R Notebook sandbox: Playing with Correlation

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Correlation

Review of Basic Differences Testing

Variance

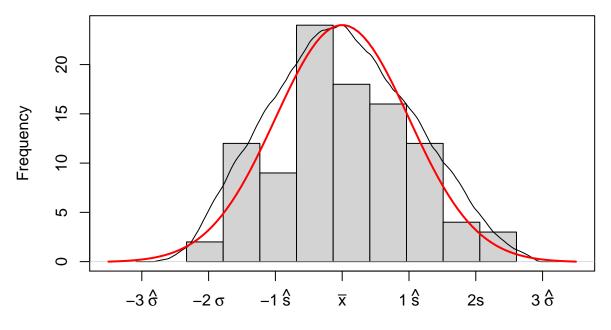
Variance measures the spread of a variable. Consider the following three examples:

```
library(humanVerseWSU);
# You need R tools for this to work: https://cran.r-project.org/bin/windows/Rtools/
# You may want to see if you have the latest version...
# library(devtools);
# detach(package:humanVerseWSU);
# install_github("MonteShaffer/humanVerseWSU");
# Choose (3) None to minimize headaches ....
normalDiagnosticPlot = function(x, normalityTest=TRUE,
                                   showDensity=TRUE,
                                   showNormal=TRUE,
                                   showSDs=FALSE,
                                   showAxis=TRUE
                               )
  {
 xx = na.omit(x);
 x.stats = doStatsSummary(x);
  # x.table = table(x);
  # library(KernSmooth); # install.packages("KernSmooth", dependencies=TRUE);
 # bin.count = dpih(xx);
```

```
# mybreaks = 100 * bin.count;
  mxlim = c(x.stats\$mean - 3.5 * x.stats\$sd,
            x.stats\$mean + 3.5 * x.stats\$sd);
  h = hist(xx, breaks="Sturges", plot=F);
  mylim = c(0, max(h\$counts));
  myMain = paste0( "Histogram (mean: ",
                  round(x.stats$mean,digits=3),
                  ", sd: ",
                  round(x.stats$sd,digits=3),
                  ")"
                  );
mxlab = "";
  if(normalityTest)
    isNormal = NULL;
    if(x.stats$shapiro.is.normal$`0.10`) { isNormal = 0.10; }
    if(x.stats$shapiro.is.normal$`0.05`) { isNormal = 0.05; }
    if(x.stats$shapiro.is.normal$`0.01`) { isNormal = 0.01; }
    isNormalResult = FALSE;
    if(!is.null(isNormal)) { isNormalResult = TRUE;}
    if(is.null(isNormal)) { isNormal = 0.05;}
    mxlab = paste0("Shapiro Normality test at (alpha = ",
                isNormal, ") is ... ",isNormalResult);
    }
### Histogram
  hist(xx, breaks="Sturges", xlim=mxlim, ylim=mylim,
      xlab=mxlab, xaxt='n', main=myMain);
  if(showDensity)
    {
    par(new=T); # overlay
  ### Density Plot (remember first reading?)
    plot( density(xx, kernel="epanechnikov") ,
            xlim=mxlim,
            main="",
            xlab="",
            ylab="",
            xaxt='n',
            yaxt='n'
        );
    }
  if(showNormal)
```

```
par(new=T); # overlay
  ### Normal Curve
    xt = seq(-3.5, 3.5, length=100);
           yt = dnorm(xt);
    plot( xt, yt,
         type="1",
          lwd=2,
          col = "red",
          axes=F,
          xlab="",
         ylab=""
       );
    }
  if(showSDs)
    {
  ### vertical lines at sd's of data ...
    abline(v=x.stats$mean,lwd=4,col="blue");
      abline(v=x.stats$mean - 1 * x.stats$sd , col="green",lwd=3);
      abline(v=x.stats$mean + 1 * x.stats$sd , col="green",lwd=3);
      abline(v=x.stats$mean - 2 * x.stats$sd , col="green",lwd=2);
      abline(v=x.stats$mean + 2 * x.stats$sd , col="green",lwd=2);
      abline(v=x.stats$mean - 3 * x.stats$sd , col="green",lwd=1);
      abline(v=x.stats$mean + 3 * x.stats$sd , col="green",lwd=1);
    }
  if(showAxis)
  ### axis labels showing the ability to use expression
    axis(1, at = -3:3, labels = c(expression("-3"~hat(sigma)), expression("-2"~sigma), expression("-
        \#axis(1, at = -3:3, labels = c("-3s", "-2s", "-1s", "hat(mu)", "1s", "2s", "3s"))
    }
  }
x.0.1 = rnorm(100,0,1); normalDiagnosticPlot(x.0.1);
```

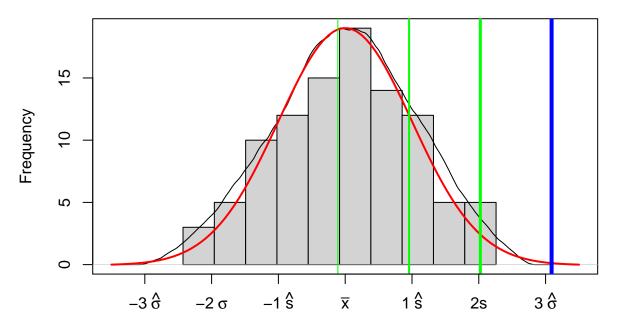
Histogram (mean: 0.123, sd: 0.911)



Shapiro Normality test at (alpha = 0.01) is ... TRUE

x.3.1 = rnorm(100,3,1); normalDiagnosticPlot(x.3.1, showSDs=TRUE);

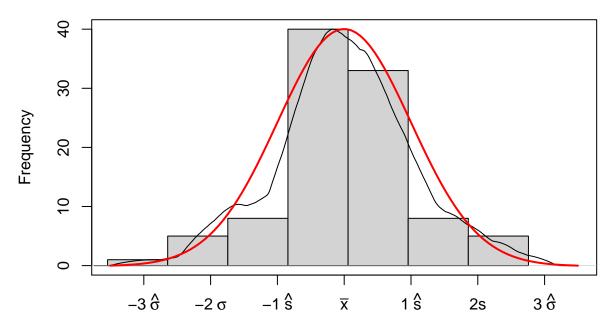
Histogram (mean: 3.09, sd: 1.067)



Shapiro Normality test at (alpha = 0.01) is ... TRUE

x.9.1 = rnorm(100,9,1); normalDiagnosticPlot(x.9.1);

Histogram (mean: 8.936, sd: 1.111)

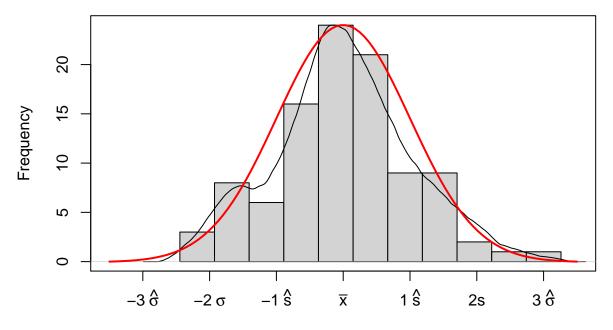


Shapiro Normality test at (alpha = 0.01) is ... TRUE

Notice that they all have about the same "spread" yet have different mean values.

x.0.1b = rnorm(100,0,1); normalDiagnosticPlot(x.0.1b);

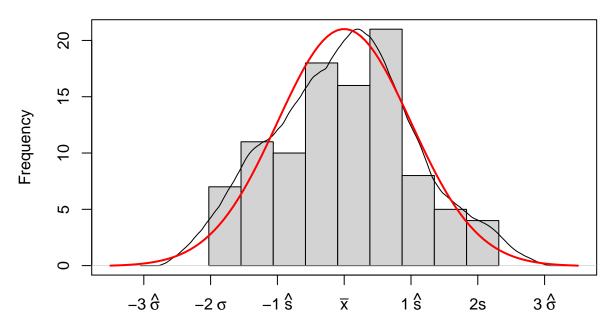
Histogram (mean: -0.143, sd: 0.964)



Shapiro Normality test at (alpha = 0.01) is ... TRUE

x.0.2 = rnorm(100,0,2); normalDiagnosticPlot(x.0.2);

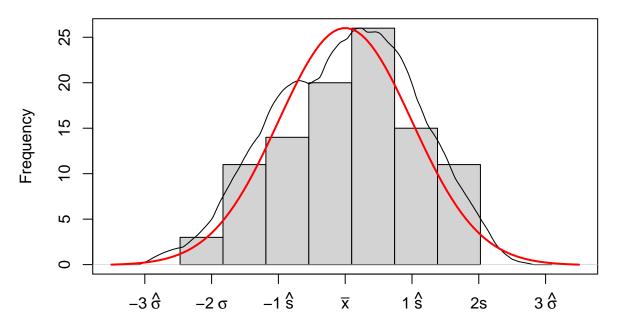
Histogram (mean: 0.202, sd: 2.073)



Shapiro Normality test at (alpha = 0.01) is ... TRUE

x.0.3 = rnorm(100,0,3); normalDiagnosticPlot(x.0.3);

Histogram (mean: -0.303, sd: 3.113)



Shapiro Normality test at (alpha = 0.01) is ... TRUE

Notice that they all have different amounts of "spread" yet have about the same mean values.

Student's t-test: Compare means of two independent samples

TODO: Review the one-sided and two-sided t-test https://en.wikipedia.org/wiki/Student%27s_t-test. Demonstrate your knowledge of those two tests by applying it to one or more of the x-data prepared above (e.g., 'x.0.1').

```
\#t.test(x.0.1, x.0.1b, paired=FALSE, var.equal=FALSE);
```

Welch's t-test: Compare means of two independent samples

We cannot always assume the two samples we want to compare have the same variance. With Student's t-test the pooled variance is used. For Welch's t-test, a different form is used https://en.wikipedia.org/wiki/Welch $\%27s_t$ -test.

```
t.test(x.0.1, x.0.1b, paired=FALSE, var.equal=FALSE);
```

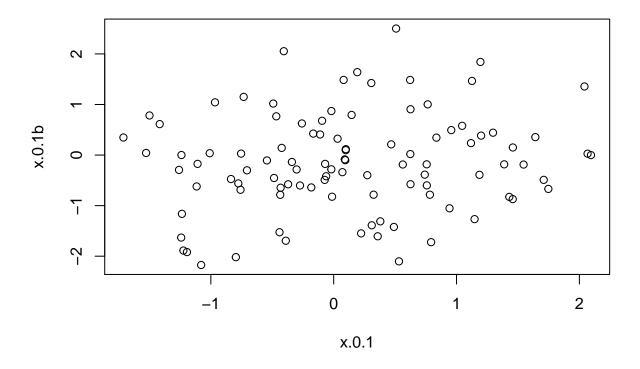
```
##
## Welch Two Sample t-test
##
## data: x.0.1 and x.0.1b
## t = 2.0071, df = 197.37, p-value = 0.04611
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
## 0.004642271 0.527730796
## sample estimates:
## mean of x mean of y
```

Interdependence of data

Above we review when two "independent" samples may have different means or variances. But what happens when the two samples have some degree of "codependency" or "interdependence"?

