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Resilient fault-tolerant anti-synchronization for stochastic delayed reaction–diffusion neural networks with semi-Markov jump parameters

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ABSTRACT

This paper deals with the anti-synchronization issue for stochastic delayed reaction–diffusion neural networks subject to semi-Markov jump parameters. A resilient fault-tolerant controller is utilized to ensure the anti-synchronization in the presence of actuator failures as well as gain perturbations, simultaneously. Firstly, by means of the Lyapunov functional and stochastic analysis methods, a mean-square exponential stability criterion is derived for the resulting error system. It is shown the obtained criterion improves a previously reported result. Then, based on the present analysis result and using several decoupling techniques, a strategy for designing the desired resilient fault-tolerant controller is proposed. At last, two numerical examples are given to illustrate the superiority of the present stability analysis method and the applicability of the proposed resilient fault-tolerant anti-synchronization control strategy, respectively.

1. Introduction

Neural network is a complex nonlinear system composed of numerous neurons, which possesses distinguished information storage capacity and associative memory function. Recent years have witnessed a rapidly growing interest in neural networks because of their potential applications in system identification (Billings, 2013), medical diagnosis (Kordylewski, Graupe, & Liu, 2001), data mining (Singh & Chauhan, 2009), and many other fields. Generally, time delay is unavoidable in most applications of neural networks, and its existence is usually one of the main sources of oscillation, divergence, and chaos. Therefore, the study of delayed neural networks (DNNs) is of practical significance, and a great amount of impressive results have been proposed in the literature; see, e.g., He, Ji, Zhang, and Wu (2016), Lee, Park, Park, Kwon, and Lee (2014), Song, Yu, Zhao, Liu, and Alsaadi (2018), Yan, Huang, Fan, Xia, and Shen (2020) and Zhang, Han, and Zeng (2017). For nonlinear systems, one of the most interesting phenomena is anti-synchronization, which depicts a dynamic behavior that the state curves of coupled systems possess the same amplitude but opposite phase. It has been shown that the anti-synchronization capability of chaotic nonlinear systems can be successfully applied to secure encryption (Lynnyk & Čelikovský, 2010) and secure communication (Mahmoud & Abo-Dahab, 2018). Over the past ten years, much attention has been directed to the anti-synchronization control of DNNs. For example, a class of DNNs with unknown parameters and external disturbances were studied in Ahn (2009), and an adaptive $\mathcal{H}_\infty$ control scheme was proposed in terms of linear matrix inequalities (LMIs). Chaotic DNNs with stochastic perturbations were examined in Ren and Cao (2009), where a memory controller was employed to guarantee the exponential anti-synchronization by combining the Halanay inequality with stochastic analysis techniques. When both time delays and reaction–diffusion factors are involved, non-fragile and pinning anti-synchronization control strategies were proposed in Hou, Huang, and Ren (2019) and Tai, Teng, Zhou, Zhou, and Wang (2019), respectively.

In practical engineering applications, neural networks sometimes need to be modeled as switched systems consisting of finite subsystems and a switching signal. Generally speaking, switched systems can be divided into arbitrary switched systems and restricted switched ones based on the switching signal. As a class of typical restricted switched systems, Markov jump systems have drawn particular attention from various fields and been recognized to be very suitable for describing practical systems with a sudden change of the structure or parameters. In the

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context of DNNs with Markov jump parameters, many results have been reported during the past two decades; see, e.g., Cheng, Park, Karimi, and Shen (2017), Prakash, Balasubramaniam, and Lakshmanan (2016), Saravanakumar, Ali, Ahn, Karimi, and Shi (2016), Senan, Syed Ali, Vadivel, and Arik (2017), Wu, Shi, Su, and Chu (2013), Yang, Cao, and Lu (2013) and Zhang and Wang (2008). However, an assumption for obtaining these results is that the sojourn time needs to obey the exponential distribution, which is a memoryless distribution. In other words, the transition probability is assumed to be only related to the current state of the system, but not to the past or the future state. This imposes a certain limitation on the applications of the DNNs with Markov jump parameters. To deal with the problem, a few research activities have been focused on semi-Markov DNNs, where the sojourn time is not limited to the exponential distribution and the transition probability of the system varies with time. Particularly, the synchronization problems of DNNs subject to semi-Markov jump parameters have been examined in Syed Ali, Vadivel, and Kwon (2019), Wan, et al. (2019) and Wei, Park, Karimi, Tian, and Jung (2017), where some sufficient conditions on the synchronization analysis and control synthesis in mean square were proposed. However, to the authors' knowledge, anti-synchronization related results of delayed reaction–diffusion neural networks (DRDNNs) with semi-Markov jump parameters have not been reported yet.

In many actual control systems, actuator failures occur frequently and they may cause unpredictable adverse consequences, such as degrading the controller performance and even destroying the controller components. As a practical means of settlement, fault-tolerant control scheme has been introduced to the control community over the past few decades. It was shown in Gao, Shen, Yang, and Gong (2017) and Shen, Jiang, and Shi (2017) that such a control scheme can automatically compensate for the effects of failures to keep the system stability and recover the system performance as much as possible so that the system works stably and reliably. In 2016, a fault-tolerant controller was designed in Chen, Huang, and Fu (2016) for the fault estimation issue of a class of Markov jump systems with time delay and Lipschitz nonlinearity. Later, the fault-tolerant passive control issue of fuzzy systems subject to semi-Markov parameters was studied in Shen, Chen, Wu, Cao, and Park (2019) on the basis of event-triggered mechanisms. On the other hand, in the hardware implementation, a digital controller often has gain uncertainties to some extent because of restrictions in accessible process memory, imprecisions of analog–digital conversion, rounding errors of digital calculators, etc (Yang, Guo, Che, & Guan, 2016). It is thus necessary to design resilient (or non-fragile) controllers to ensure the control effects. A type of nonlinear multi-agent system with both stochastic disturbances and Markovian switching topologies was evaluated in Yan, Sang, Fang, and Zhou (2018), where a resilient dynamic output-feedback controller was designed for the $\mathcal{L}_2 - \mathcal{L}_\infty$ consensus. In Wang, Ma, and Zhang (2018), the resilient estimation issue for a class of semi-Markov jump descriptor systems was investigated, and an approach for ensuring the $\mathcal{H}_\infty$ stability of the estimation error system was developed. For the controller designs of DRDNNs with Markov jump parameters, it is noteworthy that actuator failures and gain perturbations have not been considered simultaneously. This constitutes another motivation for our research.

Based on the above discussions, the anti-synchronization control issue is investigated for semi-Markov jump DRDNNs subject to possible actuator failures and gain perturbations under the master–slave synchronization configuration. As Selvaraj, Sakthivel, and Kwon (2018), the slave neural network is assumed to be disturbed by stochastic noises and modeled by stochastic functional differential equations. The aim is to design a resilient fault-tolerant controller to ensure that the master and slave DRDNNs are exponentially anti-synchronized in mean square. The main contributions of this work are as follows: 1. The considered master–slave DRDNN models, which not only involve the semi-Markov jump parameters but also the stochastic disturbances, are more general than the common DRDNN ones. 2. A mean-square exponential stability condition is presented for the resulting anti-synchronization error system with the application of the Lyapunov functional and some stochastic analysis methods. It is shown that the analysis result improves an existing result given in Shi and Ma (2012). 3. A strategy for the design of the desired resilient fault-tolerant controller is proposed via eliminating the higher-order nonlinearities caused by the coupling of the Lyapunov matrices, the gain-perturbation matrices, and the uncertainties of the actuator failures. It is shown that the required control gains can be obtained via solving a few LMIs, which are able to be checked easily by the popular computing software MATLAB.

The organization of this paper is as follows: In Section 2, the master–slave DRDNN models with semi-Markov jump parameters are proposed and some necessary preliminaries are prepared. In Section 3, a mode-dependent Lyapunov functional (LF) and some stochastic analysis methods are used to analyze the exponential stability for the resulting anti-synchronization error system. In Section 4, the design strategy of the desired resilient fault-tolerant anti-synchronization controller is presented. In Section 5, two numerical examples are given to illustrate the validity and superiority of the present analysis and synthesis methods. Finally, a conclusion is given in Section 6.

Notation. In the present study, we use $\mathcal{E}\{\cdot\}$ to denote the mathematical expectation operator, $|\cdot|$ to represent the Euclidean vector norm, $\mathbb{N}$, $\mathbb{N}_+$, $\mathbb{R}^n$, and $\mathbb{R}^{m\times n}$ to represent the set of nonnegative integers, the set of positive integers, the normal n-dimensional Euclidean space, and the family of $m \times n$ real matrices, respectively. Denote by I and 0 the unity and zero matrices, respectively. For a square real matrix M, denote by $tr(M)$ its trace, by $\mathscr{S}(M)$ the sum of M and M^T, by $*$ the symmetry blocks, and by $\lambda_{\max}(M)$ and $\lambda_{\min}(M)$ its maximum and minimum eigenvalues respectively. Let $\mathcal{C}^m(Z, \mathbb{R}^n)$ be the set of m-times continuously-differentiable functions which map Z into $\mathbb{R}^n$, and $L^2_{\mathcal{F}_0}([-\bar{\tau}, 0] \times \Omega, \mathbb{R}^n)$ be the set of all $\mathcal{F}_0$-measurable $\mathcal{C}([-\bar{\tau}, 0] \times \Omega, \mathbb{R}^n)$- random variables $\chi = \{\chi(s, x) : -\bar{\tau} \le s \le 0\}$ such that $\sup_{-\bar{\tau}\le s\le 0} \mathcal{E}\left\{|\chi(s, x)|^2\right\} < \infty$, where $\Omega = \{x = [x_1 \ \cdots \ x_q]^T \mid \phi_i \le x_i \le \psi_i,\ \phi_i,\ \psi_i \in \mathbb{R},\ i = 1, \ldots, q\}$ is a closed set in $\mathbb{R}^q$. The boundary of Ω is defined as $\partial\Omega$.

