Applied Quantitative Investment Management

Lecture 5: Instrument Pricing

Anton Vorobets

Agenda

- Full walkthrough of Chapter 4 and its Python code
- Instrument pricing (Sections 4.1 - 4.5):
  - **Bonds:** nominal, inflation-linked, and credit
  - **Equity:** fundamental and factor models
  - **Derivatives:** demystifying and presenting additional nuances
  - **Dynamic strategies including a delta hedging case study**
  - **Illiquid alternatives:** fundamental perspectives
- Multi-asset pricing case study (Section 4.6)

Bond pricing

- Bonds are characterized by having coupon payments:

$$P = \sum_{t \in \{t_1, t_2, \ldots, t_N\}} (1 + r_t)^{-t} c_t + (1 + r_{t_N})^{-t_N} C$$

- Common types of bonds:
  - Nominal (government bonds)
  - Inflation-linked (government bonds) bonds (also called linkers)
  - Credit bonds (corporate or government bonds in foreign currency)

Nominal bonds

- Pricing formula:

$$P = \sum_{t \in \{t_1, t_2, \ldots, t_N\}} (1 + r_t)^{-t} c_t + (1 + r_{t_N})^{-t_N} C$$

- Discount factor (price of a zero-coupon bond with principal 1):

$$(1 + r_t)^{-t} = D_t$$

- "Bills" (zero-coupon) versus "notes" and "bonds" (semiannual bullet bonds)

Yield to maturity

- Defined as the same rate applied to all coupon payments:

$$P = \sum_{t \in \{t_1, t_2, \ldots, t_N\}} (1 + y_{t_N})^{-t} c_t + (1 + y_{t_N})^{-t_N} C$$

- Also known as the “internal rate of return”
- Can coincide with zero-coupon rates if the curve is flat (very rare)
- Zero-coupon rates make it possible to price all bonds consistently.

  They can be extracted from coupon bond prices (STRIPS)

Inflation-linked bonds

- Coupon payments adjusted by the inflation

$$P = \sum_{t \in \{t_1, t_2, \ldots, t_N\}} D_t \bar{c}_t + D_{t_N} \bar{C}$$

$$\bar{c}_t = \frac{CPI_t}{CPI} c_t \qquad \bar{C} = \frac{CPI_{t_N}}{CPI} C$$

- Often has a principal protection to protect the principal against deflationary scenarios

Real yields and breakeven inflation

- Breakeven inflation is the difference between the yield to maturity on a nominal bond and inflation-linked bond with same maturity date
- A measure of the expected yearly inflation rate over some specific horizon
- Also possible to extract from OTC inflation swaps
- The real rate and breakeven inflation are often key risk factors tracked by multi-asset investment managers, see:

  https://antonvorobets.substack.com/p/multi-asset-macro-model

Credit bonds

- Bonds with a default probability such as corporate bonds or government in foreign (“hard”) currency
- Similar pricing form but including a spread over the yield of risk-free government bonds

$$P = \sum_{t \in \{t_1, t_2, \ldots, t_N\}} (1 + y_{t_N} + s_{t_N})^{-t} c_t + (1 + y_{t_N} + s_{t_N})^{-t_N} C$$

- The spread is compensation for default risk compared to a “hard” currency government bond

Equity models

- Fundamental models

$$P = \sum_{t \in \{t_1, t_2, \ldots, t_N\}} (1 + r_t + p)^{-t} d_t + (1 + r_{t_N+1} + p)^{-T_N+1} \frac{d_{t_N+1}}{r^\star + p - g}$$

- Factor models

$$R_{i,t} = f_i \left(F_t\right) + \varepsilon_{i,t}$$

$$R_{i,t} = \alpha_i + \beta_i^T F_t + \varepsilon_{i,t}$$

Demystifying derivatives

- Forward price

$$F_T = S_0 e^{r_T T} - \sum_{t \in \{t_1, t_2, \ldots, t_N\}} d_t e^{r_t (T-t)} \qquad F_T = S_0 e^{(r_T - q)T}$$

- European style options

$$c\left(S_0, T, K, \sigma_{T,K}, r_T, q\right) = e^{-r_T T}\left[F_T N\left(d_1\right) - K N\left(d_2\right)\right]$$

$$p\left(S_0, T, K, \sigma_{T,K}, r_T, q\right) = e^{-r_T T}\left[K N\left(-d_2\right) - F_T N\left(-d_1\right)\right]$$

$$d_1 = \frac{1}{\sigma_{T,K}\sqrt{T}}\left[\ln\left(\frac{F_T}{K}\right) + \frac{1}{2}\sigma_{T,K}^2 T\right] \quad \text{and} \quad d_2 = d_1 - \sigma_{T,K}\sqrt{T}$$

