想法 2025-09-23

1. 系统方程

$$\begin{aligned}\dot{x}(t) &= \bar{A}_{\zeta(t)}x(t) + B_{\zeta(t)}u(t) + C_{\zeta(t)}\omega(t) \\ y(t) &= D_{\zeta(t)}x(t)\end{aligned} \tag{1}$$

其中，$\begin{aligned}x(0) &= 0 \\ \zeta(0) &= \zeta_0\end{aligned}$。

$$\bar{A}_{\zeta(t)} = A_{\zeta(t)} + \Delta A_{\zeta(t)} \tag{2}$$

其中，$\Delta A_{\zeta(t)} = E_{\zeta(t)}\Delta(t)F_{\zeta(t)}$，且$\Delta^T(t)\Delta(t) \leqslant I$。

2. 扰动不可测

构建一个 Luenberger 形式的扰动观测器：

$$\hat{\omega}(t) = \eta(t) + L_{\zeta(t)}x(t) \tag{3}$$

其中，

$$\dot{\eta}(t) = -L_{\zeta(t)}C_{\zeta(t)}\eta(t) - L_{\zeta(t)}\left(C_{\zeta(t)}L_{\zeta(t)} + \bar{A}_{\zeta(t)}\right)x(t) - L_{\zeta(t)}B_{\zeta(t)}u(t) \tag{4}$$

定义扰动估计误差：

$$e_\omega(t) = \omega(t) - \hat{\omega}(t) \tag{5}$$

假设扰动的变化缓慢（$\dot{\omega}(t) \approx 0$），求导得：

$$\begin{aligned}\dot{e}_\omega(t) &= \dot{\omega}(t) - \dot{\hat{\omega}}(t) \\ &= 0 - \left(\dot{\eta}(t) + L_{\zeta(t)}\dot{x}(t)\right) \\ &= L_{\zeta(t)}C_{\zeta(t)}(\eta(t) + L_{\zeta(t)}x(t) - \omega(t)) \\ &= L_{\zeta(t)}C_{\zeta(t)}(\hat{\omega}(t) - \omega(t)) \\ &= -L_{\zeta(t)}C_{\zeta(t)}e_\omega(t)\end{aligned} \tag{6}$$

所以，原系统(1)改写为：

$$\dot{x}(t) = \bar{A}_{\zeta(t)}x(t) + B_{\zeta(t)}u(t) + C_{\zeta(t)}(e_\omega(t) + \hat{\omega}(t)) \tag{7}$$

3. 控制增益

$$u(t) = K_{\zeta(t)}x(t_S) - G_{\zeta(t)}\hat{\omega}(t),\ \ t \in [t_s, t_{s+1}) \tag{8}$$

4. 事件触发

我们强行规定事件触发的时间间隔必须大于d，所以把(8)式改写为：

$$u(t) = \begin{cases} K_{\zeta(t)}x(t - \theta(t)) - G_{\zeta(t)}\hat{\omega}(t) & t \in [t_s, t_s + d) \\ K_{\zeta(t)}[x(t) + e(t)] - G_{\zeta(t)}\hat{\omega}(t) & t \in [t_s + d, t_{s+1}) \end{cases} \tag{9}$$

其中，

$$\theta(t)=t-t_s\leqslant d,\ e(t)=x(t_s)-x(t) \tag{10}$$

我们把(7)式也改写为：

$$\dot{x}(t)=\begin{cases}\overline{A}_{\zeta(t)}x(t)+C_{\zeta(t)}(e_\omega(t)+\hat{\omega}(t)) & \\ \quad +B_{\zeta(t)}(K_{\zeta(t)}x(t-\theta(t))-G_{\zeta(t)}\hat{\omega}(t)) & t\in[t_s,t_s+d) \\ \overline{A}_{\zeta(t)}x(t)+C_{\zeta(t)}(e_\omega(t)+\hat{\omega}(t)) & \\ \quad +B_{\zeta(t)}[K_{\zeta(t)}[x(t)+e(t)]-G_{\zeta(t)}\hat{\omega}(t)] & t\in[t_s+d,t_{s+1})\end{cases} \tag{11}$$