2. Preliminaries

Consider the following DRDNN with semi-Markov jump parameters:

$$\frac{dz_1(t,x)}{dt} = \sum_{k=1}^{q} \frac{\partial}{\partial x_k}\left(D_k \frac{\partial z_1(t,x)}{\partial x_k}\right) - A_{\gamma(t)} z_1(t,x) + W_{1\gamma(t)} f(z_1(t,x)) + W_{2\gamma(t)} f(z_1(t-\tau(t),x)), \tag{1}$$

where $z_1(t,x) = [z_{11}(t,x), \ldots, z_{1n}(t,x)]^T$ with $z_{1i}(t,x) \in \mathcal{C}^2(\mathbb{R} \times \Omega, \mathbb{R})$ denoting the state variable of the ith neuron at t and $x = [x_1, \ldots, x_q]^T$; $D_k = diag\{D_{1k}, \ldots, D_{nk}\}$ with $D_{ik} \ge 0$ denoting the transmission diffusion coefficient along the ith neuron, $A_{\gamma(t)} = diag\{A_{1\gamma(t)}, \ldots, A_{n\gamma(t)}\}$ with $A_{i\gamma(t)} > 0$ being the rate at which the ith neuron resets the potential to the resting state in the case when disconnected from the network and external inputs, $W_{1\gamma(t)} = (w^{ij}_{1\gamma(t)})_{n\times n}$ and $W_{2\gamma(t)} = (w^{ij}_{2\gamma(t)})_{n\times n}$ with $w^{ij}_{1\gamma(t)}$ and $w^{ij}_{2\gamma(t)}$ representing the connection weights of neurons; $f(z_1(t,x)) = [f_1(z_{11}(t,x)) \cdots f_n(z_{1n}(t,x))]$ with $f_j\left(z_{1j}(t,x)\right)$

representing the activation function of the jth neuron which is odd for realizing anti-synchronization as Zhao, et al. (2015) and Wang, et al. (2016), and $\tau(t)$ is a variant time delay subject to $0 < \tau(t) \le \bar{\tau}$, $\dot{\tau}(t) \le \mu < 1$, where both $\bar{\tau}$ and μ are known non-negative scalars; $\gamma(t)_{t\ge 0}$ is a continuous-time semi-Markov process taking values in a limited set $\mathcal{M} = \{1, \ldots, m | m \in \mathbb{N}_+\}$; $t_l (l \in \mathbb{N})$ represents the lth transition point of $\gamma(t)$; for $l \in \mathbb{N}_+$, γ_l stands for the indicator of the system mode and $h_l = t_l - t_{l-1}$ the corresponding sojourn time with density function $e_{\gamma(t)}(h)$. The evolution of the semi-Markov process is governed by Yu and Zhang (2019):

$$\begin{cases} Pr\{\gamma_{l+1} = j, h_{l+1} \le h + \theta | \gamma_l = i, h_{l+1} > h\} \\ \quad = \xi_{ij}(h)\theta + o(\theta), \quad i \ne j, \\ Pr\{\gamma_{l+1} = j, h_{l+1} > h + \theta | \gamma_l = i, h_{l+1} > h\} \\ \quad = 1 + \xi_{ii}(h)\theta + o(\theta), \quad i = j, \end{cases} \tag{2}$$

where $o(\theta)$ stands for the little-o notation, which is defined by $\lim_{\theta\to 0}(o(\theta)/\theta) = 0$; $\xi_{ij}(h)$ corresponds to the transition rate from mode i at time t to mode j at time $t + h$ with $\xi_{ij}(h) \ge 0$ (for $i \ne j$) and $\xi_{ii}(h) = -\sum_{j=1, j\ne i}^{m} \xi_{ij}(h)$.

Remark 1. In a continuous-time Markov process, the sojourn time is exponentially distributed and the transition rates are constants, while for a semi-Markov process, the sojourn time can obey the Weibull distribution, which includes exponential distribution as its special case. It is noteworthy that the transition rates of a semi-Markov process vary with time, which leads to difficulties in the anti-synchronization analysis.

Throughout this paper, the master–slave synchronization scheme shall be adopted. We take DRDNN (1) to be a master system, and the slave system is modeled by the following stochastic functional differential equations:

$$\begin{aligned} dz_2(t,x) = &\left[\sum_{k=1}^{q} \frac{\partial}{\partial x_k}\left(D_k \frac{\partial z_2(t,x)}{\partial x_k}\right) - A_{\gamma(t)} z_2(t,x)\right. \\ &+ W_{1\gamma(t)} f(z_2(t,x)) + W_{2\gamma(t)} f(z_2(t-\tau(t),x)) \\ &\left.+ B_{\gamma(t)} u^f_{\gamma(t)}(t,x)\right] dt + \sigma(t, z_1(t,x) + z_2(t,x), \\ &z_1(t-\tau(t),x) + z_2(t-\tau(t),x)) d\omega(t), \end{aligned} \tag{3}$$

where $z_2(t,x) = [z_{21}(t,x), \ldots, z_{2n}(t,x)]^T$ with $z_{2i}(t,x)$ representing the ith neuron state; $\sigma(t, z_1(t,x) + z_2(t,x), z_1(t-\tau(t),x) + z_2(t-\tau(t),x))$ denotes a noise intensity matrix satisfying $\sigma(t,0,0) = 0$; $\omega(t)$ stands for a vector Brownian motion (Wang, et al., 2016) with $\mathcal{E}\{d\omega(t)\} = 0$ and $\mathcal{E}\{d\omega^2(t)\} = dt$; for any $\gamma(t) \in \mathcal{M}$, $B_{\gamma(t)} \in \mathbb{R}^{n\times p}$ is a constant matrix, and $u^f_{\gamma(t)}(t,x)$ corresponds to the control input with possible actuator failures, which is given as follows:

$$u^f_{\gamma(t)}(t,x) = \mathcal{G}_{\gamma(t)} u_{\gamma(t)}(t,x), \tag{4}$$

where $\mathcal{G}_{\gamma(t)}$ is the actuator fault matrix (Ye & Zhao, 2014), which is defined as $\mathcal{G}_{\gamma(t)} = diag\{g_{1\gamma(t)}, \ldots, g_{p\gamma(t)}\}$ with $0 \le \check{g}_{j\gamma(t)} \le g_{j\gamma(t)} \le \hat{g}_{j\gamma(t)} \le 1$ $(j = 1, \ldots, p)$. Here $\check{g}_{j\gamma(t)}$ and $\hat{g}_{j\gamma(t)}$ $(j = 1, \ldots, p)$ are known constants.

Remark 2. Note that $\mathcal{G}_{\gamma(t)}$ reflects actuator effectiveness. When $\mathcal{G}_{\gamma(t)} = I$ (i.e., $u^f_{\gamma(t)}(t,x) = u_{\gamma(t)}(t,x)$), there is no failure in the $\gamma(t)$-th actuator. When $0 \le \mathcal{G}_{\gamma(t)} < I$, there is a loss of efficiency in the $\gamma(t)$-th actuator, and moreover, if $\mathcal{G}_{\gamma(t)} = 0$, then $u^f_{\gamma(t)}(t,x)$ is outage or stuck.

Let us define

$$\mathcal{G}_{0\gamma(t)} = diag\{g^{[\gamma(t)]}_{01}, \ldots, g^{[\gamma(t)]}_{0p}\}, \quad \mathcal{G}_{1\gamma(t)} = diag\{g^{[\gamma(t)]}_{11}, \ldots, g^{[\gamma(t)]}_{1p}\}$$

with $g^{[\gamma(t)]}_{0j} = \left(\hat{g}_{j\gamma(t)} + \check{g}_{j\gamma(t)}\right)/2$ and $g^{[\gamma(t)]}_{1j} = \left(\hat{g}_{j\gamma(t)} - \check{g}_{j\gamma(t)}\right)/2$ $(j = 1, \ldots, p)$. Then, fault matrix $\mathcal{G}_{\gamma(t)}$ can be rewritten as

$$\begin{aligned} &\mathcal{G}_{\gamma(t)} = \mathcal{G}_{0\gamma(t)} + \mathcal{G}_{1\gamma(t)} \Lambda_{\gamma(t)},\ \Lambda_{\gamma(t)} = diag\left\{\lambda_{1\gamma(t)}, \ldots, \lambda_{p\gamma(t)}\right\}, \\ &\quad -1 \le \lambda_{j\gamma(t)} \le 1, \quad j = 1, \ldots, p. \end{aligned} \tag{5}$$

In the present study, the case of gain perturbations will also be considered. Without loss of generality (Yang et al., 2016), it is assumed that

$$u_{\gamma(t)}(t,x) = \bar{K}_{\gamma(t)}\left(z_1(t,x) + z_2(t,x)\right), \tag{6}$$

where $\bar{K}_{\gamma(t)} = K_{\gamma(t)} + \Delta K_{\gamma(t)}(t)$, $K_{\gamma(t)}$ is the control gain that needs to be determined and $\Delta K_{\gamma(t)}(t)$ denotes the possible gain perturbation, which is described as $\Delta K_{\gamma(t)}(t) = \mathscr{L}_{\gamma(t)} F_{\gamma(t)}(t) \mathscr{M}_{\gamma(t)}$. Here, $F_{\gamma(t)}(t)$ is an unknown matrix function which meets $F^T_{\gamma(t)}(t) F_{\gamma(t)}(t) \le I$; $\mathscr{L}_{\gamma(t)}$ and $\mathscr{M}_{\gamma(t)}$ are known constant matrices.

Now we are in a position to give the boundary conditions and initial conditions of systems (1) and (3), which are as follows:

$$z_i(t,x) = 0, \quad (t,x) \in [0, +\infty) \times \partial\Omega, \tag{7}$$

$$z_i(s,x) = \varphi_i(s,x), \quad (s,x) \in \left[-\bar{\tau}, 0\right] \times \Omega, \tag{8}$$

where $\varphi_i(s,x) \in L^2_{\mathcal{F}_0}([-\bar{\tau}, 0] \times \Omega, \mathbb{R}^n)$, $i = 1, 2$.