Dynamic strategies case study

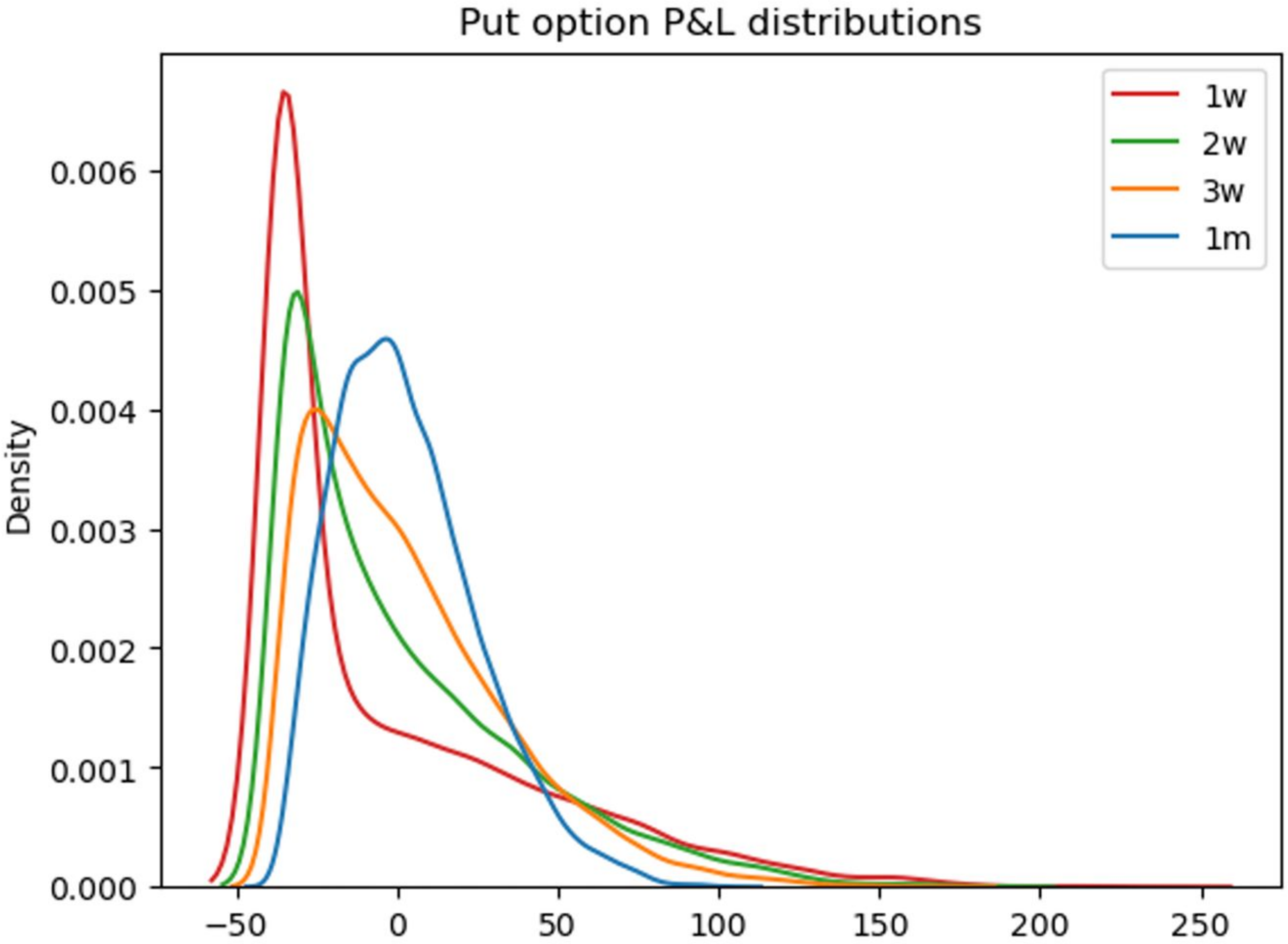
- Put option instantaneous P&L:

$$dp = \frac{\partial p}{\partial t}dt + \frac{\partial p}{\partial S}dS + \frac{\partial p}{\partial \sigma}d\sigma + \frac{\partial p}{\partial r}dr + \frac{1}{2}\frac{\partial^2 p}{\partial S^2}dS^2 + \varepsilon$$

