Bond Pricing

How should we value securities such as stocks and bonds?

- Let's look at a really simple security:

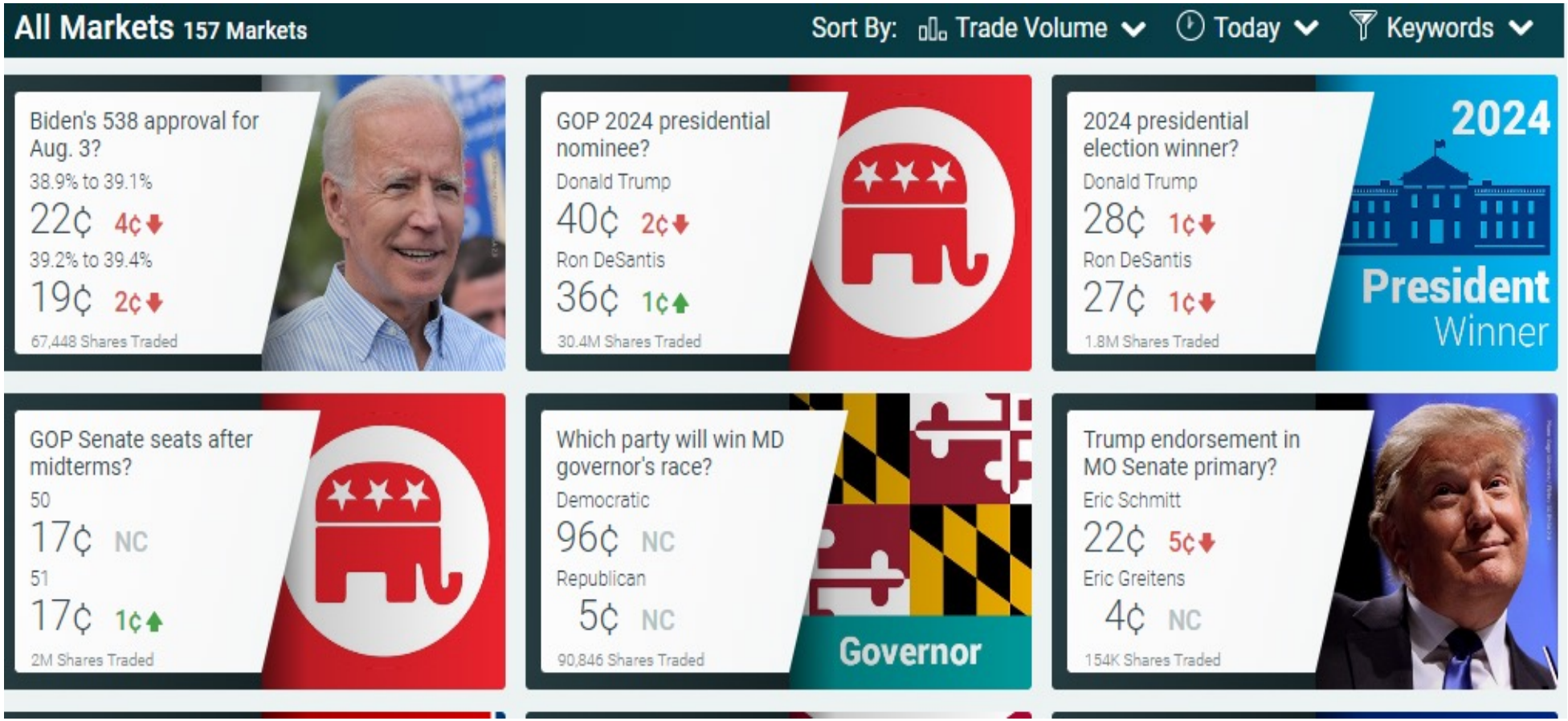
- Suppose I offered you a security with the following cash flows:
  - $1.00 if **Barbie** wins the Oscar for Best Picture
  - $0.00 if it doesn't

- How much would you be willing to pay for the contract?

Valuation of the security

- It depends on the probability that you place on winning
  - If you believe it has a 50% chance of winning, you would pay $1.00 * 50% =$0.50
  - If you believe it has a 10% chance of winning, you would pay $1.00*10% =$0.10
- These types of markets exist, and they are pretty accurate
  - Intrade (has been shut down)
  - Iowa Electronic Market
  - Betfair
  - PredictIt

Some examples of recent markets



Reference: *https://www.predictit.org/markets*

How accurate are these markets? Why?

PLACE YOUR BETS!

On terrorism, on politics, on your future. Why markets will have a say in almost everything By Bill Saporito

Markets are hard to beat and even harder to manipulate. On Dec. 11, 2003, InTrade's contract on Saddam Hussein's capture suddenly began to move. "We noticed that that contract started trading from 9 to 30 for no reason," says Mike Knesevitch, communications director. "Something was happening." In fact, someone may well have been trading on inside information. Two days later, Saddam was in custody.

