Mathematical Modeling of Confined Space Airborne Virus Transmission

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1 Introduction

Since the beginning of the COVID-19 break in Wuhan China, much research has been done in the field of modeling the outbreak on a bigger scale such as cities, states, and countries. Many models have been constructed in analyzing spreads over larger communities. However, the environment in which the infection happens can also be an important factor when considering the behavior of transmission, especially in confined spaces such as dormitories, prisons, and classrooms. Accurate mathematical models are needed to understand airborne infection in these settings for future epidemiologic studies and improvement in designing building parameters.

This article focuses on using quantitative models to simulate the airborne disease infection in a university classroom. Infections in the classroom can be very dangerous in terms of condensed sittings and discussions during the class. The exhaled air produced by an infected individual will increase the density of the virus given time in a confined space. The longer a healthy individual is exposed in a virus-dense environment gives more probability for this person to be infected. The famous Wells-Riley equation[5] has been commonly used to estimate indoor infections. However, one critical assumption of the Well-Riley equation is that the virus is assumed to be uniformly distributed in a fixed confined space. In reality, this is not often the case. One of the most important aspects of virus transmission is the relative distance between an infected individual and a healthy individual. This article focuses on improving Well-Riley's model by incorporating the distance factor and present the simulation in different dimensions to provide more insight into building safer classrooms.

2 Revising Wells Riley Equation:

Most infection risk models can be divided into two categories: the threshold model and the stochastic model. The threshold model assumes that the infection will not happen on a healthy individual until the infected does intake exceeds a certain threshold. The idea is quite straightforward in[2], where the individual has a relatively low probability of being infected as the concentration of virus that is breathed in or surrounding the healthy individual does not pass a certain threshold. Yet, this is often hard to determine and quantified when observing a new virus which there is not enough data yet to be researched. The stochastic model[2] assumes a certain probability to happen to given the density of a virus around the healthy individual. Wells Riley Model is a classical stochastic approach for estimating confined space virus transmission. The probability of an individual getting infected is depended on the amount of time which this person stays in a virus-rich environment. In order to estimate the probability of a healthy individual getting infected, Riley developed this equation.

$$P = \frac{D}{S} = 1 - e^{-I*q*p*t/Q} \tag{1}$$

P is the probability. D is the number of currently infected individuals. S is the number of potential hosts(healthy individuals) in the confined space. I is the number of infected individual, p is the breathing rate of an average adult(m^3/s), q is the quanta generation rate for the infected individual of this particular virus(quanta/s) and Q is the ventilation rate. t is the total exposure time. Together, the equation gives a Poisson distribution describing the probability of a healthy individual gets infected. This equation describes the natural idea which says that if there more infected sources presented in a fixed environment with a extending time, the probability that any healthy individual to be affected will increase at the same rate depending on the ventilation rate of the fixed environment. However, this derivation disregards the relative position of the healthy individual to the infected individual in this room.

When deriving this equation, Riley[5] made two critical assumptions. 1) the room air is well mixed, 2) a steady-state virus concentration only varies with ventilation rate. The first assumption gives that the infectious particles, namely quanta, generated by an infected individual have an equal chance to spread to anywhere in this room uniformly. Meaning that the amount of virus that is generated by an infected individual sitting at the corner of the classroom will have the same spread pattern as the spread of an infected individual who's sitting at the center of the classroom. For example, if the infector A is sitting at the center of the classroom and the infector B is sitting at the corner of the classroom. Then by Riley's assumption, they should have the same amount of influence over the healthy individual C who might sitting very close the infector A. The second assumption implies that the concentration of virus density stays the same over a fixed time interval in a given confined environment like a classroom. In this article, we use a mathematical model to tackle these two assumptions by accurately reflecting the change of virus density by distance and does not rely on the steady-state assumption as the overall density of virus should change with respect to time.