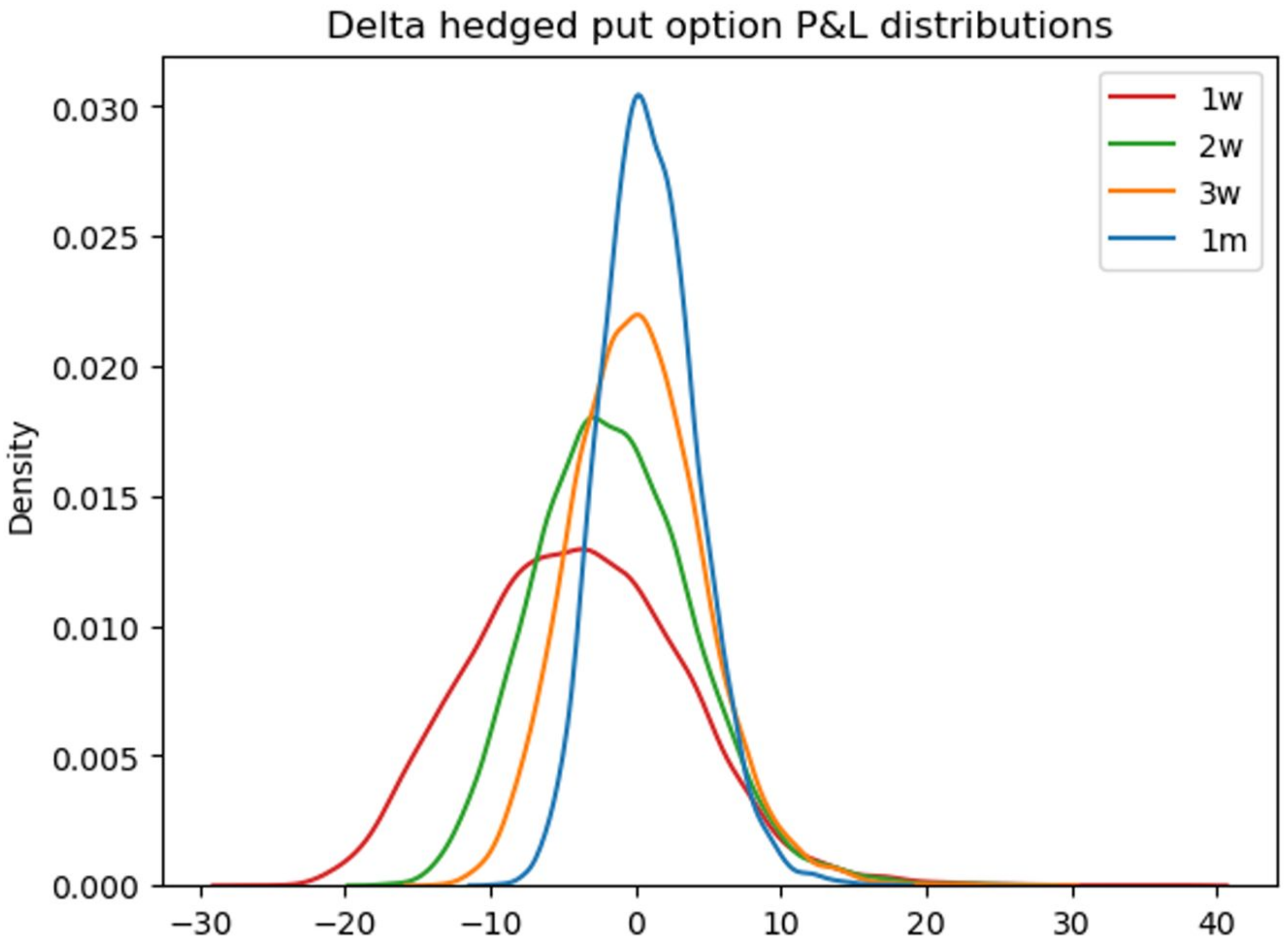
- Delta hedged and held to expiry:

$$d\Pi = \frac{\partial \Pi}{\partial t}dt + \frac{\partial \Pi}{\partial r}dr + \frac{1}{2}\frac{\partial^2 \Pi}{\partial S^2}dS^2 + \varepsilon$$

