Applied Quantitative Investment Management

Lecture 3: Investment Simulation Framework

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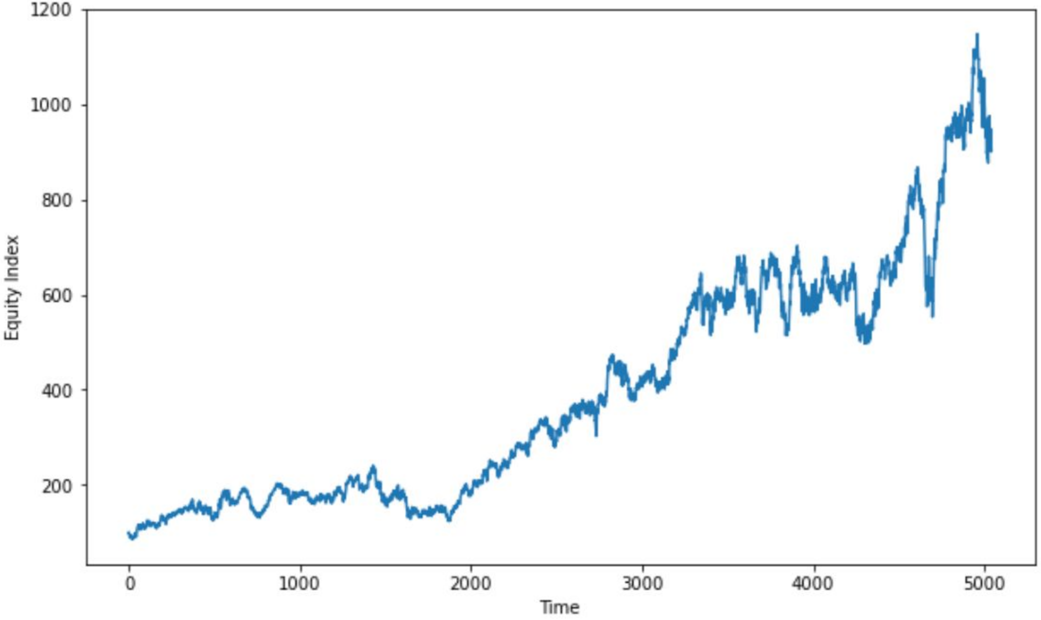
Agenda

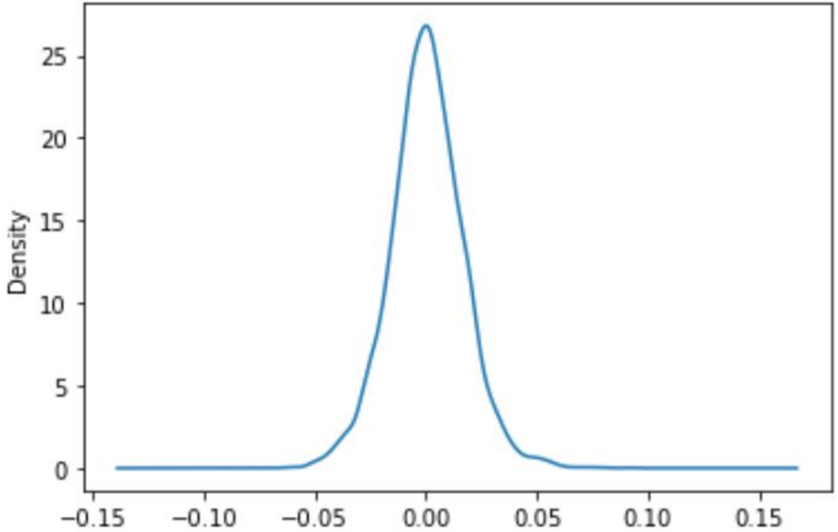
- Overview of the multi-asset investment simulation framework
- Stationary transformations (Section 3.1)
- Computing simulated risk factors (Section 3.3)
- Simulation evaluation (Section 3.4)

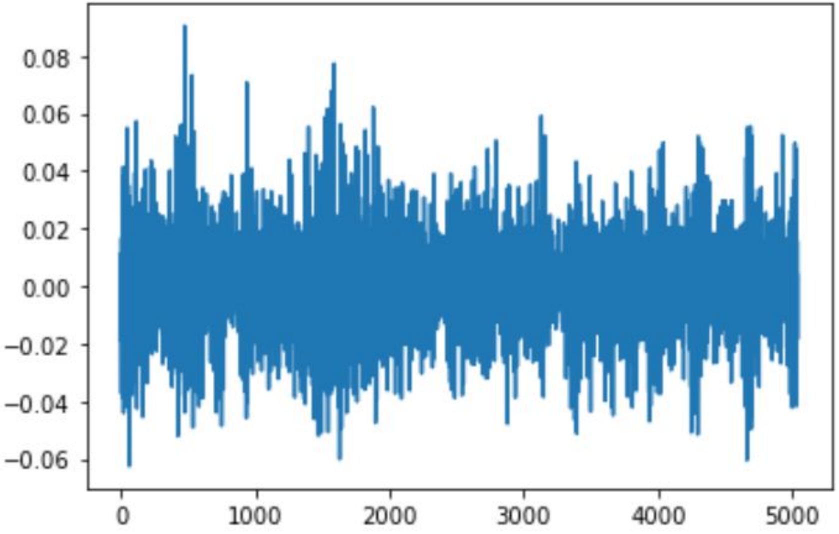
Next lecture:

- Projection of stationary transformations (Sections 1.2 and 3.2)
- Better backtesting of CVaR versus variance optimization (Section 3.5)

