Monopolistic Screening in Ride-Pooling Platforms

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Abstract

While research interest in ride-sharing has grown in the past decade, the problem of ride-pooling has garnered relatively little attention. Ride-pooling is defined as rides purchased on a ride-sharing platform where with some probability a low-quality ride is delayed by pooling multiple riders into a single ride-share service sold by a driver. I solve for the optimal profit structure of a monopolist platform which hosts buyers (riders) who seek to purchase rides from sellers (drivers). I show that the market is two-sided and that optimal pooling is necessary for the platform's optimal profit.

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1 Introduction

The problem of accurately pricing taxi services has existed for many years (Salanova et al., 2011). With the advent of ride-share platforms (like Uber and Lyft), I believe the problem of determining optimal prices for ride-share platforms should be further researched. The advent of Uber's introduction of UberPool[1]—a service that allows users to pay a lower cost by allowing the driver to pick-up and/or drop-off other riders along their route, which can delay their own arrival — warrants further research into this question. Here I highlight two arguments which support this logic.

First, the heavy regulations imposed on the taxi industry disproportionately affect lower-income individuals[2] by restricting the overall volume of rides and placing price floors through minimum fares (Frankena and Pautler, 1984). The market failures that are induced by taxi regulations through restrictions on the number of eligible taxi firms and vehicles[3] have been forced into the limelight through the explosive growth of ride-share platforms (Cramer and Krueger, 2016). Second, I agree with Arnott (1996), who proposes that taxi travel should be subsidized since public transit is not a perfect substitute for taxi travel[4], while taxi travel is a reasonable substitute for car ownership. Arnott argues that there would be positive environmental and traffic-reduction externalities from encouraging a transition from car ownership to a greater reliance on taxi travel. Together, the circumvention of the taxi industry's market failures through ride-share platforms and the possible social welfare that could be generated from an increased utilization of ride-share platforms form a strong argument for the importance of finding the optimal prices for ride-share services.

The core function of these ride-sharing platforms is to act as a matching agent in the demand and supply of a taxi-service. A typical transaction is as follows: a rider selects a pick-up location and a destination and is presented with two prices[5] corresponding to the choice of a pooled ride, at a lower cost, versus a regular ride. Then the platform matches the rider with a nearby driver, or rejects the rider if no drivers are available. Therefore, I will model this interaction as a *two-sided market*, with riders as the buyers and drivers as the sellers. I will first consider a system of drivers and sellers at three equidistant locations. The problem to resolve is as follows: what should the platform set as the low-quality price, high-quality price,

[1] I also note that Lyft later released LyftLine, which is the analogue service to UberPool. See https://www.uber.com/ride/uberpool/ and https://www.lyft.com/line for more.

[2] In other words, those who do not own cars, nor have the income to support regular usage of taxi services.

[3] Primarily, this has been implemented through the use of the medallion system in urban centers, where the municipality sells a limited number of licenses, or medallions, which are required to operate a taxi under the municipality's jurisdiction (Frankena and Pautler, 1984).

[4] I would argue that taxi travel is a higher quality service than car ownership, since there are no parking time-value costs associated with taxi travel, although this might not be outweighed by the benefit of not waiting for a taxi

[5] For Uber, UberX (non-shared trip) and UberPool (shared trip) are the two most popular types of rides. UberBlack and UberXL are other, less popular, options.

and platform rate[6] in order to maximize revenue[7]. The platform's profit is the quantity of rides purchased times the average price of a ride times the platform's rate. I find that the monopolist must seek to optimally pool riders who purchase a low-quality service, and that the market for ride-sharing is two-sided. Lastly, I outline the optimal profit structure of the platform.

1.1 Literature Review

Ride-share platforms have been studied in a variety of capacities. Cohen et al. (2016)[8] finds large consumer surpluses[9] generated from the utilization of Uber, especially at low surge prices. The labor dynamics of Uber drivers also yielded some unexpected results in Angrist et al. (2017), where the authors found that drivers were surprisingly lease-averse, and would avoid purchasing a lease from Uber in return for reduced fees per ride, even if such an agreement would have been in the driver's favor, for the duration of the study. This supports the idea that drivers prioritize the flexibility of the labor contract with Uber, rather than their expected wages. Camerer et al. (1997) suggests that taxi drivers would stop working once they hit a predetermined target the drivers had set for themselves, instead of working when hourly earnings could be higher. In contrast, Chen and Sheldon (2016) find that drivers tend to respond to "surge prices", a term Uber uses for when demand for rides is high, and driver supply is low (within a certain geographic radius). Again, this literature further highlights the inefficiencies of the standard taxi model.

Throughout this paper, I define two-sided markets as markets in which agents are buyers and sellers[10] who interact within a platform whose structure[11] results in a Nash equilibrium distinct from results in standard Walrasian[12] equilibrium models (Demange and Gale, 1985). Demange and Gale find two key results about two-sided markets: first, buyers cannot manipulate their collective demand in any feasible way to increase their payoffs while second, in contrast to buyers, sellers can increase payoffs through collusion. The result

[6]The platform's rate is a number $r \in (0, 1)$, where after a rider is charged x, the driver receives $x(1-r)$, and the platform receives xr.

[7]I solve for maximum revenue rather than profit, since I assume that marginal costs are small.

[8]Metcalfe, one of the authors in Cohen et al. (2016), popularized the economic theory of network effects, which the theory of two-sdided markets refined.

[9]Particularly in the context of the "last-mile" problem, I would expect.

[10]As a classic example, we can consider how video game platforms must attract both video game consumers to purchase the platform (the video game console e.g. a Microsoft Xbox) and video game developers to create content for their platform. Ultimately the interaction between end-users is taxed primarily as membership cost for game developers, and console cost for consumers (Rochet and Tirole, 2006).

[11]Thus not only the price levels set by the platform matters, as in the Walrasian equilibrium. There are two common definitions, according to Rochet and Tirole (2006). First, we can examine whether or not the pricing structure is neutral or not, and second whether or not there exists cross-group externalities (see background section for more).

[12]We can think of Walrasian equlibria as equilibria in the neoclassical sense. That is, where we can model both the supply and demand for a good or service, and equate the two expressions to solve for the equilibria. However, in the case where multiple equilibria exist, it is not possible to determine which equlibria will be dominated.

is a Nash equilibrium where sellers can force the maximum price equilibrium[13]. According to Rochet and Tirole (2006), "the failure of the Coase theorem is necessary but not sufficient for two-sidedness" (p. 645). Tirole uses the Coase theorem as a benchmark because two-sided markets requires that "end-users cannot reach an efficient outcome through bargaining" (p. 649), while that condition is not sufficient to demonstrate two-sidedness. However, in the case of ride-share platform pricing, we can derive two-sidedness from the asymmetric price setting, since riders know the price of rides while drivers do not. Therefore, efficient bargaining is impossible and so we have the sufficient condition for a non-neutral platform price structure that Rochet and Tirole (2006) demands.

Several papers have applied the two-sided market theory to pricing structures similar to the ride-share pricing theory I will explore. Weyl (2010) generalizes two-sided market theory to multi-sided platforms, and provides insights into the role that market size plays in the pricing dynamics of platforms. Weyl's theory could be applied to the case where there are many locations, each with their own network of buyers and sellers. So with n locations, we have $2n$ sides to the market, each facing prices set by the platform. Einav et al. (2016) applies two-sidedness to peer-to-peer markets like Ebay. Kojima and Pathak (2009) develops a theory for understanding rapid changes in pricing. If surge pricing plays an important role in the supply of drivers and the platform's profit, then Kojima's work could be used to model this dynamic pricing problem, since Chen and Sheldon (2016) confirm that drivers respond strongly to surge prices. Lastly, the queueing-theoretic approach in Banerjee et al. (2015) to the ride-share problem is closely linked the problem I will address in this paper. I will attempt to build on the model developed in Banerjee et al. by extending their framework to the case where additional riders may be picked up along the route, at the cost of a greater expected arrival for riders, but at a discounted price. My aim is to determine what the optimal discount is, given the platform's constrained maximization problem.