事件触发条件：

$$t_{s+1}=\min\{t\geqslant t_s+d|(x(t)-x(t_S))^T\Lambda_{\zeta(t)}(x(t)-x(t_S))\geqslant\sigma x^T(t)\Lambda_{\zeta(t)}x(t)+\delta\hat{\omega}^T(t)\hat{\omega}(t)\}$$

5. 马尔科夫跳变概率

$$Pr\{\zeta(t+\iota)=j|\zeta(t)=i\}=\begin{cases}\pi_{ij}\iota+o(\iota) & i\neq j\\ 1+\pi_{ii}\iota+o(\iota) & i=j\end{cases} \tag{12}$$

- 跳出（$i\neq j$）
- 驻留（$i=j$）

概率和为 1：

$$\pi_{ii}(\iota)=-\sum_{j\neq i}\pi_{ij}(\iota) \tag{13}$$

令$\zeta(t)=i$，我们替换后得到：

$$\begin{aligned}\dot{x}(t)&=\overline{A}_ix(t)+B_iu(t)+C_i(e_\omega(t)+\hat{\omega}(t))\\ \dot{e}_\omega(t)&=-L_iC_ie_\omega(t)\\ y(t)&=D_ix(t)\\ u(t)&=K_ix(t_S)-G_i\hat{\omega}(t),\ \ t\in[t_s,t_{s+1})\\ \hat{\omega}(t)&=\eta(t)+L_ix(t)\end{aligned} \tag{14}$$

其中，$x(0)=0$， $\zeta(0)=\zeta_0$，

$\overline{A}_i=A_i+\Delta A_i$

$\Delta A_i=E_i\Delta(t)F_i$ ， $\Delta^T(t)\Delta(t)\leqslant I$

$t_{s+1}=\min\{t\geqslant t_s+d|(x(t)-x(t_S))^T\Lambda_i(x(t)-x(t_S))\geqslant\sigma x^T(t)\Lambda_ix(t)+\delta\hat{\omega}^T(t)\hat{\omega}(t)\}$

$\dot{\eta}(t)=-L_iC_i\eta(t)-L_i\left(C_iL_i+\overline{A}_i\right)x(t)-L_iB_iu(t)$

6. 增广系统

定义$\tilde{x}(t)=\begin{bmatrix}x(t)\\ e_\omega(t)\end{bmatrix}$

6.1 情况 1：$t \in [t_s, t_s + d)$

$$u(t) = K_i x(t - \theta(t)) - G_i \hat{\omega}(t) \tag{15}$$

代入得：

$$\dot{\tilde{x}}(t) = \begin{bmatrix} \bar{A}_i x(t) + B_i K_i x(t - \theta(t)) + (C_i - B_i G_i)\hat{\omega}(t) + C_i e_\omega(t) \\ -L_i C_i e_\omega(t) \end{bmatrix} \tag{16}$$

其中，$\theta(t) = t - t_s \leqslant d$。

6.2 情况 2：$t \in [t_s + d, t_{s+1})$

$$u(t) = K_i (x(t) + e(t)) - G_i \hat{\omega}(t) \tag{17}$$

代入得：

$$\dot{\tilde{x}}(t) = \begin{bmatrix} \left(\bar{A}_i + B_i K_i\right) x(t) + B_i K_i e(t) + (C_i - B_i G_i)\hat{\omega}(t) + C_i e_\omega(t) \\ -L_i C_i e_\omega(t) \end{bmatrix} \tag{18}$$

其中，$e(t) = x(t_s) - x(t)$。

7. 构建新的 Lyapunov-Krasovskii 泛函 (LKF)

定义 LKF 为

$$V(t,i) = \begin{cases} V_\alpha(t,i), & t \in [t_s, t_s + d) \\ V_\beta(t,i), & t \in [t_s + d, t_{s+1}) \end{cases} \tag{19}$$

其中，

$$\begin{aligned} V_\alpha(t,i) &= \sum_{m=1}^{4} V_m(t,i) \\ V_\beta(t,i) &= V_1(t,i) \end{aligned} \tag{20}$$