Set $z_1(t,x) + z_2(t,x) = \zeta(t,x)$ $(= [\zeta_1(t,x), \ldots, \zeta_n(t,x)]^T)$. Then we are able to write the anti-synchronization error system as

$$\begin{aligned} d\zeta(t,x) = &\left[\sum_{k=1}^{l} \frac{\partial}{\partial x_k}\left(D_k \frac{\partial \zeta(t,x)}{\partial x_k}\right) - \left(A_{\gamma(t)} - B_{\gamma(t)}\mathcal{G}_{\gamma(t)}\right.\right. \\ &\left.\times \left(K_{\gamma(t)} + \Delta K_{\gamma(t)}(t)\right)\right) \zeta(t,x) \\ &\left.+ W_{1\gamma(t)} g(\zeta(t,x)) + W_{2\gamma(t)} g(\zeta(t-\tau(t),x))\right] dt \\ &+ \sigma\left(t, \zeta(t,x), \zeta\left(t-\tau(t),x\right)\right) d\omega(t), \end{aligned} \tag{9}$$

where $g\left(\zeta(t,x)\right) = f\left(\zeta(t,x) - z_1(t,x)\right) + f\left(z_1(t,x)\right)$.

The activation functions and the noise intensify matrix are assumed to meet the following conditions, which are widely used in the literature (see, e.g., Liu, Wang, Liang, & Liu, 2009; Wen, Huang, Zeng, Chen, & Li, 2015; Yan, Huang, & Cao, 2019).

Assumption 1. For any $\alpha_1, \alpha_2 \in \mathbb{R}$, $\alpha_1 \ne \alpha_2$, activation function $f_i(\cdot) (i = 1, \ldots, n)$ satisfies

$$\beta_{1i} \le \frac{f_i(\alpha_1) - f_i(\alpha_2)}{\alpha_1 - \alpha_2} \le \beta_{2i},$$

where β_{1i} and β_{2i} are known constant scalars.

Assumption 2. There exist positive scalars σ_1 and σ_2 such that

$$\begin{aligned} &\mathrm{tr}\Big(\sigma^T\left(t, \zeta(t,x), \zeta\left(t-\tau(t),x\right)\right) \sigma\left(t, \zeta(t,x), \zeta\left(t-\tau(t),x\right)\right)\Big) \\ &\le \sigma_1 \zeta^T(t,x)\zeta(t,x) + \sigma_2 \zeta^T\left(t-\tau(t),x\right) \zeta\left(t-\tau(t),x\right). \end{aligned} \tag{10}$$

Noticing that $f\left(z(t,x)\right)$ is an odd function, one can obtain from Assumption 1 that

$$\beta_{1i} \le \frac{g_i(\zeta_i(t,x))}{\zeta_i(t,x)} \le \beta_{2i}, \quad i = 1, \ldots, n. \tag{11}$$

At the end of this section, let us give the definition on the exponential anti-synchronization and prepare several useful lemmas:

Definition 1. Semi-Markov jump DRDNNs (1) and (3) are said to be exponentially anti-synchronized in mean square if the resulting error system in (9) is mean-square exponentially stable, i.e., there exist positive constants κ and H such that

$$\mathcal{E}\left\{\int_{\Omega} |\zeta(t,x)|^2 dx\right\} \le He^{-\kappa t}.$$

Lemma 1 (Wang, Kuo, & Hsu, 1986). *Let $\mathcal{W}_a$ and $\mathcal{W}_b$ be two symmetric matrices of the same dimension. If $\mathcal{W}_b \geq 0$, then*

$$\lambda_{\min}(\mathcal{W}_a)tr(\mathcal{W}_b) \leq tr(\mathcal{W}_a\mathcal{W}_b) \leq \lambda_{\max}(\mathcal{W}_a)tr(\mathcal{W}_b).$$

Lemma 2 (Zhou, Xu, Shen, & Zhang, 2013). *Let $z(x) \in \mathcal{C}^1(\Omega, \mathbb{R})$ with $z(x)/_{\partial\Omega} = 0$, then*

$$\int_\Omega z^2(x)dx \leq \left(\frac{\psi_k - \phi_k}{\pi}\right)^2 \int_\Omega \left(\frac{\partial z(x)}{\partial x_k}\right)^2 dx, \quad k = 1, \ldots, q.$$

Lemma 3 (Zhang, Xu, & Zou, 2008). *Suppose that (11) is satisfied. Then*

$$\zeta^T(t,x)\mathcal{T}_1 R\mathcal{T}_2\zeta(t,x) - \zeta^T(t,x)R(\mathcal{T}_1 + \mathcal{T}_2)g(\zeta(t,x)) + g^T(\zeta(t,x))Rg(\zeta(t,x)) \leq 0$$

holds true for any diagonal matrix $R > 0$, where $\mathcal{T}_1 = diag\,(\beta_{11}, \ldots, \beta_{1n})$ and $\mathcal{T}_2 = diag\,(\beta_{21}, \ldots, \beta_{2n})$.

Lemma 4 (Zhou & Khargonekar, 1988). *For any real matrices $\mathcal{W}_a$ and $\mathcal{W}_b$ of suitable dimensions, there exists a constant $\varepsilon > 0$ such that*

$$\mathscr{S}(\mathcal{W}_a\mathcal{W}_b) \leq \varepsilon^{-1}\mathcal{W}_a\mathcal{W}_a^T + \varepsilon\mathcal{W}_b^T\mathcal{W}_b.$$

Lemma 5 (Zhou, Park, & Ma, 2016). *Suppose there are a real constant ς and real matrices Θ, $\mathcal{U}_i$, $\mathcal{V}_i$, and S_i $(i = 1, \ldots, p)$ satisfying*

$$\begin{bmatrix} \Theta & \mathcal{U}_\mathcal{V} \\ * & \mathcal{S} \end{bmatrix} < 0,$$

where $\mathcal{U}_\mathcal{V} = [\mathcal{U}_1 + \varsigma\mathcal{V}_1, \ldots, \mathcal{U}_p + \varsigma\mathcal{V}_p]$, and $\mathcal{S} = diag\{\mathscr{S}(-\varsigma S_1), \ldots, \mathscr{S}(-\varsigma S_p)\}$. Then, one can obtain

$$\Theta + \sum_{i=1}^{p}\mathscr{S}(\mathcal{U}_i S_i^{-1}\mathcal{V}_i^T) < 0.$$

Lemma 6 (Xie, Fu, & de Souza, 1992). *Let Θ, $\mathcal{F}$, $\mathcal{W}_a$, and $\mathcal{W}_b$ be real matrices of suitable dimensions. Then*

$$\Theta + \mathscr{S}(\mathcal{W}_a\mathcal{F}\mathcal{W}_b) < 0$$

holds for $\mathcal{F}^T\mathcal{F} \leq I$ if and only if there is a scalar $\varepsilon > 0$ such that

$$\Theta + \varepsilon^{-1}\mathcal{W}_a\mathcal{W}_a^T + \varepsilon\mathcal{W}_b^T\mathcal{W}_b < 0.$$

3. Mean-square stability analysis

When $u_{\gamma(t)}(t,x) \equiv 0$, system (9) can be rewritten as the following stochastic semi-Markov jump system:

$$d\zeta(t,x) = h_{\gamma(t)}(t, \zeta(t,x), \zeta(t-\tau(t),x))dt + \sigma\,(t, \zeta(t,x), \zeta\,(t-\tau(t),x))\,d\omega(t), \tag{12}$$

where

$$h_{\gamma(t)}(t, \zeta(t,x), \zeta(t-\tau(t),x)) = \sum_{k=1}^{q}\frac{\partial}{\partial x_k}\left(D_k\frac{\partial\zeta(t,x)}{\partial x_k}\right) - A_{\gamma(t)}\zeta(t,x) + W_{1\gamma(t)}g(\zeta(t,x)) + W_{2\gamma(t)}g(\zeta(t-\tau(t),x)). \tag{13}$$

For system (12), we give a sufficient condition in the following theorem:

Theorem 1. *System (12) is exponentially stable in mean square, if, for any $i \in \mathcal{M}$, there exist scalars $\rho_i > 0$, matrices $Q_1 > 0$, $Q_2 > 0$, diagonal matrices P_i, $R_1 > 0$, and $R_2 > 0$ such that the following LMIs hold:*

$$P_i \leq \rho_i I, \tag{14}$$

$$\Xi_i = \begin{bmatrix} \Xi_{11}^{[i]} & 0 & \Xi_{13}^{[i]} & P_iW_{2i} \\ * & \Xi_{22}^{[i]} & 0 & \frac{R_2(\mathcal{T}_1+\mathcal{T}_2)}{2} \\ * & * & Q_2 - R_1 & 0 \\ * & * & * & (\mu-1)Q_2 - R_2 \end{bmatrix} < 0, \tag{15}$$

where

$$\begin{aligned}
\Xi_{11}^{[i]} &= -2P_iD_\pi - \mathscr{S}(P_iA_i) + \sum_{j=1}^{m}\bar{\xi}_{ij}P_j + Q_1 + \rho_i\sigma_1 I - \mathcal{T}_1R_1\mathcal{T}_2, \\
\Xi_{13}^{[i]} &= P_iW_{1i} + \frac{R_1\,(\mathcal{T}_1+\mathcal{T}_2)}{2}, \\
\Xi_{22}^{[i]} &= (\mu-1)Q_1 - \mathcal{T}_1R_2\mathcal{T}_2 + \rho_i\sigma_2 I, \\
\bar{\xi}_{ij} &= \varepsilon\left\{\xi_{ij}(h)\right\} = \int_0^\infty \xi_{ij}(h)e_i(h)dh, \\
D_\pi &= diag\left\{\sum_{k=1}^{q}\left(\frac{\pi}{\psi_k-\phi_k}\right)^2 D_{1k}, \ \ldots, \ \sum_{k=1}^{q}\left(\frac{\pi}{\psi_k-\phi_k}\right)^2 D_{nk}\right\}.
\end{aligned}$$