Rather Independent

```
plot(x.0.1, x.0.1b);
```



```
cor(x.0.1, x.0.1b, method="pearson"); # default

## [1] 0.1417192
    cor(x.0.1, x.0.1b, method="kendall");

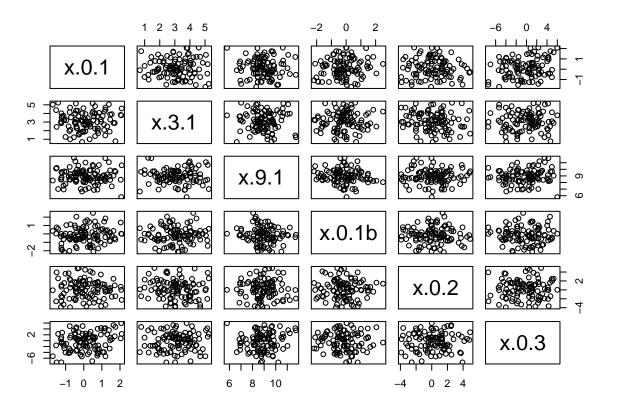
## [1] 0.07151515
    cor(x.0.1, x.0.1b, method="spearman");

## [1] 0.1058146

cor.test(x.0.1, x.0.1b, method="pearson"); # default

## ## Pearson's product-moment correlation
## ## data: x.0.1 and x.0.1b
```

```
## t = 1.4173, df = 98, p-value = 0.1596
## alternative hypothesis: true correlation is not equal to 0
## 95 percent confidence interval:
## -0.05626511 0.32897978
## sample estimates:
##
        cor
## 0.1417192
cor.test(x.0.1, x.0.1b, method="kendall");
## Kendall's rank correlation tau
##
## data: x.0.1 and x.0.1b
## z = 1.0543, p-value = 0.2918
## alternative hypothesis: true tau is not equal to 0
## sample estimates:
         tau
## 0.07151515
cor.test(x.0.1, x.0.1b, method="spearman");
##
   Spearman's rank correlation rho
##
## data: x.0.1 and x.0.1b
## S = 149016, p-value = 0.2942
\mbox{\tt \#\#} alternative hypothesis: true rho is not equal to 0
## sample estimates:
##
         rho
## 0.1058146
x.df = data.frame(cbind(x.0.1, x.3.1, x.9.1, x.0.1b, x.0.2, x.0.3));
    cor(x.df);
##
                x.0.1
                            x.3.1
                                        x.9.1
                                                   x.0.1b
                                                                x.0.2
                                                                            x.0.3
          1.00000000 -0.00962073 -0.02285625 0.14171921 -0.04764616 0.24738443
## x.0.1
## x.3.1 -0.00962073 1.00000000 -0.01351532 0.03666927 -0.12923814 0.04086212
## x.9.1 -0.02285625 -0.01351532 1.00000000 -0.13086408 -0.01378132 0.12040782
## x.0.1b 0.14171921 0.03666927 -0.13086408 1.00000000 -0.01192729 -0.03776085
## x.0.2 -0.04764616 -0.12923814 -0.01378132 -0.01192729 1.00000000 0.01999602
## x.0.3 0.24738443 0.04086212 0.12040782 -0.03776085 0.01999602 1.00000000
plot(x.df);
```

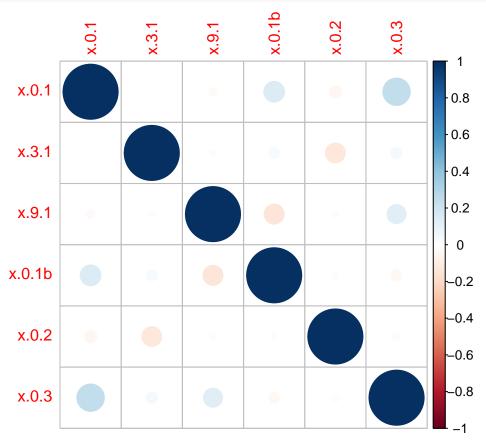


symnum(cor(x.df)); # remove noise based on cutpoints

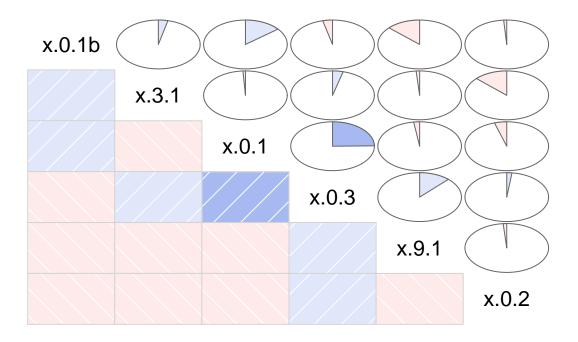
```
## x.0.1 x.3 x.9 x.0.1b x.0.2 x.0.3
## x.0.1 1
## x.3.1
## x.9.1
## x.0.1b
## x.0.2
## x.0.3
## attr(,"legend")
## [1] 0 ' ' 0.3 '.' 0.6 ',' 0.8 '+' 0.9 '*' 0.95 'B' 1
   symnum( cor(x.df),
           diag = TRUE,
           corr = TRUE,
           cutpoints=c(0.1,0.4,0.7,0.9),
           symbols = c(" ",".","*","**","***")
         x.0.1 x.3 x.9 x.0.1b x.0.2 x.0.3
## x.0.1 1
## x.3.1
## x.9.1
## x.0.1b .
## x.0.2
## x.0.3 .
                                   1
## attr(,"legend")
```

```
## [1] 0 ' ' 0.1 '.' 0.4 '*' 0.7 '**' 0.9 '***' 1
    corrplot::corrplot( (cor(x.df)) );

# https://www.statmethods.net/advgraphs/correlograms.html
library(corrgram); # install.packages("corrgram", dependencies=TRUE);
```



My title

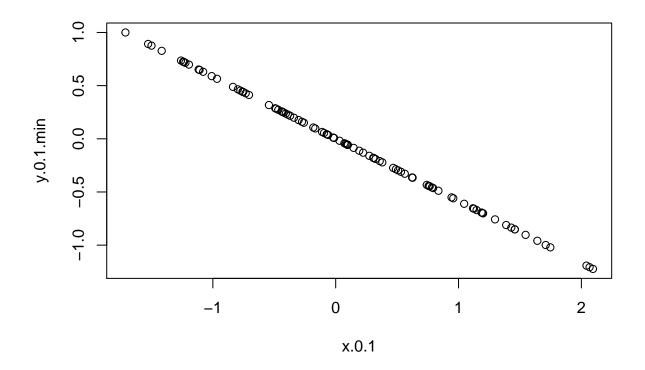


```
library(Hmisc); # install.packages("Hmisc", dependencies=TRUE);
   rcorr(as.matrix(x.df), type="pearson");
         x.0.1 x.3.1 x.9.1 x.0.1b x.0.2 x.0.3
## x.0.1
        ## x.3.1 -0.01 1.00 -0.01 0.04 -0.13 0.04
## x.9.1 -0.02 -0.01 1.00 -0.13 -0.01 0.12
## x.0.1b 0.14 0.04 -0.13
                           1.00 -0.01 -0.04
## x.0.2 -0.05 -0.13 -0.01 -0.01 1.00 0.02
## x.0.3 0.25 0.04 0.12 -0.04 0.02 1.00
## n= 100
##
##
## P
         x.0.1 x.3.1 x.9.1 x.0.1b x.0.2 x.0.3
##
               0.9243 0.8214 0.1596 0.6378 0.0131
## x.0.1
## x.3.1 0.9243
                      0.8938 0.7172 0.2000 0.6865
## x.9.1 0.8214 0.8938
                            0.1944 0.8918 0.2328
                                   0.9062 0.7092
## x.0.1b 0.1596 0.7172 0.1944
## x.0.2 0.6378 0.2000 0.8918 0.9062
## x.0.3 0.0131 0.6865 0.2328 0.7092 0.8435
# corrplot::corrplot(
# sweep
# symnum
```

Notice that since the data was randomly generated "independently", the "correlation" is close to zero.

Functionally Dependent

```
# requires functions-standardize.R of humanVerseWSU
y.0.1.min = standardizeToMin(x.0.1);
plot(x.0.1, y.0.1.min);
```



```
cor(x.0.1, y.0.1.min, method="pearson"); # default

## [1] -1

cor(x.0.1, y.0.1.min, method="kendall");

## [1] -1

cor(x.0.1, y.0.1.min, method="spearman");

## [1] -1

cor.test(x.0.1, y.0.1.min, method="pearson"); # default

##

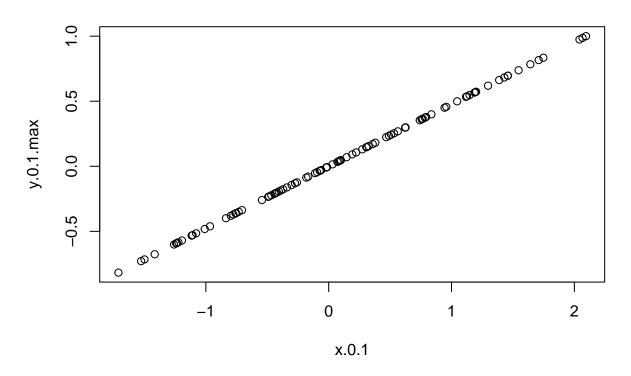
## Pearson's product-moment correlation

##

## data: x.0.1 and y.0.1.min

## t = -Inf, df = 98, p-value < 2.2e-16</pre>
```

```
## alternative hypothesis: true correlation is not equal to 0
## 95 percent confidence interval:
## -1 -1
## sample estimates:
## cor
## -1
cor.test(x.0.1, y.0.1.min, method="kendall");
##
## Kendall's rank correlation tau
##
## data: x.0.1 and y.0.1.min
## z = -14.742, p-value < 2.2e-16
\mbox{\tt \#\#} alternative hypothesis: true tau is not equal to 0
## sample estimates:
## tau
## -1
cor.test(x.0.1, y.0.1.min, method="spearman");
##
   Spearman's rank correlation rho
##
##
## data: x.0.1 and y.0.1.min
## S = 333300, p-value < 2.2e-16
\mbox{\tt \#\#} alternative hypothesis: true rho is not equal to 0
## sample estimates:
## rho
## -1
y.0.1.max = standardizeToMax(x.0.1);
plot(x.0.1, y.0.1.max);
```



```
cor(x.0.1, y.0.1.max, method="pearson"); # default
## [1] 1
cor(x.0.1, y.0.1.max, method="kendall");
## [1] 1
cor(x.0.1, y.0.1.max, method="spearman");
## [1] 1
cor.test(x.0.1, y.0.1.max, method="pearson"); # default
##
##
  Pearson's product-moment correlation
##
## data: x.0.1 and y.0.1.max
## t = Inf, df = 98, p-value < 2.2e-16
\#\# alternative hypothesis: true correlation is not equal to 0
## 95 percent confidence interval:
## 1 1
## sample estimates:
## cor
cor.test(x.0.1, y.0.1.max, method="kendall");
```

##

```
Kendall's rank correlation tau
##
## data: x.0.1 and y.0.1.max
## z = 14.742, p-value < 2.2e-16
## alternative hypothesis: true tau is not equal to 0
## sample estimates:
## tau
##
     1
cor.test(x.0.1, y.0.1.max, method="spearman");
##
##
   Spearman's rank correlation rho
##
## data: x.0.1 and y.0.1.max
## S = 0, p-value < 2.2e-16
## alternative hypothesis: true rho is not equal to 0
## sample estimates:
## rho
##
    1
```

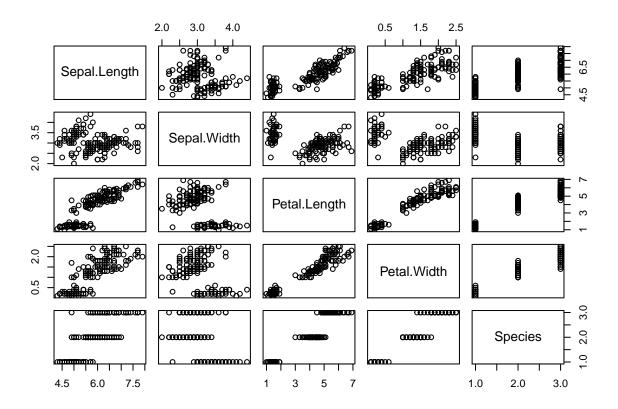
QUESTION(s) to PONDER: Why does one have a negative slope and the other have a positive slope? Does one (1) mean "perfect correlation"? How does that relate to being a linear combination of a vector basis?

Somewhere in Between

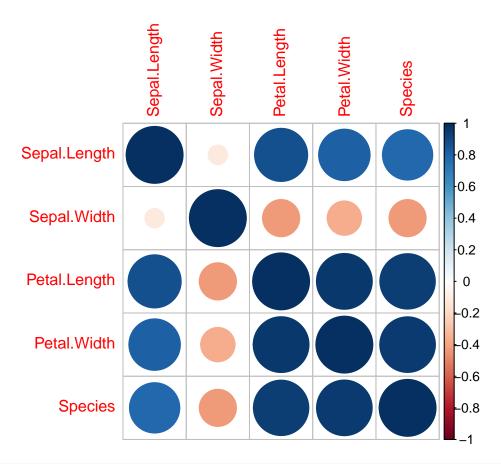
```
iris.df = iris;
iris.df$Species = as.numeric(iris.df$Species); # if not, non-numeric, throws an error ...
round( cor(iris.df), digits=2);
```

Iris

```
##
                Sepal.Length Sepal.Width Petal.Length Petal.Width Species
## Sepal.Length
                         1.00
                                    -0.12
                                                   0.87
                                                               0.82
                                                                        0.78
                                                                       -0.43
## Sepal.Width
                        -0.12
                                     1.00
                                                  -0.43
                                                              -0.37
## Petal.Length
                         0.87
                                    -0.43
                                                   1.00
                                                               0.96
                                                                        0.95
## Petal.Width
                         0.82
                                    -0.37
                                                   0.96
                                                                1.00
                                                                        0.96
## Species
                         0.78
                                    -0.43
                                                   0.95
                                                               0.96
                                                                        1.00
plot(iris.df);
```