- Why are they so accurate?
  - These types of markets "are extremely efficient at aggregating information and tapping into the collective wisdom of a group of traders, and groups are almost always smarter than the smartest people in them." from *The New Yorker* 03-24-2003

Links to valuation of other assets

- This is precisely how we will value other assets like stocks, bonds, and firms
- More specifically,
  - We will look at the cash flows that these assets generate and the likelihood of the cash flows occurring
  - Since the cash flows from these alternative assets may occur well into the future, we will need to discount them at some appropriate rate

Objectives

- Describe the links between bond price, coupon rate, time to maturity, and yield to maturity
- Calculate the price of a bond given the coupon rate, time to maturity, and the yield to maturity
- Calculate the yield to maturity given the price of a bond, the time to maturity, and the coupon rate
- Suggested sample problems:

  10^{th} edition: Chapter 8, problems 1-6, 14-18, 21-23, 28

  11^{th} edition: Chapter 8, problems 1-6, 17-21, 24-26, 31

  12^{th} edition: Chapter 8, problems 1-6, 17-21, 24-26, 31

Bond Terminology

- **Bond** – Security that obligates the issuer to make specified payments to the bondholder over a specified time period
  - i.e., a loan or IOU
  - Typical issuers are companies and governments.
- **Face Value** (Also called Par Value or Maturity Value or Principal) – Payment at the maturity of the bond.
  - The face value is almost always $1000
- **Coupon** – The interest payments (in dollars) made to the bondholder
  - Sometimes we talk about the **Coupon Rate**, which is the coupon expressed in % of face value
- **Maturity** – Years until the face value must be repaid. A bond's maturity declines over time.

5,000

THE UNITED STATES OF AMERICA

C. FOR VALUE RECEIVED PROMISES TO PAY TO THE BEARER THE SUM OF

FIVE THOUSAND DOLLARS 17710

ON THE DUE DATE, AND TO PAY INTEREST ON THE PRINCIPAL SUM FROM THE DATE HEREOF, AT THE RATE SPECIFIED HEREON. THIS NOTE AND INTEREST COUPONS ARE PAYABLE AT THE DEPARTMENT OF THE TREASURY, WASHINGTON, D.C., OR AT ANY FEDERAL RESERVE BANK OR BRANCH. THIS NOTE IS ONE OF A SERIES OF NOTES, AUTHORIZED BY THE SECOND LIBERTY BOND ACT, AS AMENDED, ISSUED PURSUANT TO THE DEPARTMENT OF THE TREASURY CIRCULAR REFERRED TO HEREON, AND IS NOT SUBJECT TO CALL FOR REDEMPTION PRIOR TO MATURITY. THE INCOME DERIVED FROM THIS NOTE IS SUBJECT TO ALL TAXES IMPOSED UNDER THE INTERNAL REVENUE CODE OF 1954. THIS NOTE IS SUBJECT TO ESTATE, INHERITANCE, GIFT OR OTHER EXCISE TAXES, WHETHER FEDERAL OR STATE, BUT IS EXEMPT FROM ALL TAXATION NOW OR HEREAFTER IMPOSED ON THE PRINCIPAL OR INTEREST HEREOF BY ANY STATE, OR ANY OF THE POSSESSIONS OF THE UNITED STATES, OR BY ANY LOCAL TAXING AUTHORITY. THIS NOTE IS ACCEPTABLE TO SECURE DEPOSITS OF PUBLIC MONEYS. IT IS NOT ACCEPTABLE IN PAYMENT OF TAXES.

WASHINGTON, D. C., AUGUST 16, 1976.

William E. Simon
SECRETARY OF THE TREASURY

8%
TREASURY NOTE
SERIES B-1986
DATED AUGUST 16, 1976
DUE AUGUST 15, 1986
CUSIP 912827 FW 7
INTEREST PAYABLE FEBRUARY 15 AND AUGUST 15
CIRCULAR No. 19-76

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BUREAU OF ENGRAVING AND PRINTING. 1974

THE UNITED STATES OF AMERICA

WILL PAY TO BEARER ON AT THE DEPARTMENT OF THE TREASURY, WASHINGTON, OR AT A DESIGNATED AGENCY, INTEREST THEN DUE ON **FEB. 15, 1986** **$200.00** $5,000 Treasury Note, Series B-1986