2 Background

While much of the platform pricing research has explored related problems, I have not found a model that adequately captures the mechanisms which drive the difference between the price of a regular taxi-service and a discounted pooled service. First, I will consider the simplest model that relies heavily on the classic vertical price differentiation literature (I use the notation in chapter 2 of Tirole (2015)). We can consider that a ride-share platform monopolist offers two services: a low quality service analogous to the pooled taxi service, and a high quality service analogous to the regular taxi service. Each service completes the same objective, which is to transport an individual from point A to point B. Let the pooled service have quality s_1, and the regular service have quality s_2, where $s_1 < s_2$. Let a consumer's taste for quality be $\theta > 0$,

[13] This results resolves the difficulty in choosing among several Walrasian equilibria.

where θ is derived from a distribution of tastes according to some density $f(\theta)$ with cumulative distribution function $F(\theta)$, where $\theta \in [0, \infty)$, and $F(0) = 0$ and $F(\infty) = 1$ as expected. We can consider that $F(\theta)$ is the fraction of consumers with taste parameter less than θ, and $\frac{1}{\theta}$ is the marginal rate of substitution[14] for consumers between income and quality. Therefore, a consumer's preferences are:

$$U = \begin{cases} s - \frac{p}{\theta}, & \text{if he buys a good with quality } s \text{ at price } p \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

The demand for a good with quality s and price p is simply the number of consumers, N, times the fraction of consumers for which $\theta s \geq p$, or equivalently

$$D(p) = N\left[1 - F\left(\frac{p}{s}\right)\right]$$

Using the same logic, we can derive the demand functions in the case with two qualities[15]:

$$D_1(p_1, p_2) = N\left[1 - F\left(\frac{p_2 - p_1}{s_2 - s_1}\right)\right]$$

$$D_2(p_1, p_2) = N\left[F\left(\frac{p_2 - p_1}{s_2 - s_1}\right) - F\left(\frac{p_1}{s_1}\right)\right]$$

However, there is much this model does not capture. For one, Uber provides the service to ride-share buyers through drivers, so there emerges a problem of endogeneity, in which changes in the demand and supply of drivers and riders cause changes in the utility the other party receives.[16] For this reason, I will build on the existing literature of two-sided markets.

2.1 Two-Sided Markets

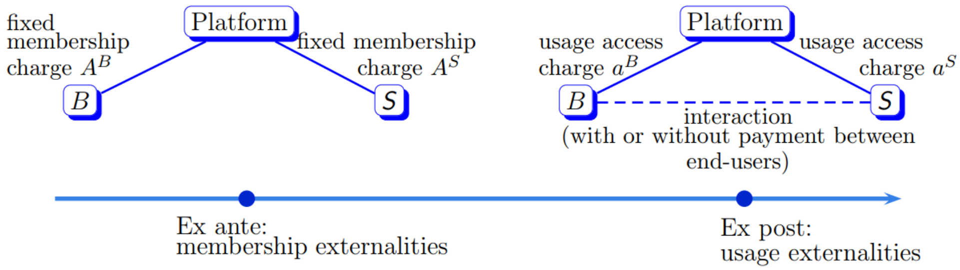
Rochet and Tirole (2003) and Rochet and Tirole (2006) focus on the distinction between usage and membership fees. Membership fees can only be enforced ex ante transactions, as outlined in Figure 1. For instance, in the example of video game consoles, the ex ante membership fee for the buyer is the console, while for the seller of video games they might have to purchase developer kits from the console seller. Ex post, per interaction fees can be charged to the buyer and seller respectively. Hence, in the case of video game platforms, every game purchased by the consumer would have a usage fee that the platform would receive.

[14] As θ increases, $\frac{1}{\theta}$ decreases, thus the consumer substitutes income for quality more.

[15] Proof is in the appendix.

[16] For instance, more drivers implies that the price of taxi-services is lower, which in turn increases demand. Thus there are cross-group network externalities we need to internalize in the model.

Figure 1: Diagram from Rochet and Tirole (2003) depicting the difference between membership and usage fees. Borrowed with permission from authors.



An interesting aspect of these markets is that the buyer or the seller can be subsidized in order for the product (from the platform's perspective) to maximize profit[17]. Buyers may have their their initial membership charge be subsidized by the sellers ($A^B \leq 0$), since then the number of buyers N^B will be greater, increasing platform profit. I will derive the platform's first order conditions for maximizing profit. I will use the notation as described in Table 1.

Definition 2.1. *A platform charging per-interaction charges a^B and a^S is one-sided if the volume of transactions V depends only on the aggregate price level $a = a^B + a^S$. Otherwise, a platform is two-sided (Rochet and Tirole, 2003).*

This difference in aggregate price levels from the one-sided market, to two-sidedness, is that the pricing structure, or which side of the market should have the pricing burden, matters. In cases where the pricing structure is neutral —where how the prices are distributed among the sides does not matter —the economic outcome will be the same, for any allocation of prices. Rochet and Tirole (2006) note that bilateral electricity trading is an example of a one-sided market. If the market is run by bilateral contracts between consumers and producers, where each side of the market pays a fee to the transmission system (to load up the electricity generated by producers, and then to distribute that electricity to consumers), then the buyer and seller only care about the aggregate fee ($a^B + a^S = a$) paid to the transmission system, since otherwise prices in the bilateral contract would adjust. Further, while the firm can be viewed as a two-sided platform —where it charges workers a fee a^S to access the firm in order to sell widgets to buyers who pay a^B per widget —then the firm is two-sided, since changing the pricing structure (say raising a^B and lowering a^S), would lead to inefficiencies since the end-users cannot bargain to compensate the workers for the change increase in consumer surplus. However, in practice, the competitive firm is viewed as a de-facto one-sided market, since changing either a^B or a^S would lead to more consumers and workers going to the firm or them leaving the firm for a different firm, thus the pricing structure is de-facto neutral.

[17] See Figure 2 for examples.

Figure 2: Diagram from Rochet and Tirole (2003) outlining various two-sided markets. Borrowed with permission from authors.

Product	Loss leader/break-even segment/subsidized segment	Profit-making segment/ subsidizing segment
	Software	
Video games	consumers (consoles)	software developers
Streaming media	consumers	servers
Browsers	users	Web servers
Operating systems (Windows; Palm, Pocket PC)	application developers (development tools, support, functionality, . . .)	clients
Text processing	reader/viewer	writer
	Portals and media	
Portals	"eyeballs"	advertisers
Newspapers	readers	advertisers
(Charge-free) TV networks	viewers	advertisers
	Payment systems	
Credit and differed debit cards (Visa, MasterCard, Amex, . . .)	cardholders	merchants
Online debit cards	merchants	cardholders
	Others	
Social gatherings	celebrities in social happenings	other participants
Shopping malls	consumers (free parking, cheap gas)	shops
Discount coupon books (Want Advertiser)	consumers	merchants
(Legacy) Internet	Web sites	dial-up consumers
Real estate	buyers	sellers

An alternative definition of two-sidedness is whether or not cross-group externalities exist —that is, the net utility for side i increases with an increase in the number of users on side j. See section 3 in Rochet and Tirole (2006) for possible complications with this definition. However, the two definitions can be often be used interchangeably.

Proposition 2.1. *(Armstrong, 2006) Given a platform that attracts two groups with the following utilities:*

$$u_1 = a_1 n_2 - p_1 \; ; \; u_2 = a_2 n_1 - p_2$$

where p_i are the platform's prices to the two groups and a_i reflects the utility from interacting with the other group. If f_i is the platform per-agent cost and $\phi_i(u_i)$ is an increasing function, then the profit-maximizing prices are:

$$p_1 = f_1 - a_1 n_2 + \frac{\phi_1(u_1)}{\phi_1(u_1)} \; ; \; p_2 = f_2 - a_2 n_1 + \frac{\phi_2(u_2)}{\phi_2(u_2)}$$

Proof. Let the number of individuals that join a group be $n_i = \phi_i(u_i)$. Then the firm's profit is: $\pi =$

$n_1(p_1 - f_1) + n_2(p_2 + f_2)$. Since $p_i = a_i n_j - u_i$, we can write:

$$\begin{aligned}\pi &= n_1(a_1 n_2 - u_1 - f_1) + n_2(a_2 n_1 - u_2 - f_2) \\ &= \phi_1(u_1)(a_1 n_2 - u_1 - f_1) + \phi_2(u_2)(a_2 n_1 - u_2 - f_2)\end{aligned}$$

Thus the platform's profits in terms of the groups' utilities is:

$$\pi(u_1, u_2) = \phi_1(u_1)(a_1\phi_2(u_2) - u_1 - f_1) + \phi_2(u_2)(a_2\phi_1(u_1) - u_2 - f_2) \tag{2}$$

Then the first order conditions are:

$$\begin{aligned}\frac{\partial \pi}{\partial u_1} &= \phi_1^{'}(u_1)(a_1\phi_2(u_2) - u_1 - f_1) + \phi_2(u_2)a_2\phi_1^{'}(u_1) = 0 \\ \frac{\partial \pi}{\partial u_2} &= \phi_2^{'}(u_2)(a_2\phi_1(u_1) - u_2 - f_2) + \phi_1(u_1)a_1\phi_2^{'}(u_2) = 0\end{aligned}$$

We can use the first order conditions to finish the proof, by noting that $p_1 = a_1 n_2 - u_1$ and likewise for p_2.