且：

$$\begin{aligned} V_1(t,i) &= e^{2\alpha t} \tilde{x}^T(t) \tilde{P}_i \tilde{x}(t) = e^{2\alpha t} \begin{bmatrix} x(t) \\ e_\omega(t) \end{bmatrix}^T \begin{bmatrix} P_{11,i} & P_{12,i} \\ \star & P_{22,i} \end{bmatrix} \begin{bmatrix} x(t) \\ e_\omega(t) \end{bmatrix} \\ V_2(t,i) &= \theta_d(t) \int_{t-\theta(t)}^{t} e^{2a\mu} \dot{x}^T(\mu) U \dot{x}(\mu) d\mu \\ V_3(t,i) &= \theta_d(t) \int_{t-\theta(t)}^{t} e^{2a\mu} \eta_1^T(\mu) \bar{Q} \eta_1(\mu) d\mu \\ V_4(t,i) &= \theta_d(t) e^{2\alpha t} \eta_2^T(t) F \eta_2(t) \end{aligned} \tag{21}$$

其中，

$$\theta_d(t)=d-\theta(t),\eta_1(\mu)=col\{x(\mu),x(\mu-\theta(\mu))\}$$

$$\eta_2(t)=\mathrm{col}\left\{x(t),x(t-\theta(t)),\int_{t-\theta(t)}^{t}x(\mu)\mathrm{d}\mu\right\}$$

$$\mathrm{F}=\begin{bmatrix}\mathcal{S}\left(\frac{H_1}{2}\right) & -H_1+H_2 & H_3\\ * & \mathcal{S}\left(-H_2+\frac{H_1}{2}\right) & H_4\\ * & * & \mathcal{S}\left(\frac{H_5}{2}\right)\end{bmatrix} \tag{22}$$

且

$$\bar{Q}=\begin{bmatrix}Q_1 & Q_2\\ * & Q_3\end{bmatrix}>0 \tag{23}$$

定义：

$$\xi(t)=col\left\{x(t),e_\omega(t),x(t-\theta(t)),\int_{t-\theta(t)}^{t}x(\mu)d\mu\right\} \tag{24}$$

所以

$$V_1(t,i)=e^{2\alpha t}\begin{bmatrix}x(t)\\ e_\omega(t)\end{bmatrix}^T\begin{bmatrix}P_{11,i} & P_{12,i}\\ \star & P_{22,i}\end{bmatrix}\begin{bmatrix}x(t)\\ e_\omega(t)\end{bmatrix}=e^{2\alpha t}\xi^T(t)\begin{bmatrix}P_{11,i} & P_{12,i} & 0 & 0\\ * & P_{22,i} & 0 & 0\\ * & * & 0 & 0\\ * & * & * & 0\end{bmatrix}\xi(t)$$

$$\begin{aligned}V_4(t,i)&=\theta_d(t)e^{2\alpha t}\eta_2^T(t)F\eta_2(t)\\ &=\theta_d(t)e^{2\alpha t}\xi^T(t)\begin{bmatrix}\mathcal{S}\left(\frac{H_1}{2}\right) & 0 & -H_1+H_2 & H_3\\ * & 0 & 0 & 0\\ * & * & \mathcal{S}\left(-H_2+\frac{H_1}{2}\right) & H_4\\ * & * & * & \mathcal{S}\left(\frac{H_5}{2}\right)\end{bmatrix}\xi(t)\end{aligned} \tag{25}$$

所以：

$$\begin{aligned}&V_1(t,i)+V_4(t,i)\\ &=e^{2\alpha t}\xi^T(t)\begin{bmatrix}P_{11,i}+\theta_d(t)\mathcal{S}\left(\frac{H_1}{2}\right) & P_{12,i} & \theta_d(t)(-H_1+H_2) & \theta_d(t)H_3\\ * & P_{22,i} & 0 & 0\\ * & * & \theta_d(t)\mathcal{S}\left(-H_2+\frac{H_1}{2}\right) & \theta_d(t)H_4\\ * & * & * & \theta_d(t)\mathcal{S}\left(\frac{H_5}{2}\right)\end{bmatrix}\xi(t)\end{aligned} \tag{26}$$