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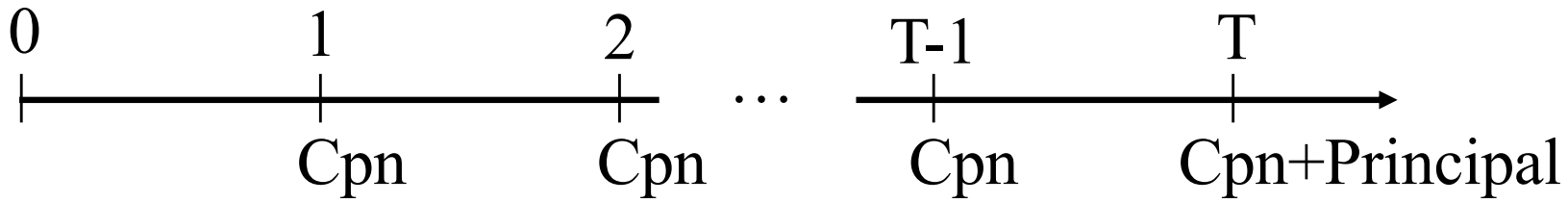
The fundamental principle of valuation

- Value of ***any*** asset is the PV of future cash flows
- Value of a bond = PV of future coupons and principal
- Value of a stock = PV of future dividends
- To value stocks and bonds we need:
  - Size of future cash flows
  - Timing of future cash flows
  - A discount rate

Bond Valuation

- Two sources of cash flows for bonds:
  - Coupon payments (an annuity)
  - Principal repayment (a lump sum)



- The value of a bond is the sum of the PVs of these two parts

Bond Pricing Formula

- We can derive an explicit formula for the price of a bond:
  - r is the discount rate
  - CPN is the coupon payment
  - T is the time to maturity

- This gives us the following formula:

$$\text{Price} = \text{PV of Coupon Payments} + \text{PV of Principal}$$

$$\text{Price} = \frac{\text{CPN}}{(1+r)} + \frac{\text{CPN}}{(1+r)^2} + \cdots \frac{\text{CPN} + \text{Principal}}{(1+r)^{\text{T}}}$$

$$= \frac{\text{CPN}}{r}\left(1 - \frac{1}{(1+r)^T}\right) + \frac{\text{Principal}}{(1+r)^{\text{T}}}$$

Bond Pricing: Example

- What is the price of an 8% annual coupon bond, with a $1,000 face value and 4 years to maturity? Assume a required return of 6.8%.

Important Note

- The coupon rate is not the discount rate used in the calculation of the bond price
  - The coupon rate tells us what cash flow the bond will produce.
  - The discount rate (or required rate of return) tells us the current rate earned on investments of equal risk.
  - Since both the coupon rate and the discount rate are listed as percentages, this misconception is common.

Bond Pricing: Example (continued)

What is the price of the bond if the coupons are paid semi-annually?

Bond information:

8% annual coupon rate, but paid semi-annually

Face value $1,000

Time to maturity 4 years

Annual required return of 6.8% (attention)

Calculating the Yield to Maturity

- If you are given the price of a bond and the coupon rate, the yield to maturity can be found by solving for r

$$\text{Price} = \frac{\text{CPN}}{r}\left(1 - \frac{1}{(1+r)^T}\right) + \frac{\text{Principal}}{(1+r)^T}$$

- There are two approaches to solve for the yield:
  - Trial and Error
  - Use a financial calculator

Bond yields: Example

What is the YTM of a 6 1/8% annual coupon bond, with a $1,000 face value, which matures in 4 years? The market price of the bond is $983.76.

Bond yields: Example 2

What is the YTM of a 7.5% annual coupon bond, with a $1,000 face value, which matures in 6 years? The market price of the bond is $1057.2.