Proof. For any $t \geq 0$, by Lemma 1 one can write that

$$\begin{aligned}
&\mathrm{tr}\Big(\sigma^T\,(t, \zeta(t,x), \zeta\,(t-\tau(t),x))\,P_i\sigma\,(t, \zeta(t,x), \zeta\,(t-\tau(t),x))\Big) \\
&= \mathrm{tr}\Big(P_i\sigma\,(t, \zeta(t,x), \zeta\,(t-\tau(t),x))\,\sigma^T\,(t, \zeta(t,x), \zeta\,(t-\tau(t),x))\Big) \\
&\leq \lambda_{max}\,(P_i)\,\mathrm{tr}\Big(\sigma^T\,(t, \zeta(t,x), \zeta\,(t-\tau(t),x))\,\sigma\,(t, \zeta(t,x), \zeta\,(t-\tau(t),x))\Big).
\end{aligned}$$

It follows from (10) and (14) that

$$\begin{aligned}
&\mathrm{tr}\Big(\sigma^T\,(t, \zeta(t,x), \zeta\,(t-\tau(t),x))\,P_i\sigma\,(t, \zeta(t,x), \zeta\,(t-\tau(t),x))\Big) \\
&\leq \rho_i\sigma_1\zeta^T(t,x)\zeta(t,x) + \rho_i\sigma_2\zeta^T\,(t-\tau(t),x)\,\zeta\,(t-\tau(t),x)\,, \\
&\quad i = 1, \ldots, n.
\end{aligned} \tag{16}$$

Utilizing the integration by parts formula and noticing the Dirichlet boundary condition in (7), for any $i \in \{1, \ldots, n\}$ and $k \in \{1, \ldots, q\}$, one has

$$\int_\Omega \zeta_i(t,x)\frac{\partial}{\partial x_k}\left(D_{ik}\frac{\partial\zeta_i(t,x)}{\partial x_k}\right)dx = -\int_\Omega D_{ik}\left(\frac{\partial\zeta_i(t,x)}{\partial x_k}\right)^2 dx,$$

which, together with Lemma 2, suggests

$$\int_\Omega \zeta_i(t,x)\frac{\partial}{\partial x_k}\left(D_{ik}\frac{\partial\zeta_i(t,x)}{\partial x_k}\right)dx \leq -\left(\frac{\pi}{\psi_k-\phi_k}\right)^2 D_{ik}\int_\Omega \zeta_i^2(t,x)dx.$$

Thus, for diagonal matrix P_i, the following inequality can also be established:

$$2\int_\Omega \zeta^T(t,x)P_i\sum_{k=1}^{q}\frac{\partial}{\partial x_k}\left(D_k\frac{\partial\zeta(t,x)}{\partial x_k}\right)dx \leq -2\int_\Omega \zeta^T(t,x)P_iD_\pi\zeta(t,x)dx. \tag{17}$$

Furthermore, let $\Gamma = diag\{\beta_1, \ldots, \beta_n\}$ with $\beta_i = max\{|\beta_{1i}|, |\beta_{2i}|\}$, $i = 1, \ \ldots, \ n$. Then, from (11) one has

$$g^T\,(\zeta(t,x))\,g\,(\zeta(t,x)) \leq \zeta^T(t,x)\Gamma^2\zeta(t,x), \tag{18}$$

$$\zeta^T(t,x)\mathcal{T}_1R_1\mathcal{T}_2\zeta(t,x) - \zeta^T(t,x)R_1\,(\mathcal{T}_1 + \mathcal{T}_2)\,g\,(\zeta(t,x)) + g^T\,(\zeta(t,x))\,R_1g\,(\zeta(t,x)) \leq 0, \tag{19}$$

$$\zeta^T(t-\tau(t),x)\mathcal{T}_1R_2\mathcal{T}_2\zeta(t-\tau(t),x) - \zeta^T(t-\tau(t),x)R_2\,(\mathcal{T}_1 + \mathcal{T}_2)\,g\,(\zeta(t-\tau(t),x)) + g^T\,(\zeta(t-\tau(t),x))\,R_2g\,(\zeta(t-\tau(t),x)) \leq 0, \tag{20}$$

where the second and third inequalities follow from Lemma 3.

Next, define a mode-dependent LF as

$$V_{\gamma(t)}(t,\zeta(t,x)) = V_{1\gamma(t)}(t,\zeta(t,x)) + V_{2\gamma(t)}(t,\zeta(t,x)), \tag{21}$$

where

$$V_{1\gamma(t)}(t,\zeta(t,x)) = \int_\Omega \zeta^T(t,x)P_{\gamma(t)}\zeta(t,x)dx,$$

$$V_{2\gamma(t)}(t,\zeta(t,x)) = \int_\Omega\int_{t-\tau(t)}^t \zeta^T(\nu,x)Q_1\zeta(\nu,x)d\nu dx + \int_\Omega\int_{t-\tau(t)}^t g^T(\zeta(\nu,x))\,Q_2 g(\zeta(\nu,x))\,d\nu dx.$$

Then, it follows that

$$\begin{aligned}&\lambda_{min}(P_{\gamma(t)})\int_\Omega \zeta^T(t,x)\zeta(t,x)dx\\ &\le V_{\gamma(t)}(t,\zeta(t,x))\\ &\le \lambda_{max}(P_{\gamma(t)})\int_\Omega \zeta^T(t,x)\zeta(t,x)dx\\ &\quad+\lambda_{max}(Q_1)\int_\Omega\int_{t-\tau(t)}^t \zeta^T(\nu,x)\zeta(\nu,x)d\nu dx\\ &\quad+\lambda_{max}(Q_2)\int_\Omega\int_{t-\tau(t)}^t g^T(\zeta(\nu,x))\,g(\zeta(\nu,x))\,d\nu dx.\end{aligned} \tag{22}$$

With the aid of the extended Itô's formula (Mao, 1996), one can write

$$\mathcal{E}\{e^{\kappa t}V_{\gamma(t)}(t,\zeta(t,x))\} = \mathcal{E}\{V_{\gamma(t)}(0,\zeta(0,x))\} + \mathcal{E}\left\{\int_0^t e^{\kappa s}[\kappa V_{\gamma(s)}(s,\zeta(s,x)) + \mathfrak{L}V_{\gamma(s)}(s,\zeta(s,x))]ds\right\}, \tag{23}$$

where

$$\mathfrak{L}V_{\gamma(s)}(s,\zeta(s,x)) = \lim_{\theta\to0^+}\frac{1}{\theta}\left\{\mathcal{E}\left\{V_{\gamma(s+\theta)}(s+\theta,\zeta(s+\theta,x))|(s,\gamma(s),\zeta(s,x))\right\} - V_{\gamma(s)}(s,\zeta(s,x))\right\}.$$

According to (2), for $\gamma(s)=i$ one has

$$\begin{aligned}&\mathfrak{L}V_{1i}(s,\zeta(s,x))\\ &=\lim_{\theta\to0^+}\frac{1}{\theta}\Bigg\{\sum_{j=1,j\ne i}^m \Pr\{\gamma_{l+1}=j,h_{l+1}\le h+\theta|\gamma_l=i,\\ &\qquad h_{l+1}>h\}V_{1j}(s+\theta,\zeta(s+\theta,x))\\ &\qquad+\Pr\{\gamma_{l+1}=i,h_{l+1}>h+\theta|\gamma_l=i,h_{l+1}>h\}\\ &\qquad\times V_{1i}(s+\theta,\zeta(s+\theta,x)) - V_{1i}(s,\zeta(s,x))\Bigg\}\\ &=\lim_{\theta\to0^+}\frac{1}{\theta}\Bigg\{\sum_{j=1,j\ne i}^m\big[\xi_{ij}(h)\theta+o(\theta)\big]V_{1j}(s+\theta,\zeta(s+\theta,x))\\ &\qquad+[1+\xi_{ii}(h)\theta+o(\theta)]V_{1i}(s+\theta,\zeta(s+\theta,x))\\ &\qquad-V_{1i}(s,\zeta(s,x))\Bigg\}\\ &=\sum_{j=1}^m\xi_{ij}(h)V_{1j}(s,\zeta(s,x)) + \lim_{\theta\to0^+}\frac{1}{\theta}\{V_{1i}(s+\theta,\zeta(s+\theta,x))\\ &\qquad-V_{1i}(s,\zeta(s,x))\}.\end{aligned} \tag{24}$$

Noting that

$$\begin{aligned}&\lim_{\theta\to0^+}\frac{1}{\theta}\mathcal{E}\big\{V_{1i}(s+\theta,\zeta(s+\theta,x)) - V_{1i}(s,\zeta(s,x))\big\}\\ &=\frac{\partial}{\partial s}V_{1i}(s,\zeta(s,x)) + \frac{\partial}{\partial\zeta}V_{1i}(\zeta(s,x))h_i(s,\zeta(s,x),\zeta(s-\tau(s),x))\\ &\quad+\frac{1}{2}\mathrm{tr}\Big(\sigma^T(s,\zeta(s,x),\zeta(s-\tau(s),x))\frac{\partial^2}{\partial\zeta^2}\\ &\quad\times V_{1i}(\zeta(s,x))\sigma(s,\zeta(s,x),\zeta(s-\tau(s),x))\Big),\end{aligned}$$

one obtains from (13) and (24) that

$$\begin{aligned}&\mathcal{E}\big\{\mathfrak{L}V_{1i}(s,\zeta(s,x))\big\}\\ &=\int_\Omega\sum_{j=1}^m\bar{\xi}_{ij}\zeta^T(s,x)P_j\zeta(s,x)dx\\ &\quad+2\int_\Omega\zeta^T(s,x)P_i\sum_{k=1}^q\frac{\partial}{\partial x_k}\left(D_k\frac{\partial\zeta(s,x)}{\partial x_k}\right)dx\\ &\quad+2\int_\Omega\zeta^T(s,x)P_i\Big[-A_i\zeta(s,x)+W_{1i}g(\zeta(s,x))\\ &\quad+W_{2i}g(\zeta(s-\tau(s),x))\Big]dx\\ &\quad+\int_\Omega\mathrm{tr}\Big(\sigma^T(s,\zeta(s,x),\zeta(s-\tau(s),x))\\ &\quad\times P_i\sigma(s,\zeta(s,x),\zeta(s-\tau(s),x))\Big)dx.\end{aligned}$$

