DSE 210: Probability and Statistics using Python

Worksheet 3 — Multiple events, conditioning, and independence

1. A fair (six-sided) die is rolled three times. What is the probability that at least one of the rolls is a 6?

2. A fair coin is tossed 10 times. What is the probability that there are at least two heads?

3. A student must choose exactly two of the following three electives: art, French, or mathematics. The probability that he chooses art is 5/8, the probability he chooses French is 5/8, and the probability that he chooses both art and French is 1/4.

   (a) What is the probability that he chooses mathematics?
   (b) What is the probability that he chooses either art or French?

4. For a bill to come before the president of the United States, it must be passed by both the House of Representatives and the Senate. Assume that, of the bills presented to the two bodies, 60% pass the House, 80% pass the Senate, and 90% pass at least one of the two. Calculate the probability that the next bill presented to the two groups will come before the president.

5. In a fierce battle, not less than 70% of the soldiers lost one eye, not less than 75% lost one ear, not less than 80% lost one hand, and not less than 85% lost one leg. What is the minimal possible percentage of those who simultaneously lost one ear, one eye, one hand, and one leg?

6. A coin is tossed three times. What is the probability that there are exactly two heads, given that:

   (a) the first outcome is a head?
   (b) the first outcome is a tail?
   (c) the first two outcomes are both heads?
   (d) the first two outcomes are both tails?
   (e) the first outcome is a head and the third outcome is a tail?

7. A card is drawn at random from a standard deck. What is the probability that:

   (a) it is a heart, given that it is red?
   (b) it is higher than a ten, given that it is a heart (interpret J, Q, K, A as having numeric value $11, 12, 13, 14$)?
   (c) it is a jack, given that it is higher than a 10?

8. If $\Pr(B^c) = 1/4$ and $\Pr(A|B) = 1/2$, what is $\Pr(A \cap B)$?

9. A die is rolled twice. What is the probability that the sum of the two rolls is > 7, given that:

   (a) the first roll is a 4?
   (b) the first roll is a 1?
   (c) the first roll is > 3?

    (d) the first roll is < 5?

10. From a deck of five cards numbered 2, 4, 6, 8, and 10, respectively, a card is drawn at random and replaced. This is done three times. What is the probability that the card numbered 2 was drawn exactly two times, given that the sum of the numbers on the three draws is 12?

11. In a particular boarding school, every student is associated with exactly one of the four houses Gryffindor, Hufflepuff, Ravenclaw, and Slytherin. The fraction of students in these houses is 1/3, 1/4, 1/6, and 1/4, respectively. Suppose that 1/2 the Gryffindors are good at the Dark Arts, along with 1/3 of the Hufflepuffs, 1/2 of the Ravenclaws and 2/3 of the Slytherins. What is the overall fraction of students at this school who are good at the Dark Arts?

12. A particular car manufacturer has three factories F_1, F_2, F_3 making 25%, 35%, and 40%, respectively, of its cars. Of their output, 5%, 4%, and 2%, respectively, are defective. A car is chosen at random from the manufacturer's supply.

    (a) What is the probability that the car is defective?

    (b) Given that it is defective, what is the probability that it came from factory F_1?

13. Suppose that there are equal numbers of men and women in the world, and that 5% of men are colorblind whereas only 1% of women are colorblind. A person is chosen at random and found to be colorblind. What is the probability that the person is male?

14. A doctor assumes that his patients has one of the three diseases d_1, d_2, or d_3, each with probability 1/3. He carries out a test that will be positive with probability 0.8 if the patient has d_1, with probability 0.6 if the patient has d_2, and with probability 0.4 if the patient has d_3.

    (a) What is the probability that the test will be positive?

    (b) Suppose that the outcome of the test is positive. What probabilities should the doctor now assign to the three possible diseases?

15. A coin is tossed three times. Consider the following five events:

    • A: Heads on the first toss
    • B: Tails on the second toss
    • C: Heads on the third toss
    • D: All three outcomes the same
    • E: Exactly one head

    Which of the following pairs of events are independent?

    (i) A and B (ii) A and D (iii) A and E (iv) D and E.

16. You randomly shuffle a standard deck and deal two cards. Which of the following pairs of events are independent?

    (i) $A = \{$first card is a heart$\}, B = \{$second card is a heart$\}$

    (ii) $A = \{$first card is a heart$\}, B = \{$first card is a 10$\}$

    (iii) $A = \{$first card is a 10$\}, B = \{$second card is a 9$\}$

    (iv) $A = \{$first card is a heart$\}, B = \{$second card is a 10$\}$

17. A student applies to UCLA and UCSD. He estimates that he has a probability of 0.5 of being accepted at UCLA and a probability of 0.3 of being accepted at UCSD. He further estimates that the probability that he will be accepted by both is 0.2.

    (a) What is the probability that he is accepted at UCSD if he is accepted at UCLA?
    (b) Is the event "accepted at UCLA" independent of the event "accepted at UCSD"?

18. Each of the four engines on an airplane functions correctly on a given flight with probability 0.99, and the engines function independently of each other. Assume that the plane can make a safe landing if at least two of its engines are functioning correctly. What is the probability of a safe landing?

19. You are dealt ten cards, one at a time, from the top of a randomly shuffled deck.

    (a) What is the probability that the tenth card you get is an ace?
    (b) What is the probability that the tenth card is an ace, given that the third card is an ace?
    (c) What is the probability that the third card is an ace, given that the tenth card is an ace?

20. Prove that if events A and B are independent, then so are:

    (a) A and B^c.
    (b) A^c and B^c.