3 Model Construction

The most efficient way of transmitting a virus in a confined space is through airborne infection. World Health Organization has advised keeping a 1.5-2 meters distance to keep people from being infected by COVID-19. Notably, the distance between healthy and infected individuals can be a key factor in determining the probability of infection. However, previous research[6] has shown that having a 2-meters relative distance does not guarantee safety from being infected. Some infective particles can travel up to 10 meters from the emission source and particles suspended in the air remain viable and infectious for hours [7]. Thus we decided to use distance coefficient $D(x_d, y_d)$ to calculate the relative distance between the infected and healthy individuals.

This distance coefficient takes a few assumptions on the spread of infectious particles. Firstly, there should be no physical barrier that blocks the airborne infectious particles from its natural spreading behavior. In other words, the position vector u $= < v_x, v_y >$ on the XY plane of the infectious particle does not change by an outside force. But the position vector on the z-direction is subject to change by gravitation force, meaning the density of infectious particles, quanta q, will decrease as the distance between the infected individual and healthy individual increases. The second assumption is that all individuals in the space should have equal levels of strength, immunity, quanta generation, and any other physical parameters of a human being. The third assumption is the neglection of the loss of infectious particles due to temperature, humidity, or other factors that are small compared to the loss of ventilation rate.

After the assumptions have been made, we will calculate the cumulative density (quanta/time step) that is received by a healthy individual, which is the summation of all infectious particles that are generated by all the infected individuals multiplied by the distance coefficient in a confined space.

$$Q(a_i) = \sum_{b_i}^{b_n} q * D(x_d, y_d) \tag{2}$$

a_i here represents the healthy individual. b_i represents all the infected individuals. Now, if we substitute the equation (2) into the (1), we will have the equation(3).

$$P = \frac{D}{S} = 1 - e^{-Q(a_i)*p*t/Q}$$

So this updated Wells-Riley equation will take into account how distance factors in considering the influence of infectious particles.

We tried a range of different equations to achieve the phenomenon of the inverse relationship between distance and density of the virus. And we find out the one that is closest to the many COVID-19 cases in universities is the following equation.

$$D(x_d, y_d) = \frac{1}{x_d^2 + y_d^2} \tag{3}$$

We generated a matrix to represent the general layout of the classroom and randomized the position of patient zero in a classroom. The standard ventilation rate is dependent on the number of students in a classroom. The ventilation per person in a building is at an average of 8.5L/s. We calculated the ventilation rate of the classroom in an hour as V($m^3/h =$ number of students * 8.5 /1000 * 3600(Clark, 2013). We then assumed the distance between individual student's seats is around 1 meter and there are a total of 60 students in a classroom of size 10m x 12m and used the quanta emission rate of influenza(515/h) as an example[4]. We made a few assumptions: 1. Class meets every day from Monday to Friday and there is no interaction between students on weekends. 2. The accumulated density of viruses in the air will be zeroed out before every day students come into class, as the ventilation system will likely to keep functioning 24 hours per day. After these assumptions were made, we then started the simulation and found out the average amount of days for the entire classroom of students to be infected under the influenza virus is around 8 working days, which is around 2 weeks of normal class in typical university settings without any measures of containment. For COVID-19, a high quanta generation rate(more than 100 quanta/h) can be reached even by performing vocalizations in light activities for infectious COVID-19 carriers([1]). Although we do not have an exact quanta generation rate for indoor COVID-19. However, in a clustered environment where classroom collaboration and discussion are positively supported, it is likely to see that the quanta generation rate would be higher.

In the data provided by CDC regarding COVID-19 outbreaks in universities, since the beginning of the in-person class on August 9th at a university in North Carolina(less than 10 total cases reported), there were more than 670 cases reported in faculty, students and staff members till the total transition to online teaching on August 19th. [8] More than 600 hundred cases are reported during the 10 days period, which is a significant increase in the amount of infected individuals.

4 Application and Analysis

We further examine the relationship between different parameters and the percentage of student being infected after one week. In the Figure 1, we assumed a medium risk virus where quanta generate rate is around 300. We fixed the number of students in a classroom and ran the model based on different ventilation rates. On the Y axis we have percentage of student getting infected after one week. And on the X axis is the varied ventilation rate. For each different ventilation rate, we ran 100 experiments and calculate the confidence interval and mean for that particular ventilation rate. In the Figure 1, we see that though there are some fluctuates for the relationship, as the ventilation rate increases, the percentage of students being infected after one week would dramatically decreases. Moreover the confidence interval shrinks as the ventilation rate gets bigger. In other words, we increase the ventilation rate per person and as the ventilation rate passes the certain threshold. It would have a very steady positive influence in controlling the spread of virus.

