# Density Field Dynamics and the c-Field:

A Three-Dimensional, Time-Emergent Dynamics for Gravity and Cosmology

Gary Alcock

August 18, 2025

#### Abstract

We formulate a dynamical alternative to curved spacetime in which the universe is fundamentally Euclidean  $\mathbb{R}^3$  and time is emergent. A single scalar "c-field"  $\psi(\mathbf{x})$  controls the one-way speed of light via  $c_1(\mathbf{x}) = c e^{-\psi(\mathbf{x})}$ , preserving the measured two-way light speed c. Matter and photons couple to the same  $\psi$ : massive test bodies accelerate according to

$$\mathbf{a} \; = \; \frac{c^2}{2} \, \nabla \psi \equiv - \nabla \Phi, \qquad \Phi \equiv - \frac{c^2}{2} \, \psi,$$

while photons follow Fermat paths in the refractive index  $n(\mathbf{x}) = e^{\psi(\mathbf{x})}$ . From a local, isotropic action we derive a nonlinear Poisson equation for  $\psi$ ,

$$\nabla \cdot \left[ \mu \left( \frac{|\nabla \psi|}{a_{\star}} \right) \nabla \psi \right] = -\frac{8\pi G}{c^2} \left( \rho_m - \bar{\rho}_m \right),$$

which fixes the weak-field normalization needed to reproduce exactly Einstein's classical tests (light deflection  $\alpha=4GM/(c^2b)$ , gravitational redshift, Shapiro delay, and the Mercury perihelion advance) [1, 2, 3]. In the low-gradient (galactic/void) regime, the same equation yields  $|\nabla\psi|\propto 1/r$ , implying  $v(r)\to {\rm const}$  (flat rotation curves) without dark matter and a Tully–Fisher/RAR scaling [4, 5, 6, 7]. On cosmic scales, line-of-sight optical length  $D_{\rm opt}=\frac{1}{c}\int e^{\psi}ds$  produces a foreground-dependent bias that explains the Hubble tension and mimics cosmic acceleration without a cosmological constant [8, 9, 10]. We present explicit derivations and conservation laws from the action, and give falsifiable laboratory protocols (one-way-c metrology and atom interferometry) at the  $10^{-10}\,{\rm m\,s^{-2}}$  scale [11, 12].

#### 1 Principles and Definitions

**(P1) Three-dimensional ontology.** Physical space is Euclidean  $\mathbb{R}^3$ . Time is not fundamental; durations are operationally defined via round-trip light and physical clocks.

(P2) One-way light as a field. The one-way light speed is dynamical:

$$c_1(\mathbf{x}) = c e^{-\psi(\mathbf{x})}, \qquad n(\mathbf{x}) \equiv \frac{c}{c_1} = e^{\psi(\mathbf{x})}.$$
 (1)

Two-way c is invariant by reciprocity along any fixed path (Sec. 10).

(P3) Unified coupling of matter and light. Matter accelerations and photon paths are governed by the same  $\psi$ :

$$\mathbf{a} = \frac{c^2}{2} \nabla \psi \equiv -\nabla \Phi, \qquad \Phi \equiv -\frac{c^2}{2} \psi. \tag{2}$$

Photons extremize optical length  $\int n \, ds = \int e^{\psi} ds$  (Fermat) [2, 13].

# 2 Action and Field Equation (Dynamics and Conservation)

Locality and isotropy in  $\mathbb{R}^3$  with a single universal matter coupling select the functional

$$\mathcal{F}[\psi] = \int d^3x \left[ \frac{a_{\star}^2}{8\pi G} \mathcal{W} \left( \frac{|\nabla \psi|^2}{a_{\star}^2} \right) + \frac{c^2}{2} \psi \left( \rho_m - \bar{\rho}_m \right) \right], \tag{3}$$

where  $\rho_m$  is the rest-mass density,  $\bar{\rho}_m$  its coarse-grained mean (to enforce large-scale homogeneity),  $a_{\star}$  is a universal acceleration scale, and  $\mu(\cdot) \equiv \mathcal{W}'(\cdot)$  is a *single* crossover function. Variation gives the *nonlinear Poisson equation* 

$$\nabla \cdot \left[ \mu \left( \frac{|\nabla \psi|}{a_{\star}} \right) \nabla \psi \right] = -\frac{8\pi G}{c^2} \left( \rho_m - \bar{\rho}_m \right). \tag{4}$$

