Applied Quantitative Investment Management

Lecture 2: Stylized Market Facts

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Agenda

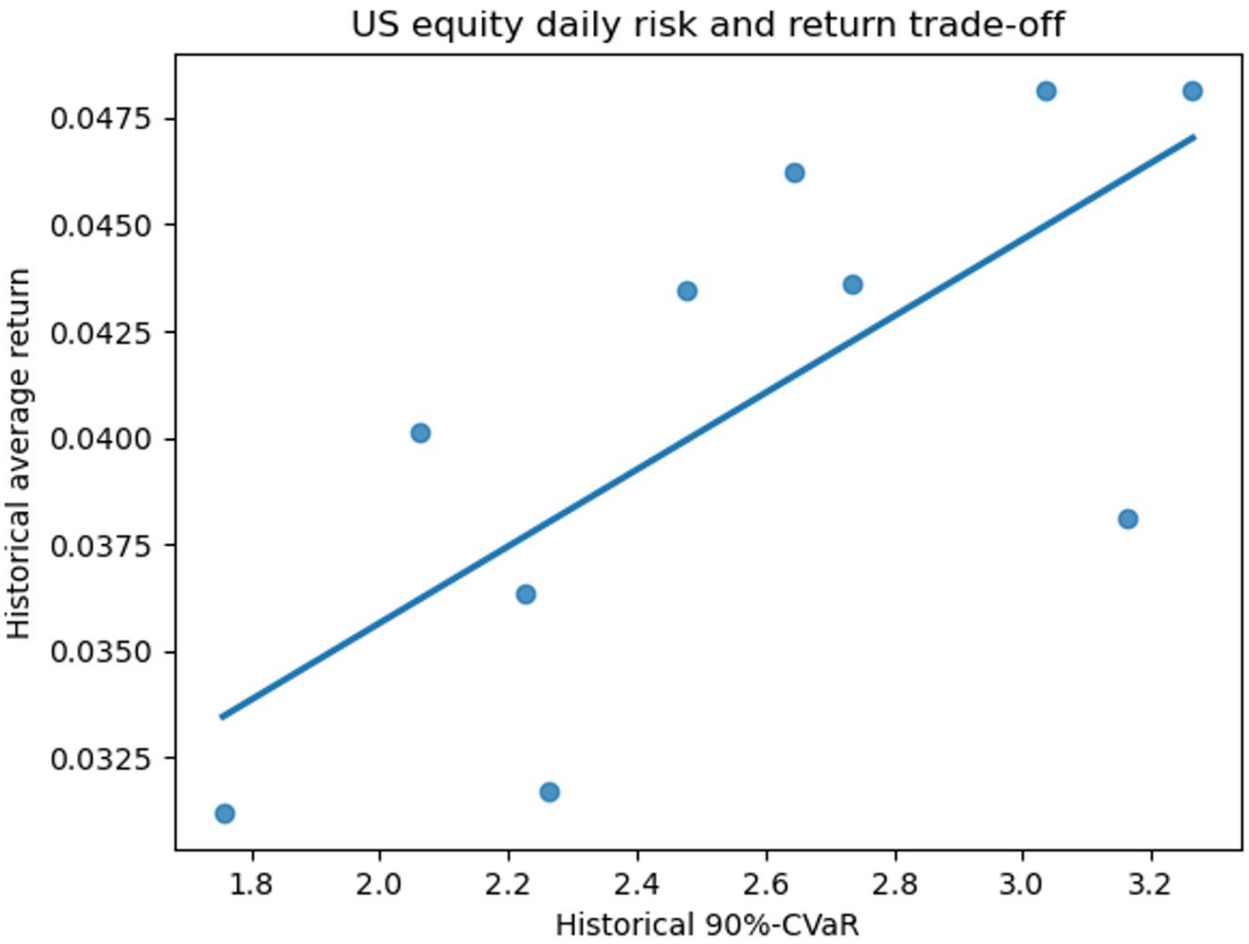
- Chapter 2 walkthrough, including accompanying Python code.
- Risk and return trade-off (Section 2.1)
- Risk clustering (Section 2.2)
- Introduction to options and the volatility risk premium (Section 2.3)
- Skewness, kurtosis, and other interesting insights (Section 2.4)
- Implications for modeling and analysis (Section 2.5)
- Naive backtesting of CVaR versus variance optimization (Section 2.6)

The Capital Asset Pricing Model (CAPM)

$$\mathbb{E}\left[R_i\right] = R_f + \beta_i \left(\mathbb{E}\left[R_m\right] - R_f\right)$$

$$\beta_i = \frac{\mathrm{Cov}\left(R_i, R_m\right)}{\mathrm{Var}\left(R_m\right)} = \rho_{i,m} \frac{\sigma_i}{\sigma_m}$$

Generalized CAPM



The daily reality

Index	Mean	Volatility	Skewness	Kurtosis	90%-CVaR
Materials	0.044%	1.502%	-0.020	9.469	2.735%
Energy	0.048%	1.821%	-0.247	13.885	3.265%
Financial	0.038%	1.817%	0.313	17.526	3.163%
Industrial	0.043%	1.338%	-0.163	10.649	2.477%
Technology	0.048%	1.633%	0.273	10.139	3.036%
Consumer Staples	0.031%	0.964%	-0.097	10.945	1.756%
Utilities	0.036%	1.224%	0.207	14.821	2.225%
Health Care	0.040%	1.130%	-0.021	12.033	2.062%
Consumer Discretionary	0.046%	1.428%	-0.226	8.958	2.644%
S&P 500	0.032%	1.221%	-0.153	12.863	2.261%

Table 2.1: Daily return statistics for US equity indices.

The monthly reality

Index	Mean	Volatility	Skewness	Kurtosis	90%-CVaR
Materials	0.859%	6.174%	-0.513	6.429	11.766%
Energy	0.948%	7.517%	-0.541	8.456	13.610%
Financial	0.682%	6.824%	-0.341	9.502	13.157%
Industrial	0.874%	5.725%	-0.782	7.713	10.999%
Technology	0.928%	6.586%	-0.468	5.191	13.118%
Consumer Staples	0.632%	3.700%	-0.713	6.476	7.376%
Utilities	0.731%	4.878%	-0.923	7.487	9.477%
Health Care	0.796%	4.470%	-0.563	6.579	8.594%
Consumer Discretionary	0.935%	5.986%	-0.534	6.575	11.597%
S&P 500	0.613%	4.725%	-0.980	7.597	9.474%