By (16) and (17), it follows that

$$\begin{aligned}\mathcal{E}\big\{\mathfrak{L}V_{1i}(s,\zeta(s,x))\big\}\le&\int_\Omega\zeta^T(s,x)\Big(-2P_iD_\pi-2P_iA_i\\ &+\sum_{j=1}^m\bar{\xi}_{ij}P_j+\rho_i\sigma_1 I\Big)\zeta(s,x)dx\\ &+2\int_\Omega\zeta^T(s,x)P_iW_{1i}g(\zeta(s,x))dx\\ &+2\int_\Omega\zeta^T(s,x)P_iW_{2i}g(\zeta(s-\tau(s),x))dx\\ &+\rho_i\sigma_2\int_\Omega\zeta^T(s-\tau(s),x)\zeta(s-\tau(s),x)dx.\end{aligned} \tag{25}$$

Similarly, the following inequality can be derived:

$$\begin{aligned}\mathcal{E}\big\{\mathfrak{L}V_{2i}(s,\zeta(s,x))\big\}=&\int_\Omega\zeta^T(s,x)Q_1\zeta(s,x)dx-(1-\dot{\tau}(s))\\ &\times\int_\Omega\zeta^T(s-\tau(s),x)\,Q_1\zeta(s-\tau(s),x)\,dx\\ &+\int_\Omega g^T(\zeta(s,x))\,Q_2 g(\zeta(s,x))\,dx\\ &-(1-\dot{\tau}(s))\int_\Omega g^T(\zeta(s-\tau(s),x))\\ &\times Q_2 g(\zeta(s-\tau(s),x))\,dx.\end{aligned} \tag{26}$$

From (19)–(21), (25), and (26) one can write

$$\begin{aligned}\mathcal{E}\big\{\mathfrak{L}V_i(s,\zeta(s,x))\big\}\le&\int_\Omega\zeta^T(s,x)\Big(-2P_iD_\pi-2P_iA_i\\ &+\sum_{j=1}^m\bar{\xi}_{ij}P_j+Q_1+\rho_i\sigma_1 I-\mathcal{T}_1R_1\mathcal{T}_2\Big)\zeta(s,x)dx\\ &+\int_\Omega\zeta^T(s,x)\big[2P_iW_{1i}+R_1(\mathcal{T}_1+\mathcal{T}_2)\big]\\ &\times g(\zeta(s,x))\,dx\\ &+2\int_\Omega\zeta^T(s,x)P_iW_{2i}g(\zeta(s-\tau(s),x))dx\\ &+\int_\Omega\zeta^T(s-\tau(s),x)\big[\rho_i\sigma_2 I+(\mu-1)Q_1\end{aligned}$$

$$
\begin{aligned}
& - \mathcal{T}_1 R_2 \mathcal{T}_2 \big] \zeta\left(s-\tau(s), x\right) dx \\
& + \int_\Omega \zeta^T\left(s-\tau(s), x\right) R_2\left(\mathcal{T}_1+\mathcal{T}_2\right) \\
& \times g\left(\zeta\left(s-\tau(s), x\right)\right) dx \\
& + \int_\Omega g^T\left(\zeta\left(s, x\right)\right)\left(Q_2-R_1\right) g\left(\zeta\left(s, x\right)\right) dx \\
& + \int_\Omega g^T\left(\zeta\left(s-\tau(s), x\right)\right)\left[(\mu-1) Q_2-R_2\right] \\
& \times g\left(\zeta\left(s-\tau(s), x\right)\right) dx \\
= & \int_\Omega \bar{\zeta}^T(s, x) \Xi_i \bar{\zeta}(s, x) dx \\
\leq & \lambda_{max}(\Xi_i) \int_\Omega \bar{\zeta}^T(s, x) \bar{\zeta}(s, x) dx,
\end{aligned} \tag{27}
$$

where

$$
\begin{aligned}
\bar{\zeta}^T(s, x) = & \left[\zeta^T(s, x)\ \zeta^T(s-\tau(s), x)\ g^T\left(\zeta\left(s, x\right)\right)\right. \\
& \times \left. g^T\left(\zeta\left(s-\tau(s), x\right)\right)\right]^T .
\end{aligned}
$$

Now, by (18), (22), (23), and (27), one has

$$
\begin{aligned}
& \min_{1\leq i\leq n} \lambda_{min}(P_i) e^{\kappa t} \mathcal{E}\left\{\int_\Omega |\zeta(t, x)|^2\, dx\right\} \\
\leq & \max_{1\leq i\leq n} \mathcal{E}\left\{\lambda_{max}(P_i) \int_\Omega \zeta^T(0, x)\zeta(0, x) dx\right\} \\
& + \mathcal{E}\left\{\lambda_{max}(Q_1) \int_\Omega \int_{-\tau(0)}^0 \zeta^T(\nu, x)\zeta(\nu, x) d\nu dx\right\} \\
& + \mathcal{E}\left\{\lambda_{max}(Q_2) \int_\Omega \int_{-\tau(0)}^0 g^T\left(\zeta(\nu, x)\right) g\left(\zeta(\nu, x)\right) d\nu dx\right\} \\
& + \max_{1\leq i\leq n} \mathcal{E}\left\{\int_\Omega \int_0^t e^{\kappa s} \kappa \lambda_{max}(P_i) \zeta^T(s, x)\zeta(s, x) ds dx\right\} \\
& + \mathcal{E}\left\{\int_\Omega \int_0^t \int_{s-\tau(s)}^s e^{\kappa s} \kappa \lambda_{max}(Q_1) \zeta^T(\nu, x)\zeta(\nu, x) d\nu ds dx\right\} \\
& + \mathcal{E}\left\{\int_\Omega \int_0^t \int_{s-\tau(s)}^s e^{\kappa s} \kappa \lambda_{max}(Q_2) g^T\left(\zeta(\nu, x)\right) g\left(\zeta(\nu, x)\right) d\nu ds dx\right\} \\
& + \max_{1\leq i\leq n} \lambda_{max}\left(\Xi_i\right) \mathcal{E}\left\{\int_\Omega \int_0^t e^{\kappa s} \zeta^T(s, x)\zeta(s, x) ds dx\right\} \\
\leq & \max_{1\leq i\leq n} H_{1i} \mathcal{E}\left\{\int_\Omega \int_0^t e^{\kappa s} \left|\zeta(s, x)\right|^2 ds dx\right\} \\
& + \max_{1\leq i\leq n} H_{2i} \mathcal{E}\left\{\int_\Omega \sup_{-\bar{\tau}\leq\nu\leq 0} \left|\varphi_1(\nu, x)+\varphi_2(\nu, x)\right|^2 dx\right\},
\end{aligned} \tag{28}
$$

where

$$
\begin{aligned}
H_{1i} = & \kappa\lambda_{max}(P_i) + (e^{\kappa\bar{\tau}}-1)\left[\lambda_{max}(Q_1)+\lambda_{max}(Q_2)\Gamma^2\right] + \lambda_{max}\left(\Xi_i\right), \\
H_{2i} = & \lambda_{max}(P_i) + \bar{\tau}\lambda_{max}(Q_1) + \bar{\tau}\lambda_{max}(Q_2)\Gamma^2 \\
& + \left(\lambda_{max}(Q_1)+\lambda_{max}(Q_2)\Gamma^2\right) \frac{1}{\kappa}(e^{\kappa\bar{\tau}}-1)(1-e^{-\kappa\bar{\tau}}).
\end{aligned}
$$

Noting (15), one can choose $\kappa > 0$ small enough such that $H_{1i} < 0$ $(i = 1, \ldots, m)$. Let

$$
H_0 = \frac{\max_{1\leq i\leq n} H_{2i} \mathcal{E}\left\{\int_\Omega \sup_{-\bar{\tau}\leq\nu\leq 0} \left|\varphi_1(\nu, x)+\varphi_2(\nu, x)\right|^2 dx\right\}}{\min_{1\leq i\leq n} \lambda_{min}(P_i)}.
$$

Then, (28) yields

$$
\mathcal{E}\left\{\int_\Omega |\zeta(t, x)|^2 dx\right\} \leq H_0 e^{-\kappa t}.
$$

Therefore, by Definition 1, system (12) is exponentially stable in mean square. In this way, the proof is completed. □

Remark 3. Compared with the reported results on semi-Markovian jump systems, the criterion given in Theorem 1 can be applied to test the stability of stochastic partial functional differential equations. It is also worth mentioning that, in the proof of Theorem 1, the derivation of infinitesimal operator $\mathfrak{L}V_{\gamma(s)}$ is directly based on the transition probabilities of the semi-Markov process, which is more clear and concise than existing ways in the literature (see, e.g., Syed Ali et al., 2019; Wan, et al., 2019; Wei et al., 2017).

