Nitsche : ? $\underline{u}^h \in \underline{V}^h$:

$$a(\underline{u}^h, \underline{v}^h) - \int_{\Gamma_c^1} \frac{\theta}{\gamma_N} \sigma(\underline{u}^h)\underline{n} \cdot \sigma(\underline{v}^h)\underline{n} + \qquad \text{(i)}$$

$\hookrightarrow K$ $\qquad \to B$

$$+ \int_{\Gamma_c^1} \frac{1}{\gamma_N} \left[\gamma_N [\![u_n^h]\!] - \sigma_n(\underline{u}^h)\right]_+ \left(\gamma_N [\![v_n^h]\!] - \theta\sigma_n(v^h)\right) \qquad \text{(ii)}$$

$\to$ KKT

$$+ \int_{\Gamma_c^1} \frac{1}{\gamma_N} \left[\gamma_N \, \underline{d}_t(\underline{u}^h) - \underline{\sigma}_t(\underline{u}^h)\right]_{S^h(\underline{u}^h)} \cdot \left(\gamma_N [\![\underline{v}_t^h]\!] - \theta\, \underline{\sigma}_t(\underline{v}^h)\right) \text{(iii)}$$

$\hookrightarrow$ FRI

$$= L(\underline{v}^h) \qquad \forall \underline{v}^h \in \underline{V}^h$$

where $\quad S^h(u^h) = \begin{cases} S_T^{1,h}, & \text{Tresca} \\ \mathcal{F}\left[\gamma_N [\![u_n^h]\!] - \sigma_n(\underline{u}^h)\right]_+ & \text{Coulomb.} \end{cases}$

$\qquad P_{1n}^{\gamma}(\underline{u})$

Define $\quad P_{\theta\gamma}^{n}(\underline{u}^h) := \gamma_N [\![u_n^h]\!] - \theta\sigma_n(\underline{v}^h) \quad$ (scalar)

$\qquad \underline{P_{\theta\gamma}^{t}}(\underline{u}^h) := \gamma_N \, \underline{d}_t(\underline{u}^h) - \theta\underline{\sigma}_t(\underline{u}^h)$

$\uparrow$ cpt_stress_interf

So the friction integral becomes:

$$\text{(iii)} = \int_{\Gamma_c^1} \frac{1}{\gamma_N} \left[\underline{P_{1\gamma}^{t}}(\underline{u}^h)\right]_{(\mathcal{F}[P_{1\gamma}^{n}(\underline{u})]_+)} \cdot \underline{P_{\theta\gamma}^{t}}(\underline{u}^h)$$

OK (it works)

Jacobian:

elasticity (linear)

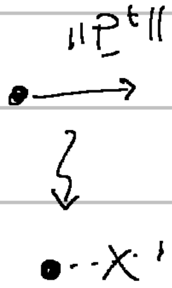
$$J(\underline{U}) = K - B + KKT(\underline{U}) + FRI(\underline{U})$$

(i) (linear)

Re:

$$\underline{F}_0(\underline{u}^h) := \int_{\Gamma_c^1} \frac{1}{\gamma_N}\left[\underline{P}^t_{1\gamma}(\underline{u}^h)\right]_{(\mathcal{F}[P^m_{1\gamma}(\underline{u})]_+)} \cdot \underline{P}^t_{\theta\gamma}(\underline{v}^h)$$

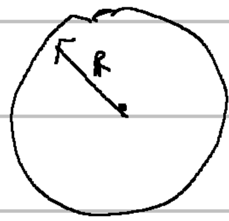
Case 1: $\overbrace{\mathcal{F}[P^m_{1\gamma}(\underline{u})]_+}^{\text{radius of the ball}} < 0 \implies$ OPEN



$$\implies \left[\frac{1}{\gamma_N}[P^t_{1\gamma}(\underline{u}^h)]\right]_0 = \underline{0}$$

$$\implies \frac{\partial F_i}{\partial u_k} = 0 \qquad (\text{open})$$

Case 2: $\mathcal{F}[P^m_{1\gamma}(\underline{u})]_+ > 0$ CLOSED



$$\implies \underline{F}(\underline{u}^h) = \int_{\Gamma_c^1} \frac{1}{\gamma_N}[\underline{P}^t_{1\gamma}(\underline{u}^h)]_{B(\mathcal{F}P^m_{1\gamma}(\underline{u}^h))} \cdot \underline{P}^t_{\theta\gamma}(\underline{v}^h).$$

Case 2.1: $\|\underline{P}^t_{1\gamma}(\underline{u}^h)\|_2 < \overbrace{\mathcal{F} P^m_{1\gamma}(\underline{u}^h)}^{>0}$ STICK

$$\implies \underline{F}_0(\underline{u}^h) = \int_{\Gamma_c} \frac{1}{\gamma_N} \underline{P}^t_{1\gamma}(\underline{u}^h)\cdot \underline{P}^t_{\theta\gamma}(\underline{v}^h)$$

$$\frac{\partial F_{0i}}{\partial u_k} \stackrel{\text{linear}}{=} \int_{\Gamma_c}\frac{1}{\gamma_N} \underline{P}^t_{1\gamma}(\underline{\varphi}_k)\cdot \underline{P}^t_{\theta\gamma}(\underline{\varphi}_i) =: B$$