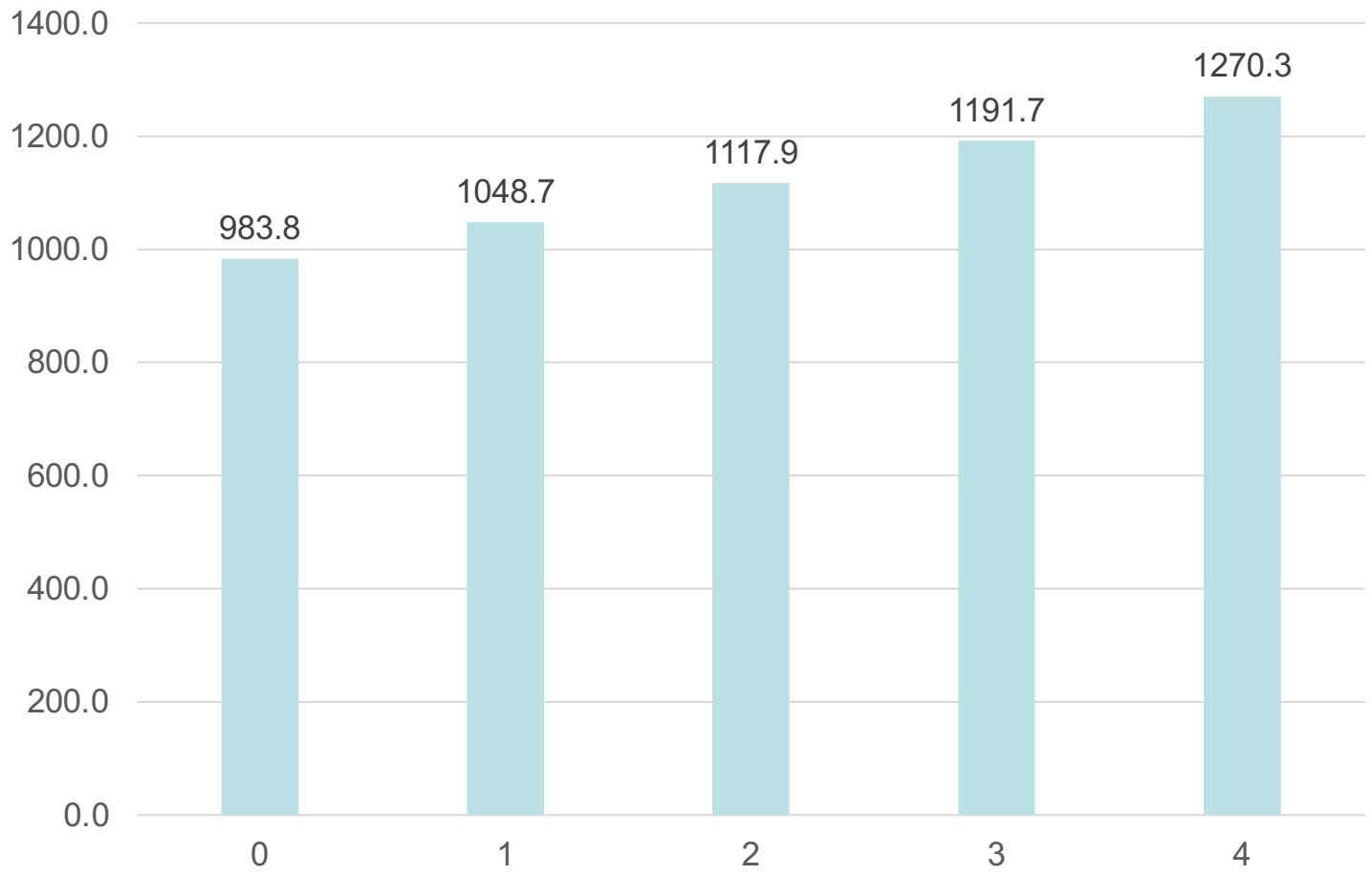
What does the YTM tell us?

- In simpler terms, it's the rate of return measuring the total performance of a bond (coupon payments as well as capital gain or loss) from the time of purchase until maturity.

More on the interpretation of YTM

- In the previous example, suppose you invested $983.76 in a savings account at 6.6%
  - What's your final wealth after 4 years?
- Alternatively, suppose you hold the bond until maturity. Reinvest coupons at 6.6% per year.
  - What's your final wealth after 4 years?

What is your final wealth if you invest $983.76 for 4 years at 6.6%?