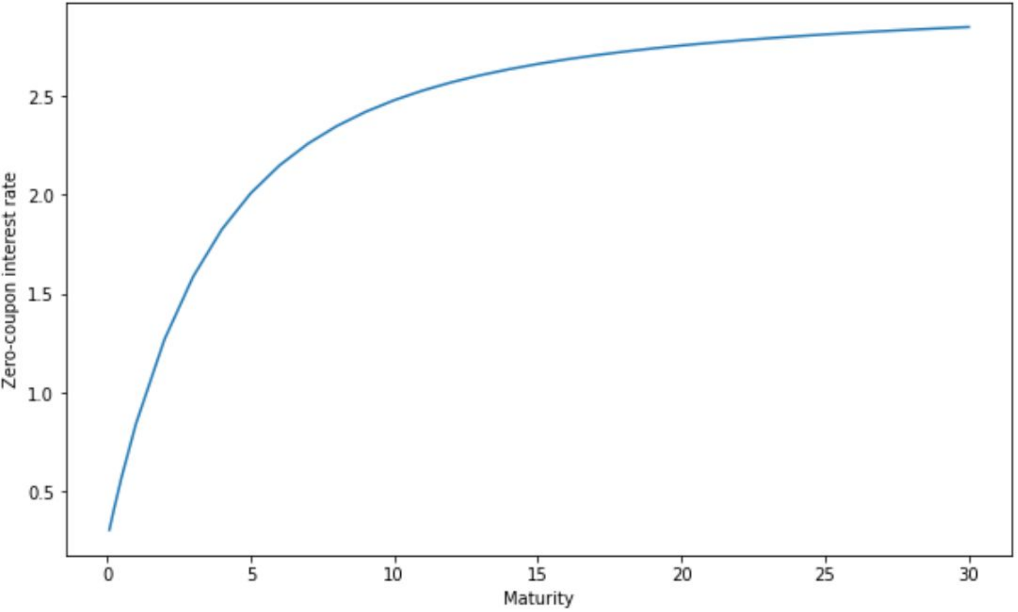
Stationary transformations

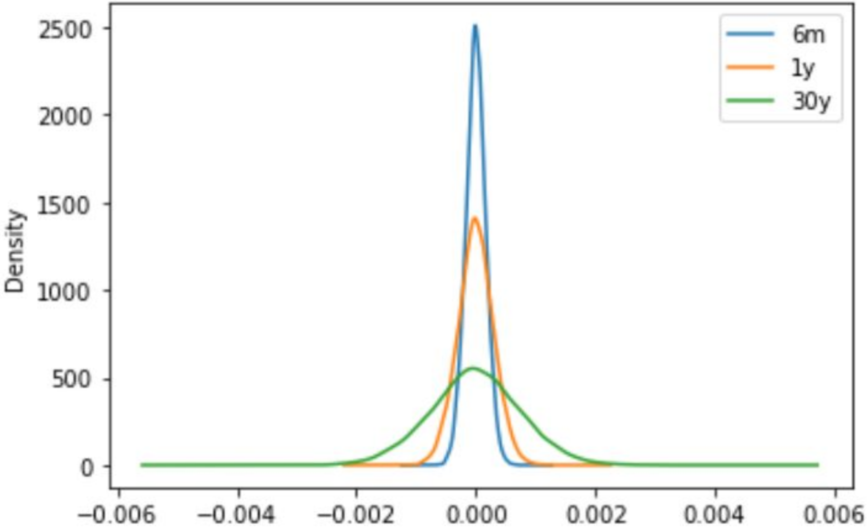


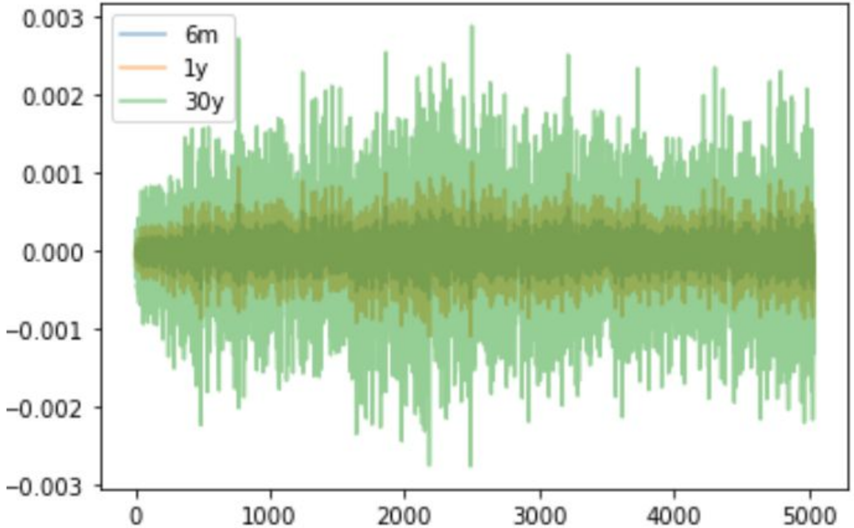




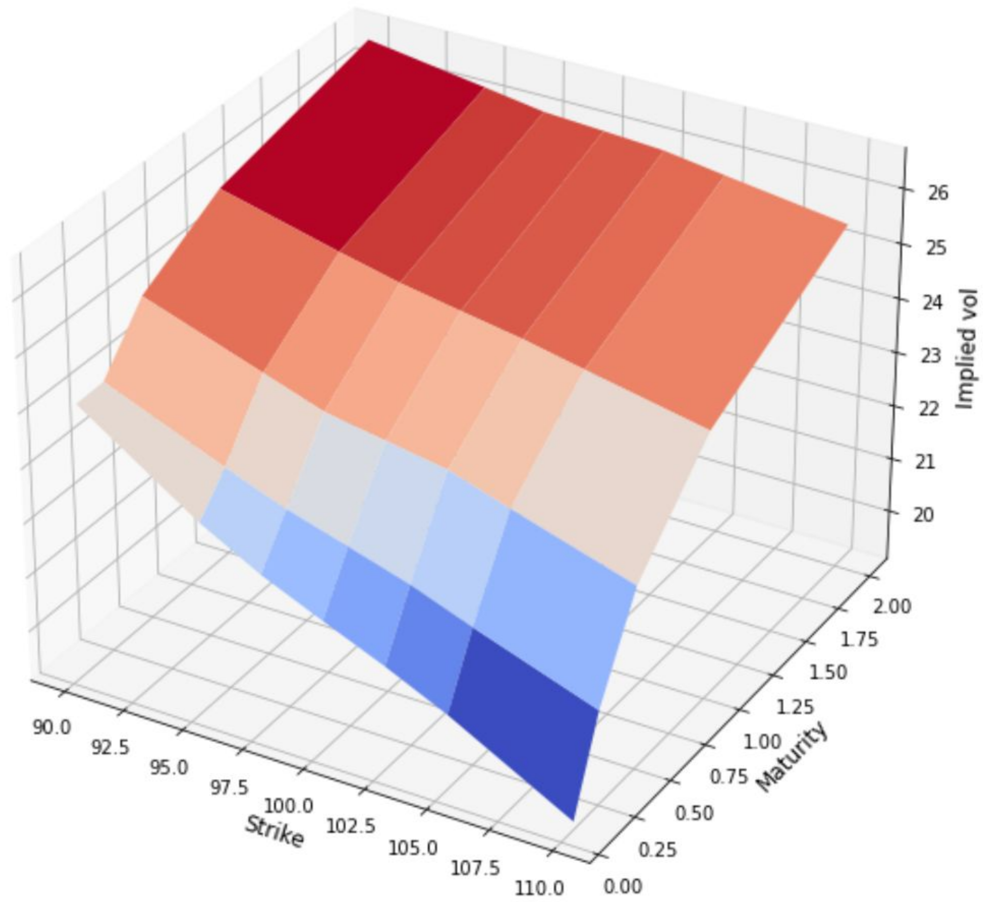
Stationary transformations

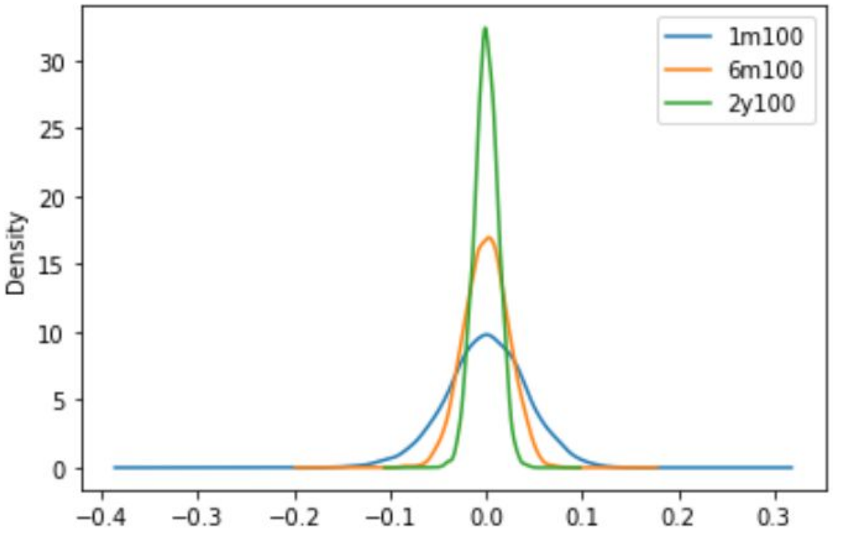


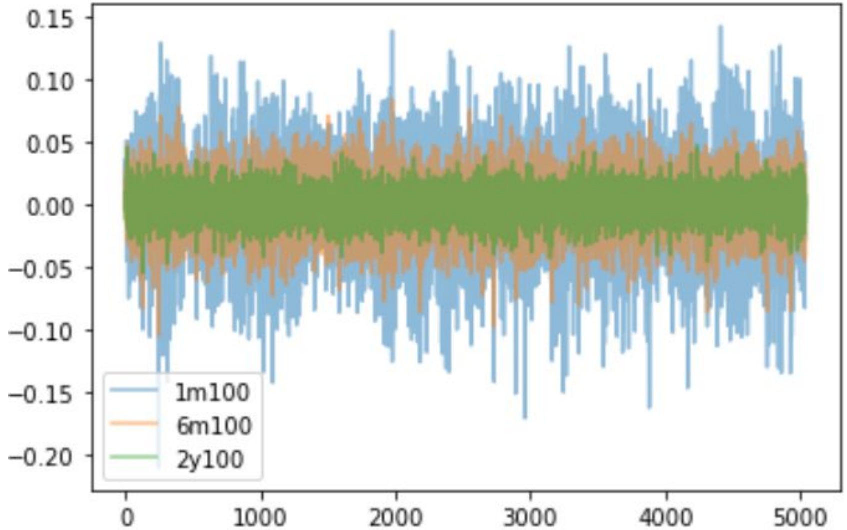




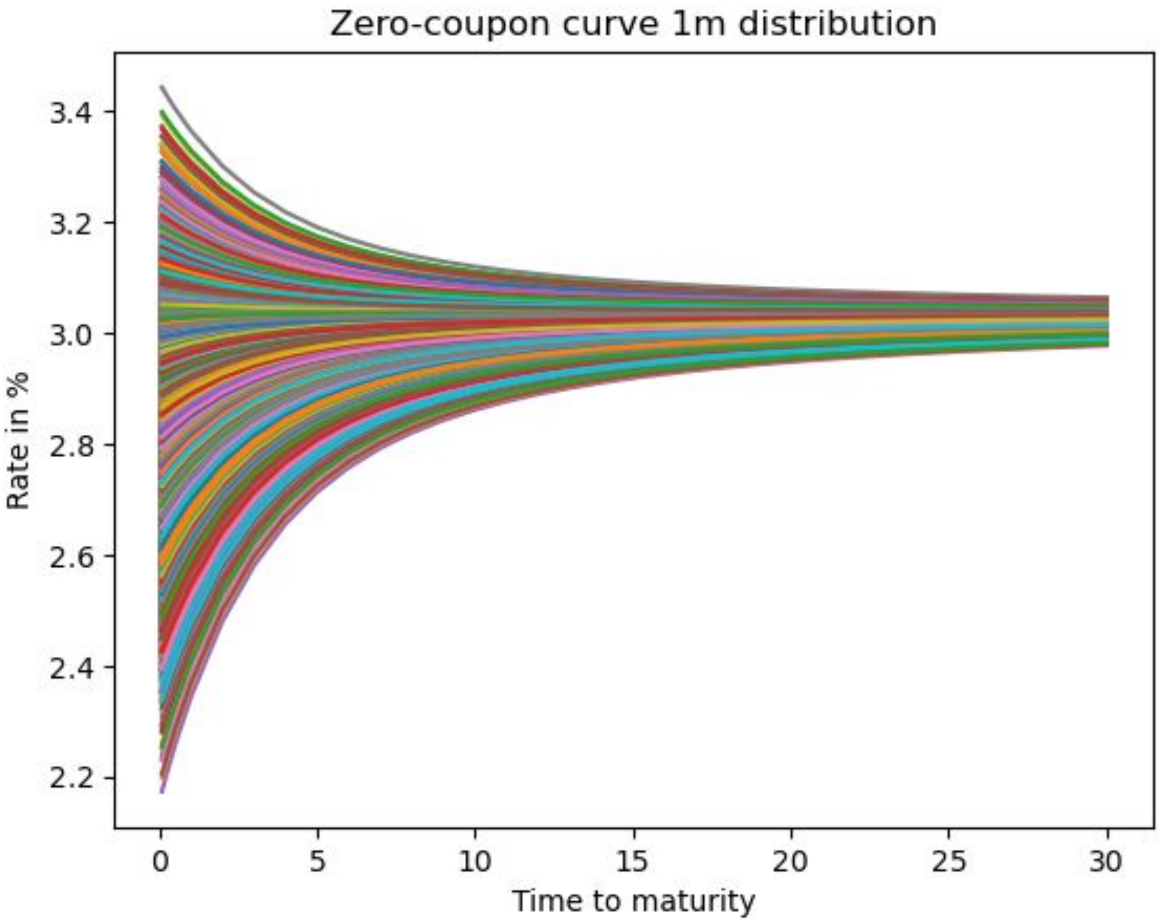
Stationary transformations



