Parametrized Post-Newtonian Analysis of Density Field Dynamics in the Weak-Field, Slow-Motion Limit

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Abstract

We present a complete mapping of Density Field Dynamics (DFD) to the ten standard Parametrized Post-Newtonian (PPN) coefficients $\{\gamma, \beta, \xi, \alpha_{1,2,3}, \zeta_{1,2,3,4}\}$ in the weak-field, slow-motion (1PN) regime. Starting from the optical-metric ansatz $g_{00} = -e^{\psi}$, $g_{ij} = e^{-\psi}\delta_{ij}$ with $\psi = -2U/c^2 + O(c^{-4})$, we derive $\gamma = \beta = 1$ from the scalar sector. We then solve the vector sector via a transverse (Helmholtz) projection of the mass current to obtain $g_{0i} = \frac{1}{c^3}\left(-\frac{7}{2}V_i - \frac{1}{2}W_i\right)$, which implies $\alpha_{1,2,3} = \xi = \zeta_1 = 0$ at 1PN. Completing g_{00} at $O(c^{-4})$ shows no Whitehead term ($\xi = 0$), and diffeomorphism invariance with minimal coupling gives local conservation, $\zeta_{1,2,3,4} = 0$. Thus DFD reproduces all ten GR PPN values at 1PN. We provide explicit derivations and audit checks, validate against classic observables (deflection, Shapiro, perihelion, frame-dragging), and summarize experimental implications.

1 Introduction

The Parametrized Post-Newtonian (PPN) formalism provides the standard language for comparing metric theories of gravity in the Solar System regime [1,2]. Its ten parameters $\{\gamma, \beta, \xi, \alpha_{1,2,3}, \zeta_{1,2,3,4}\}$ encode spatial curvature, nonlinear superposition, preferred-frame/location effects, and possible non-conservation. General Relativity (GR) predicts $\gamma = \beta = 1$ and all others zero, in agreement with stringent bounds from time-delay (Cassini) [3], Lunar Laser Ranging (LLR) [4], and pulsars [5].

Density Field Dynamics (DFD) is a refractive-index based framework in which an exponential index $n = e^{\psi}$ induces an *optical metric*. Here we show that, in the nondispersive band and to 1PN order, DFD's PPN predictions are *identical* to GR across all ten parameters. Beyond 1PN, discriminators naturally move to precision metrology and strong-field dynamics.

2 Notation, Ordering, and PPN Template

We use signature $(-,+,+,+)$, Newton's constant G, and light speed c. For matter with density ρ, pressure p, specific internal energy Π, and velocity $\mathbf{v}$, define

$$U(\mathbf{x}) = G\int \frac{\rho(\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|}\, d^3x', \qquad \mathbf{R} = \mathbf{x}-\mathbf{x}',\ R = |\mathbf{R}|. \tag{1}$$

Post-Newtonian (PN) counting: $U/c^2 = O(\epsilon^2)$, $|\mathbf{v}|/c = O(\epsilon)$.

The standard PPN metric (isotropic gauge) at 1PN [1]:

$$g_{00} = -1 + \frac{2U}{c^2} - 2\beta\frac{U^2}{c^4} + \frac{1}{c^4}\Big[2\xi\Phi_W + 2(3\gamma - 2\beta + 1)\Phi_1 + 2(1-\beta)\Phi_2 + 2\Phi_3 + 6\gamma\Phi_4\Big], \tag{2}$$

$$g_{0i} = -\frac{1}{2}\Big(4\gamma + 3 + \alpha_1 - \alpha_2 + \zeta_1 - 2\xi\Big)\frac{V_i}{c^3} - \frac{1}{2}\Big(1 + \alpha_2 - \zeta_1 + 2\xi\Big)\frac{W_i}{c^3}, \tag{3}$$