□

Armstrong (2006) gives an overview of some of the dynamics of these two-sided markets. However, Rochet and Tirole (2006) note that industries in which payments between end-users are essential, e.g. platforms where software is bought and sold, the canonical model,

$$U^i = (b^i - a^i)N^j + B^i - A^i \tag{3}$$

is insufficient as it does not capture the endogenous fraction of interactions that actually take place: it merely states that there are $N^B N^S$ *potential* interactions. First I will derive the optimal price structure in equation 3, and then I will derive the optimal price structure in the case where there are payments between end-users.

Table 1: Summary of Two-Sided Markets Notation in Rochet and Tirole (2006)

Symbol	Description
A^i	Fixed fee to agent i
B^i	Fixed benefit/cost to agent i
C^i	Fixed cost to the platform
c	Marginal cost per interaction to platform
a^i	Per-transaction usage fee to agent i
b^i	Per-transaction benefit to agent i
N^i	Number of agents of type i
p^i	Price charged to agent i

Proposition 2.2. *(Rochet and Tirole, 2006) Given utility, $U^i = (b^i - a^i)N^j + B^i - A^i$, the condition for the optimal price structure for $p^B + p^S = p$ is:*

$$-\frac{\left(1 - \frac{\partial D^B}{\partial N^S} \cdot \frac{\partial D^S}{\partial N^B}\right)}{p - c} = \frac{\frac{\partial D^B}{\partial p^B}}{D^B} + \frac{\frac{\partial D^B}{\partial p^B}\frac{\partial D^S}{\partial N^B}}{D^S} = \frac{\frac{\partial D^S}{\partial p^S}}{D^S} + \frac{\frac{\partial D^S}{\partial p^S}\frac{\partial D^B}{\partial N^S}}{D^B} \tag{4}$$

Proof. (Following the outlined proof in Rochet and Tirole (2006)) The number of users that decide to join the platform, for side i, is:

$$N^i = \Pr(U^i \geq 0)$$

Let the per-interaction price be defined as:

$$p^i \equiv a^i + \frac{A^i - C^i}{N^j} \tag{5}$$

Then we can derive the demand functions by considering the proportion of side i whose utility U^i is greater than the price they face, p^i.[18]

$$D^i(p^i, N^j) \equiv N^i = \Pr(b^i + \frac{B^i - C^i}{N^j} \geq p^i) \tag{6}$$

This system of equations has a solution for N^i, where N^i for $i \in \{B, S\}$ is a function of $n^i(p^i, p^j)$. So given the expression for the platform's profit:

$$\pi = (A^B - C^B)N^B + (A^S - C^S)N^S + (a^B + a^S - c)N^B N^S \tag{7}$$

which can be constructed by summing up the platform's profit from the fixed membership fees, $A^i - C^i$, with the variable fees $a^B + a^S - c$ times the volume of interactions $N^B N^S$. Re-writing equation 5 we have that $N^j \equiv \frac{A^i - C^i}{p^i - a^i}$, therefore we can write equation 7 as:

$$\begin{aligned}
\pi &= \frac{(A^B - C^B)(A^S - C^S)}{p^S - a^S} + \frac{(A^S - C^S)(A^B - C^B)}{p^B - a^B} \\
&+ \frac{(a^B + a^S - c)(A^B - C^B)(A^S - C^S)}{(p^B - a^B)(p^S - a^S)} \\
&= \frac{(A^S - C^S)(A^B - C^B)(p^B + p^S - c)n^B(p^B, p^S)n^S(p^B, p^S)}{(p^B - a^B)(p^S - a^S)} \\
&= (p^B + p^S - c)n^B(p^B, p^S)n^S(p^B, p^S)
\end{aligned} \tag{8}$$

Therefore, the optimal price structure is obtained from maximizing the volume of transactions, given the aggregate price level $p^B + p^S = p$ is:

$$V(p) = \max\{n^B(p^B, p^S)n^S(p^B, p^S) \mid p^B + p^S = p\} \tag{9}$$

Therefore, the Lerner index $\eta \equiv \frac{-pV'(p)}{V(p)}$ is computed as follows, for each price:

[18] Same approach as in Tirole (2015).

$$\eta = \frac{\frac{\partial n^B}{\partial p^B}}{n^B} + \frac{\frac{\partial n^S}{\partial p^B}}{n^S}$$

$$\eta = \frac{\frac{\partial n^S}{\partial p^S}}{n^S} + \frac{\frac{\partial n^B}{\partial p^S}}{n^B}$$

Therefore the optimal price structure depends on the following condition:

$$-\frac{1}{p-c} = \frac{\frac{\partial n^B}{\partial p^B}}{n^B} + \frac{\frac{\partial n^S}{\partial p^B}}{n^S} = \frac{\frac{\partial n^S}{\partial p^S}}{n^S} + \frac{\frac{\partial n^B}{\partial p^S}}{n^B} \tag{10}$$

Using the partial derivatives of n^i equation 10 can be re-written to obtain the desired equation 4. □

2.2 Contract Theory

In this section I will develop intuition for the theory of monopolistic screening in the context of the ride-pooling problem (Demange and Gale, 1985). At the heart of economics, is the concept that gains can be achieved through bi-lateral trade agreements. Without contracts, it can be more difficult or unfeasible to acquire those unrealized trade gains. A key conflict in trade is that asymmetric information creates incentive problems: adverse selection and moral hazard. Hidden Information (adverse selection) is when agents do not truthfully reveal the state of the world. Hidden Action (moral hazard) is when agents due not deliver on promises due to imperfect monitoring.

Example 2.1. *Consider a state-contingent Arrow-Debreu delivery contract where the seller is obliged to sell a high-quality good in states where the seller's cost is low and a low-quality good when the cost is high. Then the seller need not deliver on this promise.*

Proof. Since the state of the world is only privately observed, there is a situation of asymmetric information. If the seller observes that the state of the world is such that costs are low, then the seller has an incentive to either sell a low-quality good (this is preferred to delivering a high-quality good, since the low-quality good would always be cheaper to produce (adverse selection)) or simply not deliver anything (if it is optimal to not produce anything versus producing the high-quality good (hidden action)). □

In both the simple case where there are two qualities with two prices and the case of ride-sharing, we can predict how the issue of asymmetric information will manifest itself. If it were not true that the quality-per-dollar of the high-type is higher than that of the low-type, then the utility for the high-type buyer is actually higher when they purchase the low-type good. In this case, the monopolist would only produce low-type good, since only the low-type good would be demanded. However, later I will show that the monopolist could handle this issue by pricing their goods differently, in order to avoid this issue. A similar problem could be occurring with the ride-sharing problem, where the number of high-type rides purchased is actually not optimal[19] and therefore we would have to consider the incentives of consumers to consume the low-quality

[19]Here, we imagine the monopolist maximizing profit in the usual manner, i.e. as outlined in Proposition 2.1.

product at higher rates than expected. For instance, even if we consider a state of the world where the rider's taste for quality is extremely skewed towards preferring higher-quality services, if there are zero low-quality rides purchased, then it would in fact be optimal for all high-quality buyers to switch and buy the low-quality service, since the low-quality service would have no probability of an extended ride, and therefore offer the same service as the high-quality service, but at a weakly[20] lower price.

These problems are a subset of the field of monopolistic screening (Maskin and Riley (1984) and Mussa and Rosen (1978)) where the monopolist is uninformed about the types of buyers they are facing. Monopolistic screening problems are a type of Principal-Agent problem, where the Principal (the monopolist, in this case) enters a contract with an Agent (a buyer) who possesses private information. I will follow the notation in Tadelis and Segal (2005).

Assume that the seller (the Principal) may sell a quantity $x \in X \subset \mathcal{R}_{\geq 0}$ in exchange for a payment $t \in \mathcal{R}$. Then it follows that the seller's profit is

$$\max\{t - c(x), 0\}$$

where $c(\cdot)$ is their cost function. Let $v(x, \theta)$ be the buyer's utility for consuming x with type $\theta \in \Theta \subset \mathcal{R}$. [21] Since t is defined as the payment in exchange for x, the buyers utility must be:

$$\max\{v(x, \theta) - t, 0\}$$

For simplicity, we assume that the customer has two possible taste parameters $\theta_H > \theta_L > 0$ where the probability of θ_L is $p \in (0, 1)$. Further, assume that the monopolist cost function is twice continuously differentiable with $c(0) = 0, c' \geq 0, c'' \geq 0$ and similarly $v(0, \theta) = 0, v_x > 0, v_{xx} \leq 0$ and $v(x, \theta_H) > v(x, \theta_L)$ for any x. The total surplus available is:

$$S(x, \theta) = v(x, \theta) - c(x)$$

Assume that $S_x(0, \theta) > 0$ and that there exists $\bar{x}$ such that $S(x, \theta) < 0 \; \forall \; x \geq \bar{x}$.