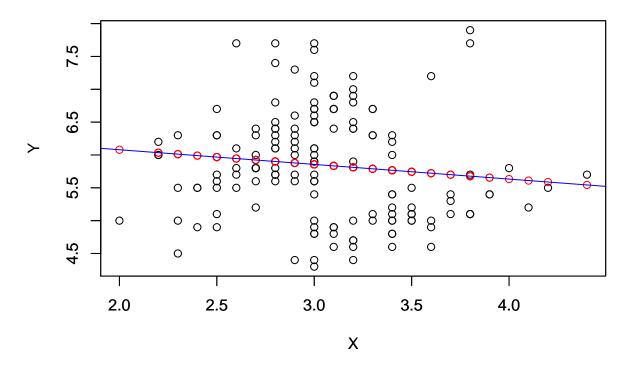


```
symnum( cor(iris.df),
            diag = TRUE,
            corr = TRUE,
            cutpoints=c(0.4,0.7,0.9),
            symbols = c(" ",".","*","**")
          );
                S.L S.W P.L P.W Sp
##
## Sepal.Length 1
## Sepal.Width
## Petal.Length *
## Petal.Width *
## Species
## attr(,"legend")
## [1] 0 ' ' 0.4 '.' 0.7 '*' 0.9 '**' 1
    corrplot::corrplot( (cor(iris.df)) );
```



```
# Let's suppose we want to consider that
# Sepal.Length is a function of Sepal.Width
Y = iris.df$Sepal.Length;
X = iris.df$Sepal.Width;
myData = data.frame(cbind(Y,X));
plot(X,Y, xlim=range(X), ylim=range(Y));
  linear.model = lm(Y~X, myData);
  linear.model.summary = summary(linear.model);
  Y.predicted = predict(linear.model);
par(new=TRUE);
  myMain = paste0("R^2=",
          round(linear.model.summary$r.squared, digits=4),
              "; adj.R^2=",
          round(linear.model.summary$adj.r.squared, digits=4)
plot(X,Y.predicted, main=myMain, ylab="", xlim=range(X), ylim=range(Y), col="red");
  abline(linear.model, col="blue");
```

R^2=0.0138; adj.R^2=0.0072

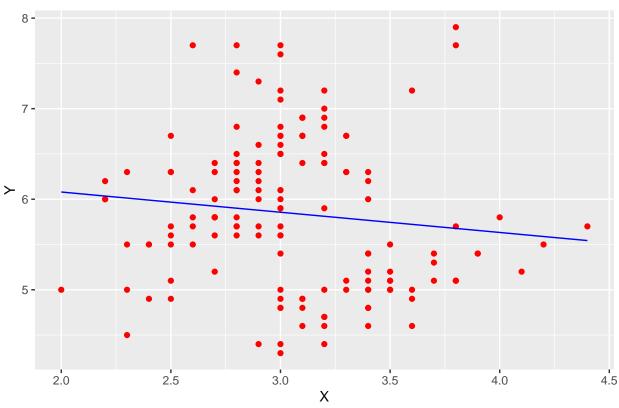


```
cor(X, Y, method="pearson"); # default
## [1] -0.1175698
cor(X, Y, method="kendall");
## [1] -0.07699679
cor(X, Y, method="spearman");
## [1] -0.1667777
cor.test(X, Y, method="pearson"); # default
##
##
   Pearson's product-moment correlation
##
## data: X and Y
## t = -1.4403, df = 148, p-value = 0.1519
\#\# alternative hypothesis: true correlation is not equal to 0
## 95 percent confidence interval:
  -0.27269325 0.04351158
## sample estimates:
##
          cor
## -0.1175698
cor.test(X, Y, method="kendall");
```

##

```
## Kendall's rank correlation tau
##
## data: X and Y
## z = -1.3318, p-value = 0.1829
## alternative hypothesis: true tau is not equal to 0
## sample estimates:
## -0.07699679
cor.test(X, Y, method="spearman");
##
## Spearman's rank correlation rho
##
## data: X and Y
## S = 656283, p-value = 0.04137
\mbox{\tt \#\#} alternative hypothesis: true rho is not equal to 0
## sample estimates:
         rho
## -0.1667777
# different visualization
library(ggplot2);
ggplot() + geom_point(aes(x = X,
              y = Y), colour = 'red') +
geom_line(aes(x = X,
y = Y.predicted), colour = 'blue') +
ggtitle('X vs Y') +
xlab('X') +
ylab('Y')
```





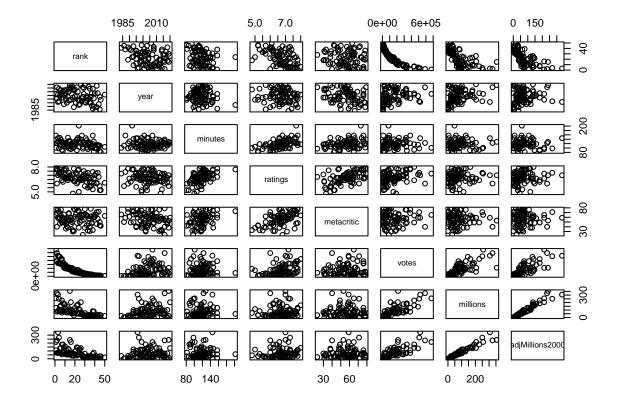
```
library(devtools);
path.github = "https://raw.githubusercontent.com/MonteShaffer/humanVerseWSU/master/";
source_url( pasteO(path.github, "humanVerseWSU/R/functions-dataframe.R") );
library(devtools);
source_url("http://md5.mshaffer.com/WSU_STATS419/denzel");
str(denzel);
```

Will & Denzel

```
## List of 4
               : chr "nm0000243"
## $ nmid
               : chr "Denzel Washington"
##
   $ countfilms:List of 2
    ..$ totalcount: num 61
    ..$ pagecount : num 50
##
   $ movies.50 :'data.frame': 50 obs. of 11 variables:
##
    ..$ rank : num [1:50] 1 2 3 4 5 6 7 8 9 10 ...
##
    ..$ title
                : chr [1:50] "American Gangster" "Training Day" "Inside Man" "The Equalizer" ...
##
                : chr [1:50] "tt0765429" "tt0139654" "tt0454848" "tt0455944" ...
##
    ..$ ttid
##
    ..$ year
                : num [1:50] 2007 2001 2006 2014 2004 ...
                 : chr [1:50] "R" "R" "R" "R" ...
##
    ..$ rated
    ..$ minutes : num [1:50] 157 122 129 132 146 138 126 118 125 115 ...
##
    ..$ genre : chr [1:50] "Biography, Crime, Drama" "Crime, Drama, Thriller" "Crime, Drama, Myste
##
```

```
..$ ratings : num [1:50] 7.8 7.7 7.6 7.2 7.7 7.3 7 6.9 7.7 6.7 ...
##
    ..$ metacritic: num [1:50] 76 69 76 57 47 76 59 53 66 52 ...
               : num [1:50] 384284 382395 332289 326479 324413 ...
##
     ..$ millions : num [1:50] 130.2 76.6 88.5 101.5 77.9 ...
##
source_url("http://md5.mshaffer.com/WSU_STATS419/will");
 str(will);
## List of 4
## $ nmid
              : chr "nm0000226"
               : chr "Will Smith"
## $ name
   $ countfilms:List of 2
##
   ..$ totalcount: num 111
    ..$ pagecount : num 50
   $ movies.50 :'data.frame': 50 obs. of 11 variables:
##
                  : num [1:50] 1 2 3 4 5 6 7 8 9 10 ...
##
    ..$ rank
                  : chr [1:50] "I Am Legend" "Suicide Squad" "Independence Day" "Men in Black" ...
##
    ..$ title
                 : chr [1:50] "tt0480249" "tt1386697" "tt0116629" "tt0119654" ...
##
    ..$ ttid
##
    ..$ year
                  : num [1:50] 2007 2016 1996 1997 2004 ...
##
                 : chr [1:50] "PG-13" "PG-13" "PG-13" "PG-13" ...
    ..$ rated
    ..$ minutes : num [1:50] 101 123 145 98 115 117 92 88 106 118 ...
##
##
                 : chr [1:50] "Action, Adventure, Drama" "Action, Adventure, Fantasy" "Action, Advent
    ..$ genre
    ..$ ratings : num [1:50] 7.2 6 7 7.3 7.1 8 6.4 6.2 6.8 6.6 ...
##
##
    ..$ metacritic: num [1:50] 65 40 59 71 59 64 49 49 58 58 ...
##
                 : num [1:50] 675193 588111 520657 507618 491489 ...
     ..$ millions : num [1:50] 256 325 306 251 145 ...
##
imdb.df = rbind(denzel$movies.50, will$movies.50);
## requires latest humanVerseWSU
## rank & votes & so on ...
imdb.rv = removeAllColumnsBut( imdb.df,
           c("rank", "year", "minutes",
             "ratings","metacritic","votes","millions") );
dim(imdb.rv); # 100 movies
## [1] 100 7
imdb.rv = removeNAsFromDataFrame(imdb.rv, c("metacritic", "millions") );
dim(imdb.rv); # 85 movies, is it worth it?
## [1] 85 7
loadInflationData(); # requires functions-inflation.R in humanVerseWSU ...
imdb.rv = standardizeDollarsInDataFrame(imdb.rv, 2000, "millions", "year", "adjMillions2000");
round( cor(imdb.rv), digits=2);
                   rank year minutes ratings metacritic votes millions
## rank
                   1.00 -0.26
                                -0.21 -0.46
                                                   -0.09 -0.88
                                                                  -0.66
                  -0.26 1.00
                               -0.08 -0.12
                                                   -0.29 0.22
## year
                                                                   0.26
                  -0.21 -0.08 1.00 0.51
## minutes
                                                   0.31 0.06
                                                                   0.01
                  -0.46 -0.12 0.51
                                        1.00
                                                   0.52 0.33
## ratings
                                                                   0.11
```

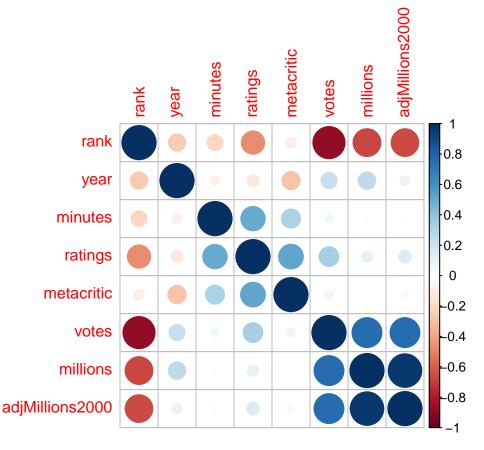
```
## metacritic
                  -0.09 -0.29
                                 0.31
                                         0.52
                                                    1.00 0.06
                                                                   0.01
## votes
                  -0.88 0.22
                                 0.06
                                         0.33
                                                    0.06 1.00
                                                                   0.77
## millions
                  -0.66 0.26
                                 0.01
                                         0.11
                                                    0.01 0.77
                                                                   1.00
## adjMillions2000 -0.66 0.08
                                 0.02
                                         0.14
                                                    0.05 0.76
                                                                   0.96
                  adjMillions2000
## rank
                            -0.66
                              0.08
## year
## minutes
                              0.02
## ratings
                              0.14
## metacritic
                              0.05
## votes
                              0.76
## millions
                              0.96
## adjMillions2000
                              1.00
plot(imdb.rv);
```



```
symnum( cor(imdb.rv),
    diag = TRUE,
    corr = TRUE,
    cutpoints=c(0.4,0.7,0.9),
    symbols = c(" ",".","*","**")
);
```

```
## rn y mn rt mt v ml a
## rank 1
## year 1
## minutes 1
```

```
## ratings . . 1
## metacritic . 1
## votes * 1
## millions . * 1
## adjMillions2000 . * ** 1
## attr(,"legend")
## [1] 0 ' ' 0.4 '.' 0.7 '*' 0.9 '**' 1
corrplot::corrplot( (cor(imdb.rv)) );
```



Can you read the results above? Are you able to describe the details? Is a correlation strong/weak? Is it positive or negative? What exactly does that all mean?

rank vs. minutes -WRITE SOMETHING HERE-

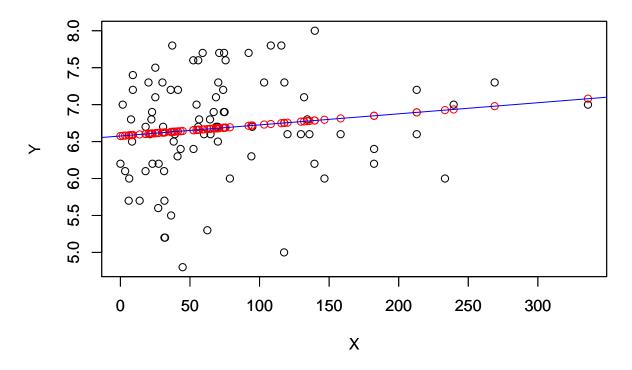
millions vs. year -WRITE SOMETHING HERE-

ratings vs. adjMillions2000 -WRITE SOMETHING HERE-

```
# Let's suppose we want to consider that
# Ratings is a function of adjMillions2000

Y = imdb.rv$ratings;
```

