Completing Local Position Invariance Tests: A Cavity–Atom Frequency Ratio Protocol

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Summary. Local Position Invariance (LPI) is a cornerstone of General Relativity, tested extensively via gravitational redshift with atomic clocks and matter [1, 2, 3, 4, 5]. However, no direct test has yet compared *cavity-stabilized optical frequencies* (photon sector) to *atomic transitions* (matter sector) across a gravitational potential. We propose a protocol to close this gap: measure the fractional slope of co-located cavity–atom frequency ratios transported between two fixed altitudes.

Observable

Define the cavity–atom ratio:

$$\frac{\Delta R^{(M,S)}}{R^{(M,S)}} \equiv \xi^{(M,S)} \frac{\Delta\Phi}{c^2}, \qquad \xi^{(M,S)} = \alpha_w - \alpha_L^{(M)} - \alpha_{\text{at}}^{(S)}. \tag{1}$$

Here the coefficients are:

- α_w: photon-sector weight, normalized to 1 in GR.
- $\alpha_L^{(M)}$: cavity length sensitivity for material M (e.g. ULE or Si).
- $\alpha_{\text{at}}^{(S)}$: atomic transition sensitivity for species S (e.g. Sr or Yb).
- $\xi^{(M,S)}$: net slope coefficient for cavity–atom ratio with material M and species S.

- GR predicts $\xi^{(M,S)} = 0$, i.e. a strict null.
- Any reproducible nonzero ξ would indicate sector-dependent deviation from LPI.

Definitions and identifiability

To isolate contributions, define:

$$\delta_{\text{tot}} \equiv \alpha_w - \alpha_L^{\text{ULE}} - \alpha_{\text{at}}^{\text{Sr}}, \qquad \delta_L \equiv \alpha_L^{\text{Si}} - \alpha_L^{\text{ULE}}, \qquad \delta_{\text{at}} \equiv \alpha_{\text{at}}^{\text{Yb}} - \alpha_{\text{at}}^{\text{Sr}}.$$

The four measured slopes across two cavity materials (ULE, Si) and two atomic species (Sr, Yb) then map to three independent combinations (Table 1).

Table 1: Mapping of measured cavity–atom ratios to sector parameters.

Measured slope	Combination	Identified parameter
ULE/Sr	$\alpha_w - \alpha_L^{\mathrm{ULE}} - \alpha_{\mathrm{at}}^{\mathrm{Sr}}$	δ_{tot}
Si/Sr	$\alpha_w - \alpha_L^{\mathrm{Si}} - \alpha_{\mathrm{at}}^{\mathrm{Sr}}$	$\delta_{\mathrm{tot}} + \delta_L$
ULE/Yb	$\alpha_w - \alpha_L^{\mathrm{ULE}} - \alpha_{\mathrm{at}}^{\mathrm{Yb}}$	$\delta_{\mathrm{tot}} + \delta_{\mathrm{at}}$
Si/Yb	$\alpha_w - \alpha_L^{\mathrm{Si}} - \alpha_{\mathrm{at}}^{\mathrm{Yb}}$	$\delta_{\mathrm{tot}} + \delta_L + \delta_{\mathrm{at}}$

Numerical scale

For Earth gravity $g \simeq 9.8\,\mathrm{m/s}^2$,

$$\frac{g\,\Delta h}{c^2} = 1.1 \times 10^{-14} \quad (\Delta h = 100\,\mathrm{m}).$$

Thus the natural scale is at 10^{-14} per 100 m altitude change, within reach of current 10^{-16} optical clock precision.

Controls and feasibility

The protocol envisions static comparisons at two fixed altitudes (e.g. basement vs. rooftop labs, or ground vs. tower). Only stationary data are analyzed, avoiding artifacts from transport in motion.

Corrections and controls:

- **Dispersion/thermo-optic:** dual-λ probing within the low-loss band, bounding $|\varepsilon_{\mathrm{disp}}| \lesssim 10\%$ [8, 9, 10].
- **Elastic sag:** orientation flips distinguish mechanical artifacts (sign-reversing) from genuine redshift (sign-preserving). In optimized silicon cavities, sag effects can be suppressed below 10^{-16} [11, 12].
- **Environmental:** vibration, temperature, pressure, and magnetic reversals, plus hardware swaps, encode residual offsets in the covariance, suppressing bias [13, 5].

Feasibility. All required components are already demonstrated separately: ultra-stable cavities at 10^{-16} [11, 12], optical clocks reaching below 10^{-18} [13, 5], and long-term LPI clock tests [2, 4, 3]. Combining these into a cavity–atom slope test is therefore technically feasible with current infrastructure.

Motivation

Existing LPI tests compare like with like: atom–atom or matter–matter systems [1, 2, 4, 5]. A cavity–atom comparison probes an untested cross-sector combination (photon vs. atomic transitions). This experiment therefore closes a missing gap in the LPI test suite. Even a null result would provide the first direct constraint on this sector and complete the phenomenological mapping of LPI across independent systems.

Falsification criterion

- GR: $\xi = 0$ at all materials/species.
- Experimental discriminator: any reproducible nonzero ξ at or above $\Delta\Phi/c^2$ would indicate violation of LPI in this sector.

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