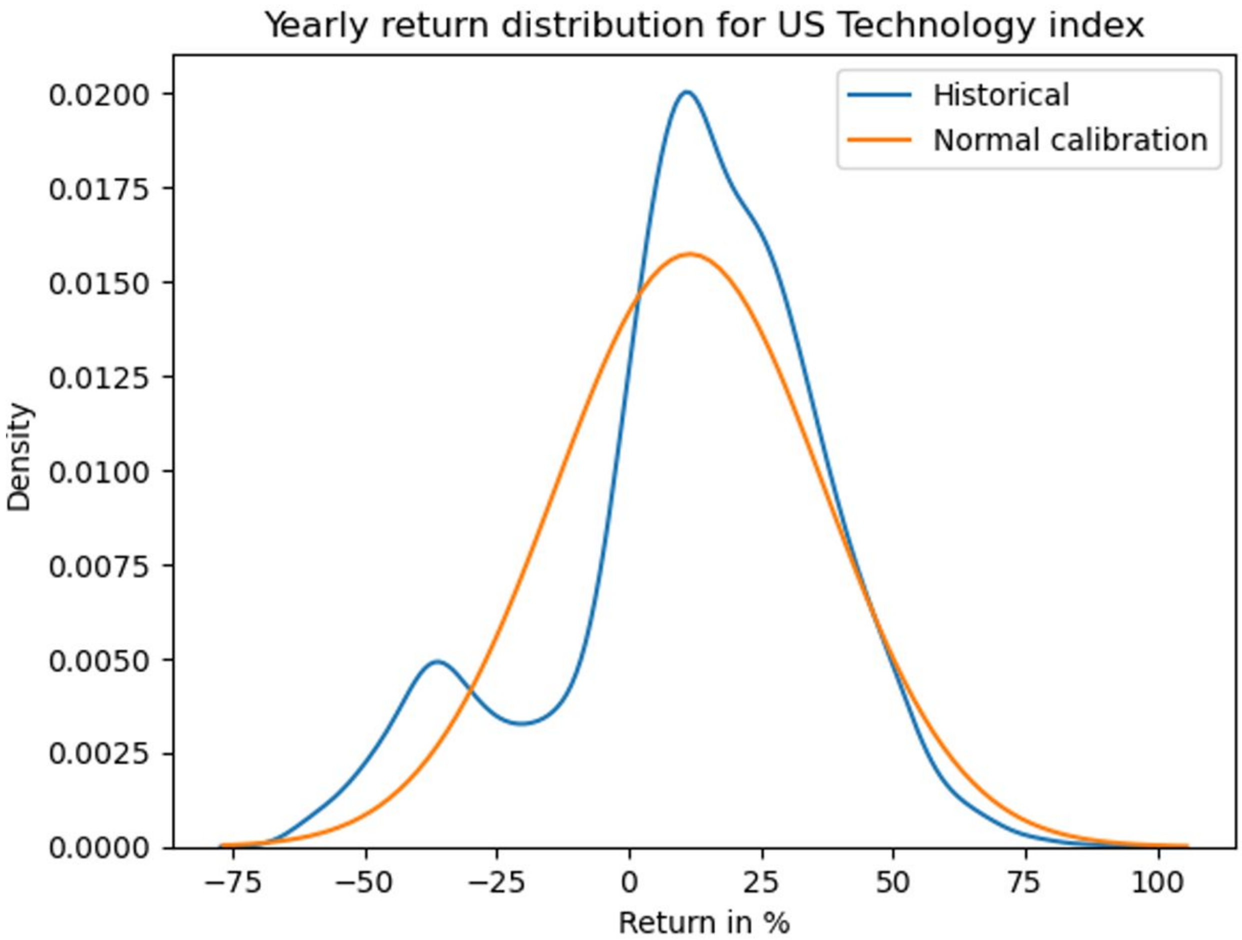
Table 2.2: Monthly return statistics for US equity indices.

The yearly reality

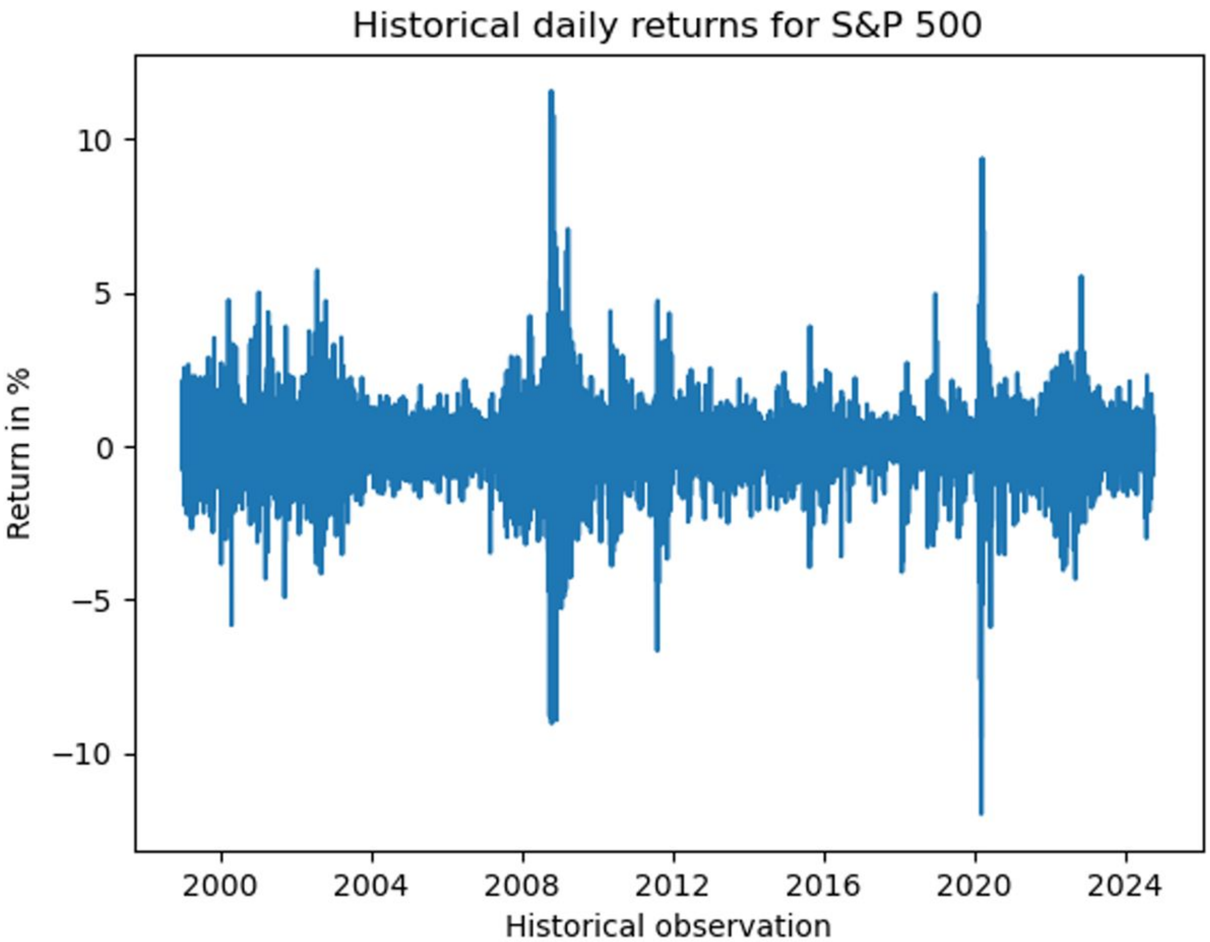
Index	Mean	Volatility	Skewness	Kurtosis	90%-CVaR
Materials	9.481%	19.089%	0.009	4.791	35.347%
Energy	11.041%	26.531%	0.201	3.192	45.323%
Financial	7.888%	23.743%	0.040	5.225	43.570%
Industrial	9.859%	19.233%	-0.130	4.700	37.393%
Technology	11.543%	25.382%	-0.588	3.260	52.990%
Consumer Staples	7.593%	11.225%	-0.694	3.862	24.169%
Utilities	8.326%	15.527%	-0.796	3.822	33.803%
Health Care	9.160%	12.983%	-0.074	3.187	23.285%
Consumer Discretionary	10.797%	19.305%	-0.016	4.200	36.436%
S&P 500	7.041%	16.677%	-0.524	3.857	33.881%

