DSE 210: Probability and Statistics using Python

Worksheet 8 — Probabilistic reasoning using Bayes' rule

1. One coin in a collection of 65 coins has two heads; the rest of the coins are fair. If a coin, chosen at random from the lot and then tossed, turns up heads six times in a row, what is the probability that it is the two-headed coin?

2. A scientist discovers a fossil fragment that he believes is either some kind of tiger (with probability 1/3) or mammoth (with probability 2/3). To shed further light on this question, he conducts a test which has the property that for tigers, it will come out positive with probability 5/6 whereas for mammoths it will come out positive with probability just 1/3. Suppose the test comes out negative. What is the probability, given the outcome of the test, that the fossil comes from a tiger?

3. Sherlock Holmes finds paw prints at the scene of a murder, and thinks that they are either from a dog, with probability 3/4, or from a small bear, with probability 1/4. He then discovers some unusual scratches on a nearby tree. The probability that a dog would produce these scratches is 1/10, while the probability that a bear would is 3/5. What is the probability, given the presence of scratches, that the animal is a bear?

4. A small boy is lost coming down Mount Washington. The leader of the search team estimates that there is a probability p that he came down on the east side and a probability $1-p$ that he came down on the west side. He has n people in his search team who will search independently, and if the boy is on the side being searched, each member will find the boy with probability u. The leader decides to have k people search the east side and the remaining $n-k$ people search the west side.

   (a) What is the probability the boy will be found if he is on the east side? On the west side? Give your answers in terms of u, k, and n.

   (b) Write down the probability of finding the boy as a function of p, u, n, and k.

   (c) What is the best choice of k, to maximize the probability of finding the boy?

5. In London, half of the days have some rain. The weather forecaster is correct 2/3 of the time: the probability that it rains, given that she has predicted rain, and the probability that it does not rain, given that she has predicted it won't rain, are both 2/3. When rain is forecast, Mr. Pickwick takes his umbrella. When rain is not forecast, he takes it with probability 1/3.

   (a) What is the probability that the forecaster predicts rain? (Hint: find an equation containing this quantity and solve for it.)

   (b) What is the probability that Pickwick has no umbrella, given that it rains?

   (c) What is the probability that he brings his umbrella, given that it does not rain?