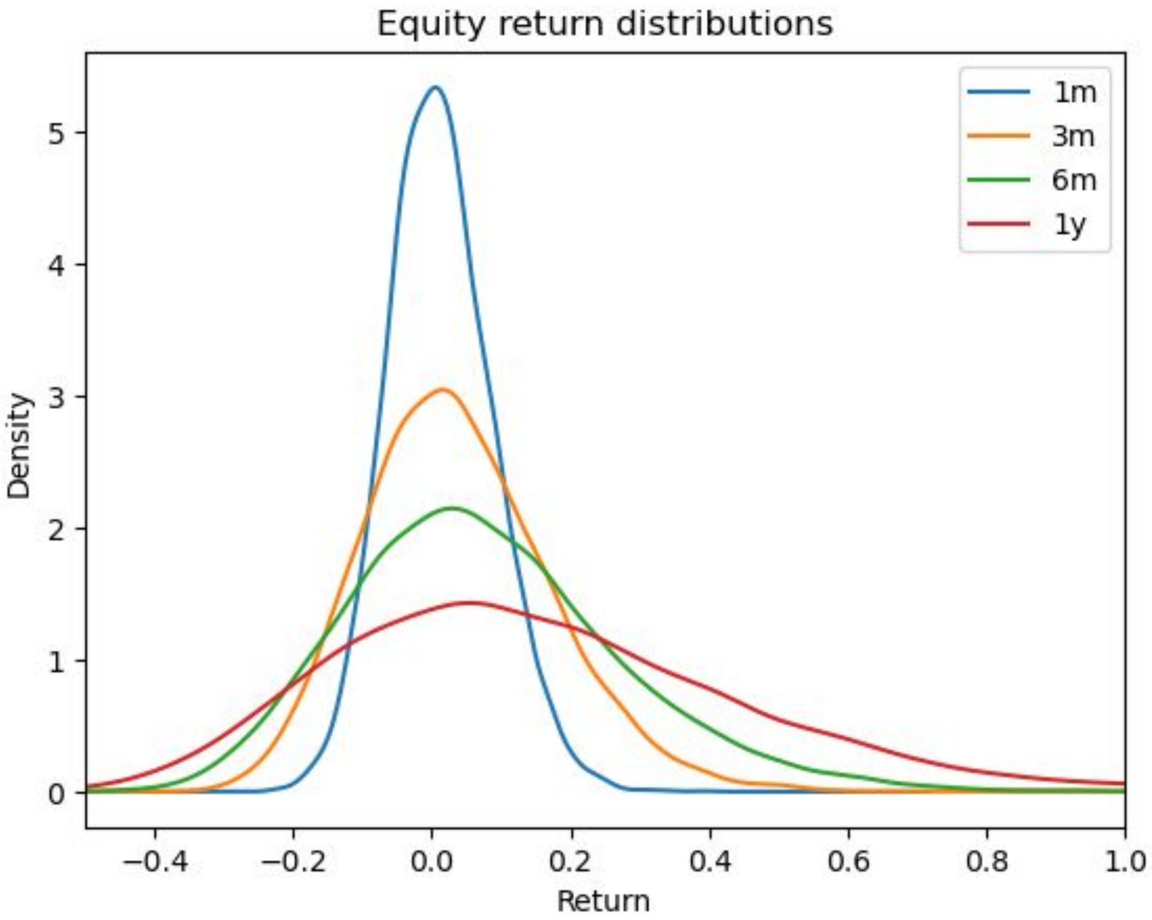




Risk factor simulation



P&L simulation



Market simulation fundamentals

- Historical data:

$$D \in \mathbb{R}^{T \times N}$$

- Stationary transformations:

$$D \mapsto ST \in \mathbb{R}^{\tilde{T} \times \tilde{N}}$$

- Simulated stationary transformations:

$$\tilde{ST} \in \mathbb{R}^{S \times \tilde{N} \times H}$$

- Simulated data:

$$\tilde{ST} \mapsto \tilde{R} \in \mathbb{R}^{S \times N \times H} \qquad R_h \in \mathbb{R}^{S \times N}$$

Market data

$$D = \begin{pmatrix} D_{1,1} & D_{1,2} & \cdots & D_{1,N} \\ D_{2,1} & D_{2,2} & \cdots & D_{2,N} \\ \vdots & \vdots & \ddots & \vdots \\ D_{T,1} & D_{T,2} & \cdots & D_{T,N} \end{pmatrix} \in \mathbb{R}^{T \times N}$$