Table 2.3: Yearly return statistics for US equity indices.

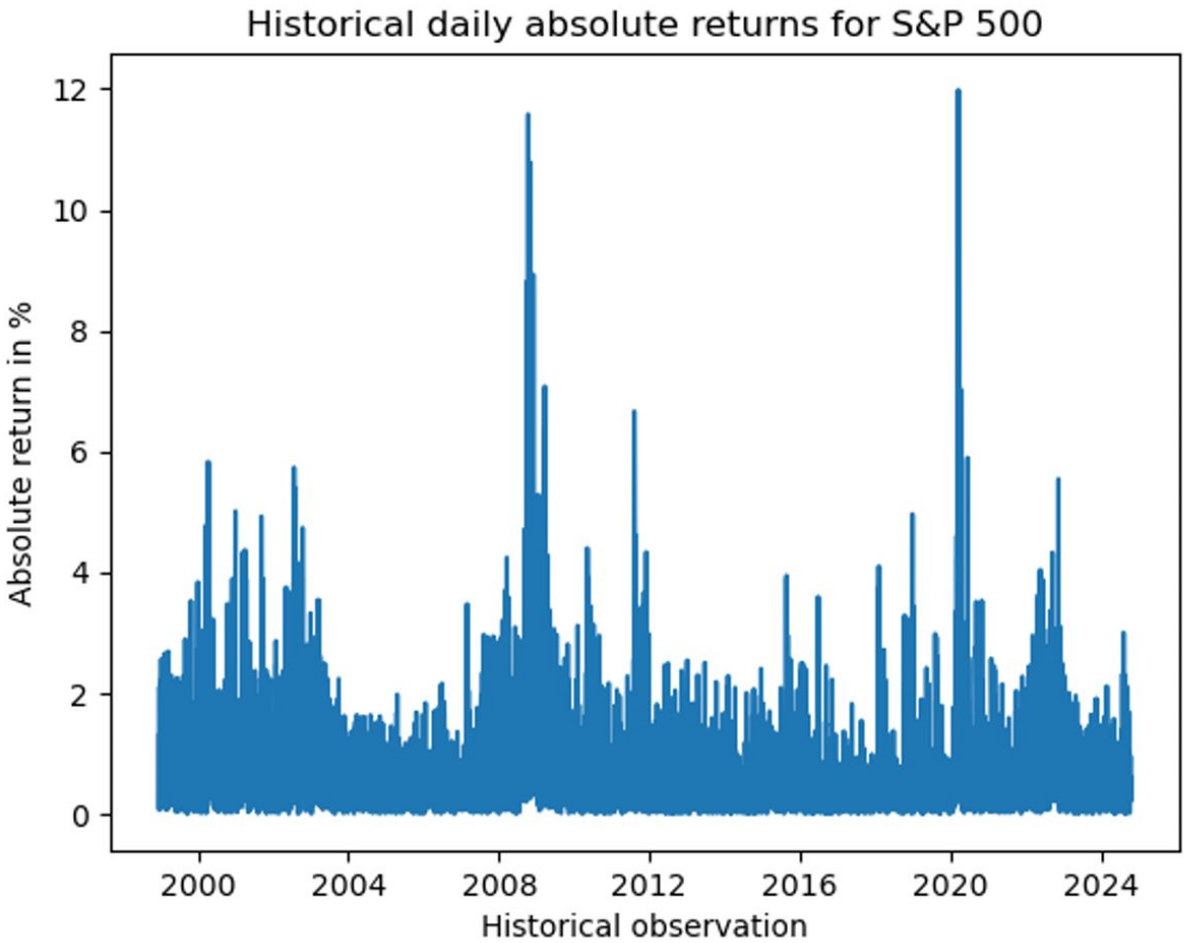
The yearly reality visualized



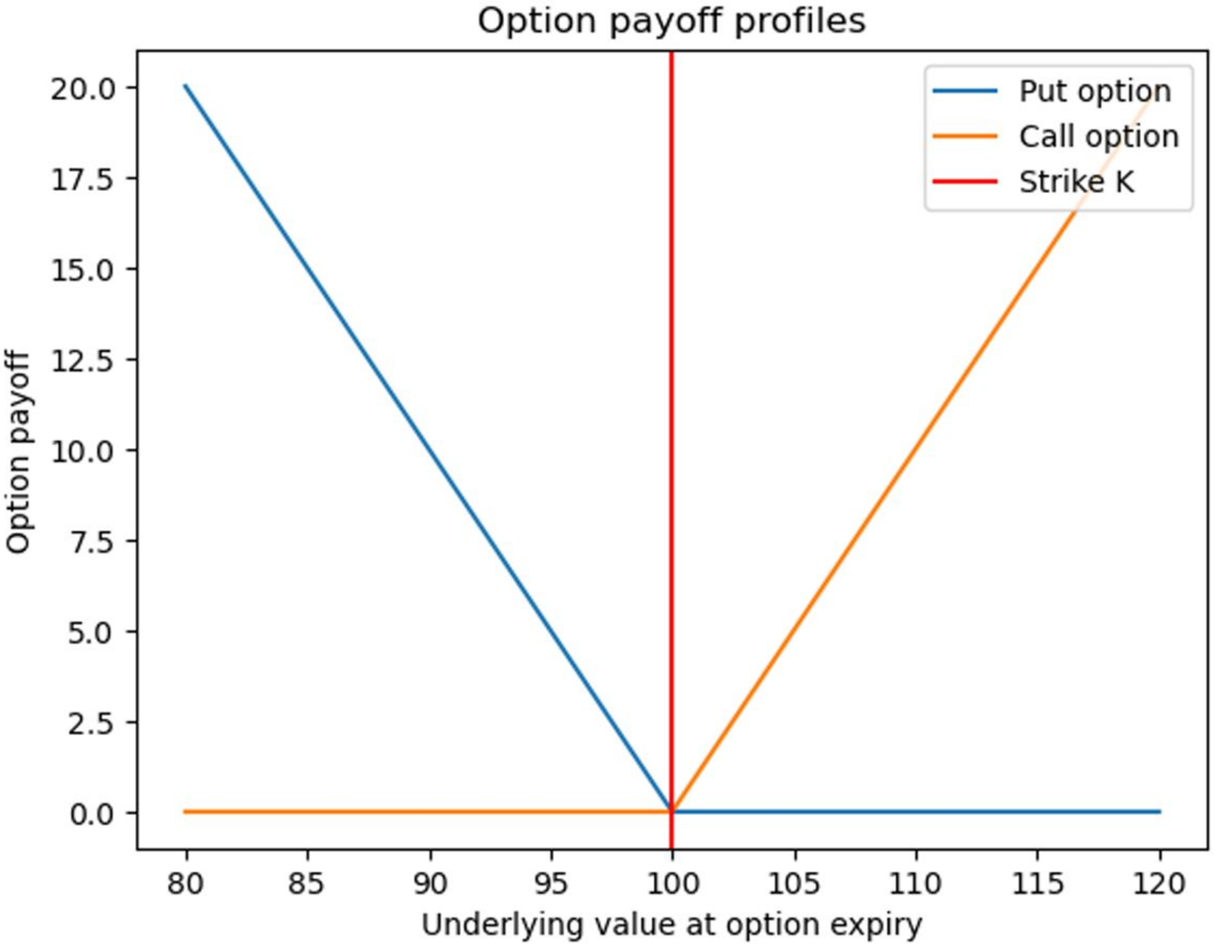
Risk clustering



Risk clustering



Options introduction



Black-Scholes

$$c\left(S_0, T, K, \sigma_{T,K}, r_T, q\right) = e^{-r_T T}\left[F_T N\left(d_1\right) - K N\left(d_2\right)\right],$$