R^2=0.0201; adj.R^2=0.0083



```
cor(X, Y, method="pearson"); # default

## [1] 0.1418799

cor(X, Y, method="kendall");

## [1] 0.1306547
```

```
cor(X, Y, method="spearman");
## [1] 0.1891664
cor.test(X, Y, method="pearson"); # default
  Pearson's product-moment correlation
##
## data: X and Y
## t = 1.3058, df = 83, p-value = 0.1952
## alternative hypothesis: true correlation is not equal to 0
## 95 percent confidence interval:
## -0.07346566 0.34458441
## sample estimates:
##
        cor
## 0.1418799
cor.test(X, Y, method="kendall");
##
## Kendall's rank correlation tau
##
## data: X and Y
## z = 1.7378, p-value = 0.08224
\mbox{\tt \#\#} alternative hypothesis: true tau is not equal to 0
## sample estimates:
##
        tau
## 0.1306547
cor.test(X, Y, method="spearman");
##
## Spearman's rank correlation rho
##
## data: X and Y
## S = 82981, p-value = 0.08294
\#\# alternative hypothesis: true rho is not equal to 0
## sample estimates:
##
        rho
## 0.1891664
# different visualization
library(ggplot2);
ggplot() + geom_point(aes(x = X,
               y = Y), colour = 'red') +
geom_line(aes(x = X,
y = Y.predicted), colour = 'blue') +
ggtitle('X vs Y') +
xlab('X') +
ylab('Y')
```



```
### 3-D plot
library(rgl);
Z = imdb.rv$millions2000;
plot3d(x=X, y=Y, z=Z,
     type="p", col="red",
      xlab="X", ylab="Y", zlab="Z",
      size=5, lwd=15, box=F
      );
# try this one ...
X = rnorm(100,0,1);
Y = rnorm(100,0,1);
Z = X+Y;
plot3d(x=X, y=Y, z=Z,
      type="p", col="red",
     xlab="X", ylab="Y", zlab="Z",
     size=5, lwd=15, box=F
      );
# what do you notice?
# how is this simular to plot(X,z-scores) on a previous notebook?
# functionally, notice that RStudio doesn't embed locally. Is there a new 3D plotting tool that does?
```

```
library(devtools);
measure.instructor = read.csv("http://md5.mshaffer.com/WSU_STATS419/measure-04343803d489fe8ee2c5f6ab97a
```

```
getOne = c("hand.length", "hand.width", "hand.elbow", "elbow.armpit", "arm.reach", "foot.length", "floot.length", "floot.lengt
n.rows = dim(measure.instructor)[1];
for(one in getOne)
      measure.instructor[one] = NA;
       }
for(i in 1:n.rows)
       measure.row = measure.instructor[i,];
       for(one in getOne)
              nidx = getIndexOfDataFrameColumns(measure.instructor, one);
              myleft = paste0(one,".left");
                     lidx = getIndexOfDataFrameColumns(measure.row, myleft);
              myright = paste0(one,".right");
                     ridx = getIndexOfDataFrameColumns(measure.row, myleft);
                     print(paste0(
                                                                   "left: ",myleft," --> ",lidx,
                                                                   " right: ",myright," --> ",ridx
                                            );
                     row.m = mean(
                                             c(as.numeric(unlist(measure.row[lidx])),
                                             as.numeric(unlist(measure.row[ridx]))),
                                           na.rm=TRUE);
              measure.instructor[i,nidx] = row.m;
               }
       }
```

Measure

```
## [1] "left: hand.length.left --> 7 right: hand.length.right --> 7"
## [1] "left: hand.width.left --> 9 right: hand.width.right --> 9"
## [1] "left: hand.elbow.left --> 11 right: hand.elbow.right --> 11"
## [1] "left: elbow.armpit.left --> 13 right: elbow.armpit.right --> 13"
## [1] "left: arm.reach.left --> 15 right: arm.reach.right --> 15"
## [1] "left: foot.length.left --> 18 right: foot.length.right --> 18"
## [1] "left: floor.kneepit.left --> 20 right: floor.kneepit.right --> 20"
## [1] "left: floor.hip.left --> 22 right: floor.hip.right --> 22"
## [1] "left: floor.armpit.left --> 25 right: floor.armpit.right --> 25"
## [1] "left: hand.length.left --> 7 right: hand.length.right --> 7"
## [1] "left: hand.width.left --> 9 right: hand.width.right --> 9"
## [1] "left: hand.elbow.left --> 11 right: hand.elbow.right --> 11"
## [1] "left: elbow.armpit.left --> 13 right: elbow.armpit.right --> 13"
## [1] "left: arm.reach.left --> 15 right: arm.reach.right --> 15"
## [1] "left: foot.length.left --> 18 right: foot.length.right --> 18"
## [1] "left: floor.kneepit.left --> 20 right: floor.kneepit.right --> 20"
```

```
## [1] "left: floor.hip.left --> 22 right: floor.hip.right --> 22"
## [1] "left: floor.armpit.left --> 25 right: floor.armpit.right --> 25"
str(measure.instructor); # lot's of columns ...
  'data.frame':
                   2 obs. of 46 variables:
                         : chr "04343803d489fe8ee2c5f6ab97a101e9" "04343803d489fe8ee2c5f6ab97a101e9"
##
   $ data collector
##
   $ person_id
                         : chr
                               "1c2408654ef5a2fe1fc962088312266c" "8a4108d8610658f282267a704288398d"
## $ side
                         : chr "right" "right"
## $ height.NA
                         : int 178 NA
                         : int 21 17
## $ head.height.NA
## $ head.circumference.NA: int 59 51
## $ hand.length.left
                        : num 19.5 NA
## $ hand.length.right
                         : int 19 13
## $ hand.width.left
                         : int 22 NA
## $ hand.width.right
                         : num 21.5 13
## $ hand.elbow.left
                         : int 45 NA
## $ hand.elbow.right
                         : int 46 NA
   $ elbow.armpit.left
                         : int 30 NA
## $ elbow.armpit.right : int 31 NA
## $ arm.reach.left
                         : num 226 NA
## $ arm.reach.right
                         : int
                                226 NA
## $ arm.span.NA
                         : int 176 NA
## $ foot.length.left
                         : num 26.5 NA
## $ foot.length.right
                         : num 26.5 NA
## $ floor.kneepit.left
                         : num 43.5 NA
## $ floor.kneepit.right : int 43 NA
## $ floor.hip.left
                         : int 94 NA
## $ floor.hip.right
                         : num 93.5 NA
## $ floor.navel.NA
                         : num 98.5 NA
## $ floor.armpit.left
                         : int 136 NA
## $ floor.armpit.right : int 134 NA
                         : chr "cm" "cm"
## $ units
## $ writing
                         : chr
                                "right" "both"
## $ eye
                         : chr "right" NA
## $ eye_color
                                "blue" "green"
                         : chr
                                "left" "both"
##
                         : chr
   $ swinging
                         : num 47 4.5
##
   $ age
## $ gender
                        : chr "male" "male"
## $ quality
                        : int 10 7
## $ minutes
                         : int
                                23 NA
## $ ethnicity
                         : chr
                                "white" "white"
                         : chr "possible ancestry may include: scottish, wea native american" "possi
## $ notes
                         : num 19.5 NaN
## $ hand.length
## $ hand.width
                                22 NaN
                         : num
                         : num 45 NaN
## $ hand.elbow
## $ elbow.armpit
                         : num 30 NaN
                         : num 226 NaN
## $ arm.reach
## $ foot.length
                         : num
                                26.5 NaN
## $ floor.kneepit
                         : num 43.5 NaN
  $ floor.hip
                         : num 94 NaN
   $ floor.armpit
                         : num 136 NaN
##
```

You should have a dataset of 10 persons. Once I merge all of the datasets of each student participant, I will provide you a larger dataset.