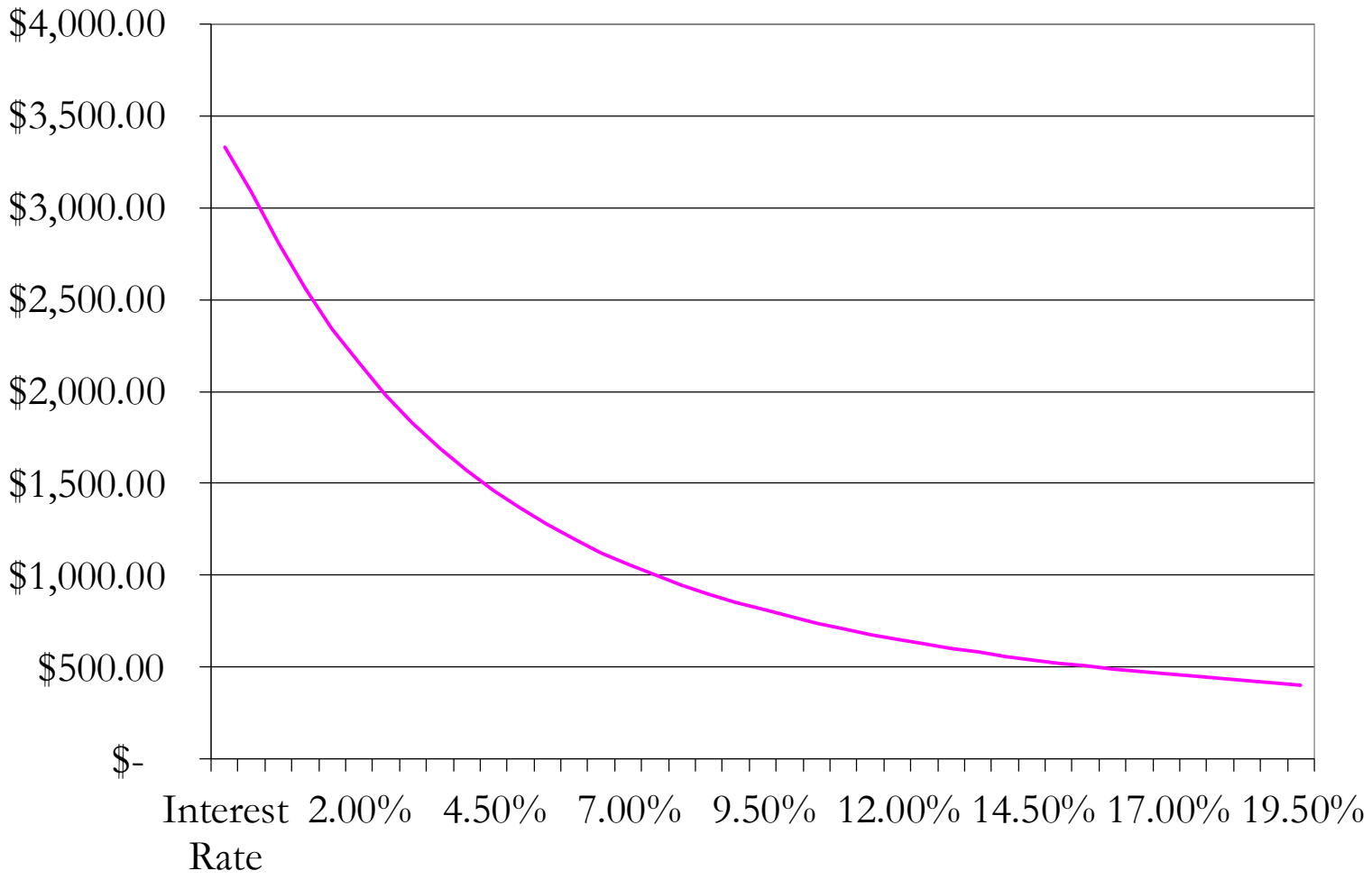
Final wealth if you hold the bond until maturity, invest coupons at 6.6%?

Time of cash flow	Cash flow amount	Future value at t=4
0		
1	61.25	74.20
2	61.25	69.60
3	61.25	65.29
4	1061.25	1061.25
	Sum	**1270.34**

Bond Price Dynamics

- After a bond is issued, the coupon rate remains constant.
- However, the yield to maturity may change as risks change
- The value of the bond will vary as the PV of the cash flows vary with different discount rates:
  - When rates go up, bond prices down.
  - When rates go down, bond prices up.

If interest rates go up, bond prices go down: A 30-year bond with 8% coupon



Excel exercise: Produce a plot for a 5-year bond with 8% coupon

Discount Rates and Bond Value

- If coupon rate < discount rate
  then the price of the bond < par value
  - a discount bond
- If coupon rate = discount rate
  then the price of the bond = par value
  - a bond ‘at par’
- If coupon rate > discount rate
  then the price of the bond > par value
  - a premium bond

Different types of bonds

- Treasury bonds
- Zero coupon bonds
- Corporate bonds
- Convertible bonds
- Puttable bonds
- Floating-rate bonds

How do you value an asset?

- It's all about cash flows and discount rates:

$$\text{Value of asset} = \sum_{t=1}^{\infty} \frac{\text{Cash Flow}_t}{(1+r)^t}$$

- The discount rate, r, captures the riskiness of the investment
  - The higher the risk of the cash flows, the higher the discount rate

Sample Problem – Ross/Westerfield/ Jaffe, Chapter 4, #65, 9/e (not in 10/e)

- Your friend is celebrating her 35th birthday today and wants to start saving for her anticipated retirement at age 65. She wants to be able to withdraw $110,000 from her savings account on each birthday for 25 years following her retirement; the first withdrawal will be on her 66th birthday. Your friend intends to invest her money in the local credit union, which offers a 9% interest per year. She wants to make equal payments on each birthday into the account starting on her 36th birthday. What amount must she deposit annually?

Valuing Stocks: The dividend discount model

Objectives

- Describe the relation between dividends, discount rates, growth, and stock prices
- Learn a simple dividend discount model
  - "Constant growth model" or "Gordon growth model"
- We will focus only on sections 9.1 – 9.2 from this chapter
- Recommended exercises:

  10^{th} edition, chapter 9: 1-7, 12-14, 16-17, 23

  11^{th} edition, chapter 9: 1-7, 12-14, 16-17, 22

  12^{th} edition, chapter 9: 1-7, 12-14, 16-17, 22

THE WALL STREET JOURNAL.