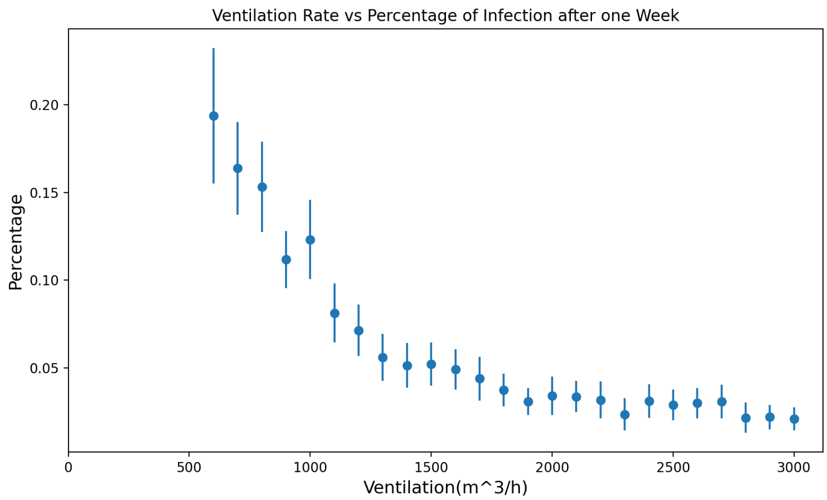

Figure 1: Percentage vs Ventilation Rate(Quanta:300)

Below in the Figure 2 is simulated based on the same experiment setting as the Figure 1, except the quanta generation rate for each infected host, which is 500 in this case.

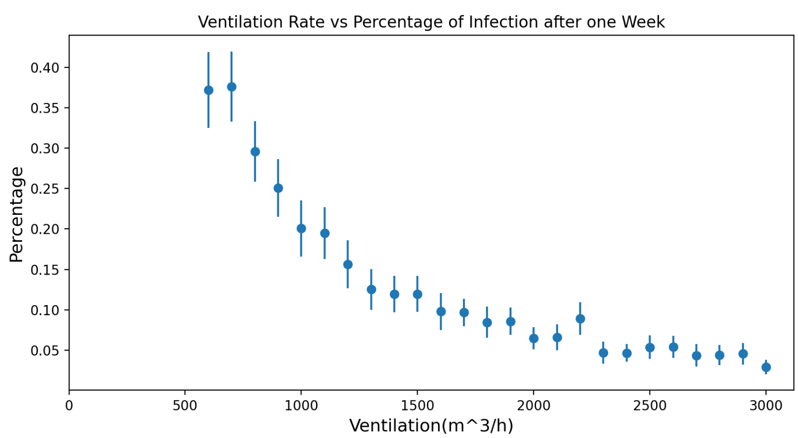

Figure 2: Percentage vs Ventilation Rate(Quanta:500)

The overall pattern still proves that a larger ventilation rate is indeed very helpful to control the spread of virus. Moreover, we observe that as the ventilation rate passes a certain threshold, we see that the absolute value of slope of this curve gets much smaller. So, from economic standpoint, it enough to have ventilation rate that is around last 1/3 tail of the curve to have enough buffer time to take positive measure in controlling the virus.

The second parameter that we controlled on is distance. We set the ventilation rate to the natural state of the classroom depending on the size of the classroom and amount of students. Then we varied the distance on the X axis and how they see changes against the percentage of students getting infected after one week. We see that the compare to the ventilation rate in Figure 1 and Figure 2. The initial value of Y axis is higher than the matching quanta part of ventilation rate. However, the curve of distance seems to decrease faster. Moreover, there is also lesser of the fluctuation of confidence interval on each distance.