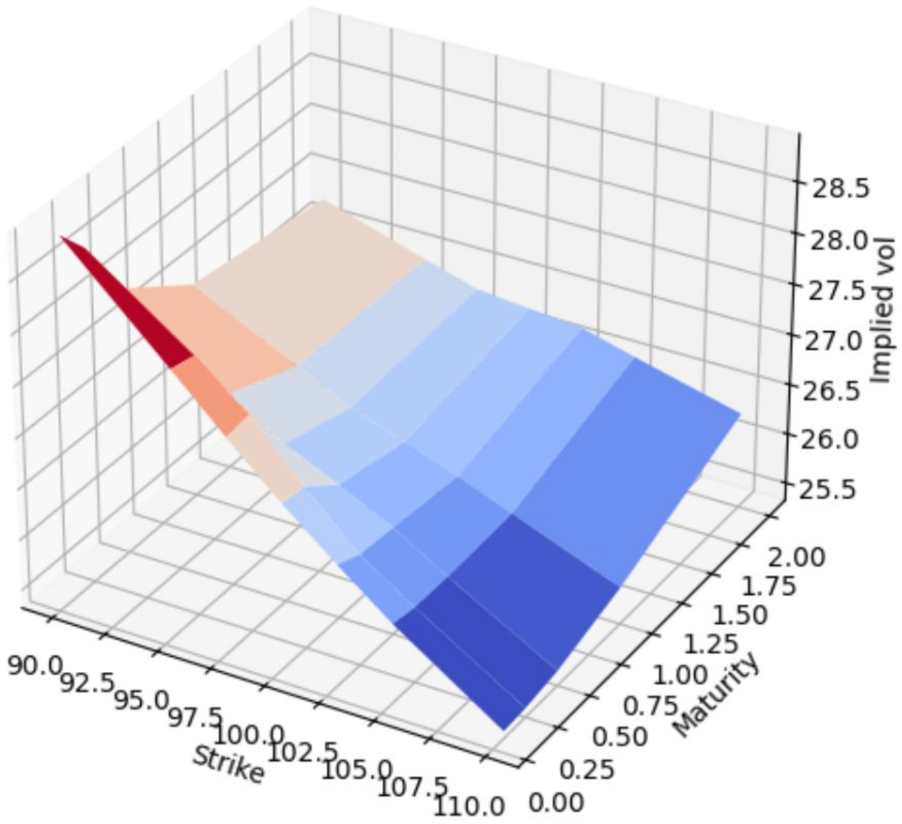
$$p\left(S_0, T, K, \sigma_{T,K}, r_T, q\right) = e^{-r_T T}\left[K N\left(-d_2\right) - F_T N\left(-d_1\right)\right],$$

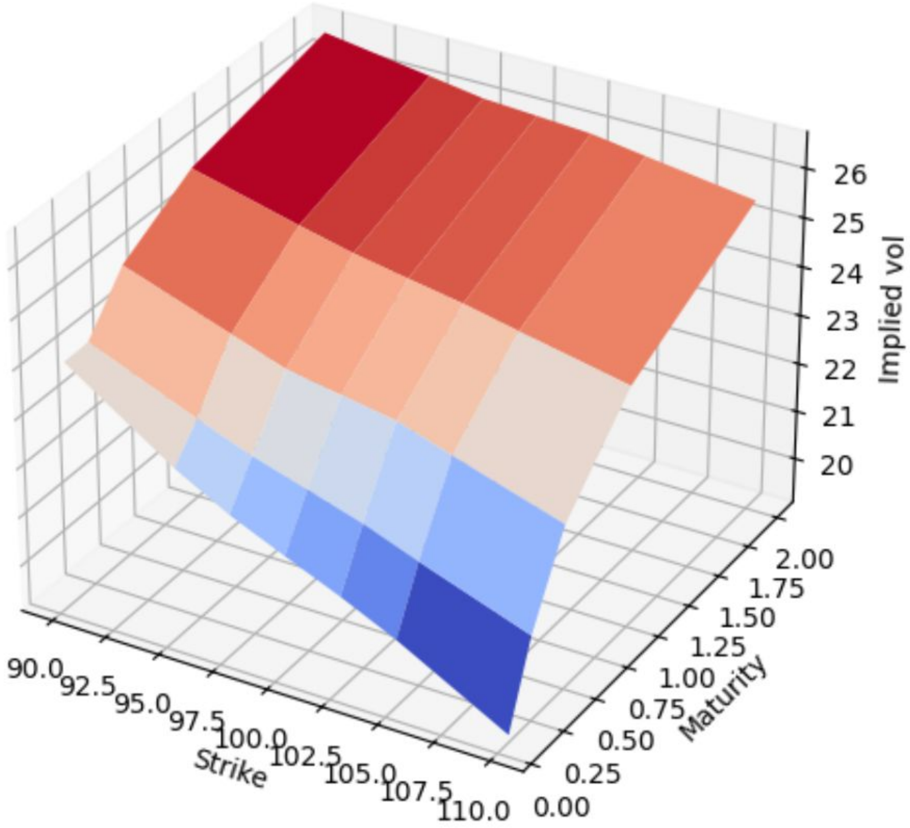
$$d_1 = \frac{1}{\sigma_{T,K}\sqrt{T}}\left[\ln\left(\frac{F_T}{K}\right) + \frac{1}{2}\sigma_{T,K}^2 T\right] \quad \text{and} \quad d_2 = d_1 - \sigma_{T,K}\sqrt{T}.$$