后续和原论文类似，易证$V(t)>0$。

8. 最小生成元的计算

$$\mathcal{L}V(t,i)=\frac{d}{dt}V(t,i)+\sum_{j=1}^{N}\pi_{ij}V(t,j)$$

8.1 情况 1：$t\in[t_s,t_s+d)$

$$\begin{aligned}
\mathcal{L}V_1(t,i)&=e^{2\alpha t}\left[2\alpha\tilde{x}^T\tilde{P}_i\tilde{x}+2\tilde{x}^T\tilde{P}_i\dot{\tilde{x}}(t)+\tilde{x}^T\left(\sum_{j=1}^{N}\pi_{ij}\tilde{P}_j\right)\tilde{x}\right]\\
&=e^{2\alpha t}\left[2\alpha\tilde{x}^T\tilde{P}\tilde{x}+\tilde{x}^T\left(\sum_{j=1}^{N}\pi_{ij}\tilde{P}_j\right)\tilde{x}+2(x^TP_{11}+e_\omega^TP_{12}^T)\left(\overline{A}x+BKx(t-\theta)+(C-BG)\hat{\omega}+Ce_\omega\right)-\right.
\end{aligned}$$

$$\begin{aligned}
\mathcal{L}V_2(t,i)&=\theta_d(t)e^{2at}\dot{x}^T(t)U\dot{x}(t)-\int_{t-\theta(t)}^{t}e^{2a\mu}\dot{x}^T(\mu)U\dot{x}(\mu)d\mu\\
&\leqslant\theta_d(t)e^{2at}\dot{x}^T(t)U\dot{x}(t)-e^{2at}e^{-2ad}\int_{t-\theta(t)}^{t}\dot{x}^T(\mu)U\dot{x}(\mu)d\mu\\
\mathcal{L}V_3(t,i)&=\theta_d(t)e^{2at}\eta_1^T(t)\overline{Q}\eta_1(t)-\int_{t-\theta(t)}^{t}e^{2a\mu}\eta_1^T(\mu)\overline{Q}\eta_1(\mu)d\mu\\
&\leqslant\theta_d(t)e^{2at}\eta_1^T(t)\overline{Q}\eta_1(t)-e^{2at}e^{-2ad}\int_{t-\theta(t)}^{t}\eta_1^T(\mu)\overline{Q}\eta_1(\mu)d\mu\\
\mathcal{L}V_4(t,i)&=-e^{2\alpha t}\eta_2^T(\boldsymbol{t})F\eta_2(\boldsymbol{t})+2\alpha\theta_d(t)e^{2\alpha t}\eta_2^T(t)F\eta_2(t)+2\theta_d(t)e^{2\alpha t}\eta_2^TF\dot{\eta}_2(t)\\
&\quad\leq-e^{2\alpha t}\eta_2^T(t)F\eta_2(t)+2\alpha\theta_d(t)e^{2\alpha t}\eta_2^T(t)F\eta_2(t)\\
&\quad+2\theta_d(t)e^{2\alpha t}x^T(t)\frac{H_1+H_1^T}{2}\dot{x}(t)+2\theta_d(t)e^{2\alpha t}x^T(t-\theta(t))(-H_1+H_2)^T\dot{x}(t)\\
&\quad+2\theta_d(t)e^{2\alpha t}\int_{t-\theta(t)}^{t}x^T(\mu)\mathrm{d}\mu H_3^T\dot{x}(t)+2\theta_d(t)e^{2\alpha t}x^T(t)H_3x(t)\\
&\quad+2\theta_d(t)e^{2\alpha t}x^T(t-\theta(t))H_4x(t)+2\theta_d(t)e^{2\alpha t}\int_{t-\theta(t)}^{t}x^T(\mu)d\mu\frac{H_5+H_5^T}{2}x(t)
\end{aligned}\tag{27}$$

构建 0 等式：

$$\begin{aligned}
&2[x^T+\dot{x}^T\mathcal{E}_1+e_\omega^T\mathcal{E}_2+\hat{\omega}^T\mathcal{E}_3+x^T(t-\theta)\mathcal{E}_4]\mathcal{Y}_i^T\\
&\times\left[-\dot{x}+\overline{A}_ix+B_iK_ix(t-\theta)+(C_i-B_iG_i)\hat{\omega}+C_ie_\omega\right]=0
\end{aligned}\tag{28}$$

8.2 情况 2：$t\in[t_s+d,t_{s+1})$