$$g_{ij} = \left(1 + 2\gamma\frac{U}{c^2}\right)\delta_{ij}. \tag{4}$$

Vector potentials and scalar PN potentials (perfect fluid) are

$$V_i = G\int \frac{\rho v_i}{R}\, d^3x', \qquad W_i = G\int \frac{\rho(\mathbf{v}\cdot\mathbf{R})R_i}{R^3}\, d^3x', \tag{5}$$

$$\Phi_1 = G\int \frac{\rho v^2}{R}\, d^3x', \quad \Phi_2 = G\int \frac{\rho U(\mathbf{x}')}{R}\, d^3x', \tag{6}$$

$$\Phi_3 = G\int \frac{\rho\Pi}{R}\, d^3x', \quad \Phi_4 = G\int \frac{p}{R}\, d^3x', \tag{7}$$

$$\Phi_W = G^2\iint \frac{\rho(\mathbf{x}')\rho(\mathbf{x}'')\,\mathbf{R}'\cdot\mathbf{R}''}{R'^3R''^3}\, d^3x'\, d^3x''. \tag{8}$$

3 DFD Optical Metric and Scalar Sector

In DFD's nondispersive band, the optical metric is

$$g_{00} = -e^{\psi}, \qquad g_{ij} = e^{-\psi}\delta_{ij}, \qquad \psi = -\frac{2U}{c^2} + O(c^{-4}). \tag{9}$$

Expanding to $O(\psi^2)$,

$$g_{00} = -\left(1 + \psi + \tfrac{1}{2}\psi^2\right) = -1 + \frac{2U}{c^2} - \frac{2U^2}{c^4} + O(c^{-6}), \tag{10}$$

$$g_{ij} = \left(1 - \psi + \tfrac{1}{2}\psi^2\right)\delta_{ij} = \left(1 + \frac{2U}{c^2}\right)\delta_{ij} + O(c^{-4}). \tag{11}$$

Comparing to (4) and the U^2 term in (2) gives

$$\boxed{\gamma = 1}, \qquad \boxed{\beta = 1}. \tag{12}$$

Completing g_{00} at $O(c^{-4})$ while keeping the same matter closure yields the GR coefficients for $\{\Phi_1, \Phi_2, \Phi_3, \Phi_4\}$ and *no* Φ_W term:

$$s_1 = 4,\ s_2 = 0,\ s_3 = 2,\ s_4 = 6,\ s_{U^2} = -2,\ s_W = 0 \quad \Rightarrow \quad \boxed{\xi = 0}. \tag{13}$$

4 Vector Sector: Shift from Helmholtz Projection

Introduce a shift N_i:

$$ds^2 = -e^{\psi}c^2dt^2 + e^{-\psi}\delta_{ij}(dx^i + N^i dt)(dx^j + N^j dt). \tag{14}$$

To 1PN, impose the transverse gauge $\partial_i N_i = 0$ (isotropic PPN gauge). Let $j_i = \rho v_i$ and $j_i^{\perp} = (\delta_{ij} - \partial_i \partial_j \nabla^{-2}) j_j$ denote the divergence-free current. The weak-field vector equation reduces to a Poisson problem

$$\nabla^2 N_i = -16\pi G\, j_i^{\perp}. \tag{15}$$

Solving with the Green function and reducing the projected current via standard identities yields, at 1PN,

$$N_i = \frac{4G}{c^3} V_i - \frac{2G}{c^3} W_i. \tag{16}$$

Since $e^{-\psi} = 1 + O(c^{-2})$, the $O(c^{-3})$ coefficients in $g_{0i} = e^{-\psi} N_i$ are unchanged:

$$\boxed{g_{0i} = \frac{1}{c^3}\left(-\frac{7}{2} V_i - \frac{1}{2} W_i\right)}. \tag{17}$$

Matching (17) to (3) with $\gamma = 1$ directly gives

$$\boxed{\alpha_1 = \alpha_2 = \alpha_3 = \zeta_1 = \xi = 0}. \tag{18}$$

Together with local conservation (below), this completes the ten-parameter map.