The weak-field normalization  $-8\pi G/c^2$  is fixed by the requirement that light bending match Einstein (Appendix A). The field stress tensor

$$T_{ij}^{(\psi)} = \frac{a_{\star}^2}{4\pi G} \left[ \mu \, \partial_i \psi \, \partial_j \psi - \frac{1}{2} \delta_{ij} \, \mathcal{W} \right] \tag{5}$$

ensures momentum conservation:  $\partial_j (T_{ij}^{(\psi)} + T_{ij}^{(m)}) = 0.$ 

**Regimes.** Choose  $\mu$  once with

$$\mu(x) \to 1 \quad (x \gg 1)$$
 and  $\mu(x) \sim x \quad (x \ll 1)$ .

Then:

- High-gradient (solar/strong):  $\mu \to 1 \Rightarrow \nabla^2 \psi = -(8\pi G/c^2)(\rho_m \bar{\rho}_m)$ .
- Low-gradient (galaxies/voids):  $\mu(x) \sim x \Rightarrow |\nabla \psi| \propto 1/r$  (spherical), yielding  $v(r) \rightarrow$  const.

#### 3 Weak-Field Limit and Newtonian Gravity

For a point mass M and  $\mu \to 1$ , solving (4) gives

$$\psi(r) = \frac{2GM}{c^2r}, \qquad \Rightarrow \qquad \mathbf{a} = \frac{c^2}{2}\nabla\psi = -\frac{GM}{r^2}\hat{\mathbf{r}}.$$
(6)

Thus Newton's inverse-square law is recovered exactly from (2)–(4), not assumed.

# 4 Light Propagation: Bending, Redshift, and Shapiro Delay

With  $n=e^{\psi}\simeq 1+\psi$  and  $\psi=2GM/(c^2r)$ :

**Deflection.** The small-angle eikonal integral (Appendix B):

$$\alpha = \int_{-\infty}^{\infty} \nabla_{\perp} \ln n \, dz = \int_{-\infty}^{\infty} \nabla_{\perp} \psi \, dz = \frac{4GM}{c^2 b}. \tag{7}$$

**Gravitational redshift.** A frequency transfer between  $r_A$  and  $r_B$  gives

$$\frac{\Delta\nu}{\nu} = \psi(r_A) - \psi(r_B) = -\frac{\Delta\Phi}{c^2}.$$
 (8)

the standard GR result [1].

**Shapiro delay.** The excess one-way time is

$$\Delta t_{1w} = \frac{1}{c} \int (n-1) \, ds \simeq \frac{1}{c} \int \psi \, ds = \frac{2GM}{c^3} \ln \frac{4r_S r_R}{b^2},$$
 (9)

giving the textbook two-way coefficient  $4GM/c^3$  [3] (Appendix C).

#### 5 Relativistic Orbits: Perihelion Advance

Test-particle dynamics follow the Lagrangian

$$L = \frac{1}{2}m e^{\psi(\mathbf{r})}(\dot{r}^2 + r^2\dot{\theta}^2) - m \Phi(\mathbf{r}), \qquad \psi = -\frac{2\Phi}{c^2}.$$
 (10)

Expanding to  $\mathcal{O}(\Phi/c^2)$  and using Binet's equation for u=1/r yields

$$\frac{d^2u}{d\theta^2} + u = \frac{GM}{\ell^2/m} + \frac{3GM}{c^2}u^2 + \cdots,$$
 (11)

hence the anomalous advance

$$\Delta \varpi = \frac{6\pi GM}{a(1 - e^2)c^2},\tag{12}$$

identical to GR (Appendix  $\mathbb{D}$ ; see also [1]).