BUSINESS | AUTOS & TRANSPORTATION | AUTOS INDUSTRY

Volkswagen to List Porsche in One of Biggest IPOs in Years

Porsche shares could be trading by the end of September, early October

Porsche models at a showroom in Berlin earlier this year.

PHOTO: LIESA JOHANNSSEN-KOPPITZ/BLOOMBERG NEWS

By *William Boston* Follow

Updated Sept. 6, 2022 10:08 am ET

What is a stock?

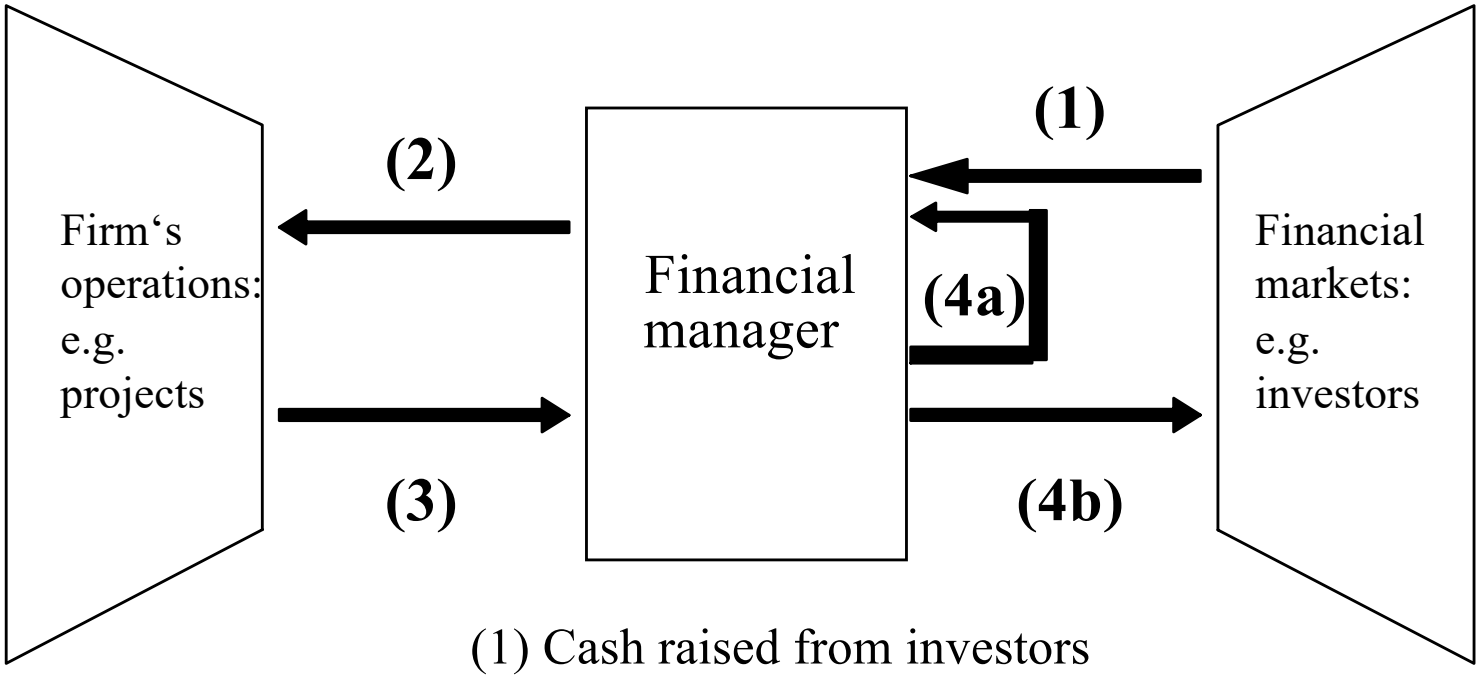
- It's an ownership stake in the company
  - Residual claimholder
  - Have voting rights

- You get cash in two ways from buying stock:
  - The company pays dividends
  - You sell your shares,
    - Either to another investor in the market
    - Or back to the company

Some definitions

- Dividends
  - Distributions from a company to its stockholders
  - You usually receive dividends in cash
- Where do they come from:
  - Firm decides to pay out a portion of earnings to stockholders
    - “Payout ratio”
  - Keeps the rest for other investments
    - “Retention ratio”
- What do you get when you own a stock?
  - (Future) Dividends D_t ; t=1,2,3,…
  - Future sale price P_t (capital gain)

Graphical Illustration of the Firm



(1) Cash raised from investors
(2) Cash invested in firm
(3) Cash generated by operations
(4a) Cash reinvested
(4b) Cash returned to investors

Price of a stock

- The stock's price is equal to:
  - Present value of the dividends plus the present value of the sales price
- Suppose we have a single holding period:
  - We buy a stock today at t=0 and sell the stock one year later at t=1
- Then the price of the stock today should be:

$$\text{Price of the stock} = \text{PV}_0 = \frac{\text{D}_1 + \text{P}_1}{1 + r}$$

What determines P_1?