	Equity Index	1m	3m	6m	1y	2y	3y	4y	5y	6y	...	cr1y	cr2y	cr3y	cr4y	cr5y	cr6y	cr7y	cr8y	cr9y	cr10y
0	100.000000	0.306974	0.415789	0.568395	0.839313	1.269176	1.587154	1.825839	2.007783	2.148673	...	3.466530	3.387351	3.322905	3.270851	3.228836	3.194803	3.167063	3.144275	3.125390	3.109597
1	98.675317	0.315879	0.424345	0.576460	0.846504	1.274975	1.591918	1.829823	2.011171	2.151599	...	3.487529	3.404820	3.337619	3.283400	3.239669	3.204264	3.175415	3.151722	3.132090	3.115674
2	99.785099	0.316697	0.425130	0.577200	0.847164	1.275507	1.592355	1.830189	2.011482	2.151868	...	3.405938	3.336940	3.280444	3.234637	3.197572	3.167498	3.142957	3.122781	3.106052	3.092056
3	98.808764	0.314546	0.423064	0.575252	0.845427	1.274106	1.591204	1.829227	2.010664	2.151161	...	3.336271	3.278973	3.231614	3.192989	3.161614	3.136092	3.115230	3.098057	3.083807	3.071878
4	96.676413	0.311534	0.420170	0.572524	0.842995	1.272145	1.589593	1.827879	2.009518	2.150171	...	3.404906	3.336081	3.279721	3.234020	3.197039	3.167032	3.142546	3.122414	3.105722	3.091757
...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
5035	918.470387	2.744650	2.756705	2.773531	2.803184	2.849691	2.883701	2.909034	2.928243	2.943064	...	1.479825	1.731878	1.926785	2.078983	2.199086	2.294901	2.372197	2.435252	2.487257	2.530611
5036	902.286066	2.746272	2.758261	2.774997	2.804487	2.850738	2.884559	2.909750	2.928851	2.943588	...	1.494460	1.744094	1.937099	2.087797	2.206707	2.301566	2.378087	2.440508	2.491989	2.534905
5037	916.604273	2.756630	2.768203	2.784355	2.812812	2.857425	2.890037	2.914321	2.932732	2.946936	...	1.390928	1.657677	1.864124	2.025434	2.152782	2.254410	2.336413	2.403318	2.458506	2.504516
5038	899.793715	2.821024	2.830010	2.842537	2.864564	2.898993	2.924087	2.942737	2.956859	2.967745	...	1.418419	1.680624	1.883503	2.041996	2.167103	2.266933	2.347481	2.413195	2.467399	2.512587
5039	903.256326	2.795153	2.805179	2.819163	2.843773	2.882294	2.910409	2.931322	2.947167	2.959386	...	1.485389	1.736523	1.930706	2.082334	2.201983	2.297435	2.374437	2.437250	2.489056	2.532243

Stationary transformations

$$ST = \begin{pmatrix} ST_{1,1} & ST_{1,2} & \cdots & ST_{1,N} \\ ST_{2,1} & ST_{2,2} & \cdots & ST_{2,N} \\ \vdots & \vdots & \ddots & \vdots \\ ST_{T,1} & ST_{T,2} & \cdots & ST_{T,N} \end{pmatrix} \in \mathbb{R}^{\tilde{T} \times \tilde{N}}$$

	Equity Index	1m	3m	6m	1y	2y	3y	4y	5y	6y	...	cr1y	cr2y	cr3y	cr4y	cr5y	cr6y	cr7y	cr8y	cr9y	cr10y
0	-0.013335	-7.420833e-06	-0.000021	-0.000040	-0.000072	-0.000116	-0.000143	-0.000159	-0.000169	-0.000176	...	-0.000210	-0.000349	-0.000441	-0.000502	-0.000542	-0.000568	-0.000585	-0.000596	-0.000603	-0.000608
1	0.011184	-6.816667e-07	-0.000002	-0.000004	-0.000007	-0.000011	-0.000013	-0.000015	-0.000016	-0.000016	...	0.000816	0.001358	0.001715	0.001951	0.002105	0.002206	0.002272	0.002315	0.002343	0.002362
2	-0.009833	1.792500e-06	0.000005	0.000010	0.000017	0.000028	0.000035	0.000038	0.000041	0.000042	...	0.000697	0.001159	0.001465	0.001666	0.001798	0.001884	0.001941	0.001978	0.002002	0.002018
3	-0.021817	2.510000e-06	0.000007	0.000014	0.000024	0.000039	0.000048	0.000054	0.000057	0.000059	...	-0.000686	-0.001142	-0.001443	-0.001641	-0.001771	-0.001856	-0.001912	-0.001949	-0.001972	-0.001988
4	-0.037304	-6.852500e-06	-0.000020	-0.000037	-0.000066	-0.000107	-0.000132	-0.000147	-0.000156	-0.000162	...	-0.001589	-0.002644	-0.003341	-0.003799	-0.004100	-0.004296	-0.004425	-0.004509	-0.004564	-0.004600
...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
5034	-0.017971	-5.269167e-06	-0.000015	-0.000029	-0.000051	-0.000082	-0.000100	-0.000112	-0.000119	-0.000123	...	-0.000135	-0.000225	-0.000285	-0.000325	-0.000351	-0.000368	-0.000380	-0.000387	-0.000392	-0.000395
5035	-0.017778	-1.351667e-06	-0.000004	-0.000007	-0.000013	-0.000021	-0.000026	-0.000029	-0.000030	-0.000031	...	-0.000146	-0.000244	-0.000309	-0.000353	-0.000381	-0.000400	-0.000412	-0.000420	-0.000426	-0.000429
5036	0.015744	-8.631667e-06	-0.000025	-0.000047	-0.000083	-0.000134	-0.000164	-0.000183	-0.000194	-0.000201	...	0.001035	0.001728	0.002189	0.002495	0.002696	0.002829	0.002917	0.002975	0.003013	0.003039
5037	-0.018510	-5.366167e-05	-0.000155	-0.000291	-0.000518	-0.000831	-0.001022	-0.001137	-0.001206	-0.001249	...	-0.000275	-0.000459	-0.000581	-0.000662	-0.000716	-0.000751	-0.000775	-0.000790	-0.000800	-0.000807
5038	0.003841	2.155917e-05	0.000062	0.000117	0.000208	0.000334	0.000410	0.000457	0.000485	0.000502	...	-0.000670	-0.001118	-0.001416	-0.001614	-0.001744	-0.001830	-0.001887	-0.001924	-0.001949	-0.001966