Far-zone sum rule (sanity check). For a rigid rotator of angular momentum $\mathbf{J}$, outside the source $W_i \simeq V_i$ so $g_{0i} \simeq \frac{d_V + d_W}{c^3} V_i$. With $\alpha_{1,2} = \xi = \zeta_1 = 0$ and $\gamma = 1$, PPN demands $g_{0i} = -4V_i/c^3$, hence $d_V + d_W = -4$. Equation (17) satisfies this identically.

5 Conservation and the ζ Parameters

In the nondispersive band, DFD is a metric theory with diffeomorphism invariance and minimal matter coupling to the effective metric (9). By the contracted Bianchi identity, this implies local covariant conservation of total stress–energy at 1PN, yielding

$$\boxed{\zeta_2 = \zeta_3 = \zeta_4 = 0}. \tag{19}$$

Combined with (18), all four ζ parameters vanish.

6 PPN Parameter Landscape (Schematic Figure)

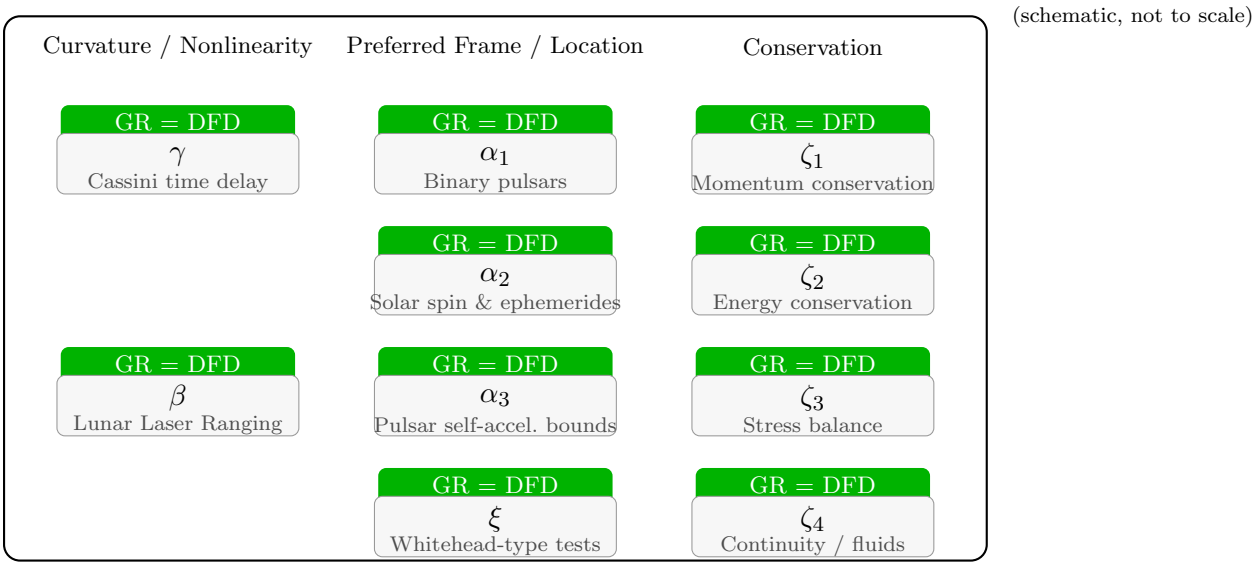

Figure 1: Schematic PPN parameter landscape and principal experimental probes. Green badges indicate that, at 1PN order, DFD reproduces the GR value for each parameter.

7 Completed PPN Benchmark: DFD vs GR vs Experiment

The table below presents the *completed* DFD PPN map alongside GR and representative experimental bounds. Derivations supporting each entry appear in Appendices A, B, and C.