Proposition 2.3. *(First Bestt Solution) If the monopolist knows the type of each consumer, then the monopolist maximizes total surplus and extracts it all by solving the problem:*

$$\max \; v(x_i, \theta_i) - c(x_i) \text{ subject to } 0 \leq x_i \leq \bar{x} \tag{11}$$

Let x_i^f be a solution, then efficient quantities are produced when:

$$v_x(x_i^f, \theta_i) = c'(x_i^f) \text{ for } i \in \{L, H\}$$

and price is set such that:

$$t_i^f = v(x_i^f, \theta_i) \text{ for } i \in \{L, H\}$$

[20] It is hard to imagine a case where the prices of the services are equivalent.

[21] Note that θ is not observed by the seller. This is the *hidden* information that leads to an adverse selection problem.

Proof. Note that $x_i^f = 0$ cannot be a solution since $S_x(0,\theta_i) = v_x(0,\theta_i) - c'(0) > 0$ which implies that more surplus could be extracted by the monopolist by increasing x_i^f. Similarly $x_i^f = \bar{x}$ cannot be a solution. Therefore, there must exist a solution x_i^f such that $v_x(x_i^f, \theta_i) = c'(x_i^f)$. Once the total surplus is maximized, the monopolist simply sets prices to extract all the consumer surplus:

$$t_i^f = v(x_i^f, \theta_i)$$

. $\square$

Now we consider the case that the types of the buyers are unobservable. The monopolist has to offer both types of buyers the same price schedule[22] and can no longer perform first-degree discrimination. If the monopolist offers the solution from Proposition 2.3, then the high-type consumer will prefer to purchase the low-type menu since

$$v(x_L^f, \theta_H) - t_L^f = v(x_L^f, \theta_H) - v(x_L^f, \theta_L) > 0 = v(x_H^f, \theta_H - t_H^F)$$

The last equality holds because the monopolist is extracting the full consumer surplus, therefore the high-type buyer cannot have utility greater than 0 from purchasing the high-type good. So clearly the first best solution is not optimal. The second best solution must be to choose a price schedule $t(x)$ to maximize expected profits.

$$\begin{aligned} &\max_{t(x)} p[t(x_L) - c(x_L)] + (1-p)[t(x_H) - c(x_H)] \\ &\qquad \text{subject to} \\ &\qquad x_i \in \arg\max_{x \geq 0} v(x, \theta_i) - t(x) \text{ for } i \in \{L, H\} \\ &\qquad v(x_i, \theta_i) - t(x_i) \geq 0 \text{ for } i \in \{L, H\} \end{aligned} \tag{12}$$

This is a very complicated problem, but we can use the Revelation Principle to simplify the problem to choosing between a set of menus $\{(x_L, t_L), (x_H, t_H)\}$. The monopolist can set up a mechanism where the buyer chooses a strategy $s \in S$ and the monopolist assigns that strategy an outcome $g(s) = (g(s), t(s))$. Then the monopolist needs only examine outcomes for which reporting the true type is optimal, and then choose the best outcome among that set. The Revelation Principle guarantees that if a mechanism can implement an outcome, then there exists a mechanism such that the same outcome is implemented *and* the buyer truthfully reports their type.

Definition 2.2. *(Mechanisms) A mechanism is defined as a pair (S, g) where $g : S \mapsto D$ and D is the set of outcomes.*

[22] Price schedules are just a set of price-quantity bundles.

Definition 2.3. *(Mechanism Implements an Outcome) A mechanism implements an outcome* $f : \Theta \mapsto D$ *if there exists* $s^* : \Theta \mapsto S$ *such that for all* $\theta \in \Theta$, $g(s^*(\theta)) = f(\theta)$ *and* $u(g(s^*(\theta)), \theta) \geq u(g(s'), \theta)$, *for all* $s' \in S$

Definition 2.4. *(Direct (Revelation) Mechanism) A direct mechanism is a pair* (Θ, g) *where the agent reports a type* $\theta \in \Theta$ *and the principal assigns an outcome* $g(\theta)$.

Definition 2.5. *(Truthful) A direct mechanism is incentive compatible (truthful) if reporting the true type is optimal.*

$$\forall \theta \in \Theta \; u(g(\theta), \theta) \geq u(g(\theta'), \theta) \; \forall \, \theta' \in \Theta$$

Proposition 2.4. *(Revelation Principe) If there exists a mechanism that implements an outcome, then there exists a truthful direct mechanism that implements the same outcome.*

Proof. Suppose that a mechanism (S, g) implements some outcome f. Then by Definition 2.3 it must be that there exists some $s^* : \Theta \mapsto S$ for all $\theta \in \Theta$, $g(s^*(\theta)) = f(\theta)$ and $u(g(s^*(\theta)), \theta) \geq u(g(s'), \theta)$, for all $s' \in S$. Note that for all $\theta' \in \Theta$ it must be that $u(g(s^*(\theta)) \geq u(g(\theta', (\theta))$. From Definition 2.3 it must also be that $g(s^*(\theta)) = f(\theta)$, therefore for all $\theta' \in \Theta$ $u(f(\theta), \theta) = u(g(s^*(\theta), \theta) \geq u(g(\theta', (\theta))$ But this implies that $u(f(\theta), \theta) \geq u(f(\theta'), \theta)$ for all $\theta' \in \Theta$. Thus by Definition 2.5 (Θ, f) is a truthful direct mechanism that implements the same outcome, as desired.

□

The Revelation Principle can simplify the monopolist's problem into the following program:

Proposition 2.5. *(Second Best Program)*

$$\begin{aligned}
&\max_{t(x)} p[t(x_L) - c(x_L)] + (1-p)[t(x_H) - c(x_H)] \\
&\qquad \text{subject to} \\
&\qquad\quad v(x_L, \theta_L) - t(x_L) \geq v(x_H, \theta_L) - t(x_H) \qquad (IC_L) \\
&\qquad\quad v(x_H, \theta_H) - t(x_H) \geq v(x_L, \theta_H) - t(x_L) \qquad (IC_H) \\
&\qquad\quad v(x_i, \theta_i) - t(x_i) \geq 0 \text{ for } i \in \{L, H\} \qquad (IR_i)
\end{aligned} \tag{13}$$

The constraints of Proposition 2.5 can be relaxed since a solution of Proposition 2.6 will also be in the solution set of the original problem. To do this, it needs to be shown that any solution of the relaxed program in 2.6 satisfies the constraints set out in 2.5. Note that the IC_L and IC_H constraints imply the following:

$$v(x_H, \theta_H) - v(x_L, \theta_H) \geq t_h - t_L \geq v(x_H, \theta_L) - v(x_L, \theta_L) \tag{14}$$

This implies that $x_H \geq x_L$. Further, IC_H must hold with equality, since otherwise the monopolist could increase profit by increasing t_H slightly. So now we can write:

$$v(x_H, \theta_H) - v(x_L, \theta_H) = t_h - t_L \geq v(x_H, \theta_L) - v(x_L, \theta_L) \tag{15}$$

Since $\theta_H > \theta_L$ and $x_H \geq x_L$, then

$$v(x_H, \theta_H) - t_H \geq v(x_L, \theta_H) - t_L \geq v(x_L, \theta_L) - t_L$$

Lastly, it needs to be shown that any solution of 2.6 also is a solution to 2.5. This follows from the fact that equation 14 implies that the constraint set of 2.5 is a subset of 2.6, and similarly equation 15 implies that the solution to 2.6 satisfy the constraints of 2.5.

Proposition 2.6. *(Relaxed Program)*

$$\begin{aligned}
&\max_{t(x)} p[t(x_L) - c(x_L)] + (1-p)[t(x_H) - c(x_H)] \\
&\quad \text{subject to} \\
&\qquad v(x_H, \theta_H) - t(x_H) = v(x_L, \theta_H) - t(x_L) \qquad (IC_H) \\
&\qquad v(x_L, \theta_L) - t(x_L) = 0 \qquad\qquad\qquad\qquad (IR_L) \\
&\qquad x_H \geq x_L
\end{aligned} \tag{16}$$

Proof. Since IR_L is a binding constraint, it follows that we can substitute t_L for $v(x_L, \theta_L)$. Then using the fact that the IC_H constraint is binding it follows that $t_H = v(x_H, \theta_H) - [v(x_L, \theta_H) - v(x_L, \theta_L)]$. The $[v(x_L, \theta_H) - v(x_L, \theta_L)]$ term can be interpreted as the **information rent** the agent derives from the asymmetric information of the problem. Essentially, it is the cost to the principal for not having access to the agent's true type. Following Tadelis and Segal (2005) the second best solution can be derived since the Kuhn-Tucker conditions are necessary and sufficient for a global optimum. □

3 Model

3.1 Model Assumptions

I assume that there are only 3 locations $\{i, j, k\}$ in the economy, and passengers seek to travel from one location to another. Further, I assume that in expectation buyers are equally likely to travel to different locations.