### 6 Galactic Dynamics: Flat Rotation Curves and Tully– Fisher

In the deep-field regime ( $|\nabla \psi| \ll a_{\star}$  with  $\mu(x) \sim x$ ), spherical symmetry gives a Gauss law from (4):

$$r^{2} \mu(|\psi'|/a_{\star}) \psi' = -\frac{4\pi G}{c^{2}} M(r).$$
 (13)

With  $\mu(x) = x$  one finds  $r^2 |\psi'| \psi' = -\frac{4\pi G a_*}{c^2} M(r)$  and hence  $|\psi'| \propto 1/r$  outside the mass. The circular speed

$$v^{2}(r) = r |\mathbf{a}| = \frac{c^{2}}{2} r |\psi'| \to v_{\text{flat}}^{2},$$
 (14)

is constant. Eliminating  $\psi'$  gives an asymptotic scaling

$$v_{\text{flat}}^4 \simeq \mathcal{C} GM \, a_{\star} \, c^2,$$
 (15)

with  $\mathcal{C}$  a number of order unity fixed by the chosen  $\mu$ . This reproduces the observed Tully–Fisher scaling and the tight radial-acceleration relation without dark halos [6, 4, 5, 7].

# 7 Cosmological Field Equation and Optical Cosmography

Equation (4) with the subtraction  $(\rho_m - \bar{\rho}_m)$  supplies the cosmological closure. Homogeneity demands  $\langle \nabla \psi \rangle = 0$  in the ensemble, but real sightlines traverse inhomogeneities:

$$D_{\text{opt}}(z,\hat{\mathbf{n}}) = \frac{1}{c} \int_0^{\chi(z)} e^{\psi(\mathbf{r})} ds \simeq \frac{\chi(z)}{c} + \frac{1}{c} \int_0^{\chi(z)} \psi(\mathbf{r}) ds.$$
 (16)

Thus the *observed* Hubble law inherits a directional bias

$$\frac{\delta H_0(\hat{\mathbf{n}})}{H_0} \approx -\frac{1}{\chi} \frac{1}{c} \int_0^{\chi} \psi(\mathbf{r}) \, ds, \tag{17}$$

predicting a correlation of local-ladder  $H_0$  with foreground large-scale structure [8]. These biases have the right sign and coherence to account for the late/early-time  $H_0$  discrepancy [9, 10].

#### 8 Emergent Time and Quantum Coupling

Operational time is defined by round-trip procedures. Quantum phases couple directly to optical length. The minimal nonrelativistic coupling consistent with (10) is

$$i\hbar \,\partial_t \Psi(\mathbf{r}, t) = -\frac{\hbar^2}{2m} \,\nabla \cdot \left( e^{-\psi(\mathbf{r})} \nabla \Psi \right) + m \,\Phi(\mathbf{r}) \,\Psi, \tag{18}$$

so an interferometer with arms sampling different  $\psi$  acquires

$$\Delta\phi = \frac{\omega_0}{c} \left( \int_{\gamma_1} e^{\psi} ds - \int_{\gamma_2} e^{\psi} ds \right) \simeq \frac{\omega_0}{c} \int (\psi_1 - \psi_2) ds. \tag{19}$$

State-of-the-art atom interferometers and optical clocks can probe the predicted  $10^{-10}$  m s<sup>-2</sup>-scale effects [11, 12].

#### 9 One-Way-c Observables (Metrology Protocols)

Two-way c is invariant along a fixed path, but differences between distinct routes expose  $\psi$ :

$$\Delta T_{1w} \equiv \frac{1}{c} \left( \int_{\gamma_{AB}} e^{\psi} ds - \int_{\gamma_{BA}} e^{\psi} ds \right) \simeq \frac{1}{c} \left( \int_{\gamma_{AB}} \psi \, ds - \int_{\gamma_{BA}} \psi \, ds \right). \tag{20}$$

Asymmetric fiber links (two heights), Mach–Zehnder with vertical separation, and triangular time transfer among three stations isolate the effect while path swapping removes instrument bias.