- Apply the PV formula again:

$$\text{Price}_1 = \text{PV}_1 = \frac{\text{P}_2 + \text{D}_2}{1 + r}$$

- Substituting this into the previous formula:

$$\text{Price}_0 = \text{PV}_0 = \frac{\text{D}_1}{1 + r} + \frac{\text{P}_2 + \text{D}_2}{(1 + r)^2}$$

General Formula for Stock Price

- We can keep repeating the process:

$$\text{Price} = \frac{D_1}{1+\text{r}} + \frac{D_2}{(1+\text{r})^2} + \frac{D_3}{(1+\text{r})^3} + \cdots + \frac{D_t + P_t}{(1+\text{r})^{\text{t}}}$$

Or equivalently,

$$\text{Price} = \frac{D_1}{1+\text{r}} + \frac{D_2}{(1+\text{r})^2} + \frac{D_3}{(1+\text{r})^3} + \cdots$$

- In other words, the price of a stock today equals the present value of the entire dividend stream that the stock will pay in the future

Why are we discounting dividends? Why not earnings?

- We discount dividends and not earnings because:
  - Only select a stock for what you can get out of it, which is dividends or the ultimate sale price

- Only a portion of earnings goes to stockholders

Estimating the Stock Price

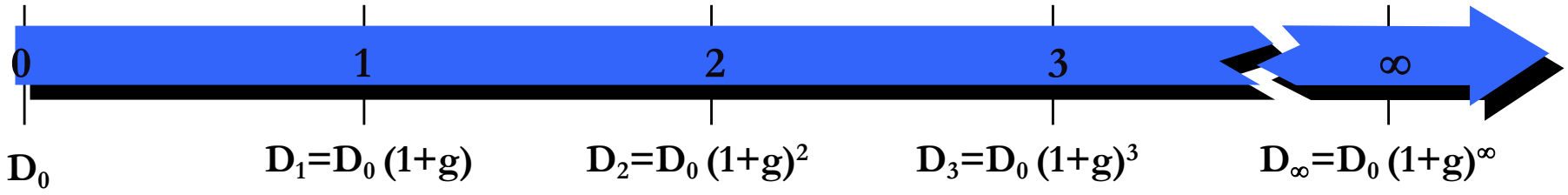
- An analyst must have two inputs to estimate a stock price:
  - Future dividend amounts
  - An appropriate discount rate
    - We will defer this discussion until later in the course
- We will focus on the problem of estimating dividends
- We will consider a simple model of stock valuation:
  - Assume constant growth in dividends

A caveat

- In practice, the valuation of stocks is difficult
  - Future cash flows are uncertain
  - Firms live forever in some cases
  - The discount rate is not easily observed
- It's much more challenging than bonds
  - We know the cash flows for bonds
  - Bonds have a finite life

The constant growth model (aka Gordon growth model)

- Can be used to value a firm in a "steady state"
  - Dividends growing at a rate that can be sustained forever
- We have the following situation:



- We can value the stock using our growing perpetuity formula:

$$P_0 = \frac{D_1}{r - g}$$

- Note that we ignore D_0 in the present value calculation; implicitly we assume that D_0 has been paid in the past

Example: Procter & Gamble

- Consider Procter & Gamble (ticker symbol PG)
  - Increased dividend every year over the last 66 years.
  - Over the last 20 years, the average annual growth rate was 8.05%
- Suppose investors expect Procter & Gamble to pay a $3.94 dividend over next year and that dividend will grow at 8.05% a year indefinitely. Suppose investors require an 11% return.
- What's the predicted price?

$$P_0 = \frac{\$3.94}{0.110 - 0.0805} = \$133.56$$

- On July 29th, 2022, the stock price closed at $138.91

Reference: *https://www.digrin.com/stocks/detail/PG/dgr10*

https://www.pginvestor.com/stock-information/splits-dividend-history/

Review problem: Constant dividend growth

You buy coffee every morning, so you wonder if you should buy the stock of your coffee shop company. After all, they charge you quite a bit for that latte. The latest dividend they have paid was **$2** per share. You expect the annual dividend to grow at **1%** per year, forever. After doing some research on *Yahoo Finance*, you realize that their dividend stream is very stable over time, almost risk-free. As an approximation, you use the yield of 30-year U.S. government bonds as the discount rate. What is the fair value of this stock?

Differential growth example

Allen, Inc., is expected to pay equal dividends at the end of each of the next two years. Thereafter, the dividend will grow at a constant annual rate of four percent, forever. The current stock price is $30. What is next year's dividend payment if the required rate of return is 12%?