Assumptions:

- Let the distance from any location to another be d.
- Let s be the utility from travelling from one location to another.
- Let p_{L_i} and p_{H_i} be the prices of the low/high quality services, respectively, charge to the buyer at location i.
- Let x_i be the probability that a passenger is picked up at the *outside* node[23]. Thus with probability x_i, a buyer of service s pays p_{L_i} but travels $2d$, a disulity.
- Let r be the platform's rate[24], a disulity for the driver.
- Assume that $F^i(d)$ is the cumulative distribution of individuals taste parameters, for buyers and sellers (i.e assuming everyone is averse to travelling d).

3.2 User Utility Preferences

First, I characterize the end-user's (i.e. the drivers and the passengers) utility preferences. Let U_i^B and U_i^S be the utility of a buyer and seller, respectively, of a ride-sharing service, s, at location i. Let $N_{J_i}^B$ be the number of rides purchased from location i with quality $J \in \{L, H\}$.

$$U_i^B = \max \begin{cases} s - d - dx_i - p_{L_i} \\ s - d - p_{H_i} \\ 0 \end{cases} \tag{17}$$

[23]We can imagine an equilateral triangle, with nodes labelled $1, 2$, and 3, then a low-quality ride might pick-up a passenger at node 1, and travel to node 2 before reaching the destination 3, rather than travelling directly from 1 to 3.

[24]The platform's rate is a number $r \in (0, 1)$, where after a rider is charged x_i, the driver receives $x_i(1 - r)$, and the platform receives $x_i r$.

$$U_i^S = \max \begin{cases} \frac{N_{H_i}^B p_{H_i}(1-r)+N_{L_i}^B p_{L_i}(1-r)-\left[dN_{H_i}^B+d(1-x_i)N_{L_i}^B+2dx_iN_{L_i}^B\right]}{N_{H_i}^B+N_{L_i}^B} \\ 0 \end{cases}$$
$$= \max \begin{cases} \frac{(1-r)p_{H_i}+(1-r)p_{L_i}-[d+d(1-x_i)+2dx_i]}{N_{H_i}^B+N_{L_i}^B} \\ 0 \end{cases} \tag{18}$$

This revenue maximization problem has several implicit constraints, which are used in order to determine the quantities demanded and supplied for the low and high quality products. Comparing one service to another service, in terms of utility preferences, form the **Incentive Compatibility** (IC) constraints, while the preference of a service to no service form the the **Individual Rationality** (IR) constraints. Given this model, I will find the optimal prices at each location, the platform rate, and the pooling probabilities of each location.

4 Analysis

4.1 Deriving the Demand Functions from User Utility Preferences

When $s - d(1-x_i) - 2dx_i - p_{L_i} \geq s - d - p_{H_i}$ the low quality service is preferred to the high quality service. This implies that at $\hat{d} = \frac{p_{H_i}-p_{L_i}}{x_i}$, a rider is indifferent between pooling or not. Similarly, a rider is indifferent to the lower quality service or not riding if: $s - d(1-x_i) - 2dx_i - p_{L_i} \geq 0 \implies \hat{d} = \frac{s-p_{L_i}}{1+x_i}$. Normalizing the population of buyers and sellers, it follows that:

$$D_{H_i}(p_{H_i}, p_{L_i}, r) = N_{H_i}^B = \left[1 - F^B\left(\frac{p_{H_i}-p_{L_i}}{x_i}\right) - F^B\left(p_{H_i}-s\right)\right] \tag{19}$$

$$D_{L_i}(p_{H_i}, p_{L_i}, r) = N_{L_i}^B = \left[F^B\left(\frac{p_{H_i}-p_{L_i}}{x_i}\right) - F^B\left(\frac{s-p_{L_i}}{1+x_i}\right)\right] \tag{20}$$

Similarly for the seller, when

$$N_{H_i}^B p_{H_i}(1-r) + N_{L_i}^B p_{L_i}(1-r) - \left[dN_{H_i}^B + d(1-x_i)N_{L_i}^B + 2dx_iN_{L_i}^B\right] \geq 0 \tag{21}$$

or at $\hat{d} = \frac{N_{H_i}^B p_{H_i}(1-r)+N_{L_i}^B p_{L_i}(1-r)}{N_{H_i}^B+(1-x_i)N_{L_i}^B+2x_iN_{L_i}^B}$ drivers prefer to drive. So it follows that:

$$D_S(p_{H_i}, p_{L_i}, r) = N_i^S = \left[1 - F^S\left(\frac{N_{H_i}^B p_{H_i}(1-r) + N_{L_i}^B p_{L_i}(1-r)}{N_{H_i}^B + (1-x_i)N_{L_i}^B + 2x_iN_{L_i}^B}\right)\right] \tag{22}$$

4.2 Optimization Constraints

To derive the constraints for the optimization program, I first make the assumption that riders at each node are equally likely to demand a ride to the other two nodes. This allows me to easily form two regularity conditions: one for the general equilibrium condition for the number of consumers at each node and a second condition about the probabilities of a ride being pooled or not. Labelling the set of possible locations as the nodes $\{i, j, k\}$, I first note the general equilibrium condition.

Proposition 4.1. *(General Equilibrium Conditions) For each node $l \in \{i, j, k\}$, the number of sellers must equal the number of buyers. Therefore, without loss of generality, the following equation must hold at each node l:*

$$N_{S_i} = N^B_{H_i} + (1 - \frac{x_j}{2} - \frac{x_k}{2})N^B_{L_i}$$

Proof. Note that for each high-type consumer, there must be exactly one seller. But, in expectation, there is less than or equal to one seller for each low-type buyer. Therefore, the general equilibria conditions must have the form:

$$N_{S_i} - N^B_{H_i} = N^B_{L_i} g(x_j, x_k)$$

where $0 \leq g(x_j, x_k) \leq 1$. I claim that $g(x_j, x_k) = 1 - \frac{x_j}{2} - \frac{x_k}{2}$. This must be correct since given a pooling probability, x_j, of the pooled rides from node j, half of them must pick up riders at i, therefore there will not be a need for $\frac{x_j}{2} N^B_{L_i}$ drivers originating from node i. A similar reasoning applies to the pooling probability x_k. Therefore $g(x_j, x_k) = 1 - \frac{x_j}{2} - \frac{x_k}{2}$, so $N_{S_i} = N^B_{H_i} + (1 - \frac{x_j}{2} - \frac{x_k}{2})N^B_{L_i}$. □

We can re-write our first constraint $g_i(p_{L_i}, p_{H_i}, r) = N^S_i - N^B_{H_i} + (1 - \frac{x_j}{2} - \frac{x_k}{2})N^B_{L_i} = 0$ as follows:

$$\begin{aligned} F^S\left(\frac{N^B_{H_i}(p_{H_i} - r) + N^B_{L_i}(p_{L_i} - r)}{N^B_{H_i} + (1 - x_i)N^B_{L_i} + 2x_i N^B_{L_i}}\right) - F^B\left(\frac{p_{H_i} - p_{L_i}}{x_i}\right) - F^B\left(p_{H_i} - s\right) \\ +(1 - \frac{x_j}{2} - \frac{x_k}{2})\left[F^B\left(\frac{p_{H_i} - p_{L_i}}{x_i}\right) - F^B\left(\frac{s - p_{L_i}}{1 + x_i}\right)\right] = 0 \end{aligned} \tag{23}$$

Proposition 4.2. *(Pooling Probabilities) Given the general equilibria conditions from Proposition 4.1, the sum of the pooling probabilities must be:*

$$\sum x_i \leq 2 \tag{24}$$

Proof. Consider if $x_j = x_k = 1$. Then it is guaranteed that rides emanating from nodes j, k will pool with riders from other nodes. For instance, if half of riders from j are heading to k, they will pick up $\frac{1}{2}N_{L_j}$ riders at i and $\frac{1}{2}N_{L_j}$ riders at k. A similar effect occurs for riders leaving from k. The result is that $N_{S_i} - N_{H_i} = N_{L_i} - (\frac{1}{2}N_{L_j} + \frac{1}{2}N_{L_k})$ drivers are needed from node i. If the number of low-type buyers at each location is the same, then there is no demand for low-type drivers from i, and thus $x_i = 0$. □

Therefore, the problem left to solve is in a revenue optimization problems with constraints as defined in Propositions 4.1 and 4.2.