You should read the "Vitrian Man" reading found in the Dropbox for this week. This would be one set of research activities you could perform for your project 1; that is, review the correlations of various components of the body. It would be a good idea to write some code below to begin studying the correlations.

```
## do some initial exploration of your measure-data
## you should be thinking about "one" good research question for PROJECT 01 ... sub-questions may be ap
## At the top-level of your "WSU_STATS419_FALL2020" repository add a folder "PROJECT-01" if you haven't
## maybe add a section to your 01_notebook in your project folder called # Correlation and perform the
personality.raw = readRDS( system.file("extdata", "personality-raw.rds", package="humanVerseWSU") );
cleanupPersonalityDataFrame = function(df)
 df = removeColumnsFromDataFrame(personality.raw, "V00");
 dim(df); # 838
 ywd.cols = c("year", "week", "day");
 ywd = convertDateStringToFormat( df$date_test,
                                   c("%Y","%W","%j"),
                                   ywd.cols,
                                   "%m/%d/%Y %H:%M"
                                 );
 ndf = replaceDateStringWithDateColumns(df, "date_test", ywd);
 ndf = sortDataFrameByNumericColumns(ndf, ywd.cols, "DESC");
 ndf = removeDuplicatesFromDataFrame(ndf, "md5_email");
 dim(ndf); # 678
 ndf;
 }
personality.clean = cleanupPersonalityDataFrame(personality.raw);
### let's examine the data in total
personality.Vs = removeColumnsFromDataFrame(personality.clean,c("md5_email","year","week","day"));
round( cor(personality.Vs), digits=2);
Personality
##
        V01
              V02
                    V03
                          V04
                                V05
                                      V06
                                            V07
                                                  V08
                                                        V09
                                                              V10
                                                                    V11
                                                                          V12
## V01 1.00 0.35 -0.19 0.07 0.20
                                     0.17 0.19
                                                 0.05
                                                       0.25
                                                             0.35
                                                                         0.46
                                                                   0.10
## V02 0.35 1.00 -0.44 0.12 0.18 0.21 0.13
                                                 0.01
                                                       0.23
                                                             0.44
                                                                   0.10
## V03 -0.19 -0.44 1.00 -0.03 -0.01 0.11 -0.10 -0.01 -0.09 -0.29
                                                                   0.03 - 0.25
## V04 0.07 0.12 -0.03 1.00 0.22 0.11 0.26 0.08
                                                      0.28
                                                            0.22
## V05 0.20 0.18 -0.01 0.22 1.00 0.39 0.18 0.23 0.50 0.27 -0.20
                                                                        0.17
## V06 0.17 0.21 0.11 0.11 0.39 1.00 0.04 0.20 0.34 0.23 -0.14 0.05
```

```
## V07 0.19 0.13 -0.10 0.26 0.18 0.04 1.00 0.13 0.14 0.27 0.15
             0.01 -0.01 0.08 0.23 0.20 0.13 1.00 0.23 0.11 -0.13
## V08
       0.05
                                                                      0.12
## V09
       0.25
             0.23 - 0.09
                        0.28 0.50 0.34
                                         0.14 0.23 1.00
                                                          0.34 - 0.15
                                                                       0.23
## V10
       0.35
             0.44 - 0.29
                        0.22 0.27 0.23
                                         0.27 0.11 0.34
                                                           1.00
                                                                 0.02
                                                                       0.33
       0.10
             0.10 0.03
                        0.07 -0.20 -0.14  0.15 -0.13 -0.15
                                                           0.02
                                                                1.00
                                                                       0.18
       0.46
             0.30 - 0.25
                        0.21 0.17 0.05 0.28 0.12 0.23
                                                          0.33 0.18
## V12
                                                                      1.00
             0.32 - 0.27
       0.24
                        0.16 0.11 0.04 0.31 0.17
                                                     0.14
                                                          0.32 0.28
## V14
       0.45
             0.65 - 0.52
                        0.16 0.25
                                   0.16 0.19 0.10
                                                     0.34
                                                           0.50
                                                                 0.03
                                                                       0.44
## V15
       0.08
             0.08 - 0.03
                        0.36 0.30 0.21
                                         0.22 0.23
                                                     0.33
                                                           0.20
                                                                 0.05
                                                                       0.15
## V16
       0.23
             0.32 - 0.07
                        0.25 0.52
                                    0.42
                                         0.16 0.25
                                                     0.59
                                                           0.39 - 0.21
                                                                       0.16
## V17
       0.22
             0.26 - 0.06
                        0.23
                              0.55
                                    0.55
                                         0.10 0.21
                                                     0.56
                                                          0.32 - 0.12
                                                                       0.16
       0.24
             0.31 - 0.11
                        0.26
                             0.39
                                         0.18 0.17
## V18
                                    0.40
                                                     0.42
                                                           0.38 - 0.06
                                                                       0.23
## V19
       0.25
             0.29 -0.18 0.25 0.15
                                   0.10 0.27
                                               0.11 0.24
                                                          0.36
                                                                 0.19
                                                                       0.35
## V20
       0.02
            0.00 0.15 -0.16 -0.22 -0.16 0.04 -0.12 -0.22 -0.04
                                                                 0.41
                                                                       0.02
             0.19
## V21
       0.19
                                                                 0.52
                                                                       0.35
       0.16
             0.13 - 0.13
                        0.39 0.27
                                   0.13 0.34 0.18
                                                    0.26
                                                           0.30
                                                                 0.08
                                                                       0.30
      0.24
             0.34 -0.11 0.15 0.52 0.51
                                         0.10 0.19 0.43
## V23
                                                          0.41 - 0.13
                                                                       0.11
## V24 -0.10
             0.00 0.19 0.27 0.28
                                   0.21
                                         0.11 0.13 0.23 0.12 -0.10 -0.08
## V25 -0.02
            0.09 0.11 -0.01 -0.11 0.02 0.11 -0.09 -0.15 -0.03 0.41
                                                                      0.00
## V26
       0.37
             0.25 - 0.11
                        0.27 0.39
                                   0.16 0.26 0.15 0.44
                                                          0.34 - 0.04
                                                                       0.32
## V27
       0.25
            0.29 - 0.17
                        0.13 0.23 0.11 0.17 0.05 0.29
                                                          0.40 0.02
                                                                      0.26
       0.10 -0.09 0.11 0.16 0.30 0.17 0.10 0.23 0.38
                                                          0.09 -0.26
            0.16 -0.02 0.02 -0.17 -0.10 0.16 -0.08 -0.11
                                                          0.00 0.55
## V29
       0.04
                                                                       0.17
             0.16 - 0.06
                        0.21 0.44 0.33
                                         0.11 0.22 0.48
## V30
       0.18
                                                           0.25 - 0.24
                                                                       0.14
            0.16 -0.10 0.30 0.11 0.04
## V31
       0.14
                                         0.33 0.10 0.13 0.22 0.22
                                                                       0.32
## V32
       0.14
            0.17 - 0.06
                        0.17 0.15 0.05
                                         0.17
                                               0.03 0.18
                                                          0.22 0.08
                                                                       0.17
## V33
       0.07
             0.10 0.02
                        0.16 0.30 0.22
                                         0.08 0.19
                                                     0.31
                                                           0.17 - 0.10
                                                                       0.04
## V34
       0.27
             0.31 - 0.21
                        0.25
                              0.18 0.07
                                         0.21 0.11
                                                     0.23
                                                           0.37
                                                                 0.13
                                                                       0.38
## V35 -0.04
             0.00 0.15
                        0.20
                             0.10 0.00 0.10 0.08 0.07
                                                           0.02
                                                                 0.15
                                                                       0.11
## V36
       0.14
             0.23 - 0.07
                        0.30
                              0.29 0.15
                                         0.17 0.14
                                                     0.28
                                                          0.27
                                                                 0.09
                                                                       0.16
## V37
       0.22
             0.27 - 0.11
                        0.12
                              0.00 - 0.05
                                         0.24 - 0.02
                                                     0.00
                                                           0.16
                                                                 0.38
                                                                       0.35
## V38
       0.19
             0.26 - 0.20
                        0.29
                              0.18 0.06
                                         0.29
                                              0.09
                                                     0.20
                                                           0.32
                                                                 0.11
                                                                       0.29
## V39
       0.16
             0.16 - 0.03
                        0.24
                              0.29 0.17
                                         0.14 0.13
                                                     0.32
                                                           0.21 - 0.06
                                                                       0.13
       0.22
             0.25 -0.13
                             0.22 0.15
                                         0.19
                                               0.03
                                                     0.25
                                                           0.30
## V40
                        0.26
                                                                 0.10
                                                                       0.24
## V41
       0.19
             0.23 - 0.14
                        0.22
                              0.03 - 0.03
                                         0.21
                                               0.00
                                                     0.07
                                                           0.20
                                                                 0.25
                                                                       0.28
                                                     0.20
       0.25
             0.24 - 0.20
                        0.26 0.15 0.06 0.20 0.07
                                                           0.30
## V42
                                                                0.11
                                                                      0.33
## V43
       0.15
            0.21 - 0.07
                        0.24 0.24 0.15
                                         0.21 0.10 0.25
                                                           0.27
                                                                 0.11
## V44
       0.24
            0.23 -0.16 0.32 0.17
                                    0.12
                                         0.23 0.15
                                                     0.27
                                                           0.35
                                                                 0.10
                                                                       0.28
             0.19 - 0.12
                        0.21
                              0.34 0.26
                                         0.15
                                               0.20
                                                     0.35
                                                           0.28
                                                                 0.03
       0.11
                                                                       0.07
## V46
       0.09 0.12 -0.01
                        0.12 0.31 0.28
                                         0.07 0.21 0.32 0.16 -0.07
                                                                       0.06
       0.11 0.15 0.00
## V47
                        0.17
                              0.42 0.33
                                         0.11 0.19
                                                     0.32
                                                          0.23 - 0.05
                                                                       0.00
## V48
       0.06 -0.04 0.10 0.09 0.18 0.10 0.04 0.15 0.26
                                                          0.08 - 0.15
                                                                       0.03
## V49
       0.26
            0.30 - 0.18
                        0.21 0.19 0.04
                                         0.23 0.16
                                                     0.17
                                                           0.25
                                                                 0.22
                                                                       0.33
            0.31 -0.21
## V50
       0.24
                        0.03 0.11 -0.01
                                         0.15 0.05 0.16
                                                           0.26
                                                                0.12
                                                                       0.19
## V51
       0.42
            0.29 - 0.21
                        0.18 0.18 0.08
                                         0.20 0.12
                                                     0.24
                                                           0.33
                                                                 0.09
                                                                       0.43
## V52
       0.12
             0.15 - 0.11
                        0.36
                              0.22
                                    0.15
                                         0.23
                                               0.15
                                                     0.23
                                                           0.25
                                                                 0.10
                                                                       0.19
## V53
       0.15
             0.23 - 0.02
                        0.17
                              0.38
                                    0.33
                                         0.07
                                               0.16
                                                     0.43
                                                           0.23 - 0.10
                                                                       0.08
                        0.22 0.35
                                    0.22
## V54
       0.25
             0.31 - 0.14
                                         0.11 0.14
                                                     0.39
                                                           0.34 - 0.02
                                                                       0.16
## V55
       0.03
             0.08 0.05
                        0.08 0.16
                                    0.18
                                         0.06 0.08
                                                     0.15
                                                          0.09 0.03
                                                                       0.01
## V56
       0.22
             0.18 - 0.09
                        0.17
                              0.26
                                    0.33
                                         0.07
                                               0.15
                                                     0.34
                                                           0.23 - 0.06
                                                                       0.12
## V57
             0.29 - 0.16
                        0.20 0.22
                                    0.15
                                         0.19
                                               0.12
                                                    0.26
                                                           0.34 0.11
       0.28
                                                                      0.30
## V58
       0.19 0.26 -0.15 0.23 0.30 0.26
                                         0.12 0.17
                                                    0.33 0.31 -0.06
                                                                      0.17
## V59
       0.21 \quad 0.23 \quad -0.18 \quad 0.25 \quad 0.11 \quad -0.01 \quad 0.30 \quad 0.05 \quad 0.14 \quad 0.28 \quad 0.23
                                                                      0.32
## V60 0.09 0.11 0.07 0.18 0.33 0.36 0.03 0.17 0.34 0.16 -0.10 0.05
```

```
V13
             V14
                   V15
                        V16
                              V17
                                    V18
                                         V19
                                               V20
                                                    V21
                                                          V22
            0.45
                 0.08 0.23 0.22 0.24 0.25 0.02 0.19 0.16 0.24 -0.10
## V01 0.24
## V02 0.32 0.65
                 0.08 0.32 0.26 0.31 0.29 0.00 0.28 0.13 0.34
## V03 -0.27 -0.52 -0.03 -0.07 -0.06 -0.11 -0.18 0.15 -0.18 -0.13 -0.11
                                                                    0.19
       0.16 0.16
                 0.36
                       0.25 0.23 0.26 0.25 -0.16 0.14
                                                        0.39
                                                               0.15
## V05
       0.11 0.25
                 0.30 0.52 0.55 0.39 0.15 -0.22 -0.09 0.27
                                                               0.52
                                                                    0.28
                       0.42 0.55 0.40 0.10 -0.16 -0.09 0.13
       0.04 0.16 0.21
                                                               0.51
                       0.16  0.10  0.18  0.27  0.04  0.23  0.34
## V07
       0.31
            0.19
                  0.22
                                                               0.10
                                                                    0.11
## V08
       0.17
            0.10
                  0.23
                       0.25
                             0.21 0.17
                                        0.11 -0.12 0.00
                                                        0.18
                                                               0.19
                                                                    0.13
## V09
       0.14
            0.34
                 0.33
                       0.59 0.56 0.42
                                        0.24 -0.22 -0.05
                                                        0.26
                                                              0.43 0.23
## V10
       0.32
            0.50
                  0.20 0.39 0.32 0.38
                                        0.36 -0.04 0.19
                                                         0.30
                                                              0.41 0.12
## V11
       0.28
            0.03
                 0.05 -0.21 -0.12 -0.06
                                        0.19 0.41 0.52
                                                        0.08 -0.13 -0.10
## V12
       0.40
            0.44 0.15 0.16 0.16 0.23
                                        0.35 0.02 0.35
                                                        0.30
                                                              0.11 -0.08
                                        0.59 0.05 0.41
## V13
       1.00 0.36 0.20 0.16 0.11
                                  0.18
                                                         0.30
                                                              0.11 0.00
       0.36
            1.00 0.16 0.36 0.29
                                   0.33
                                        0.34 - 0.07
                                                   0.27
                                                         0.22
                                                               0.36 -0.03
## V14
## V15
       0.20
            0.16
                  1.00
                       0.31
                             0.34
                                  0.25
                                        0.26 -0.10 0.13
                                                         0.38
                                                               0.26
                                                                    0.25
       0.16 0.36
                 0.31 1.00 0.58
                                  0.44
                                        0.27 -0.20 -0.12
## V16
                                                         0.21
                                                               0.49
                                                                    0.28
## V17
       0.11
            0.29
                 0.34
                       0.58 1.00
                                  0.50
                                        0.18 -0.22 -0.06
                                                         0.20
                                                               0.59
                                                                    0.19
            0.33 0.25 0.44 0.50 1.00 0.25 -0.23 0.01
## V18
       0.18
                                                        0.23
                                                               0.47
                                                                    0.21
## V19
       0.59
            0.34 0.26 0.27 0.18 0.25 1.00 -0.03 0.31
                                                         0.29
                                                               0.19
                                                                    0.07
## V20
       0.05 -0.07 -0.10 -0.20 -0.22 -0.23 -0.03 1.00 0.25 -0.09 -0.20 -0.11
       0.41 0.27 0.13 -0.12 -0.06 0.01 0.31 0.25 1.00 0.15 -0.02 -0.09
       0.30
            0.22  0.38  0.21  0.20  0.23  0.29  -0.09  0.15  1.00  0.20  0.21
## V22
                 0.26
                       0.49 0.59 0.47 0.19 -0.20 -0.02 0.20
## V23
       0.11
            0.36
                                                               1.00
## V24
       0.00 -0.03 0.25 0.28 0.19 0.21 0.07 -0.11 -0.09 0.21
                                                              0.25
                                                                    1.00
## V25
       0.11
            0.00 -0.02 -0.10 -0.07 -0.03 0.05 0.32 0.26 0.06 -0.05
                                                                    0.07
## V26
       0.22
            0.34 0.26 0.43 0.28 0.27 0.27 -0.13 0.08 0.33
                                                               0.26
                                                                    0.15
## V27
       0.25
            0.37
                 0.11
                       0.36 0.29 0.22 0.29 0.01 0.13
                                                        0.13
                                                               0.25
                                                                    0.11
## V28
       0.02 0.03 0.20 0.36 0.25 0.18 0.10 -0.21 -0.23
                                                        0.16 0.13
                                                                    0.24
## V29
       0.31
            0.08 0.02 -0.14 -0.11 -0.07 0.20 0.40 0.45 0.07 -0.09 -0.04
## V30
       0.07
            0.25
                  0.31
                       0.48 0.40 0.31
                                        0.13 -0.19 -0.11
                                                         0.21
                                                               0.36
                                                                    0.25
## V31
       0.37
            0.20 0.21
                       0.16 0.09
                                  0.13 0.33 0.05 0.29
                                                         0.29
                                                               0.07
                                                                    0.13
## V32
       0.23
            0.20
                 0.18
                       0.22 0.15
                                  0.13 0.27 0.03 0.12
                                                        0.19
                                                               0.15
                                                                    0.10
       0.00
            0.18 0.25 0.34 0.32
                                  0.22 0.14 -0.05 -0.05
## V33
                                                         0.13
                                                               0.24
                                                                    0.19
## V34
       0.39
            0.38
                 0.21
                       0.21 0.17
                                  0.23
                                        0.40 -0.01 0.28
                                                         0.28
                                                               0.20
                                                                    0.03
## V35
       0.12 -0.02 0.18 0.08 0.01 0.05 0.16 0.06 0.12 0.16
                                                              0.00
                                                                    0.26
## V36
       0.23 0.25
                 0.29 0.31 0.30 0.32 0.27 -0.03 0.11
                                                        0.29
                                                               0.29
                                                                    0.23
## V37
       0.40
            0.24
                 0.04 -0.01 -0.03 0.03 0.29 0.15 0.44
                                                        0.08 -0.02 -0.05
       0.31
            0.33
                  0.24
                       0.26
                             0.17
                                  0.22
                                        0.35 -0.03
                                                   0.25
                                                         0.31
                                                               0.21
                                                                    0.11
## V38
## V39
            0.21
                 0.22 0.35 0.29 0.27
                                        0.20 -0.09 0.00 0.26
                                                               0.26
       0.11
                                                                    0.17
                       0.27
                             0.20
                                  0.22 0.34 -0.02 0.20
## V40
       0.30
            0.31
                 0.27
                                                        0.25
                                                               0.22
                                                                    0.13
## V41
       0.34
            0.23
                 0.11 0.08 0.04
                                  0.07
                                        0.28 0.11 0.41
                                                        0.13
                                                              0.03
                                                                    0.04
## V42
       0.27
            0.32
                 0.24
                       0.19
                            0.20
                                   0.25
                                        0.30 -0.02 0.26
                                                         0.30
                                                               0.18
                                                                    0.09
## V43
       0.26
            0.25
                  0.24
                       0.30 0.23
                                  0.23
                                        0.28 0.00 0.14
                                                         0.25
                                                               0.22
                                                                    0.11
## V44
       0.26
            0.30
                  0.23
                       0.29 0.22
                                   0.20 0.36 -0.02 0.23
                                                         0.32
                                                               0.20
                                                                    0.13
                                   0.23
## V45
       0.15
            0.23
                  0.33
                       0.41 0.37
                                        0.25 -0.06 0.04
                                                         0.28
                                                               0.35
                                                                    0.17
## V46
       0.05
            0.19
                  0.26
                       0.34 0.34
                                  0.23
                                        0.12 -0.09 -0.05
                                                         0.21
                                                               0.30
                                                                    0.23
                                  0.32 0.16 -0.15 -0.07
## V47
       0.09
            0.19
                 0.25
                       0.37 0.42
                                                        0.21
                                                               0.43
                                                                    0.21
## V48
       0.00
            0.06
                 0.12 0.25 0.16 0.10 0.04 -0.10 -0.11
                                                        0.05
                                                               0.12
                                                                    0.19
                                        0.44 0.02 0.35
## V49
       0.51
            0.35
                  0.22
                       0.20 0.14
                                  0.17
                                                         0.31
                                                               0.12
                                                                    0.03
## V50
            0.36
                 0.08
                       0.23 0.13
                                  0.15
                                        0.30 0.13 0.26
       0.31
                                                         0.09
                                                               0.13
                                                                    0.00
## V51
       0.32 0.39 0.10 0.22 0.20 0.22 0.28 -0.01 0.24 0.24 0.18
                                                                    0.01
## V52
       0.23 0.17 0.38 0.26 0.22 0.18 0.30 -0.03 0.15 0.33
                                                              0.21
                                                                    0.18
## V53 0.06 0.26 0.28 0.49 0.47 0.32 0.17 -0.12 -0.05 0.20 0.41 0.18
```

```
## V54 0.14 0.38 0.30 0.45 0.40 0.29 0.19 -0.07 0.03 0.19
                 0.21
                       0.21 0.20 0.15 0.13 0.02 0.01 0.14
## V55
       0.07
            0.10
                                                              0.19
                                                                    0.23
## V56
       0.10
            0.25
                  0.29
                       0.39 0.38 0.40 0.18 -0.12 0.02 0.19
                                                               0.39
## V57
       0.34
            0.39
                  0.24
                       0.31 0.24
                                  0.27 0.34 0.00 0.23
                                                         0.24
                                                              0.23
                                                                    0.12
## V58
       0.15
            0.35
                  0.28
                       0.35
                             0.33
                                  0.35 0.21 -0.15
                                                   0.02
                                                         0.26
                                                               0.35
                                                                    0.19
            0.29
                       0.14 0.07
                                  0.12 0.35 0.05 0.38 0.27
## V59
      0.42
                  0.19
                                                               0.07
                                                                    0.06
## V60 -0.03
            0.16
                  0.27
                        0.40
                             0.40
                                   0.32 0.10 -0.14 -0.12
                                                        0.20
                                                               0.33
                              V29
##
        V25
             V26
                   V27
                        V28
                                    V30
                                         V31
                                              V32
                                                    V33
                                                          V34
                                                                V35
                                                                     V36
## V01 -0.02
            0.37
                  0.25
                        0.10 0.04 0.18 0.14 0.14
                                                   0.07
                                                        0.27 - 0.04
                                                                    0.14
## V02 0.09 0.25
                 0.29 -0.09 0.16 0.16 0.16 0.17 0.10 0.31
                                                               0.00
                                                                    0.23
## V03 0.11 -0.11 -0.17 0.11 -0.02 -0.06 -0.10 -0.06 0.02 -0.21
                                                               0.15 - 0.07
## V04 -0.01 0.27
                  0.13
                       0.16 0.02 0.21 0.30 0.17
                                                        0.25
                                                   0.16
                                                               0.20
                                                                    0.30
## V05 -0.11
            0.39
                 0.23
                       0.30 - 0.17
                                  0.44 0.11 0.15 0.30
                                                        0.18
                                                               0.10
                                                                    0.29
## V06 0.02 0.16 0.11
                                                        0.07
                       0.17 -0.10 0.33 0.04 0.05 0.22
                                                               0.00
                                                                    0.15
## V07 0.11
            0.26
                  0.17
                       0.10 0.16
                                  0.11
                                        0.33 0.17
                                                   0.08
                                                         0.21
                                                               0.10
                                                                    0.17
## V08 -0.09
            0.15
                  0.05
                       0.23 - 0.08
                                  0.22
                                        0.10
                                             0.03
                                                   0.19
                                                         0.11
                                                               0.08
                                                                    0.14
## V09 -0.15
            0.44
                 0.29 0.38 -0.11 0.48
                                        0.13 0.18 0.31
                                                         0.23
                                                               0.07
                                                                    0.28
## V10 -0.03 0.34
                  0.40 0.09 0.00 0.25
                                        0.22 0.22 0.17
                                                         0.37
                                                               0.02
                                                                    0.27
## V11 0.41 -0.04
                 0.02 -0.26  0.55 -0.24  0.22  0.08 -0.10
                                                        0.13
                                                              0.15
                                                                    0.09
## V12
      0.00 0.32
                 0.26 0.10 0.17 0.14 0.32 0.17 0.04
                                                        0.38
                                                               0.11
                                                                    0.16
## V13 0.11 0.22 0.25 0.02 0.31 0.07 0.37 0.23 0.00 0.39
                                                              0.12
                                                                    0.23
## V14 0.00
            0.34
                 0.37 0.03 0.08 0.25
                                       0.20 0.20 0.18
                                                        0.38 -0.02
## V15 -0.02
            0.26
                  0.11 0.20 0.02 0.31
                                        0.21 0.18 0.25
                                                              0.18
                                                        0.21
                                                                    0.29
                  0.36
                       0.36 -0.14 0.48
## V16 -0.10
            0.43
                                        0.16
                                             0.22 0.34
                                                         0.21
                                                               0.08
                                                                    0.31
## V17 -0.07
                 0.29 0.25 -0.11 0.40 0.09 0.15 0.32 0.17
            0.28
                                                               0.01
                                                                    0.30
## V18 -0.03
            0.27
                  0.22 0.18 -0.07 0.31
                                        0.13 0.13 0.22 0.23
                                                               0.05
                                                                    0.32
## V19 0.05 0.27
                  0.29 0.10 0.20 0.13
                                        0.33 0.27 0.14
                                                         0.40
                                                               0.16
                                                                    0.27
      0.32 - 0.13
                  0.01 -0.21 0.40 -0.19
                                        0.05 0.03 -0.05 -0.01
                                                               0.06 - 0.03
## V21 0.26 0.08 0.13 -0.23 0.45 -0.11
                                        0.29 0.12 -0.05
                                                        0.28
                                                              0.12 0.11
## V22 0.06 0.33 0.13 0.16 0.07 0.21
                                        0.29 0.19 0.13
                                                        0.28
                                                               0.16
                                                                    0.29
## V23 -0.05
            0.26
                  0.25
                       0.13 -0.09 0.36
                                        0.07 0.15 0.24
                                                         0.20
                                                               0.00
                                                                    0.29
## V24
       0.07
            0.15
                  0.11 0.24 -0.04 0.25
                                        0.13 0.10 0.19
                                                         0.03
                                                               0.26
                                                                    0.23
      1.00 -0.19
                  0.04 -0.20 0.37 -0.17
                                        0.06 0.08 -0.14
                                                         0.03
                                                               0.10
                                                                    0.04
            1.00
                 0.25 0.33 -0.10 0.38
                                        0.22 0.18 0.25
## V26 -0.19
                                                         0.31
                                                               0.10
                                                                    0.26
## V27
       0.04
            0.25
                  1.00
                       0.10 0.03 0.19
                                        0.15
                                             0.23
                                                   0.07
                                                         0.23
                                                               0.07
                                                                    0.13
## V28 -0.20 0.33 0.10 1.00 -0.24 0.31 0.16 0.16 0.24
                                                        0.05
                                                              0.14
                                                                    0.09
## V29 0.37 -0.10 0.03 -0.24 1.00 -0.19 0.24 0.12 -0.07
                                                        0.16
                                                              0.15
## V30 -0.17
            0.38 0.19 0.31 -0.19 1.00 0.12 0.13 0.28
                                                        0.14
                                                               0.07
                                                                    0.24
      0.06
            0.22
                  0.15
                       0.16 0.24 0.12
                                        1.00
                                             0.27
                                                   0.14
                                                         0.37
                                                               0.35
                                                                    0.38
## V32 0.08
            0.18
                 0.23 0.16 0.12 0.13
                                       0.27
                                             1.00 0.21
                                                         0.40
                                                               0.22
                                                                    0.26
                  0.07 0.24 -0.07 0.28
## V33 -0.14
            0.25
                                        0.14 0.21
                                                   1.00
                                                        0.22
                                                               0.14
                                                                    0.39
## V34
       0.03
            0.31
                 0.23 0.05 0.16 0.14
                                        0.37 0.40 0.22
                                                        1.00
                                                               0.16
                                                                    0.41
## V35
       0.10
            0.10
                  0.07
                       0.14 0.15 0.07
                                        0.35 0.22 0.14
                                                         0.16
                                                              1.00
                                                                    0.23
## V36
      0.04
            0.26
                 0.13 0.09 0.12 0.24
                                        0.38 0.26 0.39
                                                              0.23
                                                        0.41
                                                                    1.00
       0.15
            0.17
                 0.13 -0.05 0.40 -0.06
                                        0.41 0.14 -0.08
                                                        0.28
                                                               0.21
                                                                    0.10
## V38 0.04
            0.31
                  0.19
                       0.16 0.13 0.18
                                        0.41
                                              0.29
                                                   0.30
                                                              0.19
                                                         0.44
                                                                    0.45
## V39 -0.06
            0.36
                  0.10
                       0.22 -0.07 0.28
                                        0.19
                                              0.20
                                                   0.45
                                                         0.30
                                                               0.13
                                                                    0.45
## V40
      0.00
            0.27
                  0.26 0.14 0.08 0.24
                                        0.38 0.41
                                                   0.24
                                                         0.49
                                                               0.19
                                                                    0.41
## V41
      0.10
            0.16 0.20 0.05 0.28 -0.01
                                        0.45 0.17
                                                   0.00
                                                        0.34
                                                              0.22
                                                                    0.17
## V42 -0.04
            0.28
                  0.23
                       0.13 0.14 0.16
                                        0.39
                                              0.32
                                                   0.23
                                                         0.46
                                                               0.18
                                                                    0.38
## V43 0.06
            0.22
                 0.22 0.14 0.08 0.22
                                        0.33 0.41
                                                   0.23
                                                         0.42
                                                              0.19
                                                                    0.34
## V44 -0.01 0.29 0.15 0.13 0.11 0.18 0.43 0.30 0.22 0.41
                                                              0.18
                                                                    0.43
## V45 0.04 0.19 0.16 0.16 0.03 0.29 0.26 0.26 0.44 0.32 0.10
                                                                    0.42
## V46 -0.01 0.14 0.10 0.19 -0.07 0.22 0.21 0.23 0.36 0.22 0.18 0.34
```

```
## V47 -0.04 0.19 0.12 0.15 -0.01 0.27 0.12 0.28 0.47 0.24
## V48 -0.07
            0.22
                 0.09 0.45 -0.18 0.22 0.03 0.21
                                                   0.31
                                                         0.14
                                                               0.05
                                                                     0.18
## V49 0.08
            0.30
                  0.17 0.11 0.24 0.13 0.48 0.39
                                                   0.20
                                                         0.53
                                                               0.26
                                                                     0.38
## V50 0.11
            0.18
                  0.38
                       0.06 0.15 0.10 0.23 0.34
                                                    0.13
                                                         0.38
                                                               0.10
                                                                     0.20
## V51 -0.03
             0.28
                  0.27
                        0.06 0.10
                                   0.13
                                        0.27
                                              0.21
                                                    0.14
                                                          0.45
                                                               0.07
                                                                     0.28
## V52 0.04
                  0.13 0.11 0.13 0.18 0.44 0.23
                                                   0.29
            0.27
                                                         0.39
                                                               0.20
                                                                     0.39
                        0.19 - 0.04
## V53 -0.05
            0.23
                  0.17
                                  0.36
                                        0.16 0.28
                                                   0.51
                                                         0.26
                                                               0.11
## V54 -0.05
            0.30
                  0.19
                        0.20 - 0.01
                                   0.34
                                        0.23
                                              0.29
                                                    0.42
                                                         0.36 - 0.02
                                                                     0.46
## V55
      0.12
            0.10
                  0.05
                        0.06 0.02
                                   0.18
                                        0.11
                                              0.15
                                                    0.24
                                                         0.12
                                                               0.26
                                                                     0.24
                                   0.24
## V56 -0.05
            0.15
                  0.14
                       0.14 - 0.04
                                        0.18 0.21
                                                   0.32
                                                         0.24
                                                               0.10
                                                                     0.33
## V57 0.07
            0.29
                  0.37
                        0.08 0.13
                                   0.22
                                        0.31
                                              0.39
                                                    0.29
                                                         0.54
                                                               0.13
                                                                     0.39
## V58 -0.07
             0.21
                       0.13 - 0.06
                                   0.29
                                        0.21
                                              0.21
                  0.15
                                                   0.42
                                                         0.31
                                                               0.13
                                                                     0.43
## V59 0.07
             0.28
                  0.18
                        0.06 0.28
                                   0.10
                                        0.51
                                              0.27
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                                                         0.42
                                                               0.26
                                                                     0.36
## V60 -0.02
                                   0.32
                                                    0.42
                                                         0.15
             0.17
                  0.15
                        0.26 - 0.12
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                                              0.22
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##
        V37
             V38
                   V39
                         V40
                              V41
                                    V42
                                          V43
                                               V44
                                                     V45
                                                           V46
                                                                V47
                                                                      V48
## V01 0.22
             0.19
                  0.16
                        0.22
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                                   0.25
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                                                    0.11
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                  0.16
                       0.25 0.23 0.24
                                        0.21
                                             0.23 0.19
## V02 0.27
            0.26
                                                         0.12
                                                               0.15 - 0.04
## V03 -0.11 -0.20 -0.03 -0.13 -0.14 -0.20 -0.07 -0.16 -0.12 -0.01
                                                               0.00
## V04 0.12 0.29
                 0.24 0.26 0.22 0.26 0.24 0.32 0.21
                                                         0.12
                                                               0.17
                                                                     0.09
## V05
      0.00
            0.18
                 0.29
                       0.22 0.03 0.15
                                        0.24 0.17
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                                                               0.42
                                                                     0.18
                       0.15 -0.03 0.06 0.15 0.12 0.26
## V06 -0.05
            0.06 0.17
                                                         0.28
                                                               0.33
                                                                     0.10
## V07 0.24
            0.29
                 0.14
                       0.19 0.21
                                  0.20 0.21 0.23
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                                                               0.11
                 0.13
## V08 -0.02
            0.09
                        0.03 0.00
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                        0.25
                             0.07
                                   0.20
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                                              0.27
## V09
       0.00
             0.20
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                                                    0.35
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                                                               0.32
                                                                     0.26
## V10
                       0.30 0.20 0.30
      0.16
            0.32 0.21
                                        0.27
                                             0.35
                                                   0.28 0.16
                                                               0.23
                                                                     0.08
## V11
       0.38
            0.11 - 0.06
                       0.10
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                                   0.11
                                        0.11 0.10
                                                    0.03 -0.07 -0.05 -0.15
## V12
       0.35
            0.29
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## V13
       0.40
             0.31
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                                   0.32
                                        0.25 0.30
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## V14
                                                         0.19
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                                                                     0.06
## V15 0.04
            0.24
                  0.22
                        0.27
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                                   0.24
                                        0.24
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                                                                     0.12
## V16 -0.01
             0.26
                  0.35
                        0.27
                             0.08
                                   0.19
                                         0.30
                                              0.29
                                                    0.41
                                                          0.34
                                                               0.37
                                                                     0.25
## V17 -0.03
            0.17
                  0.29
                        0.20
                             0.04
                                  0.20
                                        0.23
                                              0.22
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                                                         0.34
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                                                                     0.16
## V18
       0.03
            0.22
                  0.27
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       0.29
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                             0.28 0.30
                                        0.28 0.36
                                                   0.25
## V19
                                                         0.12
                                                               0.16
                                                                     0.04
## V20
       0.15 -0.03 -0.09 -0.02
                             0.11 - 0.02
                                        0.00 -0.02 -0.06 -0.09 -0.15 -0.10
       0.44
            0.25 0.00 0.20 0.41 0.26
                                        0.14 0.23 0.04 -0.05 -0.07 -0.11
## V21
       0.08 0.31 0.26
                       0.25 0.13 0.30
                                        0.25 0.32 0.28 0.21 0.21 0.05
## V23 -0.02
            0.21 0.26
                       0.22 0.03 0.18
                                        0.22 0.20 0.35 0.30
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                                                                     0.12
## V24 -0.05
            0.11
                 0.17
                        0.13
                             0.04 0.09
                                        0.11 0.13
                                                   0.17 0.23
                                                               0.21
                                                                     0.19
## V25
      0.15
            0.04 - 0.06
                       0.00 0.10 -0.04
                                        0.06 -0.01 0.04 -0.01 -0.04 -0.07
                       0.27 0.16 0.28
                                        0.22 0.29
       0.17
             0.31 0.36
                                                   0.19 0.14 0.19
## V27
       0.13
            0.19
                  0.10
                       0.26 0.20 0.23
                                        0.22 0.15 0.16 0.10 0.12
                                                                    0.09
## V28 -0.05
            0.16 0.22
                       0.14 0.05
                                   0.13
                                        0.14 0.13 0.16
                                                         0.19
                                                               0.15
                                                                     0.45
## V29 0.40
            0.13 - 0.07
                        0.08 0.28
                                  0.14 0.08 0.11 0.03 -0.07 -0.01 -0.18
## V30 -0.06
            0.18
                 0.28
                        0.24 - 0.01
                                   0.16
                                        0.22 0.18 0.29
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                                                               0.27
## V31
      0.41
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                  0.19
                        0.38
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                                        0.33
                                              0.43
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## V32
       0.14
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## V33 -0.08
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            0.30
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## V34
       0.28
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## V35
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                                              0.18
                                                   0.10
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                                                               0.12
                                                                     0.05
## V36
       0.10
            0.45
                  0.45
                       0.41
                             0.17
                                   0.38
                                        0.34 0.43
                                                   0.42 0.34
                                                               0.42
                                                                     0.18
## V37
       1.00 0.31 -0.05 0.22 0.47
                                   0.31
                                        0.19 0.22 -0.03 -0.03 -0.10 -0.10
## V39 -0.05 0.32 1.00 0.32 0.07 0.27 0.27 0.28 0.38 0.28 0.44 0.29
```

```
## V40 0.22 0.46 0.32 1.00 0.28 0.46 0.47 0.41 0.29 0.29 0.32 0.19
                  0.07
                        0.28 1.00 0.33 0.23 0.29
                                                    0.10 0.03 -0.08 -0.02
## V41
       0.47
             0.26
       0.31
             0.43
                  0.27
                        0.46 0.33
                                   1.00
                                         0.42 0.38
                                                    0.31
                                                          0.27
                                                                0.24 0.12
## V43
       0.19
             0.39
                  0.27
                        0.47
                             0.23
                                   0.42
                                         1.00 0.37
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                                                          0.36
                                                                0.31
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       0.22
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                        0.41 0.29
                                   0.38
                                         0.37
                                               1.00
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                                                          0.31
                                                                0.22
                                                                      0.02
## V45 -0.03
             0.34
                  0.38
                        0.29 0.10
                                   0.31
                                         0.37 0.32 1.00
                                                          0.46
                                                                0.48
                                                                      0.21
                                         0.36 0.31
## V46 -0.03
             0.25
                  0.28
                        0.29 0.03
                                   0.27
                                                    0.46
                                                          1.00
                                                                0.44
## V47 -0.10
             0.23
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                        0.32 - 0.08
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                                         0.31
                                              0.22
                                                    0.48
                                                          0.44
                                                                1.00
                                                                      0.26
## V48 -0.10
             0.14
                  0.29
                        0.19 - 0.02
                                   0.12
                                         0.21
                                               0.02
                                                     0.21
                                                          0.21
                                                                0.26
                                                                      1.00
## V49
      0.41
             0.44
                  0.29
                        0.46 0.35
                                   0.39
                                         0.42
                                              0.38
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                                                          0.21
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## V50
       0.30
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                  0.13
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                              0.34
                                    0.34
                                         0.39
                                               0.28
                                                     0.25
                                                          0.16
                                                                0.19
                                                                      0.09
## V51
       0.27
             0.35
                  0.22
                        0.34
                                         0.32
                                               0.43
                             0.34
                                    0.47
                                                     0.21
                                                          0.16
                                                                0.18
                                                                      0.14
## V52
       0.20
             0.40
                  0.27
                        0.38
                              0.24
                                   0.42
                                         0.34
                                              0.36
                                                    0.40
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                                                    0.55
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                                                                0.59
## V53 -0.12
                  0.46
                        0.34 - 0.01
                                         0.35
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## V54 0.03
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                                         0.37
                                               0.37
                                                     0.50
                                                          0.41
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                                                                      0.29
## V55 -0.01
             0.17
                  0.25
                        0.22
                              0.03
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                                                          0.33
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## V56 -0.01
                  0.32
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                                         0.31
                                               0.34
             0.24
                                                     0.41
                                                          0.43
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## V57 0.20
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                  0.32
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                              0.26
                                   0.41
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                                               0.42
                                                     0.38
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                                                                0.35
                                                                      0.23
## V58 -0.02
                                         0.33
                                              0.35
             0.34
                  0.36
                        0.31
                             0.08
                                   0.32
                                                     0.45
                                                          0.44
                                                                0.43
                                                                      0.22
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## V59 0.48
             0.50
                  0.15
                        0.40
                              0.44
                                   0.42
                                         0.35
                                               0.38
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                                                                0.11
                                                                      0.05
## V60 -0.15
             0.16
                  0.37
                        0.25 -0.03
                                   0.18
                                         0.29
                                               0.21
                                                     0.47
                                                          0.49
                                                                0.52
                                                                      0.35
        V49
              V50
                   V51
                         V52
                               V53
                                     V54
                                          V55
                                                V56
                                                      V57
                                                            V58
                                                                 V59
                                                                       V60
                        0.12
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                                         0.03 0.22
## V01 0.26
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                  0.29
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                                   0.31
                                         0.08 0.18
## V02 0.30
             0.31
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                                                                0.23
                                                                      0.11
## V03 -0.18 -0.21 -0.21 -0.11 -0.02 -0.14
                                         0.05 -0.09 -0.16 -0.15 -0.18
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       0.21
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## V05
       0.19
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## V06
       0.04 - 0.01
                  0.08
                        0.15
                              0.33 0.22
                                         0.18 0.33 0.15
                                                          0.26 - 0.01
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## V07
       0.23 0.15
                  0.20
                       0.23 0.07
                                   0.11
                                         0.06 0.07 0.19 0.12
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                                                                      0.03
## V08
       0.16
            0.05
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                        0.15
                              0.16
                                   0.14
                                         0.08 0.15
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## V09
       0.17
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## V10
       0.25
             0.26
                  0.33
                        0.25 0.23 0.34
                                         0.09 0.23
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                                                                0.28
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## V11
       0.22
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                        0.10 -0.10 -0.02
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       0.33
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                        0.19 0.08 0.16
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## V12
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                                                                0.32
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## V13
       0.51
             0.31
                  0.32
                        0.23 0.06
                                   0.14
                                         0.07 0.10
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                                                          0.15
                                                                0.42 - 0.03
                                                    0.39
## V14
       0.35
             0.36
                  0.39
                        0.17
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                                   0.38
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## V15
       0.22
            0.08
                  0.10
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                                   0.30
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## V16
       0.20
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## V18
       0.17
             0.15
                  0.22 0.18 0.32 0.29
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                        0.30 0.17 0.19
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## V19
       0.44
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                  0.28
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                                                                0.35 0.10
## V20
       0.02
            0.05 - 0.14
## V21
       0.35
             0.26
                  0.24
                        0.15 -0.05 0.03 0.01 0.02 0.23
                                                          0.02
                                                                0.38 - 0.12
## V22
       0.31
            0.09 0.24
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## V23
       0.12
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## V24
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             0.00 0.01
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## V25
       0.08
             0.11 - 0.03
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                                                     0.07 - 0.07
                                                                0.07 - 0.02
                        0.27 0.23 0.30
                                                    0.29 0.21
## V26
       0.30
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## V27
       0.17
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                        0.13 0.17
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                                         0.05 0.14
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## V28
       0.11
             0.06
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                        0.11 0.19 0.20
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                  0.10
                        0.13 -0.04 -0.01
                                         0.02 -0.04 0.13 -0.06
## V29
       0.24
                                                                0.28 - 0.12
## V30
       0.13 0.10 0.13 0.18 0.36 0.34 0.18 0.24 0.22 0.29
                                                                0.10 0.32
## V31
       0.48 0.23 0.27 0.44 0.16 0.23 0.11 0.18 0.31 0.21
                                                                0.51 0.13
## V32 0.39 0.34 0.21 0.23 0.28 0.29 0.15 0.21 0.39 0.21 0.27 0.22
```