When the sojourn time obeys exponential distribution, the transition rates are independent of h (i.e., $\xi_{ij}(h) \equiv \xi_{ij}$). In this case, the semi-Markov process reduces to a normal Markov process. And we can write the following result:

Corollary 1. *System (12) with Markov jump parameters is exponentially stable in mean square, if, for any $i \in \mathcal{M}$, there are scalars $\rho_i > 0$, matrices $Q_1 > 0$, $Q_2 > 0$, diagonal matrices $P_i > 0$, $R_1 > 0$, and $R_2 > 0$, such that (14) and (15) hold.*

Remark 4. The stochastic stability of system (12) with Markov jump parameters was also considered in Shi and Ma (2012). Comparing the criterion in Corollary 1 with that in Shi and Ma (2012) (see Theorem 1 therein), by adopting similar derivation procedures as those in Xu and Lam (2007) it is not difficult to show that, the former, which involves fewer decision variables, is a necessary but not sufficient condition for the latter. Thus, the present analysis method is theoretically less conservative than that reported in Shi and Ma (2012).

4. Anti-synchronization control synthesis

Based on Theorem 1, one can develop a strategy for the resilient fault-tolerant anti-synchronization controller design, which is given by the following theorem:

Theorem 2. *Given a constant $\varsigma > 0$, semi-Markov jump DRDNNs (1) and (3) are exponentially anti-synchronized in mean square under the resilient fault-tolerant control, if, $\forall i \in \mathcal{M}$, there exist scalars $\rho_i > 0$, $\varepsilon_1 > 0$, $\varepsilon_2 > 0$, matrices $Q_1 > 0$, $Q_2 > 0$, X_i, Y_i, diagonal matrices $P_i > 0$, $R_1 > 0$, $R_2 > 0$, such that (14) and the following LMI hold:*

$$
\begin{bmatrix}
\Phi_{11}^{[i]} & 0 & \Phi_{13}^{[i]} & P_i W_{2i} & \Phi_{15}^{[i]} & \Phi_{16}^{[i]} & \Phi_{17}^{[i]} \\
* & \Phi_{22}^{[i]} & 0 & \Phi_{24} & 0 & 0 & 0 \\
* & * & \Phi_{33} & 0 & 0 & 0 & 0 \\
* & * & * & \Phi_{44} & 0 & 0 & 0 \\
* & * & * & * & \Phi_{55}^{[i]} & \varepsilon_2 \mathscr{L}_i^T & 0 \\
* & * & * & * & * & \Phi_{66}^{[i]} & 0 \\
* & * & * & * & * & * & -\varepsilon_2 I
\end{bmatrix} < 0, \tag{29}
$$

where

$$
\begin{aligned}
\Phi_{11}^{[i]} = & -2P_i D_\pi - \mathscr{S}\left(P_i A_i - B_i \mathcal{G}_{0i} Y_i\right) \\
& + \sum_{j=1}^m \bar{\xi}_{ij} P_j + Q_1 + \rho_i \sigma_1 I - \mathcal{T}_1 R_1 \mathcal{T}_2 + \varepsilon_1 \mathscr{M}_i^T \mathscr{M}_i, \\
\Phi_{13}^{[i]} = & P_i W_{1i} + \frac{R_1\left(\mathcal{T}_1+\mathcal{T}_2\right)}{2}, \quad \Phi_{15}^{[i]} = P_i B_i \mathcal{G}_{0i} \mathscr{L}_i,
\end{aligned}
$$

$$\Phi_{16}^{[i]} = P_iB_i\mathcal{G}_{0i} - B_i\mathcal{G}_{0i}X_i + \varsigma Y_i^T,$$
$$\Phi_{17}^{[i]} = P_iB_i\mathcal{G}_{1i},\ \Phi_{22}^{[i]} = (\mu-1)Q_1 - \mathcal{T}_1R_2\mathcal{T}_2 + \rho_i\sigma_2 I,$$
$$\Phi_{24} = \frac{R_2(\mathcal{T}_1+\mathcal{T}_2)}{2},\ \Phi_{33} = Q_2 - R_1,$$
$$\Phi_{44} = (\mu-1)Q_2 - R_2,\ \Phi_{55}^{[i]} = -\varepsilon_1 I + \varepsilon_2\mathscr{L}_i^T\mathscr{L}_i,$$
$$\Phi_{66}^{[i]} = -\mathscr{S}(\varsigma X_i) + \varepsilon_2 I,$$
$$\bar{\xi}_{ij} = \mathcal{E}\left\{\xi_{ij}(h)\right\} = \int_0^\infty \xi_{ij}(h)e_i(h)dh,$$
$$D_\pi = diag\left\{\sum_{k=1}^{q}\left(\frac{\pi}{\psi_k-\phi_k}\right)^2 D_{1k},\ldots,\sum_{k=1}^{q}\left(\frac{\pi}{\psi_k-\phi_k}\right)^2 D_{nk}\right\}.$$

Moreover, the desired feedback gain of (6) can be given by

$$K_i = X_i^{-1}Y_i,\quad i\in\mathcal{M}. \tag{30}$$

Proof. By Theorem 1, the mean-square exponential anti-synchronization of system (1) and system (3) under the resilient fault-tolerant control is able to be realized if (14) and

$$\begin{bmatrix} \bar{\Xi}_{11}^{[i]} & 0 & \Xi_{13}^{[i]} & P_iW_{2i} \\ * & \Xi_{22}^{[i]} & 0 & \frac{R_2(\mathcal{T}_1+\mathcal{T}_2)}{2} \\ * & * & Q_2-R_1 & 0 \\ * & * & * & (\mu-1)Q_2-R_2 \end{bmatrix} < 0 \tag{31}$$

are satisfied, where

$$\bar{\Xi}_{11}^{[i]} = \Xi_{11}^{[i]} + \mathscr{S}\left(P_iB_i\mathcal{G}_i\left(K_i + \Delta K_i(t)\right)\right).$$

Noting $\Delta K_i(t) = \mathscr{L}_iF_i(t)\mathscr{M}_i$ and $F_i^T(t)F_i(t) \le I$, one has from Lemma 4 that

$$\mathscr{S}\left(P_iB_i\mathcal{G}_i\Delta K_i(t)\right) \le \varepsilon_1^{-1}\left(P_iB_i\mathcal{G}_i\mathscr{L}_i\right)\left(P_iB_i\mathcal{G}_i\mathscr{L}_i\right)^T + \varepsilon_1\mathscr{M}_i^T\mathscr{M}_i. \tag{32}$$

In addition, in the light of (30), one can write

$$\mathscr{S}\left(P_iB_i\mathcal{G}_iK_i\right) = \mathscr{S}\left(B_i\mathcal{G}_{0i}Y_i + \left(P_iB_i\mathcal{G}_i - B_i\mathcal{G}_{0i}X_i\right)X_i^{-1}Y_i\right). \tag{33}$$

By (32) and (33), it is obvious that (31) holds true if

$$\Xi_i + diag\{\Pi_i^{[i]},\ 0,\ 0,\ 0\} + \mathscr{S}\left(\mathcal{U}_iX_i^{-1}\mathcal{V}_i^T\right) < 0, \tag{34}$$

where

$$\Pi_i^{[i]} = \mathscr{S}\left(B_i\mathcal{G}_{0i}Y_i\right) + \varepsilon_1\mathscr{M}_i^T\mathscr{M}_i + \varepsilon_1^{-1}\left(P_iB_i\mathcal{G}_i\mathscr{L}_i\right)\left(P_iB_i\mathcal{G}_i\mathscr{L}_i\right)^T,$$
$$\mathcal{U}_i = \left[\left(P_iB_i\mathcal{G}_i - B_i\mathcal{G}_{0i}X_i\right)^T,\ 0,\ 0,\ 0\right]^T,$$
$$\mathcal{V}_i = \left[Y_i,\ 0,\ 0,\ 0\right]^T.$$

Let

$$\bar{\Phi}_{15}^{[i]} = P_iB_i\mathcal{G}_i\mathscr{L}_i,\quad \bar{\Phi}_{16}^{[i]} = P_iB_i\mathcal{G}_i - B_i\mathcal{G}_{0i}X_i + \varsigma Y_i^T.$$

Then, according to Lemma 5 and Schur's complement, (34) is ensured by

$$\begin{bmatrix} \Phi_{11}^{[i]} & 0 & \Phi_{13}^{[i]} & P_iW_{2i} & \bar{\Phi}_{15}^{[i]} & \bar{\Phi}_{16}^{[i]} \\ * & \Phi_{22}^{[i]} & 0 & \Phi_{24} & 0 & 0 \\ * & * & \Phi_{33} & 0 & 0 & 0 \\ * & * & * & \Phi_{44} & 0 & 0 \\ * & * & * & * & -\varepsilon_1 I & 0 \\ * & * & * & * & * & -\mathscr{S}(\varsigma X_i) \end{bmatrix} < 0,$$

which, by (5), can be re-expressed as

$$\Phi_i + \mathscr{S}\left(\mathcal{W}_{ai}\Lambda_i\mathcal{W}_{bi}\right) < 0, \tag{35}$$

where

$$\Phi_i = \begin{bmatrix} \Phi_{11}^{[i]} & 0 & \Phi_{13}^{[i]} & P_iW_{2i} & \Phi_{15}^{[i]} & \Phi_{16}^{[i]} \\ * & \Phi_{22}^{[i]} & 0 & \Phi_{24} & 0 & 0 \\ * & * & \Phi_{33} & 0 & 0 & 0 \\ * & * & * & \Phi_{44} & 0 & 0 \\ * & * & * & * & -\varepsilon_1 I & 0 \\ * & * & * & * & * & -\mathscr{S}(\varsigma X_i) \end{bmatrix},$$
$$\mathcal{W}_{ai} = \begin{bmatrix} (P_iB_i\mathcal{G}_{1i})^T, & 0, & 0, & 0, & 0, & 0 \end{bmatrix}^T,$$
$$\mathcal{W}_{bi} = \begin{bmatrix} 0, & 0, & 0, & 0, & \mathscr{L}_i, & I \end{bmatrix}.$$

Noting $\Lambda_i^T\Lambda_i \le I$, by Lemma 6, (35) holds true if the following inequality is satisfied:

$$\Phi_i + \varepsilon_2^{-1}\mathcal{W}_{ai}\mathcal{W}_{ai}^T + \varepsilon_2\mathcal{W}_{bi}^T\mathcal{W}_{bi} < 0,$$

which, by Schur's complement again, can be re-organized as (29). This completes the proof of the theorem. □

Remark 5. During the design of the desired resilient fault-tolerant anti-synchronization controller, high-order nonlinearities appear as a result of the couplings of the Lyapunov matrices, the gain perturbation matrices, and the uncertainties of the actuator failures. How to deal with such high-order nonlinearities is not easy. In this paper, based on the analysis result and using several decoupling approaches, a strategy is developed in Theorem 2, where the desired gains can be obtained via solving several LMIs, which are checked easily by the computing software MATLAB.