4.3 Optimization Problem

Proposition 4.3. *The optimal profit is the solution $(p^*_{L_i}, p^*_{H_i}, r^*, x^*_i)$ to the following program:*

$$(p^*_{L_i}, p^*_{H_i}, r^*, x^*_i) = \underset{p_{L_i}, p_{H_i}, r}{\arg\max} \left\{ \pi_R = \sum_i \left[p_{L_i} N^B_{L_i} r + p_{H_i} N^B_{H_i} r \right] \right\} \tag{25}$$

$$\text{subject to } \; N^S_i = N^B_{H_i} + (1 - \frac{x_j}{2} - \frac{x_k}{2}) N^B_{L_i} \tag{26}$$

We will have an augmented system of equations with 13 equations and 13 unknowns. We can write the Lagrangian as follows:

$$\mathcal{L}(p_{L_i}, p_{H_i}, r, \lambda_i) = \sum_i \left[p_{L_i} N^B_{L_i} r + p_{H_i} N^B_{H_i} r \right] + \sum_i \lambda_i g_i(p_{L_i}, p_{H_i}, r) \tag{27}$$

Which results in the following first order conditions for our profit function[25]:

$$\frac{\partial \pi_R}{\partial p_{L_i}} = r N^B_{L_i} - r p_{L_i} \left[f^B \left(\frac{p_{H_i} - p_{L_i}}{x_i} \right) - f^B \left(\frac{s - p_{L_i}}{1 + x_i} \right) \right] + r p_{H_i} \left[f^B \left(\frac{p_{H_i} - p_{L_i}}{x_i} \right) \right] \tag{28}$$

$$\frac{\partial \pi_R}{\partial p_{H_i}} = r N^B_{H_i} - r p_{H_i} \left[f^B \left(\frac{p_{H_i} - p_{L_i}}{x_i} \right) \right] + r p_{L_i} \left[f^B \left(\frac{p_{H_i} - p_{L_i}}{x_i} \right) \right] \tag{29}$$

$$\frac{\partial \pi_R}{\partial r} = \sum_i \left[p_{L_i} N^B_{L_i} + p_{H_i} N^B_{H_i} \right] \tag{30}$$

$$\frac{\partial \pi_R}{\partial x_i} = r p_{L_i} \frac{\partial N^B_{L_i}}{\partial x_i} + r p_{H_i} \frac{\partial N^B_{H_i}}{\partial x_i} \tag{31}$$

Then the constrained first order conditions are:

$$\frac{\partial \mathcal{L}}{\partial p_{L_i}} = r \frac{\partial \pi_R}{\partial p_{L_i}} + \lambda_i \frac{\partial g_i(p_{L_i}, p_{H_i}, r)}{\partial p_{L_i}} \tag{32}$$

$$\frac{\partial \mathcal{L}}{\partial p_{H_i}} = r \frac{\partial \pi_R}{\partial p_{H_i}} + \lambda_i \frac{\partial g_i(p_{L_i}, p_{H_i}, r)}{\partial p_{H_i}} \tag{33}$$

$$\frac{\partial \mathcal{L}}{\partial r} = \frac{\partial \pi_R}{\partial r} + \lambda_i \frac{\partial g_i(p_{L_i}, p_{H_i}, r)}{\partial r} \tag{34}$$

$$\frac{\partial \mathcal{L}}{\partial x_i} = \frac{\partial \pi_R}{\partial x_i} + \lambda_i \frac{\partial g_i(p_{L_i}, p_{H_i}, r)}{\partial x_i} \tag{35}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_i} = g_i(p_{L_i}, p_{H_i}, r) \tag{36}$$

[25]Not including constraints.

Assuming that our pick-up probabilities, $x_i \in [0,1]$ are exogenous, Lagrange's method guarantees us a local maximum for maximizers $(p^*_{L_i}, p^*_{H_i}, r^*, \lambda^*_i)$ if and only if the Hessian matrix evaluated at $(p^*_{L_i}, p^*_{H_i}, r^*, \lambda^*_i)$ is negative.

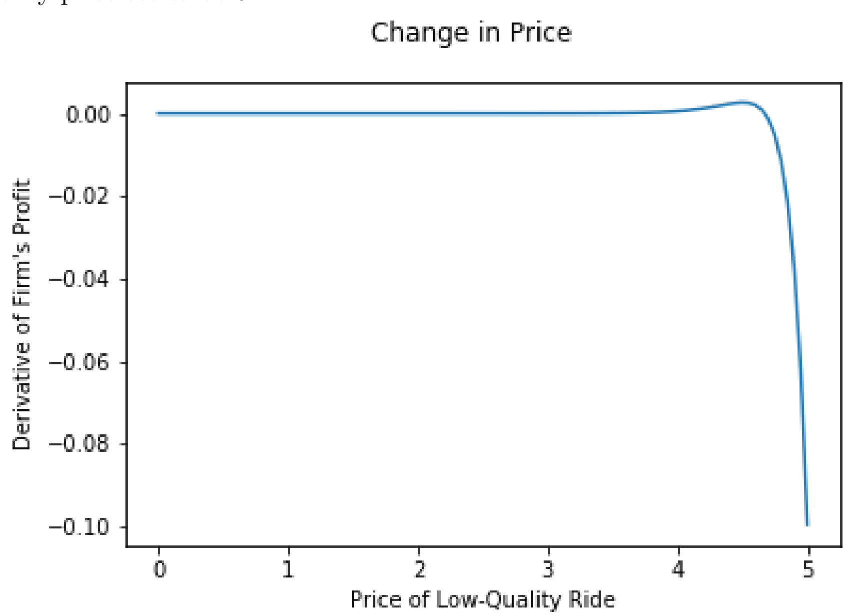
I solve for the arguments of the maximum numerically with the scientific computing package SciPy in Python. [26]

5 Numerical Results

There are two main dimensions along which the numerical results could reveal deeper intuitions about the ride-sharing problem. For one, it is necessary to see if there are nonlinearities in the solution to the optimization problem that could lead to local optima that are not in fact global optima. And further, my overarching hope is that the numerical solutions reveal how the firm should optimally manipulate the quality of the lower-quality service in order to maximize profit, in the style of Proposition 2.6.

From the numerical computation of optimal profit structure I find several results. Figure 3 confirms that the price of the low-quality service would be strictly lower than that of the high-quality service, and when $\frac{\partial \pi}{\partial p_L} = 0$ and $\frac{\partial^2 \pi}{\partial p_L^2 < 0}$, ceteris peribus, the firm would not want to continue increasing the price of the low-quality service. This result is not surprising.

Figure 3: Diagram depicting how the firm's profit changes as the price of the low-quality service increases, with high-quality price set to be 5.

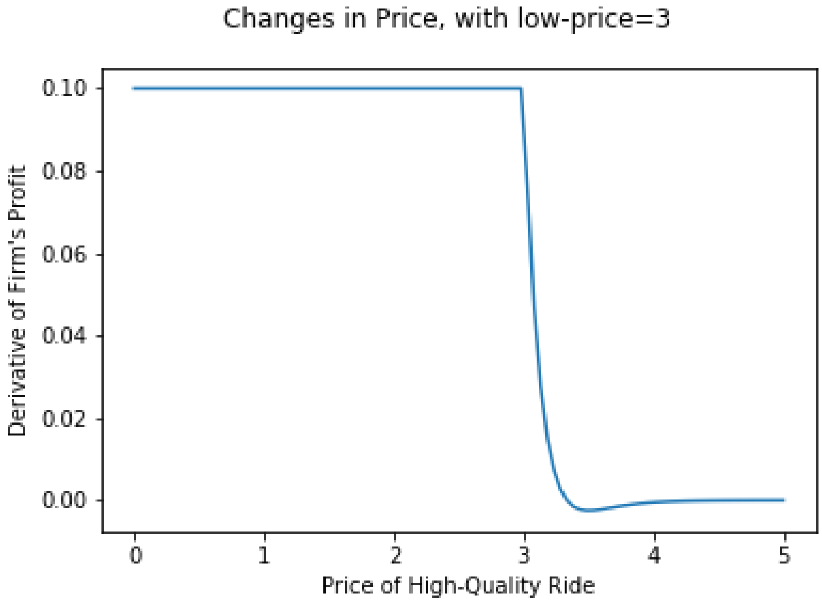


In Figure 4, since the derivative of profit with respect to price $\frac{\partial \pi}{\partial p_H}$ is weakly positive, The firm will

[26] Full code is available at: https://github.com/lmeninato/ECON396

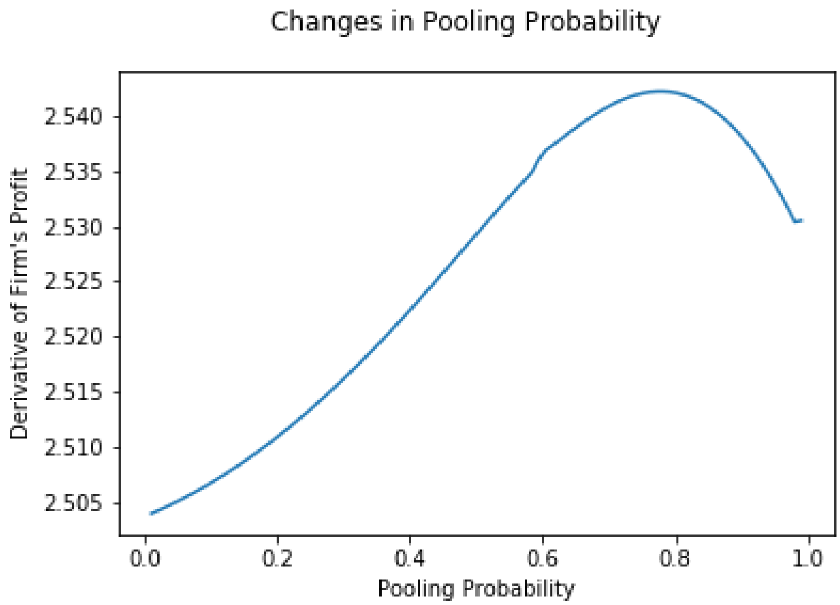
choose the highest p_H possible minus information rent, since the monopolist does not know ex ante which service a buyer would prefer.