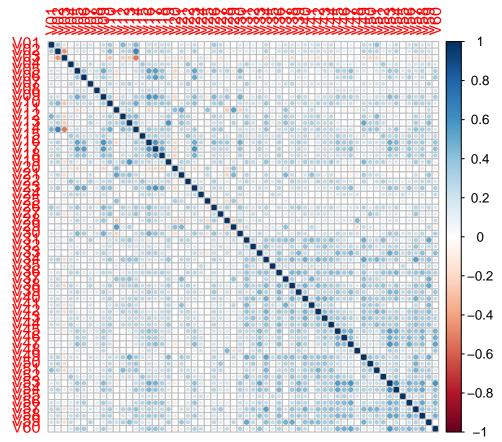
```
0.20
               0.13
                    0.14
                           0.29 0.51 0.42 0.24 0.32
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## V34
               0.38
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        0.26
## V35
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                                                                                0.15
## V36
        0.38
               0.20
                     0.28
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                                  0.41
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                                                                  0.43
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##
  V37
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  V38
        0.44
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##
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  V39
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                                                      0.32
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                                               0.22
## V40
        0.46
               0.37
                     0.34
                            0.38
                                  0.34
                                        0.41
                                                      0.28
                                                            0.48
                                                                   0.31
                                                                         0.40
                                                                                0.25
## V41
        0.35
               0.34
                     0.34
                            0.24 - 0.01
                                         0.10
                                               0.03
                                                      0.05
                                                            0.26
                                                                   0.08
                                                                         0.44 - 0.03
                                         0.30
## V42
        0.39
               0.34
                     0.47
                            0.42
                                  0.27
                                               0.14
                                                      0.33
                                                            0.41
                                                                   0.32
                                                                         0.42
                                                                                0.18
## V43
        0.42
               0.39
                     0.32
                            0.34
                                  0.35
                                         0.37
                                               0.25
                                                      0.31
                                                            0.39
                                                                   0.33
                                                                         0.35
                                                                                0.29
## V44
        0.38
               0.28
                            0.36
                                  0.32
                                         0.37
                                               0.15
                                                      0.34
                     0.43
                                                            0.42
                                                                   0.35
                                                                         0.38
                                                                                0.21
##
  V45
        0.30
               0.25
                     0.21
                           0.40
                                  0.55
                                         0.50
                                               0.29
                                                      0.41
                                                            0.38
                                                                   0.45
                                                                         0.25
                                                                                0.47
               0.16
                     0.16
                           0.29
                                  0.49
                                               0.33
                                                      0.43
                                                                   0.44
## V46
        0.21
                                         0.41
                                                            0.30
                                                                         0.13
                                                                                0.49
## V47
        0.22
               0.19
                     0.18
                            0.28
                                  0.59
                                         0.49
                                               0.23
                                                      0.51
                                                            0.35
                                                                   0.43
                                                                         0.11
                                                                                0.52
## V48
        0.13
               0.09
                     0.14
                            0.07
                                  0.31
                                         0.29
                                               0.18
                                                      0.13
                                                            0.23
                                                                   0.22
                                                                         0.05
                                                                                0.35
        1.00
                     0.34
                                  0.24
                                         0.33
                                                      0.26
## V49
               0.39
                           0.34
                                               0.18
                                                            0.46
                                                                   0.27
                                                                         0.48
                                                                                0.14
  V50
        0.39
               1.00
                     0.35
                           0.21
                                  0.22
                                         0.29
                                               0.15
                                                      0.20
                                                            0.46
                                                                   0.24
                                                                         0.27
                                                                                0.14
  V51
        0.34
               0.35
                     1.00
                           0.22
                                  0.25
                                               0.08
##
                                         0.32
                                                      0.28
                                                            0.46
                                                                   0.32
                                                                         0.32
                                                                               0.15
## V52
        0.34
               0.21
                     0.22
                            1.00
                                  0.25
                                         0.34
                                               0.14
                                                      0.30
                                                            0.32
                                                                   0.27
                                                                         0.37
                                                                                0.23
                                  1.00
                     0.25
##
  V53
        0.24
               0.22
                           0.25
                                        0.59
                                               0.31
                                                      0.50
                                                            0.41
                                                                   0.53
                                                                         0.13
                                                                                0.59
  V54
        0.33
               0.29
                     0.32
                            0.34
                                  0.59
                                         1.00
                                               0.25
                                                      0.44
                                                            0.42
                                                                         0.24
                                                                   0.50
               0.15
                            0.14
                                  0.31
                                         0.25
                                               1.00
                                                      0.26
##
  V55
        0.18
                     0.08
                                                            0.21
                                                                   0.28
                                                                         0.09
                                                                                0.36
                            0.30
                                  0.50
                                         0.44
                                               0.26
##
  V56
        0.26
               0.20
                     0.28
                                                     1.00
                                                            0.31
                                                                   0.50
                                                                         0.21
                                                                                0.45
## V57
                                         0.42
        0.46
               0.46
                     0.46
                           0.32
                                  0.41
                                               0.21
                                                      0.31
                                                            1.00
                                                                   0.40
                                                                         0.31
                                                                                0.33
## V58
        0.27
               0.24
                     0.32
                           0.27
                                  0.53
                                         0.50
                                               0.28
                                                     0.50
                                                            0.40
                                                                   1.00
                                                                         0.21
                                                                                0.47
## V59
        0.48
               0.27
                     0.32
                           0.37
                                  0.13
                                        0.24
                                               0.09
                                                      0.21
                                                            0.31
                                                                   0.21
                                                                         1.00
                                                                                0.01
## V60
        0.14
              0.14 0.15
                           0.23
                                  0.59
                                        0.43
                                               0.36
                                                      0.45
                                                            0.33
                                                                  0.47
                                                                         0.01
                                                                                1.00
#plot(personality.Vs); # too big ...
    symnum( cor(personality.Vs),
             diag = TRUE,
             corr = TRUE,
             cutpoints=c(0.15,0.30,0.60,0.90),
             symbols = c(" ",".","*","**","***")
          );
       V01 V02 V03 V04 V05 V06 V07 V08 V09 V10 V11 V12 V13 V14 V15 V16 V17 V18 V19
##
## VO1 1
## V02 *
           1
## V03 .
                1
## V04
                    1
## V05
                        1
## V06 .
                             1
## VO7 .
                                 1
## V08
                                      1
## V09
                                          1
## V10 *
                                              1
## V11
                                                  1
## V12 *
                                                       1
## V13 .
            *
                                                           1
## V14 *
                                                               1
## V15
## V16 .
                                                                        1
## V17 .
                                                                            1
```

```
## V18 .
## V19 .
                                                                                1
## V20
## V21 .
## V22 .
## V23 .
## V24
## V25
## V26 *
## V27 .
## V28
## V29
## V30 .
## V31
## V32
## V33
## V34 .
## V35
## V36
## V37 .
## V38 .
## V39 .
## V40 .
## V41 .
## V42 .
## V43 .
## V44 .
## V45
## V46
## V47
## V48
## V49 .
## V50 .
## V51 *
## V52
## V53
## V54 .
## V55
## V56 .
## V57 .
## V58 .
## V59 .
## V60
## V20 V21 V22 V23 V24 V25 V26 V27 V28 V29 V30 V31 V32 V33 V34 V35 V36 V37 V38
## VO1
## V02
## V03
## V04
## V05
## V06
## V07
## V08
## V09
## V10
```

```
## V11
## V12
## V13
## V14
## V15
## V16
## V17
## V18
## V19
## V20 1
## V21 .
           1
## V22
               1
## V23 .
                    1
## V24
## V25 *
                            1
## V26
                                1
## V27
                                    1
## V28 .
                                        1
## V29 *
                                             1
## V30
                                                 1
## V31
                                                     1
## V32
## V33
                                                             1
## V34
                                                                  1
## V35
                                                                      1
## V36
                                                                          1
## V37
                                                                              1
## V38
                                                                                  1
## V39
## V40
## V41
## V42
## V43
## V44
## V45
## V46
## V47
## V48
## V49
## V50
## V51
## V52
## V53
## V54
## V55
## V56
## V57
## V58 .
## V59
## V60
## V39 V40 V41 V42 V43 V44 V45 V46 V47 V48 V49 V50 V51 V52 V53 V54 V55 V56 V57
## VO1
## V02
## V03
```

```
## V04
## V05
## V06
## V07
## V08
## V09
## V10
## V11
## V12
## V13
## V14
## V15
## V16
## V17
## V18
## V19
## V20
## V21
## V22
## V23
## V24
## V25
## V26
## V27
## V28
## V29
## V30
## V31
## V32
## V33
## V34
## V35
## V36
## V37
## V38
## V39 1
## V40 *
## V41
                1
## V42 .
                    1
## V43 .
                        1
## V44 .
                            1
## V45 *
                                 1
## V46 .
                                     1
## V47 *
                                         1
## V48 .
                                             1
## V49 .
                                                  1
## V50
                                                      1
## V51 .
                                                          1
## V52 .
                                                              1
## V53 *
## V54 *
                                                                       1
## V55 .
                                                                           1
## V56 *
                                         *
                                                                               1
## V57 *
                                                                                   1
```