Considering the actuator failure problem and the gain perturbation problem separately, the following results can be established:

Corollary 2. *Suppose that there are no actuator failures; i.e., $\mathcal{G}_i = \mathcal{G}_{0i} = I$ $(i = 1,\ldots,n)$. Then, for a given constant $\varsigma > 0$, semi-Markov jump DRDNNs (1) and (3) are exponentially anti-synchronized in mean square under the resilient control, if, $\forall i \in \mathcal{M}$, there exist scalars $\rho_i > 0$, $\varepsilon_1 > 0$, matrices $Q_1 > 0$, $Q_2 > 0$, X_i, Y_i, diagonal matrices $P_i > 0$, $R_1 > 0$, $R_2 > 0$, such that (14) and the following LMI hold:*

$$\begin{bmatrix} \Phi_{11}^{[i]} & 0 & \Phi_{13}^{[i]} & P_iW_{2i} & \frac{P_iB_i\mathscr{L}_i}{2} & \Phi_{16}^{[i]} \\ * & \Phi_{22}^{[i]} & 0 & \Phi_{24} & 0 & 0 \\ * & * & \Phi_{33} & 0 & 0 & 0 \\ * & * & * & \Phi_{44} & 0 & 0 \\ * & * & * & * & -\varepsilon_1 I & 0 \\ * & * & * & * & * & -\mathscr{S}(\varsigma X_i) \end{bmatrix} < 0,$$

where $\Phi_{11}^{[i]}$, $\Phi_{13}^{[i]}$, $\Phi_{16}^{[i]}$, $\Phi_{22}^{[i]}$, Φ_{24}, Φ_{33}, and Φ_{44} are the same as those in Theorem 2. In this case, the required feedback gain can be provided by $K_i = X_i^{-1}Y_i$ $(i \in \mathcal{M})$.

Corollary 3. *Suppose that there are no gain perturbations; i.e., $\Delta K_i(t) = 0$ $(i = 1,\ldots,n)$. Then, for a given constant $\varsigma > 0$, semi-Markov jump DRDNNs (1) and (3) are exponentially anti-synchronized in mean square under the fault-tolerant control, if, $\forall i \in \mathcal{M}$, there exist scalars $\rho_i > 0$, $\varepsilon_1 > 0$, matrices $Q_1 > 0$, $Q_2 > 0$, X_i, Y_i, diagonal matrices $P_i > 0$, $R_1 > 0$, $R_2 > 0$, such that (14) and the following LMI hold:*

$$\begin{bmatrix} \tilde{\Phi}_{11}^{[i]} & 0 & \Phi_{13}^{[i]} & P_iW_{2i} & P_iB_i\mathcal{G}_{1i} & \Phi_{16}^{[i]} \\ * & \Phi_{22}^{[i]} & 0 & \Phi_{24} & 0 & 0 \\ * & * & \Phi_{33} & 0 & 0 & 0 \\ * & * & * & \Phi_{44} & 0 & 0 \\ * & * & * & * & -\varepsilon_1 I & 0 \\ * & * & * & * & * & \tilde{\Phi}_{66}^{[i]} \end{bmatrix} < 0,$$

Table 1
Comparisons of maximum allowed σ_2 for different methods.

σ_2	$\sigma_1 = 0.1$	$\sigma_1 = 0.2$	$\sigma_1 = 0.4$	$\sigma_1 = 0.8$	$\sigma_1 = 1.6$
Corollary 1	1.408	1.314	1.127	0.755	0.037
Theorem 1 of Shi and Ma (2012)	0.854	0.759	0.569	0.189	No solution

where

$$\tilde{\Phi}_{11}^{[i]} = -2P_iD_\pi - \mathscr{S}(P_iA_i - B_i\mathcal{G}_{0i}Y_i) + \sum_{j=1}^{m}\bar{\xi}_{ij}P_j + Q_1 + \rho_i\sigma_1 I - \mathcal{T}_1R_1\mathcal{T}_2,\ \tilde{\Phi}_{66}^{[i]} = -\mathscr{S}(\varsigma X_i) + \varepsilon_1 I,$$

and $\Phi_{13}^{[i]}$, $\Phi_{16}^{[i]}$, $\Phi_{22}^{[i]}$, Φ_{24}, Φ_{33}, Φ_{44}, and $\bar{\xi}_{ij}$ are the same as those in Theorem 2. In this case, the required feedback gain can be provided by $K_i = X_i^{-1}Y_i$ ($i \in \mathcal{M}$).

5. Numerical examples

In the section, two examples are given to illustrate the superiority of the stability analysis method and applicability of the resilient fault-tolerant anti-synchronization control strategy, respectively.

Example 1. Consider unforced Markov jump system (9) (i.e., system (12)) with two modes, where the parameters are provided by

$$D = \begin{bmatrix} 0.5 & 0.5 \\ 0.3 & 0.7 \end{bmatrix}, A_1 = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix},$$
$$A_2 = \begin{bmatrix} 0.4 & 0 \\ 0 & 0.4 \end{bmatrix},$$
$$W_{11} = \begin{bmatrix} 0.5 & 0.3 \\ 0.2 & 0.1 \end{bmatrix}, W_{21} = \begin{bmatrix} 0.1 & 0.2 \\ 0.3 & 0.4 \end{bmatrix},$$
$$W_{12} = \begin{bmatrix} 0.5 & 0.6 \\ 0.4 & 0.2 \end{bmatrix}, W_{22} = \begin{bmatrix} 0.2 & 0.3 \\ 0.4 & 0.5 \end{bmatrix},$$
$$\xi_{11} = -0.6,\ \xi_{12} = 0.6,\ \xi_{21} = 0.4,\ \xi_{22} = -0.4,$$
$$g_i(\zeta_i) = \frac{1}{2}(|\zeta_i + 1| - |\zeta_i - 1|)\ (i = 1,\ 2),\ \Omega = \{x|x \in [-1,\ 1]\}.$$

Set $\mu = 0.05$. Then, for the case that $\sigma_1 = 1.6$, it is found that the criterion in Theorem 1 in Shi and Ma (2012) fails to check whether the system is stable or not since the LMIs therein have no feasible solutions $\forall \sigma_2 > 0$, while by Corollary 1, it can be concluded that the system is exponentially stable in mean square for $\sigma_2 \in (0,\ 0.037]$. And for the case that $\sigma_1 = 0.8$, the maximum allowed σ_2 ensuring the stability is calculated as 0.755 by Theorem 1 in Shi and Ma (2012) and 0.189 by Corollary 1, respectively. More detail comparisons between Theorem 1 in Shi and Ma (2012) and Corollary 1 in this paper for different values of σ_1 are provided in Table 1, by which one can see the present analysis method is much less conservative than that proposed in Shi and Ma (2012), which further confirms the statement in Remark 4.

Example 2. Consider two-mode semi-Markov jump DRDNNs given by (1) and (3) with

$$D = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, A_1 = \begin{bmatrix} 1.0 & 0 \\ 0 & 1.0 \end{bmatrix},$$
$$A_2 = \begin{bmatrix} 0.9 & 0 \\ 0 & 0.9 \end{bmatrix}, B_1 = B_2 = \begin{bmatrix} -2 & 1 \\ 0 & -2 \end{bmatrix},$$
$$W_{11} = \begin{bmatrix} 2.0 & -0.1 \\ -5.0 & 2.8 \end{bmatrix}, W_{21} = \begin{bmatrix} -1.6 & -0.1 \\ -0.3 & -2.5 \end{bmatrix},$$
$$W_{12} = \begin{bmatrix} 0.5 & 0.6 \\ 0.4 & 0.2 \end{bmatrix}, W_{22} = \begin{bmatrix} 0.2 & 0.3 \\ 0.4 & 0.5 \end{bmatrix},$$
$$f_i(\cdot) = \tanh(\cdot)\ (i = 1,\ 2),\ \Omega = \{x|x \in [-2,\ 2]\},\ \tau(t) = 1,$$
$$\sigma(t, z_1(t,x) + z_2(t,x), z_1(t - \tau(t), x) + z_2(t - \tau(t), x)) = diag\{z_1(t,x) + z_2(t,x), z_1(t - \tau(t), x) + z_2(t - \tau(t), x)\}.$$

The initial conditions are taken as

$$z_1(s,x) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, z_2(s,x) = \begin{bmatrix} -1 \\ 2 \end{bmatrix}, (s,x) \in [-1, 0] \times \Omega,$$

and the boundary conditions are set as the type of Dirichlet. Assume the sojourn time obeys the Weibull distribution. Specifically, let $h_l \sim$ Weibull(2, 2) for $i = 1$ (i.e., $e_1(h) = 0.5h\exp(-(0.5h)^2)$) and $h_l \sim$ Weibull (1, 3) for $i = 2$ (i.e., $e_2(h) = 3h^2\exp(-h^3)$), respectively. Then we have

$$\begin{bmatrix} \xi_{11}(h) & \xi_{12}(h) \\ \xi_{21}(h) & \xi_{22}(h) \end{bmatrix} = \begin{bmatrix} -0.5h & 0.5h \\ 3h^2 & -3h^2 \end{bmatrix}.$$