$$F_T = S_0 e^{r_T T} - \sum_{t\in\{t_1,t_2,\ldots,t_N\}} d_t e^{r_t (T-t)}$$

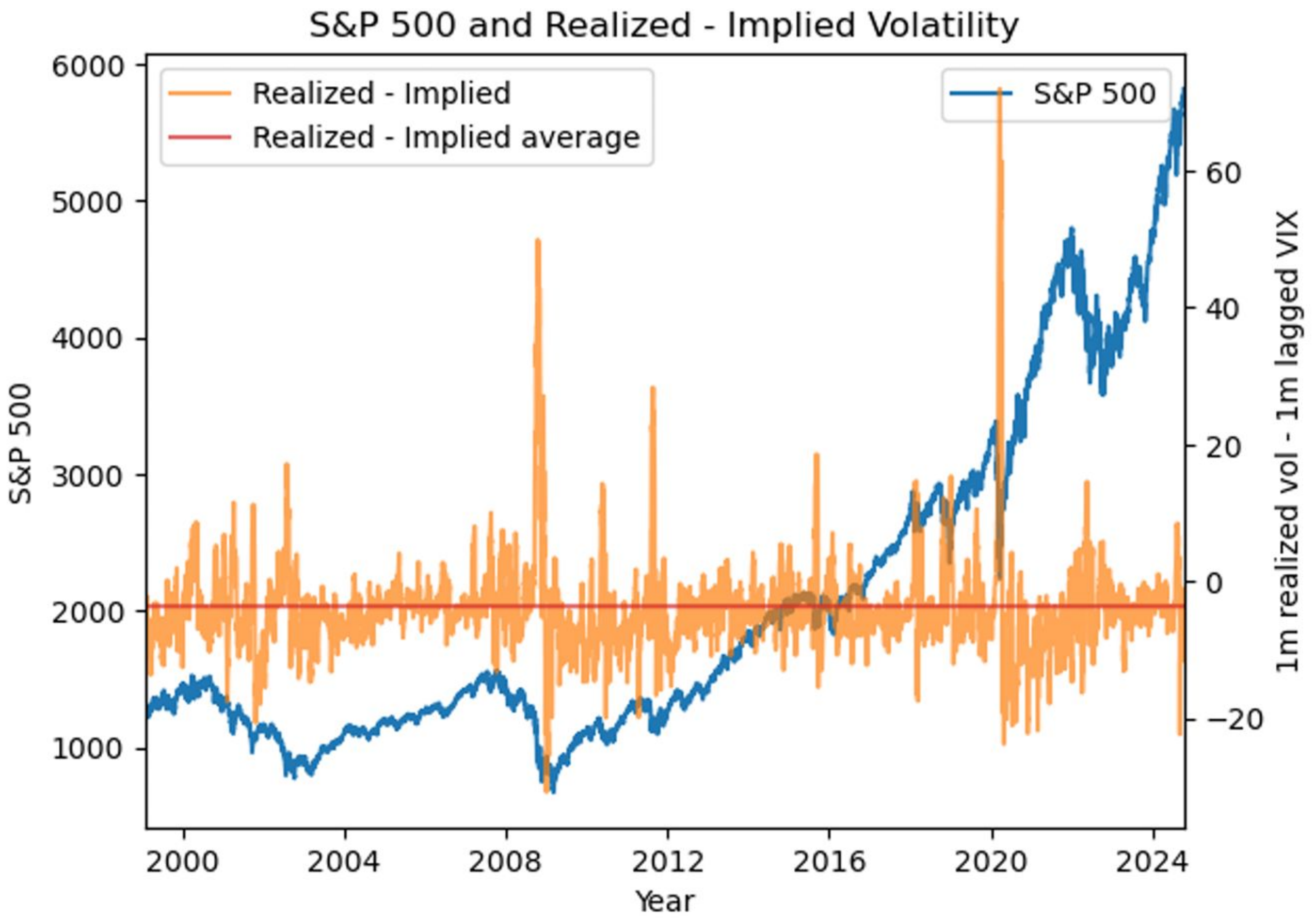
$$F_T = S_0 e^{(r_T - q)T}$$

Implied volatility surfaces

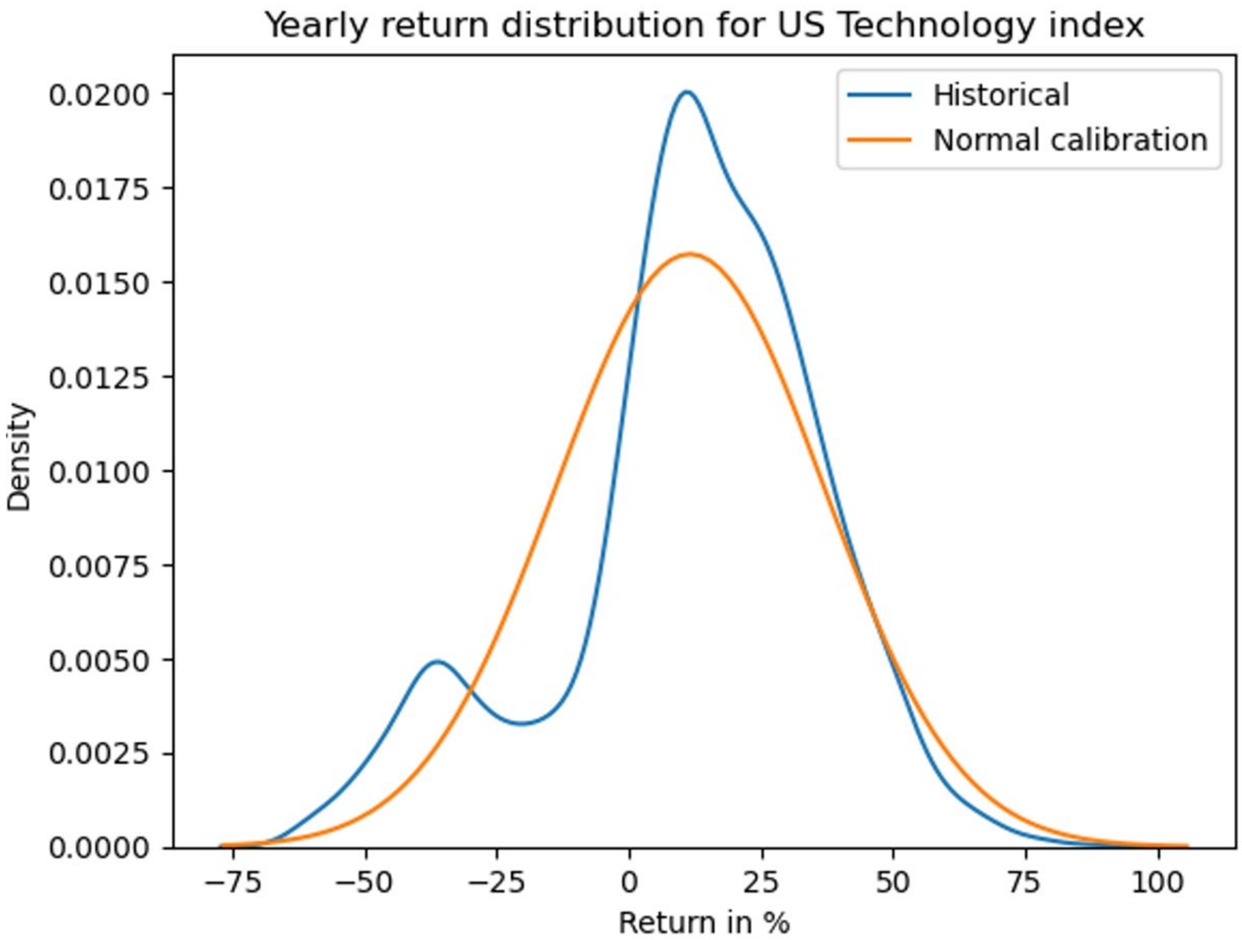


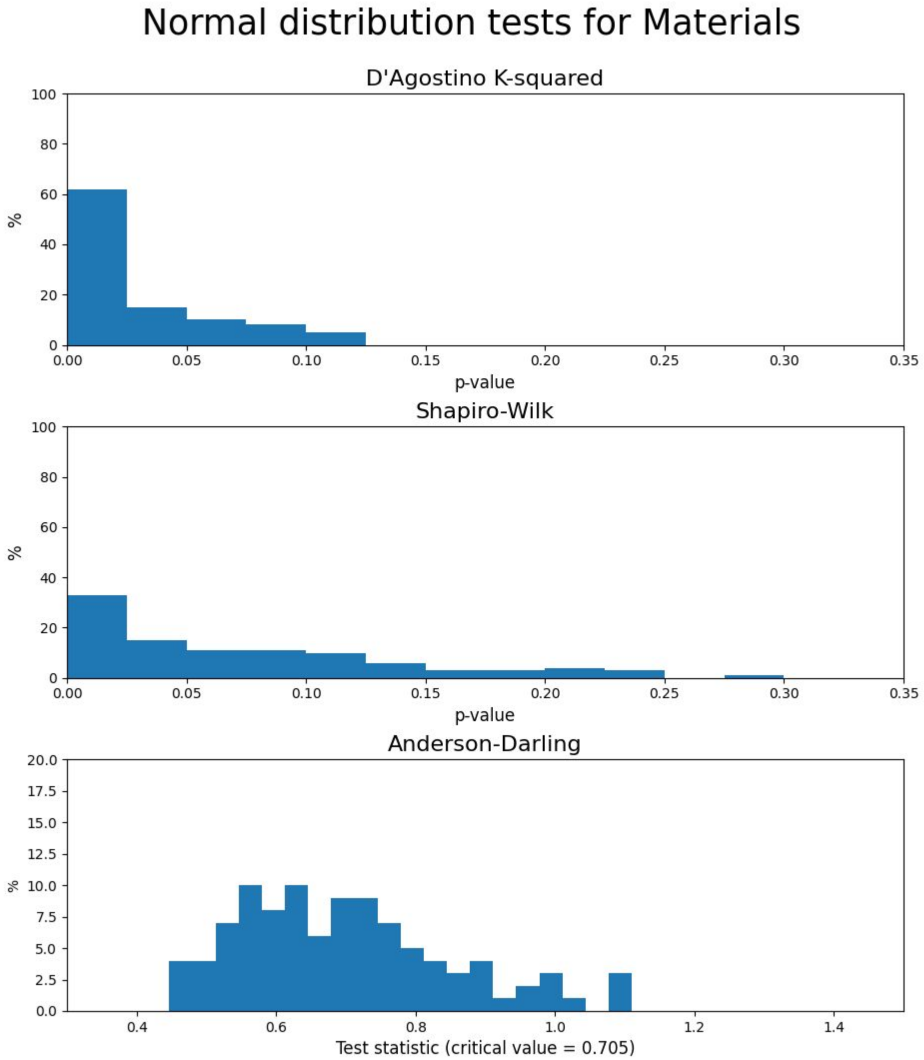


Implied versus realized volatility



Skewness and kurtosis




Normal distribution tests for Materials
D'Agostino K-squared
%
0
20
40
60
80
100
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
p-value
Shapiro-Wilk
%
0
20
40
60
80
100
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
p-value
Anderson-Darling
%
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0
0.4
0.6
0.8
1.0
1.2
1.4
Test statistic (critical value = 0.705)

