Matrix Multiplication, 67826 - Exercise 2

Due: May 29, 2025

Question 1

Show that the flip operation is not transitive. Your example should contain at most three multiplications.

Question 2

(a) Write the UVW representation of Strassen's algorithm (slide 7 in the second presentation) for computations over $\mathbb{Z}_2$.

(b) Lift the binary solution $a = b = d = 1, c = 0$ for

$$ab + cd = 3$$
$$ad + cb = 1$$

from $\mathbb{Z}_2$ to $\mathbb{Z}_4$ using Hensel lifting (slides 39-41 in the second presentation).

Question 3

(a) Translate the following algorithm from a trace representation to a UVW representation:

$a_1 b_1 c_1 + a_1 b_2 c_2 + a_2 b_2 c_3$

(b) Translate Strassen's algorithm from a UVW representation to a trace representation.

Question 4

During research, you found a trilinear aggregation algorithm that uses $\overline{S} = \{(i, j, k) \mid 0 \leq i \leq j \leq k \leq n\}$ as the set of triplets. This algorithm uses additional $\frac{105n^2}{16} + 39n + 55$ multiplications for explicit corrections, i.e., explicit computation of undesirable terms.

(a) How many multiplications does this algorithm use as a function of n?

(b) How many multiplications does this algorithm use for $n = 44$? What is the resulting ω_0? Is it better than known algorithms?

(c) Assume that the algorithm works only for n divisible by four. What is the optimal n for this algorithm? What is the resulting ω_0? (You can use Desmos or other tools for an easier search.)