By calculating the mathematical expectation we can get the transition rates as

$$\mathcal{E}\{\xi(h)\} = \begin{bmatrix} -0.8862 & 0.8862 \\ 2.7082 & -2.7082 \end{bmatrix}.$$

Let us consider the following parameters concerning the actuator failures as well as gain perturbations:

$$\mathscr{L}_1 = \begin{bmatrix} 0.2 & 0.1 \\ 0 & 0.1 \end{bmatrix}, \mathscr{L}_2 = \begin{bmatrix} 0.2 & 0.1 \\ 0 & 0.1 \end{bmatrix},$$
$$\mathscr{M}_1 = \begin{bmatrix} 0.02 & 0 \\ 0 & 0.01 \end{bmatrix}, \mathscr{M}_2 = \begin{bmatrix} 0.01 & 0.1 \\ 0 & 0.02 \end{bmatrix},$$
$$F_i(t) = sin(t),\ \check{g}_{ji} = 0.2,\ \hat{g}_{ji} = 0.8\ \ (i,\ j = 1,\ 2).$$

Then, by setting $\varsigma = 0.01$ and utilizing MATLAB to solve the LMIs in Theorem 2, the desired control gain matrices can be obtained as

$$K_1 = \begin{bmatrix} 95.1945 & 4.3730 \\ -55.3191 & 101.1654 \end{bmatrix},$$
$$K_2 = \begin{bmatrix} 48.9157 & 25.2136 \\ -36.9121 & 78.3478 \end{bmatrix}.$$

The semi-Markov jump signal is given in Fig. 1. Under the switching signal, the state trajectories of the anti-synchronization error system without control are depicted in Fig. 2, while, with the application of the designed controller, the desired fast convergence of the anti-synchronization error system is shown in Fig. 3.

In order to illustrate the control effect more clearly, let us consider the case that $x = -1.5$. In such a case, the phase-plane plot of the master DRDNN is given in Fig. 4, which shows a chaotic behavior. The state trajectories for the unforced master–slave DRDNNs and the controlled master–slave DRDNNs are depicted in Fig. 5 and Fig. 6, respectively. From the simulations, it can be observed that the designed resilient fault-tolerant controller works well for the anti-synchronization purpose.

6. Conclusions

The anti-synchronization control problem for stochastic DRDNNs with semi-Markov jump parameters has been studied

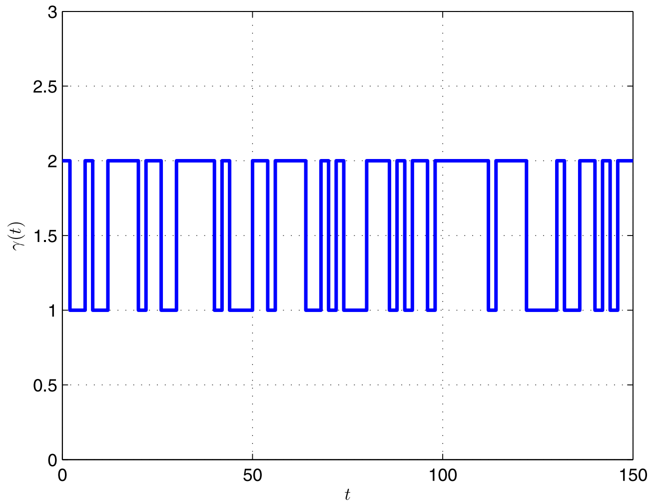

Fig. 1. Semi-Markov jump signal.

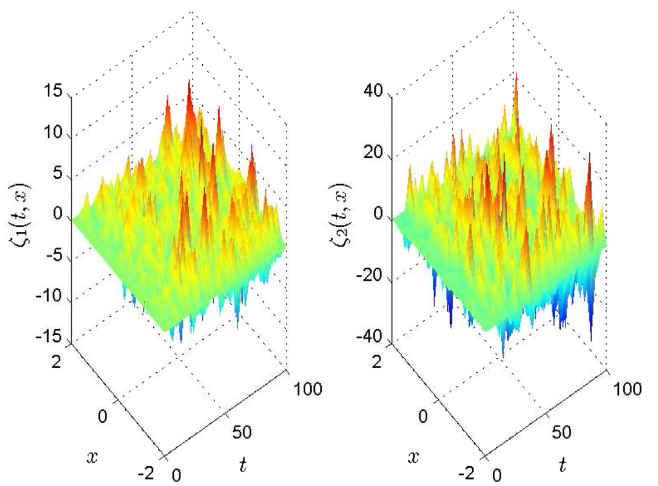

Fig. 2. State trajectories of the anti-synchronization error system without control.

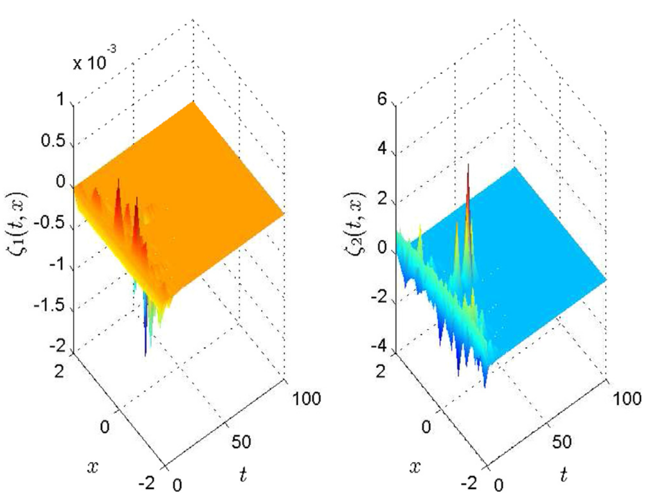

Fig. 3. State trajectories of the controlled anti-synchronization error system.

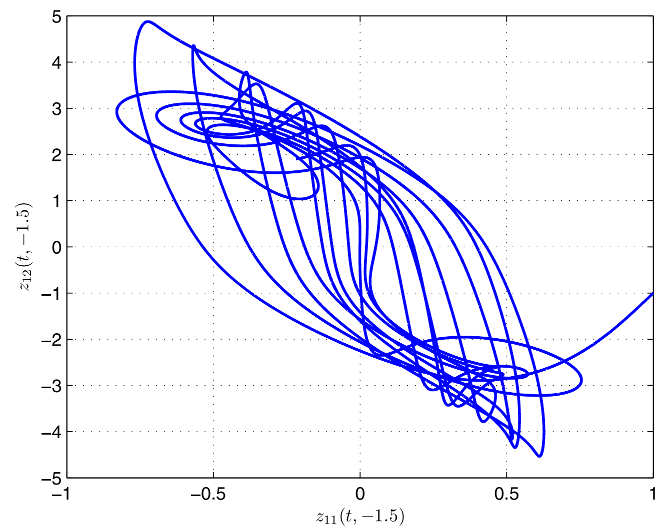

Fig. 4. Chaos behavior.

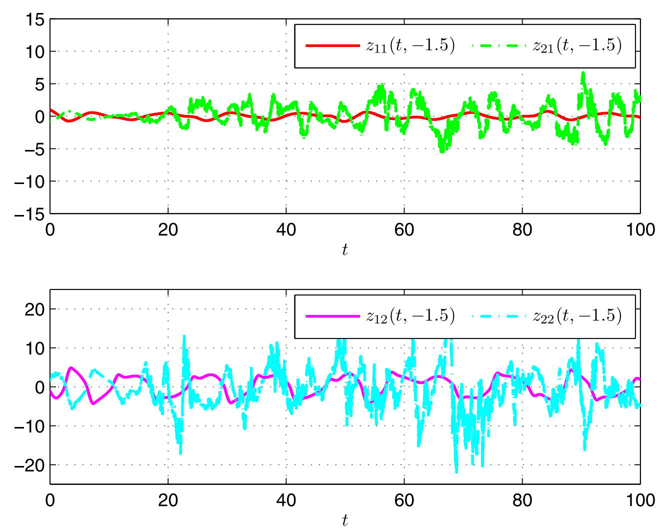

Fig. 5. State trajectories of the unforced master–slave DRDNNs.

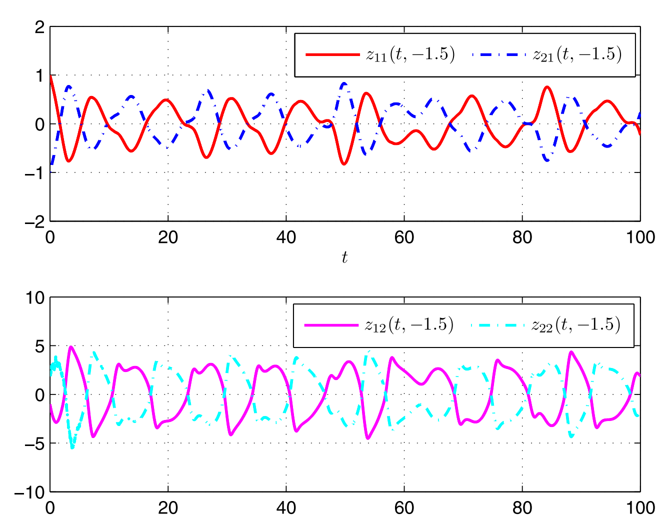

Fig. 6. State trajectories under control.

in the presence of actuator failures as well as gain perturbations. First, by using a mode-dependent LF and some stochastic analysis methods, a sufficient condition has been derived to ensure the resulting anti-synchronization error system to be exponentially stability in mean square. It has been shown that the obtained criterion improves a previously reported result. Then, based on the present analysis result and employing some decoupling techniques, a strategy for the design of a resilient fault-tolerant anti-synchronization controller has been developed. Finally, two numerical examples have been given to verify the superiority of the stability analysis method and applicability of the resilient fault-tolerant anti-synchronization control strategy, respectively.

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