# 10 Lorentz invariance, simultaneity, and experimental constraints

Conventionality of one-way c. As emphasized by Reichenbach, Edwards, and others, the *one-way* speed of light is not directly measurable without a simultaneity convention; only two-way c is empirically fixed [14, 15, 16, 17]. DFD promotes the convention parameter to a field  $\psi$  but constrains it dynamically via (4).

Two-way invariance and Michelson-Morley/Kennedy-Thorndike. For a fixed arm  $\gamma$  used in both directions, the round-trip time is

$$T_{2w} = \frac{1}{c} \int_{\gamma} e^{\psi} ds + \frac{1}{c} \int_{\gamma^{\text{rev}}} e^{\psi} ds = \frac{2}{c} \int_{\gamma} e^{\psi} ds,$$
 (21)

which is independent of the arm orientation under a rigid rotation of the apparatus if  $\psi$  is a scalar function of the ambient mass distribution on the arm scale. Thus modern Michelson–Morley tests (optical cavities/whispering galleries) remain null to current sensitivity [18, 19, 20]. Kennedy–Thorndike experiments (boost dependence) are likewise preserved because the round-trip speed along a fixed arm is path-symmetric [21, 1].

**Local Lorentz symmetry.** Locally, light rays in the optical medium  $n = e^{\psi}$  follow null geodesics of Gordon's "optical metric" [13, 2]. Hence matter and light exhibit *local* Lorentz symmetry with respect to that effective metric, explaining the excellent agreement of special-relativistic kinematics and clock comparisons (Ives–Stilwell, time dilation, etc.) while allowing *global* one-way anisotropy tied to  $\psi$ .

**GPS** and time transfer. Global navigation timing enforces a synchronization convention equivalent to isotropic two-way c in the chosen Earth-centered inertial frame [22]. DFD reproduces all round-trip observables by design; one-way anisotropy shows up only in *route-dependent* comparisons (Sec. 7), which are not tested by standard GPS common-view protocols.

**Summary.** DFD is consistent with the tightest existing tests of Lorentz invariance and light-speed isotropy because those tests are fundamentally two-way [1, 18, 19, 20]. What is new (and falsifiable) is the prediction of nonreciprocal one-way delays between distinct routes in the presence of ambient  $\nabla \psi$ .

#### 11 Discussion and Conclusion

A single scalar  $\psi$  controlling the one-way light speed unifies gravity and optics in  $\mathbb{R}^3$  with emergent time. From the action (3) we obtain a nonlinear Poisson law (4) whose weak-field normalization reproduces all Einstein classic tests exactly, and whose deep-field limit yields flat rotation curves and a Tully–Fisher/RAR scaling without dark matter. Cosmologically, line-of-sight optical length produces a foreground-dependent  $H_0$  bias (resolving the Hubble tension) and an acceleration scale  $\sim 10^{-10}\,\mathrm{m\,s^{-2}}$  without a cosmological constant. The framework is falsifiable now via precision metrology and atom interferometry. It replaces four-dimensional curvature with a dynamical one-way c, closes conservation by construction, and removes the GR–QM clash by eliminating fundamental time.

#### A Weak-Field Normalization and the Factor of Two

In the weak-field regime take  $\mu \to 1$ , so  $\nabla^2 \psi = -(8\pi G/c^2)\rho_m$ . For a point mass,  $\psi = 2GM/(c^2r)$  (up to a constant). Photons see  $n = e^{\psi} \simeq 1 + \psi = 1 + 2GM/(c^2r)$ . The eikonal bending formula requires  $\psi = -2\Phi/c^2$  with  $\nabla^2 \Phi = 4\pi G\rho_m$  to obtain  $\alpha = 4GM/(c^2b)$ . This fixes the unique  $-8\pi G/c^2$  normalization in (4); any other choice fails the Einstein factor.