Tail correlations

	0	1	2	3	4	5	6	7	8	9
0, Materials	100.0	54.8	89.7	88.5	-42.7	54.4	56.0	62.5	85.6	83.2
1, Energy	54.8	100.0	60.8	77.1	-10.9	52.3	65.0	52.4	23.2	63.1
2, Financial	89.7	60.8	100.0	91.7	-40.4	45.8	53.7	72.5	80.1	85.9
3, Industrial	88.5	77.1	91.7	100.0	-30.6	65.1	71.6	68.2	72.8	87.7
4, Technology	-42.7	-10.9	-40.4	-30.6	100.0	-35.0	-35.0	4.7	-29.6	5.4
5, C. Staples	54.4	52.3	45.8	65.1	-35.0	100.0	89.4	7.0	45.3	40.8
6, Utilities	56.0	65.0	53.7	71.6	-35.0	89.4	100.0	12.7	40.2	44.6
7, Health Care	62.5	52.4	72.5	68.2	4.7	7.0	12.7	100.0	58.4	81.7
8, C. Discretionary	85.6	23.2	80.1	72.8	-29.6	45.3	40.2	58.4	100.0	76.7
9, S&P 500	83.2	63.1	85.9	87.7	5.4	40.8	44.6	81.7	76.7	100.0

