DSE 210: Probability and Statistics using Python

Worksheet 2 — Probability spaces

1. In each of the following experiments, define the sample space Ω.
   (a) A two-sided coin is tossed, with heads on one side and tails on the other.
   (b) You can choose a color for your new car's exterior (choices: red, black, silver, blue) and interior (choices: beige, black).
   (c) A customer is asked for the month and day-of-week on which her birthday lies.
   (d) A coin is tossed 100 times in a row.
2. Let A, B, C be events defined on a sample space Ω. Use notation for union, intersection, and set difference to write expressions for each of the following combinations.
   (a) All three events occur.
   (b) At least one of the events occurs.
   (c) A and B occur, but not C.
3. Consider a sample space $\Omega = \{a, b, c\}$ with probabilities $\Pr(a) = 1/2$ and $\Pr(b) = 1/3$.
   (a) What is $\Pr(c)$?
   (b) How many distinct events can be defined on this space?
   (c) What event has probability 2/3?
4. A fair coin is tossed three times in succession. For each of the following events on sample space $\{H, T\}^3$, describe it in words and determine its probability.
   (a) $E_1 = \{HHH, HHT, HTH, HTT\}$
   (b) $E_2 = \{HHH, TTT\}$
5. Let A and B be events defined on a sample space Ω such that $\Pr(A \cap B) = 1/4$, $\Pr(A^c) = 1/3$, and $\Pr(B) = 1/2$. Here $A^c = \Omega \setminus A$ is the event that A *doesn't* happen. What is $\Pr(A \cup B)$?
6. A pair of dice are rolled. What is the probability that they show the same value?
7. Recall that a chessboard has 64 squares arranged in an 8×8 grid. A rook is a particular chess piece that is said to *attack* anything that shares either the same row or the same column. Suppose two rooks are placed at random on a chessboard (in distinct locations). What is the chance that they are attacking each other?
8. In Morse code, each letter is formed by a succession of dashes and dots. For instance, the letter S is represented by three dots and the letter O is represented by three dashes. Suppose a child types a sequence of 9 dots/dashes at random (each position is equally likely to be a dot or a dash). What is the probability that it spells out SOS?

9. A die is loaded in such a way that the probability of each face turning up is proportional to the number of dots on that face (for instance, a six is three times as probable as a two). What is the probability of getting an even number in one throw?

10. Five people of different heights are lined up against a wall in random order. What is the probability that they just happen to be in increasing order of height (left-to-right)?

11. A deck of ordinary cards is shuffled and a hand of 13 cards are dealt. What is the probability that the first and second cards are of the same suit?

12. Snow White asks three of the seven dwarfs, chosen at random, to accompany her on a trip.

    (a) What is the probability that Dopey is in this group?

    (b) What is the probability that both Dopey and Sneezy are in the group?

    (c) What is the probability that neither Dopey nor Sneezy is in the group?

13. Assume that whenever a child is born, it is equally likely to be a girl or boy. What is the probability that a randomly-chosen family with six children has exactly three girls and three boys?

14. In a true-false exam consisting of ten questions, a student answers every question by randomly guessing.

    (a) What is the probability that he gets all the questions right?

    (b) What is the probability that he gets at least 80% of the questions right?

15. How long does a sequence of random decimal digits (i.e. each digit is equally likely to be $0, 1, 2, \ldots, 9$) have to be in order for the probability of the digit 7 appearing to be at least 0.9?

16. A distribution on $[0, 1]$ has density

$$p(x) = \begin{cases} cx & \text{if } 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

    where c is some constant.

    (a) What is the numerical value of c?

    (b) What is the probability that the outcome lies in the interval $[0, 1/2]$?

    (c) What is the probability that the outcome lies in the interval $[1/3, 2/3]$?

17. A dart is thrown at the square of side-length 2 centered at the origin: $\{(x, y) : -1 \leq x, y \leq 1\}$. Suppose that it is equally likely to land at any point of the square.

    (a) What is the probability that the dart lands in the unit circle $\{(x, y) : x^2 + y^2 \leq 1\}$?

    (b) Write a computer simulation in which you repeatedly pick points (x, y) according to this distribution and determine whether they fall in the circle. Use your simulation to estimate the value of π.