DSE 210 - Yuan Hu

Worksheet 8

8.1 Let P(Two) indicates probability of getting the two heads coin from 65 coins.
P(One) indicates probability of getting a regular coin from 65 coins.

Let P(6 Heads | Two) denotes probability of getting 6 heads up when tossing a two headed coin.
P(6 Heads | One) denotes probability of getting 6 Heads up when tossing a regular coin.

P(Two | 6 Heads) denotes probability of the selected coin is a two headed coin given the result that after 6 times of tossing, head showed up 6 times.

$$P(\text{Two} \mid 6\text{Heads}) = \frac{P(6\text{Heads} \mid \text{Two}) \cdot P(\text{Two})}{P(6\text{Heads} \mid \text{Two}) \cdot P(\text{Two}) + P(6\text{Heads} \mid \text{One}) \cdot P(\text{One})}$$

$$= \frac{1 \times 1/65}{1 \times 1/65 + (\frac{1}{2})^6 \times \frac{64}{65}} \quad \boxed{= \frac{1}{2}}$$

8.2 Let P(Tiger) denotes probability that fossil comes from tiger. $P(\text{Tiger}) = \frac{1}{3}$
P(Mammal) . . . . . . fossil comes from mammals $P(\text{mammal}) = \frac{2}{3}$

Let P(Pos | Tiger) denotes probability that the test will come out Positive given a tiger sample.
P(Pos | Mammal) . . . . given a mammal sample

Hence, $P(\text{Neg} \mid \text{Tiger}) = 1 - \frac{5}{6} = \frac{1}{6}$, $P(\text{Neg} \mid \text{Mammal}) = 1 - \frac{1}{3} = \frac{2}{3}$

$$P(\text{Tiger} \mid \text{Neg}) = \frac{P(\text{Neg} \mid \text{Tiger}) \cdot P(\text{Tiger})}{P(\text{Neg} \mid \text{Tiger}) \cdot P(\text{Tiger}) + P(\text{Neg} \mid \text{Mammal}) \cdot P(\text{Mammal})} = \frac{1/6 \times \frac{1}{3}}{1/6 \times \frac{1}{3} + \frac{2}{3} \times \frac{2}{3}}$$

$$\boxed{= 1/9}$$

8.3. $P(\text{Dog}) = 3/4$, $P(\text{Bear}) = 1/4$

$P(\text{Scratches} \mid \text{Dog}) = 1/10$, $P(\text{Scratches} \mid \text{Bear}) = 3/5$

$$P(\text{Bear} \mid \text{Scratches}) = \frac{P(\text{Scratches} \mid \text{Bear}) \times P(\text{Bear})}{P(\text{Scratches} \mid \text{Bear}) \times P(\text{Bear}) + P(\text{Scratches} \mid \text{Dog}) \times P(\text{Dog})}$$

$$= \frac{3/5 \times 1/4}{3/5 \times 1/4 + 1/10 \times \frac{3}{4}} = \boxed{2/3}$$

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Worksheet 9

9.1 $h^* = \arg\max_j \pi_j \cdot P_j(x)$, $j \in \{1, 2, 3\}$

$$\pi_1 \cdot P_1(x) = \begin{cases} \frac{1}{3} \times \frac{7}{8}, & x \in [-1, 0] \\ \frac{1}{3} \times \frac{1}{8}, & x \in (0, 1] \end{cases} = \begin{cases} \frac{7}{24}, & x \in [-1, 0] \\ \frac{1}{24}, & x \in (0, 1] \end{cases}$$

$$\pi_2 P_2(x) = \begin{cases} 0, & x \in [-1, 0] \\ 1/6, & x \in (0, 1] \end{cases}$$

$$\pi_3 P_3(x) = \begin{cases} 1/4, & x \in [-1, 0] \\ 1/4, & x \in (0, 1] \end{cases}$$

$$\arg\max_j \pi_j P_j(x) = \begin{cases} 1, & x \in [-1, 0] \\ 3, & x \in (0, 1] \end{cases}$$

Hence, $h^* = \begin{cases} \cancel{\frac{7}{24}}\ 1, & x \in [-1, 0] \\ \cancel{1/4}\ 3, & x \in (0, 1] \end{cases}$

Worksheet 10

10.1 (a) For $k=3$, the optimal k-means solution of centers will be located at point $(-9, 0, 9)$

$$\text{Cost} = \|-10-(-9)\|^2 + \|-8-(-9)\|^2 + \|0-0\|^2 + \|8-9\|^2 + \|10-9\|^2$$
$$= 4.$$

(b). $k=3$, $u_1 = -10$, $u_2 = -8$, $u_3 = 0$

Iteration 1:

Assign -10 to u_1, -8 to u_2, $\{0, 8, 10\}$ to u_3

calculate: $mean_1 = -10$, $mean_2 = -8$, $mean_3 = 6$

Update: $u_1 = mean_1 = -10$, $u_2 = mean_2 = -8$, $u_3 = mean_3 = 6$

Final set of cluster centers are $-10, -8, 6$

$$\text{Cost} = \|-10-(-10)\|^2 + \|-8-(-8)\|^2 + \|0-6\|^2 + \|8-6\|^2 + \|10-6\|^2$$
$$= 0 + 0 + 36 + 4 + 16$$
$$= 56$$