Figure 4: Diagram depicting how the firm's profit changes as the price of the high-quality service increases, with low-quality price set to be 3.



Lastly, Figure 5 shows that it is optimal for the firm to pool as many low-type rides together as possible, since $\frac{\partial \pi}{\partial x_i} > 0$ over the domain of x_i. In light of the contract-theoretic background work in section 2, this result is useful for working within the two-sided market framework developed by Rochet and Tirole (2006). The difficulty in applying vertical price differentiation to two-sidedness, is that generally platforms have a simple approach to finding the optimal profit structure. First the platform must maximize the volume of transactions, and then they find optimal prices in accordance with the Lerner index. But here this was demonstrated to *not* be the case. Intuitively, the reason for this is that the firm could maximize the volume of transactions by having very inefficient pooling, such that x_i is close to 0, but contract theory in contrast would have that $x_i = 1$, since at that point the quality of low-type service has been maximally reduced and the low-type consumer surplus has been optimally extracted. With a continuous distribution of types, the Revelation Principle cannot be used to simplify this problem, but the numerical results confirm this intuitive result.

Figure 5: Diagram depicting how the firm's profit changes as the probability of a low-quality service being pooled increases.



Recalling Definition 2.1, it can be confirmed that this problem does involve a two-sided market. The model has been built to reflect that ride-sharing platforms currently charge usage fees to the driver, and not the buyer, of about 10% of each transaction. I verify that the profit does change if instead each side of the market is charged 5%. This confirms that the model satisfies Definition 2.1 (Two-Sidedness).

5.1 Monopolistic Screening

The monopolist solution to the first degree price discrimination problem in Proposition 4.3 is to offer consumers the choice between two price-quantity bundles: $(N^B_{L_i}, p_{L_i})$ and $(N^B_{H_i}, p_{H_i})$. From there, the firm can either reduce the quality of the low-type option by increasing the corresponding x for any location or maximize the volume of interactions. If the quality-per-dollar of the low type is higher than that of the high-type, then it can be optimal for high-type consumers to purchase the low-type option. In Proposition 2.2, I note that the optimal price structure for a two-sided market is obtained by maximizing the volume of transactions $V(p)$, given an aggregate price level $p^B + p^S = p$.

$$V(p) = \max\{n^B(p^B, p^S)n^S(p^B, p^S) \mid p^B + p^S = p\} \tag{37}$$

However, we must also note that in the model I outline in Section 3, the ability for the firm to alter the low-type quality is in conflict with maximizing the volume of transactions. The only mechanism the firm has to alter product quality, by either having low-type drivers pool as many rides together as possible or by having the same drivers ride as little as possible, directly impacts the volume of transactions. We can view the volume of transactions as follows:

$$V(p) = \max\{n^H(p^H, p^L, p^S)n^S(p^H, p^L, p^S) + n^L(p^H, p^L, p^S)\left[n^S(p^H, p^L, p^S) - n^H(p^H, p^L, p^S)\right]g(\hat{x})\} \quad (38)$$

Where $g(\hat{x}) \leq 1$ is a function that adjusts the number of low-type drivers. Given some data points drawn from some transformed distribution of $F^i(d)$, we could receive estimates of what $g(\hat{x})$ is. Then, similar to equation 9, we could then find the optimal price structure using the Lerner index compared to the elasticity of demand. The optimal price structure would depend on the following condition:

$$-\frac{1}{p} = \eta \quad (39)$$

where $\eta = \frac{-pV'(p)}{V(p)}$. This task is relatively straightforward if we assume that the firm is "cooperating", and thus drivers are encouraged to pool as efficiently as possible. This works if we could assume that the high-type quality is greater than or equal to the low type quality, but this is not necessarily true. For instance, consider the case that there is only one low-type buyer and an arbitrary amount of high-type buyers. Then, the low-type buyer would receive service s, but since there are no other low-type buyers, the service would not be pooled, thus they would receive utility $s - d - p_L \geq s - d - p_H$. The inequality becomes strict if we assume that $p^H > p^L$. So unfortunately, we have to consider the case where the quality-per-dollar of the low-type is higher than that of the high-type. In this case, we could consider the case that drivers do not optimally pool drivers, and instead their efficiency is tweaked around such as to maximize platform profit.

6 Conclusion

The rapid development of ride-sharing technologies threatens to drastically change how transportation functions, both as a private service and a public utility. While long-distance travel might not necessarily be usurped by this new technology, it appears likely that the final-leg of transportation will be dominated by ride-sharing platforms (Chong et al., 2011). Efficient ride-pooling models will increase ridership, creating welfare for consumers and drivers, since less drivers will be needed to accommodate the same volume of ridership. My model reveals three key intuitions about a platform in which sellers provide two vertically differentiated services to riders (buyers). First, I numerically confirm that this market satisfies Definition 2.1 of a two-sided market. Also, rather than maximize the volume of transactions directly, it is preferable that the platform maximize the volume of transactions subject to riders being pooled as much as possible, reducing the quality of the low-type service. This result confirms the contract-theoretic intuition set forth in section 2. Lastly, I give an outline of how the optimal prices could be found using Lerner index in equations 38 and 39.

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A Appendix

A.1 Derivation of demand functions for the case in equation 1

In equation (1) I only stated the demand functions for the case where there is non-zero demand for good 1. Since, if $s_2/p_2 \geq s_1/p_1$, then good 2 is always preferred to good 1, because the quality-per-dollar of good 2 is higher than good 1 (Tirole, 2015). Then the demand for good 2 would be N times the fraction of consumers who would purchase the good, or $1 - F(\frac{p_2}{s_2})$. Thus the demand for good 2 is:

$$D_2(p_1, p_2) = N\left[1 - F\left(\frac{p_2}{s_2}\right)\right]$$

In the case where some consumers buy the low quality good and some consumers buy the high quality good, we have that consumers will buy the high quality good if their taste parameter is great enough, so $\theta s_2 - p_2 \geq \theta s_1 - p_1$, or $\hat{\theta} = \frac{p_2 - p_1}{s_2 - s_1}$. Thus $1 - F(\hat{\theta})$ proportion of consumers have taste parameter greater than $\hat{\theta}$. Further, consumers with taste parameter less than $\hat{\theta}$ will purchase lower quality good unless their taste parameter is low enough such that $\theta s_1 - p_1 \leq 0$ or $\theta = \frac{p_1}{s_1}$. Therefore, the demand functions are:

$$D_2(p_1, p_2) = N\left[1 - F\left(\frac{p_2 - p_1}{s_2 - s_1}\right)\right]$$

$$D_1(p_1, p_2) = N\left[F\left(\frac{p_2 - p_1}{s_2 - s_1}\right) - F\left(\frac{p_1}{s_1}\right)\right]$$

A.2 Two-Sided Markets Background

Figure 6: Diagram from Rochet and Tirole (2003) outlining various two-sided markets. Borrowed with permission from authors.