Stationary transformation suggestions

- For **indices, stocks, currencies, and implied volatilities**, log changes:

$$ST_{t,n} = \ln D_{t,n} - \ln D_{t-1,n}$$

- For **interest rates and spreads** with maturity *m*, log changes in constant maturity zero-coupon bonds:

$$ST_{t,n} = m\left(D_{t-1,n} - D_{t,n}\right)$$

- **“Filtering” of persistent time series** with long-run values also a possibility, subsequently using the residuals for simulation

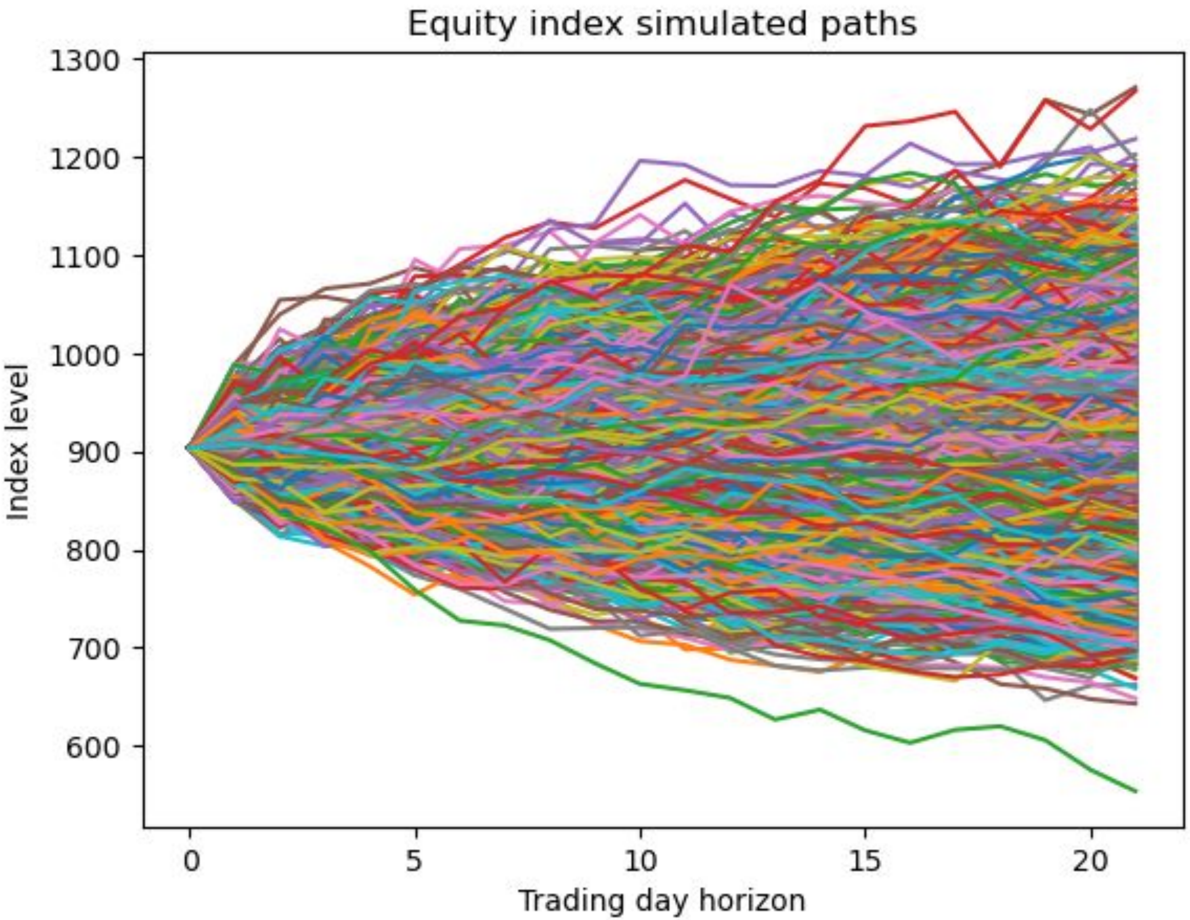
Simulated paths

Stationary transformations:

