5354\right) = 5.077$$

Simpson's $\frac{1}{3}$ rule:

$$\sum_{\substack{i=1 \\ i=odd}}^{n-1} f(x_i) = f(1) + f(3) + f(5) = 2.5544$$

$$\sum_{\substack{i=2 \\ i=even}}^{n-1} f(x_i) = f(2) + f(4) = 2.2549$$

$$\int_a^b f(x)\,dx = \frac{h}{3}\left[f(a) + 4\sum_{\substack{i=1 \\ i=odd}}^{n-1} f(x_i) + 2\sum_{\substack{i=1 \\ i=even}}^{n-1} f(x_i) + f(b)\right]$$

$$= \frac{1}{3}\left(0 + 4(2.5544) + 2(2.2549) + 0.5354\right)$$

$$= 5.0876$$

Simpson's $\frac{3}{8}$ rule:

$$\sum_{\substack{i \neq 3k \\ i=1}}^{n-1} f(x_i) = f(1) + f(2) + f(4) + f(5) = \cancel{2.6228}\ 3.465$$

$$\sum_{\substack{i=1 \\ i=3k}}^{n-1} f(x_i) = f(3) = 1.3443$$

$$\int_a^b f(x)\,dx = \frac{3h}{8}\left[f(a) + 3\sum_{\substack{i=1 \\ i \neq 3k}}^{n-1} f(x_i) + 2\sum_{\substack{i=1 \\ i=3k}}^{n-1} f(x_i) + f(b)\right]$$

$$= \frac{3}{8}\left(0 + 3(\underbrace{\cancel{2.6228}}_{3.465}) + 2(1.3443) + 0.5354)\right)$$

$$= 5.1071$$

antiderivative substitution:

$$\left[(-(6)^3 - 3(6)^2 - 6(6) - 6)e^{-6} + C\right] - \left[(-(0)^3 - 3(0)^2 - 6(0) - 0)e^{-0} + C\right]$$

$$= -0.9072 + C + 6 - C = 5.0928$$

All three methods gave results very close to the original value, but simpson's $\frac{1}{3}$ rule gave a percent relative error of approximately 0.1% making it the most precise between the two

Question #3:

let forward divided difference be denoted by f

backward divided difference be denoted by b

central divided difference be denoted by c

The general relationship:

$$\frac{f(x+\Delta x)-f(x)}{\Delta x}+\frac{f(x)-f(x-\Delta x)}{\Delta x}=\frac{f(x+\Delta x)-f(x)+f(x)-f(x-\Delta x)}{\Delta x}$$

$$=\frac{f(x+\Delta x)-f(x-\Delta x)}{\Delta x}$$

$\therefore$ $f+b=2c$

$\therefore$ To compute the forward difference: $f=2c-b$

$c=50.5$, $b=39$

$\therefore$ $f=2(50.5)-39=62$