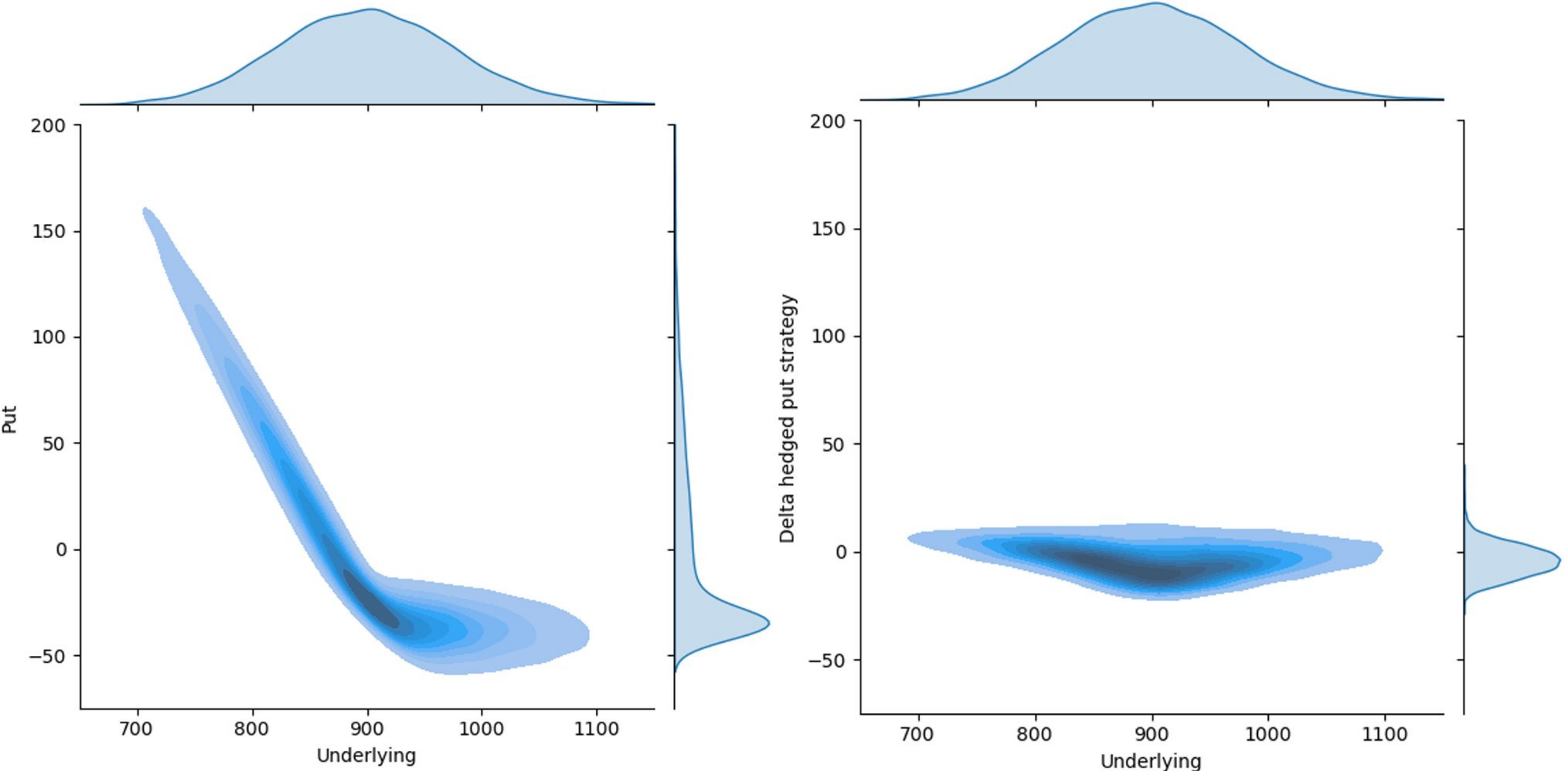
Dynamic strategies case study



Dynamic strategies case study



Dynamic strategies case study



Illiquid alternatives

- Private equity or debt, real estate, infrastructure, forest, land, etc.
- **Challenge:** few and delayed price observations, for example, quarterly observations delayed one quarter (risk of lookahead bias)
- Imputation with variational autoencoders (VAEs) is a possibility
- **Two approaches:**
  - Map to liquid factors and potentially add an illiquidity risk premium
  - Model the fundamentals similar to the dividend discount model if the exposure is very unique

Multi-asset pricing case study