$$\tilde{ST} \in \mathbb{R}^{S \times \tilde{N} \times H}$$

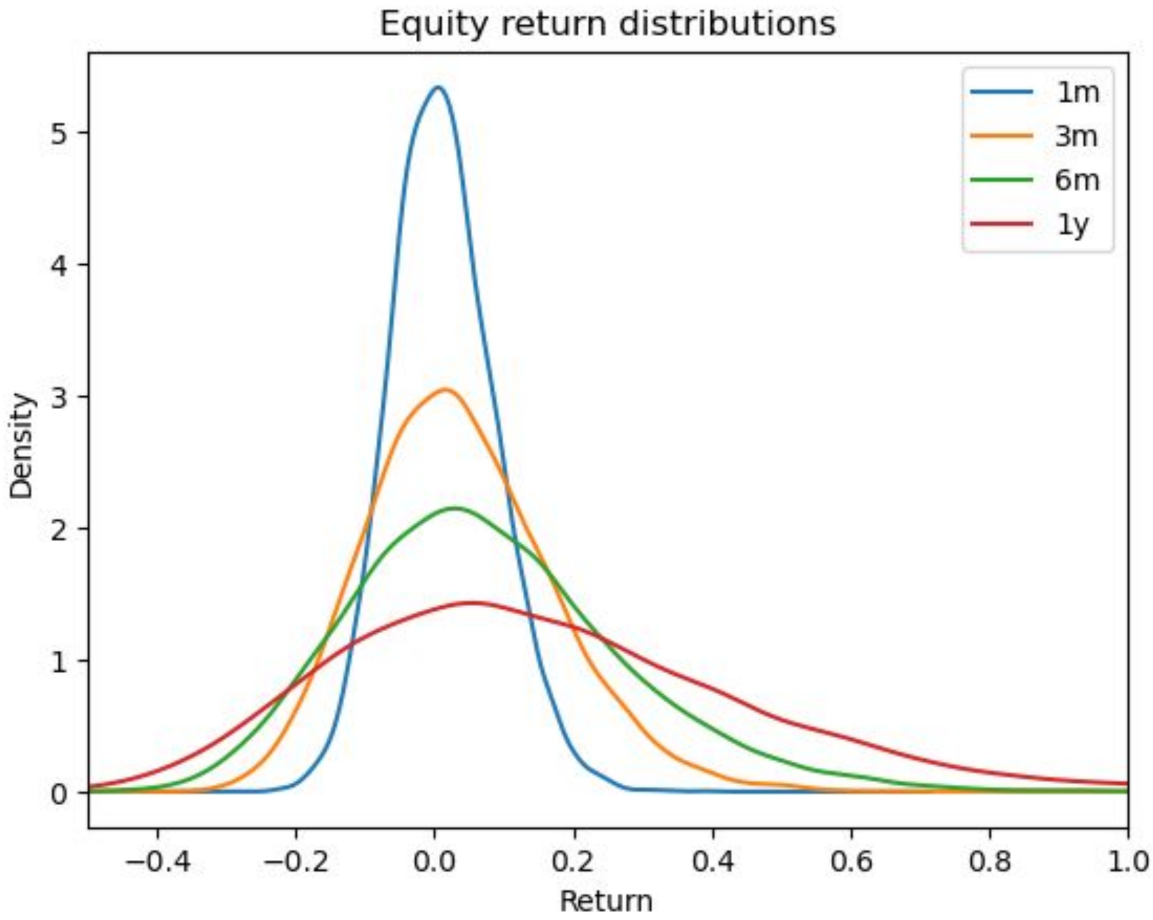
Risk factor paths:

$$\tilde{ST} \mapsto \tilde{R} \in \mathbb{R}^{S \times N \times H}$$



Specific horizons

$$R_h \in \mathbb{R}^{S \times N}$$



Computing simulated risk factors and P&L

- **“Inverse” of the stationary transformation**
- Often involves cumulative sums of log change, for example, computing the discrete return on a one-month horizon using daily log changes
- For interest rates and credit spreads, it is important to first compute the initial discount factors. The cumulative sums of our log changes are in those
- See Chapter 3.3 Python code for all the inverse risk factors computations

Simulation evaluation

- **Objective:** simulate future paths that generalize out-of-sample and capture both the cross-sectional and time series dependencies
- **Challenge:** only one real-world observation for each time step to evaluate S cross-sectional simulations
- **Reality:** No single metric to assess if the simulation is “good” for fully general time series data
- **Proposal:** Use a mix of time series and cross-sectional metrics
- **Sanity check:** visualize market paths for prices, interest rates, portfolios, etc. to see if they fall within the simulated bands

Simulation evaluation

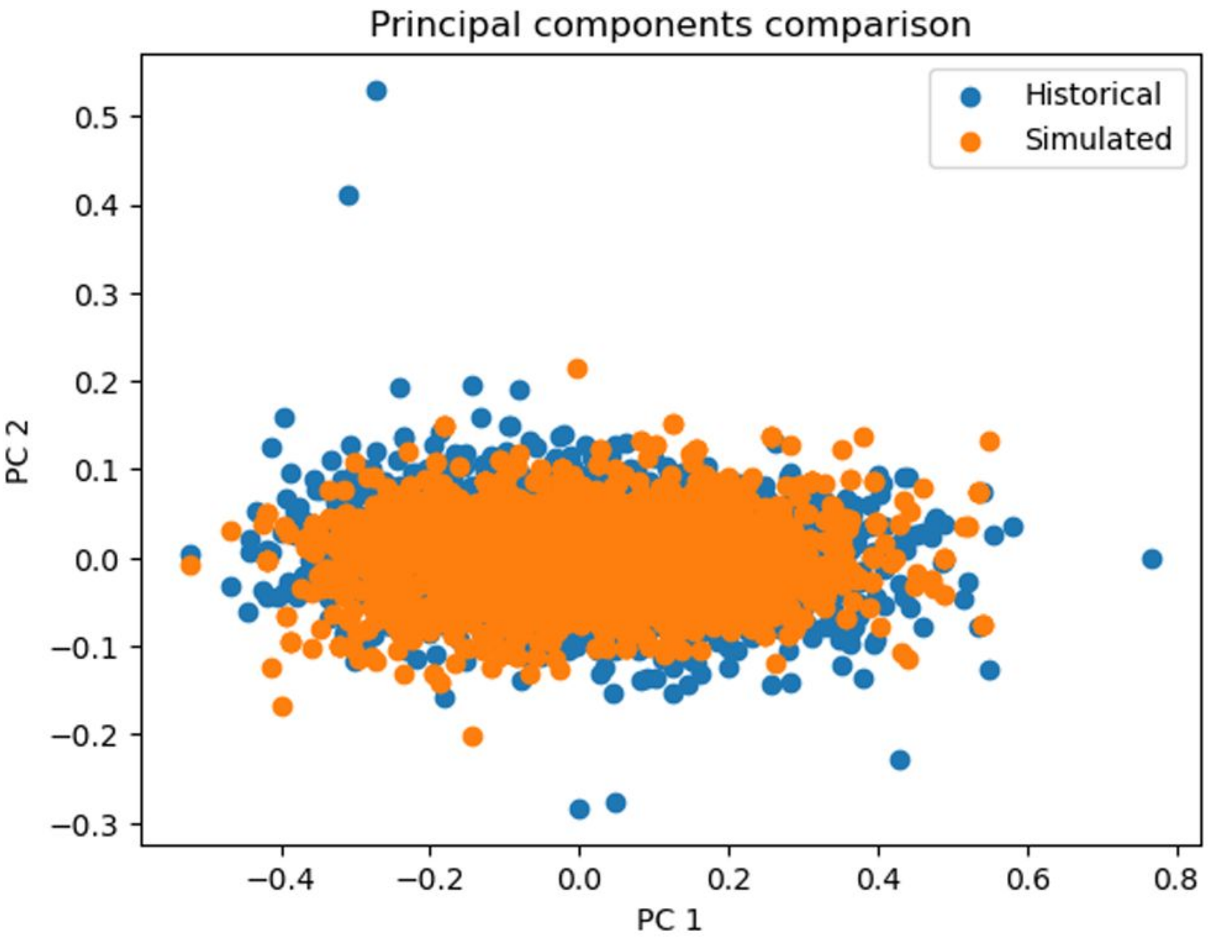
	Mean	Vol	Skew	Kurt
(0, Equity Index)	-0.009	0.048	-0.021	-0.092
(5, 2y)	-0.001	0.001	0.008	-0.031
(13, 10y)	0.000	0.000	0.008	-0.031
(33, 30y)	0.000	0.001	0.007	-0.031
(37, 1m100)	-0.003	-0.012	-0.049	0.118
(51, 6m100)	0.015	-0.008	0.018	0.065
(65, 2y100)	0.000	0.011	0.009	-0.004
(69, cr1y)	-0.001	0.003	-0.032	0.025
(73, cr5y)	-0.001	0.006	-0.031	0.027