$\hookrightarrow$ Case 2.2 $\quad \|\underline{P}_{1\gamma}^{t}(\underline{u}^h)\|_2 > \mathcal{F}\, P_{1\gamma}^{n}(\underline{u}^h)$. SLIDE

$$\Rightarrow \quad \underline{F}_0(\underline{u}^h) = \int_{\Gamma_c^1} \left(\frac{\mathcal{F}}{\gamma_N} P_{1\gamma}^{n}(\underline{u}^h) \underbrace{\frac{\underline{P}_{1\gamma}^{t}(\underline{u}^h)}{\|\underline{P}_{1\gamma}^{t}(\underline{u}^h)\|}} \right) \cdot \underline{P}_{\theta\gamma}^{t}(\underline{\varphi}_i)$$

$$\frac{\partial F_{0,i}}{\partial u_k} = \int_{\Gamma_c^1} \left(\frac{\mathcal{F}}{\gamma_N} \underset{(*)}{P_{1\gamma}^{n}(\underline{\varphi}_k)\,\hat{t}} \right) \cdot \underline{P}_{\theta\gamma}^{t}(\underline{\varphi}_i) \qquad \underline{V}$$

$$+ \int \left(\frac{\mathcal{F}}{\gamma_N} P_{1\gamma}^{n}(\underline{u}^h) \frac{\partial \hat{t}}{\partial u_k} \right) \cdot \underline{P}_{\theta\gamma}^{t}(\underline{\varphi}_i) \,,$$

where
$$\hat{t} := \frac{\underline{P}_{1\gamma}^{t}(\underline{u}^h)}{\|\underline{P}_{1\gamma}^{t}(\underline{u}^h)\|} \,,$$

$$\frac{\partial \hat{t}}{\partial u_k} = \frac{\underline{P}_{1\gamma}^{t}(\underline{\varphi}_k)\,\|\underline{P}_{1\gamma}^{t}(\underline{u})\| - \underline{P}_{1\gamma}^{t}(\underline{u}^h)\frac{\partial}{\partial u_k}\|\cdots\|}{\|\underline{P}_{1\gamma}^{t}(\underline{u})\|^2} \quad (**) \quad (***)$$

$$\left[\frac{\partial}{\partial u^k}\|\underline{P}_{1\gamma}^{t}(\underline{u})\| = \frac{\partial}{\partial u^k}\left((P_{1\gamma}^{t}(\underline{u}))_1^2 + (P_{1\gamma}^{t}(\underline{u}))_2^2 \right)^{\frac{1}{2}} \right.$$

$$= \cancel{\frac{1}{2}} \frac{1}{\|P_{1\gamma}^{t}(\underline{u})\|} \cancel{2} \left[(P_{1\gamma}^{t}(\underline{u}))_1 \frac{\partial (P_{1\gamma}^{t}(\underline{u}))_1}{\partial u_k} \right.$$

$$\left.\left. + (P_{1\gamma}^{t}(\underline{u}))_2 \frac{\partial (P_{1\gamma}^{t}(\underline{u}))_2}{\partial u_k} \right] \right]$$

$$= \frac{P_{1\gamma}^{t}(\underline{\varphi}_k)}{\|P_{1\gamma}^{t}(\underline{u})\|} - \frac{P_{1\gamma}^{t}(\underline{u}^h)}{\|P_{1\gamma}^{t}(\underline{u}_h)\|^3} \underline{P}_{1\gamma}^{t}(\underline{u}) \cdot \underline{P}_{1\gamma}^{t}(\underline{\varphi}_k)$$

$\Longrightarrow$

$$\frac{\partial F_i}{\partial u_k} = \int_{\Gamma_c^1} \left(\frac{\mathcal{F}}{\gamma_N} P_{1\gamma}^{n}(\underline{\varphi}_k) \boxed{\frac{P_{1\gamma}^{t}(\underline{u}^h)}{\|P_{1\gamma}^{t}(\underline{u}^h)\|}} \right) \cdot \underline{P_{\theta\gamma}^{t}}(\underline{\varphi}_i)$$

$$+ \int_{\Gamma_c^1} \frac{\mathcal{F}}{\gamma_N} P_{1\gamma}^{n}(\underline{u}) \left(\frac{\underline{P_{1\gamma}^{t}}(\underline{\varphi}_k)}{\boxed{\|P_{1\gamma}^{t}(\underline{u})\|}} - \frac{P_{1\gamma}^{t}(\underline{u}^h)}{\|P_{1\gamma}^{t}(u_h)\|^3} \left(\underline{P_{1\gamma}^{t}}(\underline{u}) \cdot \underline{P_{1\gamma}^{t}}(\underline{\varphi}_k) \right) \right) \cdot \underline{P_{\theta\gamma}^{t}}(\underline{\varphi}_i)$$

corresponds to $FR1(\underline{U})$