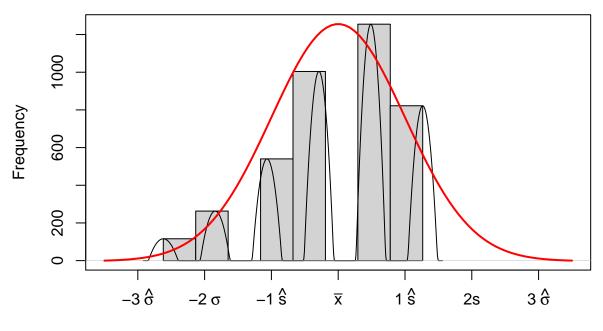
```
## V58 *
## V59
## V60 *
##
      V58 V59 V6
## V01
## V02
## V03
## V04
## V05
## V06
## V07
## V08
## V09
## V10
## V11
## V12
## V13
## V14
## V15
## V16
## V17
## V18
## V19
## V20
## V21
## V22
## V23
## V24
## V25
## V26
## V27
## V28
## V29
## V30
## V31
## V32
## V33
## V34
## V35
## V36
## V37
## V38
## V39
## V40
## V41
## V42
## V43
## V44
## V45
## V46
## V47
## V48
## V49
## V50
```



```
## let's look at all the data ...
Vs = sample( as.vector(unlist(personality.Vs)) );
head(Vs);

## [1] 3.4 2.6 3.4 2.6 1.0 4.2
normalDiagnosticPlot(Vs[1:4000]); # shapiro.test breaks if too large ...
```

