BI2-Inviscid Flow over a Bump in a Channel

Shengye Wang[1], Yaming Chen[2], Xiaogang Deng[1]
[1] College of Aerospace Science and Engineering, National University of Defense Technology, Changsha, Hunan 410073, Peoples Republic of China
[2] College of Science, National University of Defense Technology, Changsha, Hunan 410073, Peoples Republic of China

1. Code description

The current flow solver makes use of a robust 5th-order finite difference scheme WCNS-E-5 [1-4]. The WCNS-E-5 is a typical explicit fifth-order one of weighted compact nonlinear schemes (WCNSs) which were first derived by Deng [1]. To address the question of efficiency, 2nd-order scheme MUSCL [5] is also used for comparison. The procedure of discretization for both WCNS and MUSCL consists of three components: (i) cell-center to cell-node differencing, (ii) flux evaluation at the cell-center, and (iii) cell-node to cell-center interpolation of conservative variables. For this test case, Roe's flux difference scheme is used.

For finite difference scheme, a grid transformation from the Cartesian coordinates to the curvilinear coordinates is required. For the satisfaction of geometric conservation law (GCL), SCMM [6,7] is used for the calculation of grid metric and Jacobian. For temporal integration, the LU-SGS method is adopted.

2. Case summary

In this benchmark case, the L2 norm of the density residual is monitored and the residual tolerance is set to be 10^{-10}, which is shown in Figure 1. All the meshes and order refinement studies are performed on a single processor, where the average Taubench CPU time is 8.5 seconds.

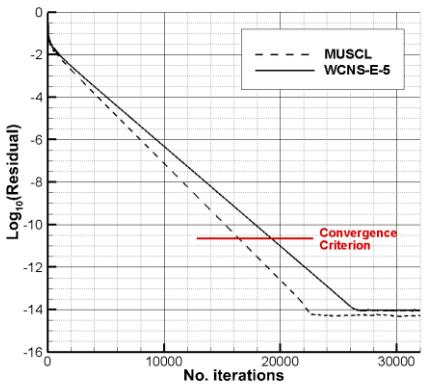

Figure 1. Convergence history for WCNS-E-5 and MUSCL on the medium (96×32) grid.

3. Meshes

The set of structured grids are generated by the python scripts provided on the web site. The plot3d type of meshes containing 24×8, 48×16, 96×32, 192×64, 384×128, 768×256 cells are chose.

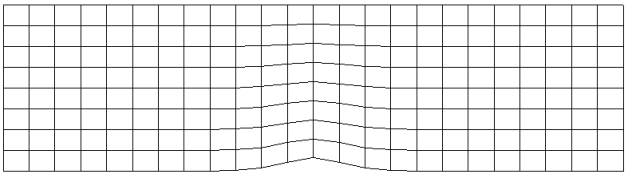

Figure 2. The coarsest (24×8) grid.

4. Results

The simulation starts with uniform flow with Mach number of 0.5. The contour plot of entropy error for WCNS-E-5 on the finest (768×256) grid is shown in Figure 3. It can be seen that the larger entropy error is located at the top and downstream of the smooth bump.

The WCNS-E-5 and MUSCL are compared in the results. Both the computational errors against the grid size and work units are shown in Figure 4. It is evident that the error decreases with the mesh size to the power as the order of the scheme. For the efficiency, Figure 4 shows that the computational cost of WCNS-E-5 is less than that of MUSCL achieving the same error, namely high-order methods can obtain a numerical solution more efficiently than low-order ones, given the same error threshold. Table 1 and Table 2 show the detail of the results.

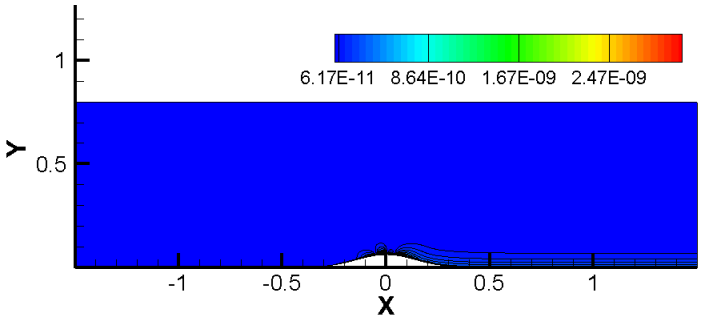

Figure 3. Contour plot of the entropy error for WCNS-E-5 on the finest (768×256) grid.

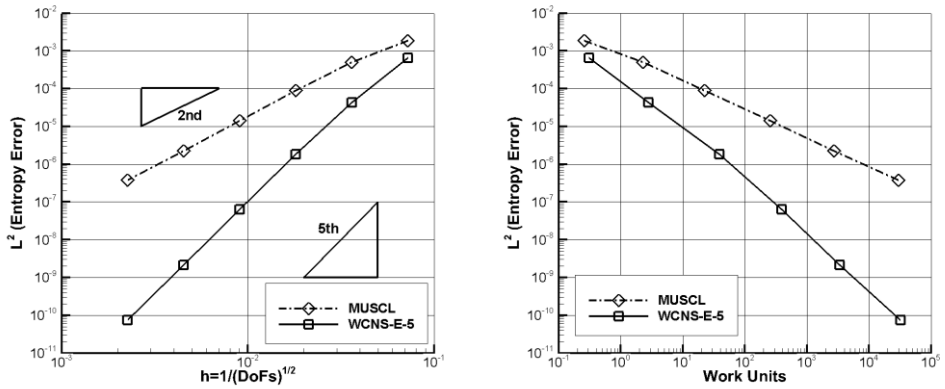

Figure 4. Entropy errors for WCNS-E-5 and MUSCL.

Table 1. Detail results obtained by WCNS-E-5.

Grid	h	L2-norm entropy error	Convergence rate	Work Unit

24x8	0.072169	6.58E-04	--	0.31
48x16	0.036084	4.25E-05	3.95	2.78
96x32	0.018042	1.82E-06	4.54	38.88
192x64	0.009021	6.29E-08	4.86	391.77
384x128	0.004511	2.13E-09	4.89	3343.42
768x256	0.002255	7.54E-11	4.82	32276.42

Table 2. Detail results obtained by MUSCL.

Grid	h	L2-norm entropy error	Convergence rate	Work Unit
24x8	0.072169	1.85E-03	--	0.26
48x16	0.036084	4.99E-04	1.89	2.29
96x32	0.018042	8.94E-05	2.48	22.21
192x64	0.009021	1.41E-05	2.67	255.39
384x128	0.004511	2.24E-06	2.65	2726.25
768x256	0.002255	3.73E-07	2.59	29940.89

5. References

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