## B Light Deflection (Full Integral)

With 
$$\psi = 2GM/(c^2r)$$
 and  $r = \sqrt{b^2 + z^2}$ ,

$$\frac{\partial \psi}{\partial b} = -\frac{2GM}{c^2} \frac{b}{(b^2 + z^2)^{3/2}}.$$

Thus

$$\alpha = \int_{-\infty}^{\infty} \frac{\partial \psi}{\partial b} \, dz = \frac{2GMb}{c^2} \int_{-\infty}^{\infty} \frac{dz}{(b^2 + z^2)^{3/2}} = \frac{2GMb}{c^2} \cdot \frac{2}{b^2} = \frac{4GM}{c^2b}.$$

### C Shapiro Delay (One-Way and Two-Way)

$$\Delta t_{1w} = \frac{1}{c} \int (n-1) \, ds \simeq \frac{1}{c} \int \psi \, ds = \frac{2GM}{c^3} \int \frac{dz}{\sqrt{b^2 + z^2}} = \frac{2GM}{c^3} \ln \frac{z + \sqrt{b^2 + z^2}}{b} \bigg|_{-L}^{+L}.$$

For  $L \gg b$ ,  $\Delta t_{1\text{w}} \simeq \frac{2GM}{c^3} \ln \frac{4L^2}{b^2}$ ; the round-trip doubles the coefficient to  $4GM/c^3$  as in GR.

### D Perihelion Advance (Derivation)

With  $L=\frac{1}{2}me^{\psi}(\dot{r}^2+r^2\dot{\theta}^2)-m\Phi$  and  $\psi=-2\Phi/c^2$ , the conserved angular momentum is  $\ell=me^{\psi}r^2\dot{\theta}$ . Eliminating  $\dot{\theta}$  and expanding  $e^{\psi}=1-2\Phi/c^2+\cdots$ , the radial Euler–Lagrange equation yields to first post-Newtonian order

$$\ddot{r} - \frac{\ell^2}{m^2 r^3} = -\Phi' + \frac{2\Phi}{c^2} \frac{\ell^2}{m^2 r^3}.$$

Writing u = 1/r and using  $(d/dt) = \dot{\theta}(d/d\theta) = (\ell/mr^2)(d/d\theta)$  gives

$$\frac{d^2u}{d\theta^2} + u = \frac{GM}{\ell^2/m} + \frac{3GM}{c^2}u^2,$$

hence  $\Delta \varpi = 6\pi GM/[a(1-e^2)c^2]$ .

#### E Optical Cosmography and $H_0$ Bias

Let  $\chi$  be the comoving *Euclidean* distance inferred in absence of  $\psi$ . The actual optical distance is  $D_{\rm opt} = \frac{1}{c} \int_0^{\chi} e^{\psi} ds$ . For statistically homogeneous  $\psi$ ,  $\langle \psi \rangle = 0$ , so  $\langle D_{\rm opt} \rangle = \chi/c$ . Fluctuations along a given line yield

$$\delta D_{\mathrm{opt}} \simeq \frac{1}{c} \int_0^{\chi} \psi \, ds, \quad \frac{\delta H_0}{H_0} \simeq -\frac{\delta D_{\mathrm{opt}}}{\chi/c} = -\frac{1}{\chi} \frac{1}{c} \int_0^{\chi} \psi \, ds,$$

predicting directional anisotropy correlated with foreground large-scale structure.

#### F One-Way-c Metrology (Protocols)

**Asymmetric fiber:** deploy two parallel fibers at heights  $h_1 \neq h_2$  between stations A and B. Measure  $T_{AB}$  and  $T_{BA}$  with active path swapping; the nonreciprocal difference is  $\Delta T_{1w} = c^{-1} (\int_{\gamma_{AB}} \psi \, ds - \int_{\gamma_{BA}} \psi \, ds)$ .

**Mach–Zehnder:** vertical arm separation  $\Delta h$  imprints  $\Delta \phi = (\omega_0/c) \int \Delta(e^{\psi}) ds$ .

**Triangular time transfer:** stations A,B,C; two loops ( $A \rightarrow B \rightarrow C \rightarrow A$  and  $A \rightarrow C \rightarrow B \rightarrow A$ ). The loop difference isolates  $\phi \psi ds$  geometry while each edge preserves two-way c.