Table 3.1: Difference between prior and simulation statistics using one state variable.

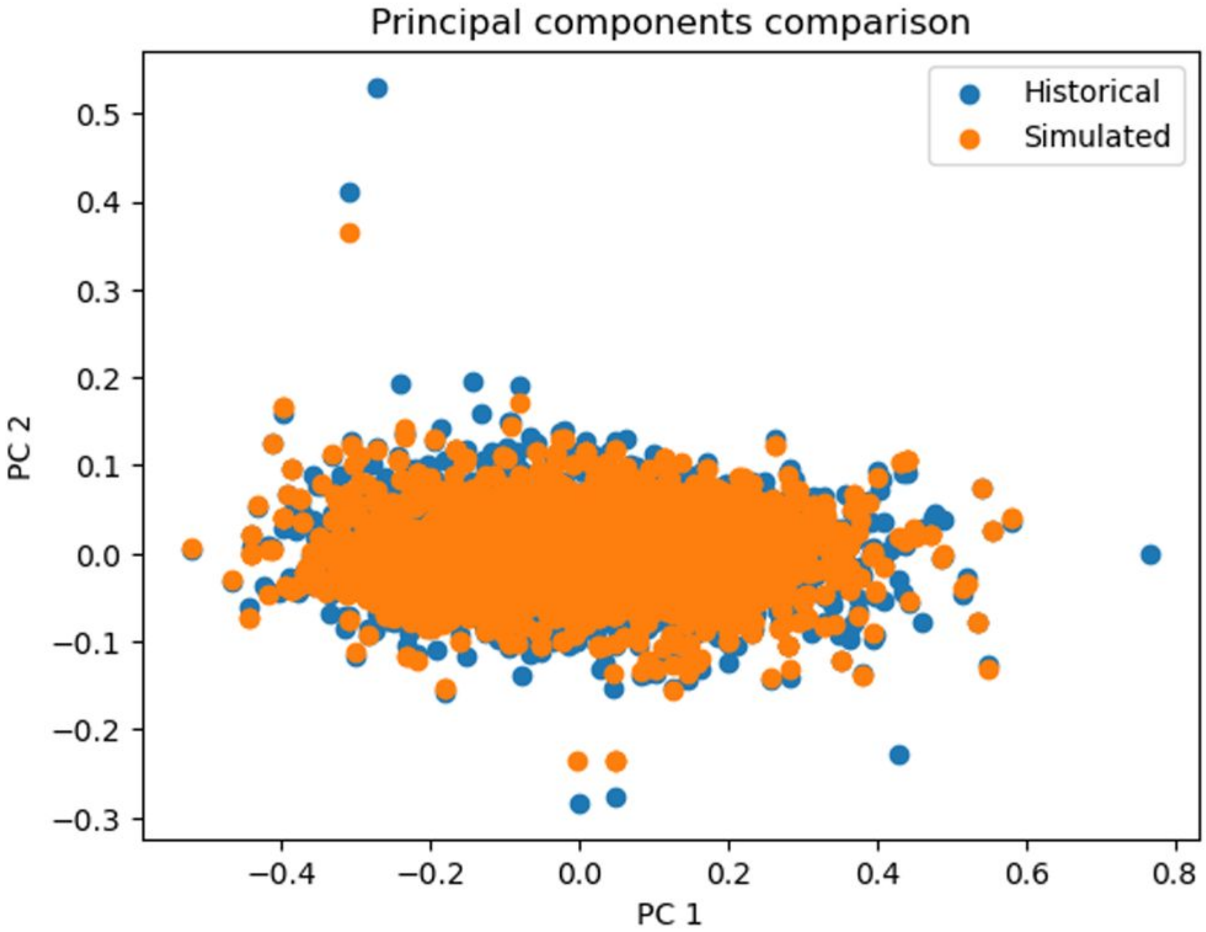
	Mean	Vol	Skew	Kurt
(0, Equity Index)	-0.004	0.013	-0.031	0.117
(5, 2y)	-0.001	0.001	-0.020	-0.008
(13, 10y)	-0.001	0.001	-0.019	-0.009
(33, 30y)	-0.001	0.001	-0.019	-0.009
(37, 1m100)	-0.022	-0.021	-0.021	-0.039
(51, 6m100)	-0.016	-0.002	0.024	-0.078
(65, 2y100)	-0.016	-0.011	0.036	-0.135
(69, cr1y)	-0.001	0.001	-0.023	0.264
(73, cr5y)	-0.001	0.000	-0.023	0.252

Table 3.2: Difference between prior and simulation statistics using two state variables.

Cross-sectional summary (one state variable)



Cross-sectional summary (two state variables)



Summary and next lecture

- Overview of the multi-asset investment simulation framework for fully general multi-asset data
- Understanding of the relationship between stationary transformations and risk factors

Next lecture:

- A good idea to get an introduction to Entropy Pooling (pages 70-73)
- Projection of stationary transformations (Sections 1.2 and 3.2)
- Better backtesting of CVaR versus variance optimization (Section 3.5)