WORKSHEET 1 SETS AND COUNTING

1. Set A is {1, 2, 3, 4, 5}.
(a) What is the formal notation for *the set of all sequences of three elements from A?*
(b) How many such sequences are there?

Solution:

(a) *The set of all sequences of three elements from A* denoted $\mathbf{A^3 = A * A * A}$

(b) There are $\mathbf{5^3 = 125}$ sequences.

2. Let A and B be sets with $|A| = 5$ and $|B| = 7$.
(a) What is the largest size $A \bigcup B$ could possibly have?
(b) What is the smallest size $A \bigcup B$ could have?
(c) What is the largest size $A \bigcap B$ could have?
(d) What is the smallest size $A \bigcap B$ could have?

Solution:

(a) The largest size of $A \bigcup B$ is **12** obtained when $A \bigcap B = \varnothing$.

(b) The smallest size of $A \bigcup B$ is **7** obtained when $A \bigcap B = 5$.

(c) The largest size of $A \bigcap B$ is **5** when A of size 5 is a subset of B of size 7.

(d) The smallest size of $A \bigcap B$ is **0** obtained when $A \bigcap B = \varnothing$

3. How many binary sequences of length 10 are there?

Solution:

There are 2^{10} binary sequences of length 10.

4. An ice-cream parlor lets you choose 3 of their 10 available flavors. How many choices do you have?

Solution:

Since the order doesn't matter, there are $\binom{\mathbf{10}}{\mathbf{3}} = \mathbf{120}$ choices.

5. You have six sports trophies, and you want to choose three of them and line them up on your mantelpiece. How many different arrangements can you make?

Solution:

Apparently, the order of trophies matters, there are $\binom{6}{3} = 20$ ways to select three trophies, and $3! = 6$ ways to permuted three trophies, so the total different arrangements can be make is $\binom{\mathbf{6}}{\mathbf{3}} \times \mathbf{6} = \mathbf{120}$.