Product	Loss leader/break-even segment/subsidized segment	Profit-making segment/ subsidizing segment
	Software	
Video games	consumers (consoles)	software developers
Streaming media	consumers	servers
Browsers	users	Web servers
Operating systems (Windows; Palm, Pocket PC)	application developers (development tools, support, functionality, . . .)	clients
Text processing	reader/viewer	writer
	Portals and media	
Portals	"eyeballs"	advertisers
Newspapers	readers	advertisers
(Charge-free) TV networks	viewers	advertisers
	Payment systems	
Credit and differed debit cards (Visa, MasterCard, Amex, . . .)	cardholders	merchants
Online debit cards	merchants	cardholders
	Others	
Social gatherings	celebrities in social happenings	other participants
Shopping malls	consumers (free parking, cheap gas)	shops
Discount coupon books (Want Advertiser)	consumers	merchants
(Legacy) Internet	Web sites	dial-up consumers
Real estate	buyers	sellers

Proposition A.1. *(Armstrong, 2006) Given a platform that attracts two groups with the following utilities:*

$$u_1 = a_1 n_2 - p_1 \ ; \ u_2 = a_2 n_1 - p_2$$

where p_i are the platform's prices to the two groups and a_i reflects the utility from interacting with the other group. If f_i is the platform per-agent cost and $\phi_i(u_i)$ is an increasing function, then the profit-maximizing prices are:

$$p_1 = f_1 - a_1 n_2 + \frac{\phi_1(u_1)}{\phi_1(u_1} \ ; \ p_2 = f_2 - a_2 n_1 + \frac{\phi_2(u_2)}{\phi_2(u_2}$$

Proof. Let the number of individuals that join a group be $n_i = \phi_i(u_i)$. Then the firm's profit is: $\pi = n_1(p_1 - f_1) + n_2(p_2 + f_2)$. Since $p_i = a_i n_j - u_i$, we can write:

$$\begin{aligned}\pi &= n_1(a_1 n_2 - u_1 - f_1) + n_2(a_2 n_1 - u_2 - f_2) \\ &= \phi_1(u_1)(a_1 n_2 - u_1 - f_1) + \phi_2(u_2)(a_2 n_1 - u_2 - f_2)\end{aligned}$$

Thus the platform's profits in terms of the groups' utilities is:

$$\pi(u_1, u_2) = \phi_1(u_1)(a_1\phi_2(u_2) - u_1 - f_1) + \phi_2(u_2)(a_2\phi_1(u_1) - u_2 - f_2) \tag{40}$$

Then the first order conditions are:

$$\frac{\partial \pi}{\partial u_1} = \phi_1^{'}(u_1)(a_1\phi_2(u_2) - u_1 - f_1) + \phi_2(u_2)a_2\phi_1^{'}(u_1) = 0$$

$$\frac{\partial \pi}{\partial u_2} = \phi_2^{'}(u_2)(a_2\phi_1(u_1) - u_2 - f_2) + \phi_1(u_1)a_1\phi_2^{'}(u_2) = 0$$

We can use the first order conditions to finish the proof, by noting that $p_1 = a_1 n_2 - u_1$ and likewise for p_2. □

Table 2: Summary of Two-Sided Markets Notation

Symbol	Description
A^i	Membership/fixed fee to agent i
a^i	Per-transaction usage fee to agent i
f_i	Marginal cost of transaction
b^i	Gross surplus to agent i
N^i	Number of agents of type i
p^i	Price charged to agent i

Proposition A.2. *(Rochet and Tirole, 2006) Given utility, $U^i = (b^i - a^i)N^j + B^i - A^i$, the condition for the optimal price structure for $p^B + p^S = p$ is:*

$$-\frac{\left(1 - \frac{\partial D^B}{\partial N^S} \cdot \frac{\partial D^S}{\partial N^B}\right)}{p - c} = \frac{\frac{\partial D^B}{\partial p^B}}{D^B} + \frac{\frac{\partial D^B}{\partial p^B}\frac{\partial D^S}{\partial N^B}}{D^S} = \frac{\frac{\partial D^S}{\partial p^S}}{D^S} + \frac{\frac{\partial D^S}{\partial p^S}\frac{\partial D^B}{\partial N^S}}{D^B} \tag{41}$$

Proof. (Following the outlined proof in Rochet and Tirole (2006)) The number of users that decide to join the platform, for side i, is:

$$N^i = \Pr(U^i \geq 0)$$

Let the per-interaction price be defined as:

$$p^i \equiv a^i + \frac{A^i - C^i}{N^j} \tag{42}$$

Then we can derive the demand functions by considering the proportion of side i whose utility U^i is greater than the price they face, p^i.[27]

$$D^i(p^i, N^j) \equiv N^i = \Pr(b^i + \frac{B^i - C^i}{N^j} \geq p^i) \tag{43}$$

This system of equations has a solution for N^i, where N^i for $i \in \{B, S\}$ is a function of $n^i(p^i, p^j)$. So given the expression for the platform's profit:

[27] Same approach as in Tirole (2015).

$$\pi = (A^B - C^B)N^B + (A^S - C^S)N^S + (a^B + a^S - c)N^B N^S \tag{44}$$

which can be constructed by summing up the platform's profit from the fixed membership fees, $A^i - C^i$, with the variable fees $a^B + a^S - c$ times the volume of interactions $N^B N^S$. Re-writing equation 42 we have that $N^j \equiv \frac{A^i - C^i}{p^i - a^i}$, therefore we can write equation 44 as:

$$\begin{aligned}\pi &= \frac{(A^B - C^B)(A^S - C^S)}{p^S - a^S} + \frac{(A^S - C^S)(A^B - C^B)}{p^B - a^B} \\ &+ \frac{(a^B + a^S - c)(A^B - C^B)(A^S - C^S)}{(p^B - a^B)(p^S - a^S)} \\ &= \frac{(A^S - C^S)(A^B - C^B)(p^B + p^S - c)n^B(p^B, p^S)n^S(p^B, p^S)}{(p^B - a^B)(p^S - a^S)} \\ &= (p^B + p^S - c)n^B(p^B, p^S)n^S(p^B, p^S)\end{aligned} \tag{45}$$

Therefore, the optimal price structure is obtained from maximizing the volume of transactions, given the aggregate price level $p^B + p^S = p$ is:

$$V(p) = \max\{n^B(p^B, p^S)n^S(p^B, p^S) \mid p^B + p^S = p\} \tag{46}$$

Therefore, the Lerner index $\eta \equiv \frac{-pV'(p)}{V(p)}$ is computed as follows, for each price:

$$\eta = \frac{\frac{\partial n^B}{\partial p^B}}{n^B} + \frac{\frac{\partial n^S}{\partial p^B}}{n^S}$$
$$\eta = \frac{\frac{\partial n^S}{\partial p^S}}{n^S} + \frac{\frac{\partial n^B}{\partial p^S}}{n^B}$$

Therefore the optimal price structure depends on the following condition:

$$-\frac{1}{p - c} = \frac{\frac{\partial n^B}{\partial p^B}}{n^B} + \frac{\frac{\partial n^S}{\partial p^B}}{n^S} = \frac{\frac{\partial n^S}{\partial p^S}}{n^S} + \frac{\frac{\partial n^B}{\partial p^S}}{n^B} \tag{47}$$

Using the partial derivatives of n^i equation 47 can be re-written to obtain the desired equation 41. □

Proposition A.3. *(Rochet and Tirole, 2006) Assuming a price-setting scenario, so bargaining does not directly take place between buyers and sellers, probability of trade $x(b, a) \in [0, 1)$ and balanced transfers $t^i(b, a^B, a^S)$ such that the expected*[28] *net surplus for side i depends only on the sum of $a^B + a^S = a$, rather than how the usage fees are allocated across the two sides:*

$$\beta^i(a) \equiv E[(b^i - a^i]x(b, a) + t^i(b, a^B, a^S)]$$

If buyers will buy if their benefit is greater than or equal to $\hat{b}^B$, then at the optimal per-transaction charge the following condition must hold:

$$\frac{dv}{da} = E_{b^S}\left[-f(\hat{b}^B(a - b^S))\frac{\partial \hat{b}^B}{\partial a}\left[a - c + \frac{1 - F^B(\hat{b}^B(a - b^S))}{f^B(\hat{b}^B(a - b^S))}\right]\right] = 0 \tag{48}$$

[28]The expectation is drawn from a *product* distribution, since β is the expectation of a random variable drawn from the distribution $F^B \times F^S$.

Proof. The demand function for side-i can be derived like before:

$$D^i(p^B, p^S) = \Pr\left[E\left[b^i x(b,a) + t^i(b, a^B, a^S)\right] - a^i X + \frac{B^i - A^i}{N^j} \geq 0\right] \tag{49}$$

with $X \equiv E[x(b,a)]$ and per-transaction price:

$$p^i = \frac{A^i - C^i}{N^j} + a^i X - E\left[b^i x(b,a) + t^i(b, a^B, a^S)\right] \tag{50}$$

then the platform's profit is:

$$\begin{aligned} \pi &= \sum_i (A^i - C^i)N^i + (a-c)XN^B N^S \\ &= [p^B + p^S + E[(b^B + b^S - c)x(b,a)]]n^B n^S \end{aligned} \tag{51}$$

Let the average surplus per interaction be $v(a)$, such that $v(a) \equiv E[(b^B + b^S - c)x(b,a)]$. Then define a function $\hat{b}^B(a - b^S) = t$ such that $t = t^S = -t^B$ is the price charged to consumers, then the sellers set t such that,

$$t = \arg\max_t \left\{ \frac{t - (a - b^S)}{1 - F^B(t)} \right\} \tag{52}$$

Total usage surplus is then:

$$v(a) = E\left[\int_{\hat{b}^B(a-b^S)}^{\infty} (b^B + b^S - c) dF^B(b^B)\right] \tag{53}$$

Differentiating with respect to a gives the desired result. ☐