Histogram (mean: 3.697, sd: 1.03)



Shapiro Normality test at (alpha = 0.05) is ... FALSE

We have too many observations (n=678) and too many variables (m=60) to be able to effectively analyze this data. We need to begin performing multivariate analysis.

In general, we treat the rows as observations and the columns as features, factors, or variables. We could transpose the dataframe and reverse rows/cols. As we proceed, remember this. We will talk about general forms of data manipulation called data reduction.

In general, we want to reduce the number of factors to consider. Or we may want to classify the subjects observed (the rows) into like groups. For the next two weeks we will consider "exploratory data analysis of multivariate data".

Correlation does not imply causation

Does zero correlation imply independence?

```
X = rnorm(100,0,1);
    Y = ( X - mean(X) ) / sd(X);
cor(X,Y); ## ARGH!

## [1] 1
    Y = X^2;
cor(X,Y); ## Getting closer

## [1] -0.03895873
    Y = ( sample(X) - mean(X) ) / sd(X);
cor(X,Y); ## Getting closer
```

```
# build a Y that is a function of X such that correlation is 0.
# it can be close to zero with isClose function
# ?isClose

# maybe review the formula for correlation
# \url{https://en.wikipedia.org/wiki/Correlation_and_dependence#Sample_correlation_coefficient}

# \url{https://math.stackexchange.com/questions/444408/why-does-zero-correlation-not-imply-independence
# \url{https://stats.stackexchange.com/questions/413326/why-does-independence-imply-zero-correlation}

# if need be, you can change the form of X, but it should be of length 100...
```