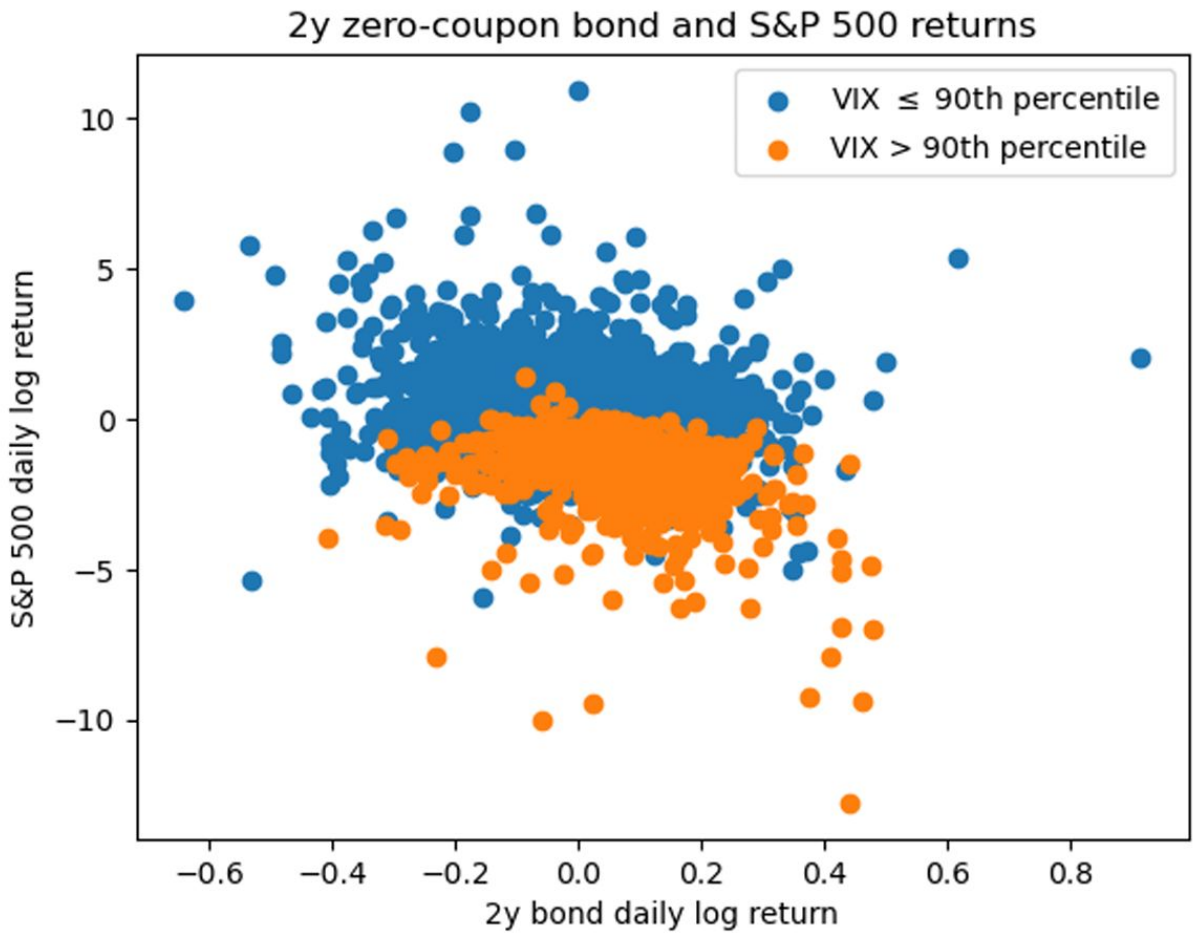
Table 2.4: Correlation matrix for the 10% worst S&P 500 scenarios.

Calm market correlations

	0	1	2	3	4	5	6	7	8	9
0, Materials	100.0	43.4	65.9	83.7	50.5	47.3	32.4	51.3	70.8	76.2
1, Energy	43.4	100.0	34.5	44.0	3.3	16.4	34.9	9.1	5.0	29.1
2, Financial	65.9	34.5	100.0	82.8	42.9	51.3	31.6	61.4	71.2	80.1
3, Industrial	83.7	44.0	82.8	100.0	57.2	55.6	36.9	57.8	75.6	88.0
4, Technology	50.5	3.3	42.9	57.2	100.0	14.6	4.4	51.1	60.7	82.3
5, C. Staples	47.3	16.4	51.3	55.6	14.6	100.0	58.8	55.3	52.7	50.0
6, Utilities	32.4	34.9	31.6	36.9	4.4	58.8	100.0	20.6	17.3	30.7
7, Health Care	51.3	9.1	61.4	57.8	51.1	55.3	20.6	100.0	69.2	72.5
8, C. Discretionary	70.8	5.0	71.2	75.6	60.7	52.7	17.3	69.2	100.0	83.0
9, S&P 500	76.2	29.1	80.1	88.0	82.3	50.0	30.7	72.5	83.0	100.0

Table 2.5: Correlation matrix for the 90% best S&P 500 scenarios.

Equity bond correlation and the VIX



Equity bond correlation and the VIX

	0	1	2	3	4
0, S&P 500	100.0	-25.3	-32.9	-32.6	-46.1
1, 13w bond	-25.3	100.0	29.8	17.9	8.4
2, 2y bond	-32.9	29.8	100.0	93.5	19.4
3, 30y bond	-32.6	17.9	93.5	100.0	18.2
4, VIX	-46.1	8.4	19.4	18.2	100.0

Table 2.6: Correlation matrix conditional on daily VIX changes being above the 90th percentile.

	0	1	2	3	4
0, S&P 500	100.0	-5.4	-19.7	-19.6	-65.8
1, 13w bond	-5.4	100.0	18.2	13.7	1.6
2, 2y bond	-19.7	18.2	100.0	92.9	13.6
3, 30y bond	-19.6	13.7	92.9	100.0	13.4
4, VIX	-65.8	1.6	13.6	13.4	100.0

Table 2.7: Correlation matrix conditional on daily VIX changes being below the 90th percentile.

