Tensor Notes

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Tensors are important in many areas of science and I, as a physicist and a astrophysical researcher, have tried to put the basics of it at these very brief "Tensor Notes".

I hope you enjoy!

Chapter 1

Introduction

1.1 The index notation

An easy way to introduce the index notation is using a matrix as it follows

$$\begin{pmatrix} v_1^{'} \\ \vdots \\ v_n^{'} \end{pmatrix} = \begin{pmatrix} A_{11} & \dots & A_{1n} \\ \vdots & & \vdots \\ A_{n1} & \dots & A_{nn} \end{pmatrix} \begin{pmatrix} v_1 \\ \vdots \\ v_n \end{pmatrix} \tag{1.1.1}$$

and, as we can see, the prime terms can be written like

$$v_n^{'} = \sum_{m=1}^{n} A_{nm} v_m$$

A similar case is the inner product,

$$\overrightarrow{v} \cdot \overrightarrow{u} = \sum_{m=1}^{n} v_m u_m$$

The inner product case here is a simple one, but soon we will go further into the inner products.

It is useful to omit the summation symbols in index notation,

$$\sum_{m=1}^{n} A_{nm} v_m \implies A_{nm} v_m$$

$$\sum_{\beta=1}^{n} \sum_{\gamma=1}^{n} A_{\alpha\beta} B_{\beta\gamma} C_{\gamma\delta} \implies A_{\alpha\beta} B_{\beta\gamma} C_{\gamma\delta}$$

Note that a summation is assumed over the indices that appear twice and no summation is assumed when indices appear only once.

1.2 Invariant vectors, covariant vectors and contravariant vectors

We can explain the basics of these two definitions using the concept of base. We have an orthogonal base and an unknown vector, if this base scales changes from meters to centimeters (divided by 100) we have three options to this vector change:

- The vector does not change, because it does not depend on the unit scale, so it is an invariant one;
- The vector changes proportionally, because it is proportional to the unit scale, so it is a covariant one;
- The vector changes inversely, because it is inversely proportional to the unit scale, so it is a contravariant one.

[1]

[1] [1],[2]

Bibliography

[1] Kees Dullemond and Kasper Peeters. Introduction to tensor calculus. Kees Dullemond and Kasper Peeters, pages 42–44, 1991.

[2] Wikipedia contributors. Covariance and contravariance of vectors — Wikipedia, the free encyclopedia.