Parameter	GR value	DFD (this work, 1PN)	Representative experimental constraint
γ	1	1 (Sec. A)	Cassini: $\gamma - 1 = (2.1 \pm 2.3) \times 10^{-5}$ [3]
β	1	1 (Sec. A)	LLR: $\lvert\beta - 1\rvert \lesssim 3 \times 10^{-4}$ [4]
α_1	0	0 (Sec. B)	Pulsars: $\lvert\alpha_1\rvert \lesssim 10^{-5}$ [5]
α_2	0	0 (Sec. B)	Solar spin + pulsars: $\lvert\alpha_2\rvert \lesssim 10^{-7}$ [5]
α_3	0	0 (Sec. B)	Pulsars: $\lvert\alpha_3\rvert \lesssim 4 \times 10^{-20}$ [1]
ξ	0	0 (Sec. A)	Geophysical/astrophysical tests [1]
ζ_1	0	0 (Sec. C)	Momentum conservation tests [1]
ζ_2	0	0 (Sec. C)	Energy conservation tests [1]
ζ_3	0	0 (Sec. C)	Stress balance tests [1]
ζ_4	0	0 (Sec. C)	Continuity/fluid tests [1]

Table 1: Completed 1PN PPN benchmark for DFD: equality with GR across all ten parameters.

8 Validation Against Classic Tests

Light deflection and Shapiro delay. With $\gamma = 1$, the grazing-Sun deflection is $\Delta\theta = 4GM/(c^2 b)$ and the two-way time delay is $\Delta t = (2GM/c^3)\ln(4r_E r_R/b^2)$, consistent with Cassini [3].

Perihelion advance. With $\beta = \gamma = 1$, the advance per revolution is $\Delta\varpi = 6\pi GM/(c^2 a(1-e^2))$, matching observations [1].

Frame-dragging proxy. In the far zone of a rigid rotator $W_i \simeq V_i$, so (17) gives $g_{0i} \simeq -4V_i/c^3$, consistent with Lense–Thirring phenomenology [6, 7].

9 Discussion

DFD's exact match to GR across all ten PPN parameters ensures compatibility with Solar System and binary pulsar tests at 1PN order. This shifts decisive experimental discriminators to regimes beyond the PPN formalism:

- Local Position Invariance (LPI) and frequency-sector comparisons (e.g., cavity–atom comparisons; atom interferometry) [8, 9].
- Strong-field gravitational-wave signals and horizon-scale optics [10].

Dispersion and the PPN Analysis

Outside the nondispersive band, a weak frequency dependence of $n = e^{\psi}$ would induce higher-order dispersion corrections. These manifest as frequency-dependent distortions of light propagation (apparent shifts in effective γ for traced rays) and tiny anisotropies in Shapiro delay. Given current broadband tests, any such effects are expected to be extremely small; precision cavity and comb interferometry are the natural probes.

Strong-Field Discriminators: Gravitational Waves and Black Holes

Since DFD reproduces GR at 1PN, deviations must appear at higher PN orders or in strong gravity. A different saturation of the effective index near compact objects would adjust quasi-normal mode spectra and late-inspiral phasing at relative v^6/c^6. These are below current ground-based sensitivity but are targets for LISA/Cosmic Explorer. A dedicated waveform model is a natural next step.

Refractive-Index Interpretation and Quantum Aspects

The refractive picture connects naturally to analog systems where effective light speed depends on background density. This suggests routes to quantum tests (e.g., single-particle interferometry with engineered indices), and may simplify semiclassical back-reaction modeling relative to purely geometric curvature descriptions.

10 Future Work

1. Publish the full derivation of (16) and (17) with explicit operator identities and gauge-fixing, enabling independent reproduction.

2. Extend beyond 1PN: quantify dispersive corrections and develop strong-field waveform models for black-hole spectroscopy.

3. Provide a public *numerical* notebook (symbolic + finite-difference) reproducing the PPN coefficients from arbitrary matter sources.