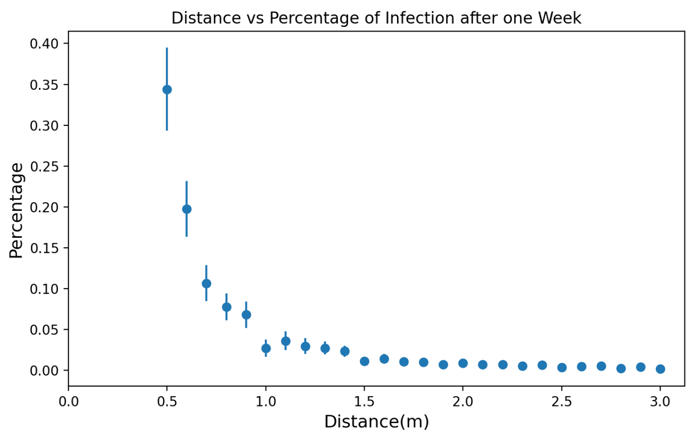

Figure 3: Percentage vs Distance Quanta(300)

Below in the Figure 4 is simulated based on the same experimental setting as the one in Figure 3, except the quanta now is around 500, which infected the entire class faster as each hosts emits more infected particles. It is also interesting to see that when the distance between students is around 0.5 meters, different quanta generation rate can result significantly different percentage of infected students for the first week. However, when the distance between students are around 2.0 meters, both simulations of experiments proves to have similar effect. However, it does not guarantees the absolute safety from being infected. Through active actions such sneeze or coughs, virus-carried droplets or particles can travel as far as 8 meters, which explained the outlier that we found in the experiments. [3]

We also see how distance between students can strongly influence the outcome of the simulation. For each different distance starting from 0.5 to 3.0 meters between individual students. Again, if we consider the slope of the curve,

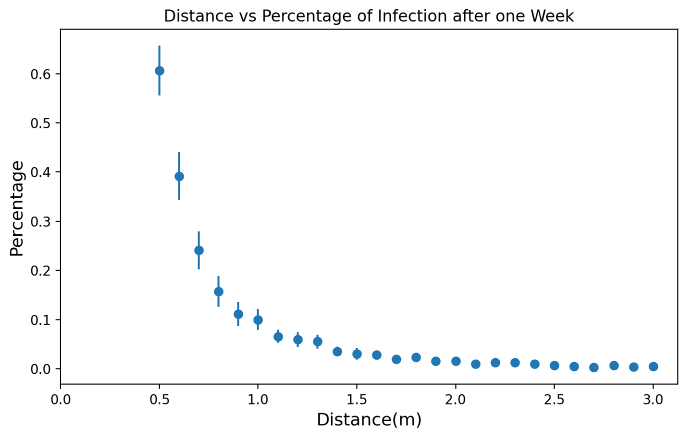

Figure 4: Percentage vs Distance Quanta(500)

we see that the the curve drops in a very steep manner between 0.5 and 1.0, but begins to be relatively flat around 2.0 meters. For university classrooms, we still high suggest at least 2.0 meter relative distance between students to have enough time to take the appropriate measure. Moreover, compared to increase of ventilation rate, the increase of distance proves to be more economical and easier to implement. And this could be the first line of defense against a highly contagious virus.

5 Conclusions and Future Works

We have shown that the traditional Wells-Riley Formula(1)does not fully describe the confined space virus transmission such as classrooms, lecture halls etc. Through incorporating the distance coefficient $D(x_d, y_d)$ into the equation (1), we generated an updated version in equation (3) where it provides a more accurate simulation for clustered sittings like classrooms.By natural tendencies, we do not expect virus to still have the same amount of activity without regarding the distance. And the study has showned that breaking large classrooms into small classroom settings can significantly decrease the speed of virus transmission. We have also constructed a systematic model where infection transmission in these kinds of environments can be quantified and analyzed by inputting different parameters for this model. In the future study, we hope to add more dynamic variations, for example, students may go to restrooms during the class or some classes may have heavier emphasis on classroom discussions where the others might not. Moreover, several classroom can be put together to construct a model for a particular floor of an academic buildings.

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