The DDM model is used in practice

- These are very simple models of the stock price
  - However, they are used in practice for “back of the envelope” calculations

Morgan Stanley

In-Depth Page 1

Equity Research
Europe

Industry

Capital Goods

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GICS SECTOR	INDUSTRIALS
Strategists' Recommended Weight	6.9%
MSCI Europe Benchmark Weight	7.9%

COMPANIES FEATURED	
Siemens (SIEGn.DE, €61.9)	Overweight
ABB (ABBn.VX, SFr 7.70)	Equal-weight–V
Schneider (SCHN.PA, €59.5)	Equal-weight
Electrolux (ELUXb.ST, SKr 162.0)	Underweight

Industry Overview

April 14, 2005

DDM and LBO Analysis: Electricals Not Undervalued

- **Seeking new ways to assess the value of leading electrical equipment stocks** We use a Dividend Discount Model to compare actual and implied earnings growth rates, while our LBO model generates an implicit IRR based on a hypothetical 70% debt-funded buyout. Our findings tend to support our existing views.
- **Valuations already discounting five-year EPS CAGR of close to 20%** Overall, the stocks do not appear especially undervalued — we expect a 2005-09 EPS CAGR of 18% against implied growth using the DDM approach of 19%; the group average IRR of 25% is unexciting. However, Siemens stands out.

Aside #1: Why do stock prices change?

- The following data is for IBM for the period 20.07.2022 – 29.07.2022

Currency in USD

Download

Date	Open	High	Low	Close*	Adj Close**	Volume
Jul 29, 2022	129.52	131.00	129.31	130.79	130.79	5,785,000
Jul 28, 2022	128.75	129.81	128.61	129.22	129.22	3,913,700
Jul 27, 2022	127.97	129.43	127.58	129.12	129.12	4,175,600
Jul 26, 2022	128.26	129.30	127.63	128.08	128.08	3,645,300
Jul 25, 2022	128.44	129.13	127.90	128.54	128.54	4,702,400
Jul 22, 2022	127.03	128.32	125.71	128.25	128.25	6,465,200
Jul 21, 2022	128.75	128.81	125.13	127.15	127.15	11,975,400
Jul 20, 2022	130.70	130.72	128.06	129.18	129.18	9,882,000

- Prices vary quite substantially over the period of 9 days.

Reference: *https://finance.yahoo.com/quote/IBM*

The dividend discount model again

- Let's consider the constant growth dividend discount model:

$$P_0 = \frac{D_1}{r - g}$$

- Stock prices could change because
  - Expected future dividends change ($\uparrow D_1 \rightarrow \uparrow$Price)
  - Expected growth changes ($\uparrow g \rightarrow \uparrow$Price)
  - Required rate of return changes ($\uparrow r \rightarrow \downarrow$Price)

- Stock prices change because new information is constantly causing investors to revise beliefs about D_1, r, and g

How long does it take for markets to react to new information?

- This is one of the most researched questions in finance
  - Hours?
  - Days?
  - Weeks?
  - Months?

Efficient response to new information

- Research shows that new information is included in stock prices within seconds
  - E.g., Busse and Green, 2002, Journal of Financial Economics
  - https://youtu.be/FC7kd6dTw9c
  - Paper based on CNBC shows like '*Squawk on the Street*'
- Is this always the case?
- More generally, is there ever a disconnect between the market price and the fundamentals of the firm?

Aside #2: Does price deviate from fundamental value?

- MCI is (was) one of the largest telecommunications firms in the world:
  - Engaged in merger negotiations with several major firms
  - Eventually acquired by Worldcom for more than $20 billion
  - Traded under the ticker symbol MCIC

- Massmutual Corporate Investors is a closed-end fund
  - Similar to a mutual fund except it trades on an exchange
  - Fund has $200 million of net assets
  - Nothing to do with the telecommunications business or MCI
  - Traded under the ticker symbol MCI

What's the story

- Stock returns should not be related
- Whenever new information came out about MCI Communications:
  - There was large abnormal trading volume in MassMutual
- Explanation:
  - Investors didn't even know the ticker symbol of the stocks they are trading!!
  - Mostly small investors made the mistake

Coding exercise: Download and plot stock prices (if enough time)

- Why do we do this?
  - Why is this important?
- What if I don’t have any coding experience?
- Will this be on the exam?
- What if Pythons is more important to me than R?
- What if I want to learn the basics of R?
  - Online tutorials, e.g., on LinkedIn Learning
  - Course “Learning R” by Barton Poulson

Coding exercise: Download and plot stock prices (continued)

- Download and save the file:
  - “coding-exercise-AAPL-prices-handout.R”
- We will look at this code file

- Next: Create a similar code file for another company that you like