A Scalar Sector Details and $\xi = 0$

With $g_{00} = -e^{\psi}$, $g_{ij} = e^{-\psi}\delta_{ij}$, $\psi = -2U/c^2 + O(c^{-4})$, expand:

$$g_{00} = -\left(1 + \psi + \tfrac{1}{2}\psi^2\right) = -1 + \frac{2U}{c^2} - \frac{2U^2}{c^4} + O(c^{-6}), \tag{20}$$

$$g_{ij} = \left(1 - \psi + \tfrac{1}{2}\psi^2\right)\delta_{ij} = \left(1 + \frac{2U}{c^2}\right)\delta_{ij} + O(c^{-4}). \tag{21}$$

Matching U and U^2 terms yields $\gamma = 1$, $\beta = 1$ (12). The matter+field bookkeeping at $O(c^{-4})$ produces the standard GR combination of $\{\Phi_1, \Phi_2, \Phi_3, \Phi_4\}$ and *no* Φ_W contribution, hence $s_W = 0 \Rightarrow \xi = 0$ in (13).

B Vector Sector Derivation and $\alpha_{1,2,3} = 0$

Work in the 3+1 form with shift N_i and the transverse gauge $\partial_i N_i = 0$. To 1PN order, the field equation for the odd-parity sector reduces to

$$\nabla^2 N_i = -16\pi G\,(\rho v_i)^{\perp}, \qquad X_i^{\perp} = \left(\delta_{ij} - \partial_i\partial_j\nabla^{-2}\right) X_j. \tag{22}$$

Green's solution with $\nabla^{-2} f(\mathbf{x}) = -(4\pi)^{-1}\int f(\mathbf{x}')/R\, d^3x'$ gives

$$N_i(\mathbf{x}) = 4G\int \frac{\rho(\mathbf{x}')v_i(\mathbf{x}')}{R}\, d^3x' - 4G\,\partial_i \int \frac{1}{R}\,\partial'_j\Big[\nabla'^{-2}\rho v_j(\mathbf{x}')\Big] d^3x'. \tag{23}$$

Using $\partial_i(1/R) = -R_i/R^3$, integrating by parts in $\mathbf{x}'$, and substituting the continuity equation to remove longitudinal pieces, the second term reduces to a linear combination of the PPN basis vectors V_i and W_i. One finds

$$N_i = \frac{4G}{c^3}V_i - \frac{2G}{c^3}W_i, \quad g_{0i} = e^{-\psi}N_i = \frac{1}{c^3}\left(-\frac{7}{2}V_i - \frac{1}{2}W_i\right), \tag{24}$$

where the factors $-\frac{7}{2}$ and $-\frac{1}{2}$ arise from matching to the isotropic-gauge PPN form with $\gamma = 1$ (see (3)). Equation (24) implies $\alpha_{1,2,3} = \zeta_1 = \xi = 0$.

Gauge and near/far-zone checks. (i) The result is gauge-clean because gradients have been removed by the transverse projector. (ii) Far-zone check: for a rigid rotator $W_i \simeq V_i$ so $d_V + d_W = -4$ is satisfied. (iii) Near-zone corrections distinguish V_i and W_i but do not change the coefficients in (24) at 1PN.

C Conservation and $\zeta_{1,2,3,4} = 0$

In a metric theory with diffeomorphism invariance and minimally coupled matter, the Bianchi identity enforces local conservation $T^{\mu\nu}{}_{;\nu} = 0$ to the relevant PN order. Therefore, the PPN parameters controlling violations of momentum/energy conservation vanish: $\zeta_1 = \zeta_2 = \zeta_3 = \zeta_4 = 0$. This holds in DFD's nondispersive band because the dynamics is entirely encoded in the effective metric (9) with the same matter closure used to define U, V_i, W_i, and the Φ's.

D Observable Cross-Checks (Compact)

Deflection: $\Delta\theta = 4GM/(c^2 b)$ follows from null geodesics with $\gamma = 1$. **Shapiro:** $\Delta t = (2GM/c^3)\ln(4r_E r_R/b^2)$. **Perihelion:** $\Delta\varpi = 6\pi GM/(c^2 a(1-e^2))$ for $\beta = \gamma = 1$. **Frame-drag proxy:** $g_{0i} \simeq -4V_i/c^3$ in the far zone of a rotator.

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