#### References

- [1] Clifford M. Will. The confrontation between general relativity and experiment. *Living Reviews in Relativity*, 17(4), 2014.
- [2] Volker Perlick. Ray Optics, Fermat's Principle, and Applications to General Relativity. Springer, 2000.
- [3] Irwin I. Shapiro. Fourth test of general relativity. *Physical Review Letters*, 13:789–791, 1964.
- [4] Mordehai Milgrom. A modification of the newtonian dynamics as a possible alternative to the hidden mass hypothesis. *Astrophysical Journal*, 270:365–370, 1983.
- [5] Jacob Bekenstein and Mordehai Milgrom. Does the missing mass problem signal the breakdown of newtonian gravity? *Astrophysical Journal*, 286:7–14, 1984.
- [6] R. Brent Tully and J. Richard Fisher. A new method of determining distances to galaxies. *Astronomy and Astrophysics*, 54:661–673, 1977.
- [7] Stacy S. McGaugh, Federico Lelli, and James M. Schombert. The radial acceleration relation in rotationally supported galaxies. *Physical Review Letters*, 117:201101, 2016.
- [8] Licia Verde, Tommaso Treu, and Adam G. Riess. Tensions between the early and the late universe. *Nature Astronomy*, 3:891–895, 2019.
- [9] Planck Collaboration. Planck 2018 results. vi. cosmological parameters. Astronomy & Astrophysics, 641:A6, 2020.
- [10] Adam G. Riess, Wenlong Yuan, Lucas M. Macri, and et al. A comprehensive measurement of the local value of the hubble constant with 1 km s<sup>-1</sup> mpc<sup>-1</sup> uncertainty from the hubble space telescope and the sh0es team. *Astrophysical Journal Letters*, 934:L7, 2022.
- [11] Achim Peters, Keng-Yeow Chung, and Steven Chu. Measurement of gravitational acceleration by dropping atoms. *Nature*, 400:849–852, 1999.
- [12] C. W. Chou, D. B. Hume, T. Rosenband, and D. J. Wineland. Optical clocks and relativity. *Science*, 329:1630–1633, 2010.
- [13] Walter Gordon. Zur lichtfortpflanzung nach der relativitätstheorie. Annalen der Physik, 377(22):421–456, 1923.
- [14] Hans Reichenbach. *Philosophy of Space and Time*. Dover (English translation), 1958. Originally 1928.
- [15] William F. Edwards. Special relativity in anisotropic space. American Journal of Physics, 31:482–489, 1963.

- [16] R. Anderson, I. Vetharaniam, and G. E. Stedman. Conventionality of simultaneity, gauge dependence and test theories of relativity. *Physics Reports*, 295(3–4):93–180, 1998.
- [17] David Malament. Causal theories of time and the conventionality of simultaneity.  $No\hat{u}s$ , 11(3):293-300, 1977.
- [18] Holger Müller, Sven Herrmann, Christian Braxmaier, Stephan Schiller, and Achim Peters. Modern michelson-morley experiment using cryogenic optical resonators. *Physical Review Letters*, 91:020401, 2003.
- [19] Ch. Eisele, A. Yu. Nevsky, and S. Schiller. Laboratory test of the isotropy of light propagation at the  $10^{-17}$  level. *Physical Review Letters*, 103:090401, 2009.
- [20] Sven Herrmann, Alexander Senger, Holger Müller, and et al. Rotating optical cavity experiment testing lorentz invariance at the  $10^{-17}$  level. *Physical Review D*, 80:105011, 2009.
- [21] Roy J. Kennedy and Edward M. Thorndike. Experimental establishment of the relativity of time. *Physical Review*, 42:400–418, 1932.
- [22] Neil Ashby. Relativity in the global positioning system. Living Reviews in Relativity, 6(1), 2003.