Stylized market fact conclusions

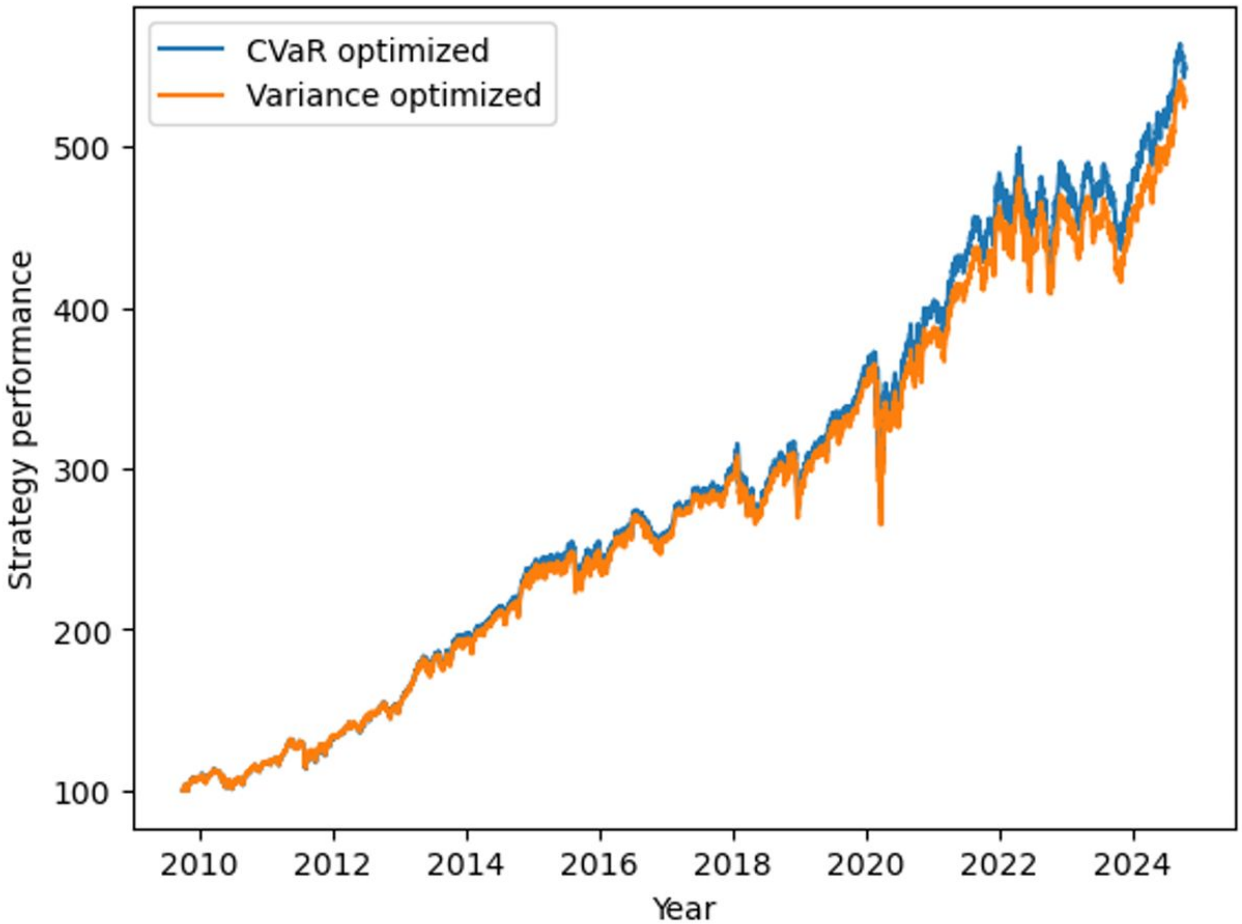
1. Return distributions do not follow nice bell-shaped curves. They are skewed and fat-tailed in complex ways.

2. Cross-sectional asset dependencies are not just linear and constant, even for plain vanilla instruments such as stocks and bonds.

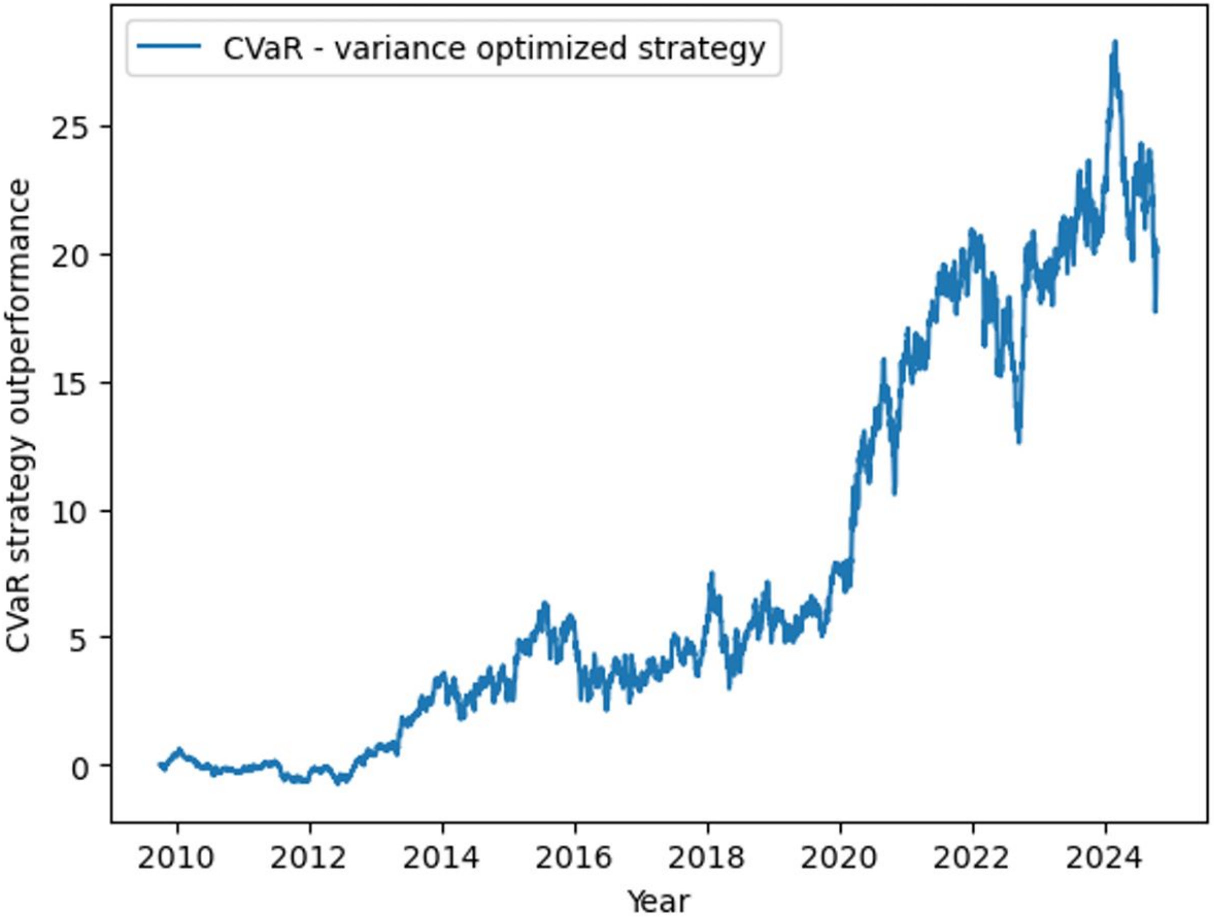
3. There is a tendency for risk to cluster, i.e., periods with high market volatility tend to be followed by periods with continued high volatility and vice versa.

4. There exists a volatility risk premium, which indicates that investors are willing to pay a premium above fair value for convexity and, hence, perceive risk as losses instead of all deviations from the mean.

Naive CVaR versus variance backtesting



Naive CVaR versus variance backtesting



Upcoming lectures

- Market simulation framework that focuses on simulating risk factors (Chapter 3) and performing subsequent pricing (Chapter 4).
- Simulation methods: Time- and State-Dependent Resampling as well as generative machine learning (Chapter 3).
- Better backtesting with synthetic data (Chapter 3).

Useful references

- The Normal Distribution Myth SSRN article: **https://ssrn.com/abstract=5283255**

- Black-Scholes model: **https://en.wikipedia.org/wiki/Black_model**

- Kalinchenko, K., S. Uryasev, and R. T. Rockafellar (2012). “Calibrating risk preferences with the generalized capital asset pricing model based on mixed conditional value-at-risk deviation”. The Journal of Risk 15.1.