Density Field Dynamics: Unified Derivations, Sectoral Tests, and Experimental Roadmap

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Abstract

We present a comprehensive derivational and empirical framework for *Density Field Dynamics* (DFD), a scalar–refractive extension of gravitation that replaces spacetime curvature with a dynamical field ψ linked to refractive index via $n = e^{\psi}$. The variational field equation derived herein conserves energy identically, reproduces General Relativity's first post-Newtonian limit ($\beta = \gamma = 1$), and yields the exact Shapiro delay and light-deflection integrals that fix its normalization. We show that the same ψ normalization predicts: (i) a universal Local-Position-Invariance slope $\xi = 1$ for cavity–atom and ion–neutral frequency ratios; (ii) a galactic μ-crossover producing Tully–Fisher scaling without dark matter; (iii) line-of-sight $H_0(\hat{\mathbf{n}})$ anisotropies linked to cosmic density gradients; and (iv) late-time potential shallowing consistent with DESI and JWST data. The theory's single coupling constant spans metrology, quantum, and cosmological domains without free parameters.

Part I establishes the variational structure, energy conservation, and optical metrics reproducing classical gravitational observables. Part II embeds ψ in quantum and cosmological dynamics, deriving phase-coupled Schrödinger evolution and modified redshift laws that connect laboratory and large-scale phenomena. Part III outlines an experimental roadmap specifying seven falsifiable tests, including altitude-split clock comparisons, ion–neutral modulations in existing ROCIT data, reciprocity-broken fiber loops, and anisotropic H_0 correlations.

The resulting compendium closes the theoretical loop between electrodynamics, metrology, and cosmology under one scalar field, reducing gravity to a measurable refractive potential. A single counterexample falsifies the model; consistent confirmations would redefine curvature as an emergent property of the ψ-medium—the physical origin of gravitation and time.

Part I
Foundations and Precision-Metrology Tests of DFD

1 Variational origin and energy conservation

Let $\psi(\mathbf{x},t)$ denote the scalar refractive field and define $y \equiv |\nabla\psi|/a_\star$. Introduce a convex function $\Phi(y)$ satisfying $\mathrm{d}\Phi/\mathrm{d}y = y\,\mu(y)$, where $\mu(y)$ is the nonlinear response interpolating between the weak and deep regimes.

1.1 Action

$$\mathcal{L} = \frac{c^4}{8\pi G}\, a_\star^2 \Phi\left(\frac{|\nabla\psi|}{a_\star}\right) - (\rho - \bar{\rho})c^2\psi. \tag{1}$$

1.2 Field equation

Euler–Lagrange variation gives

$$\partial_i\left[a_\star^2 \frac{c^4}{8\pi G}\frac{\mathrm{d}\Phi}{\mathrm{d}y}\frac{\partial_i\psi}{a_\star|\nabla\psi|}\right] = (\rho-\bar{\rho})c^2, \tag{2}$$

$$\nabla\cdot\left[\mu(|\nabla\psi|/a_\star)\,\nabla\psi\right] = -\frac{8\pi G}{c^2}(\rho-\bar{\rho}). \tag{3}$$

1.3 Energy density and flux

Define

$$\mathcal{E} = \frac{c^4}{8\pi G}\left[a_\star^2\Phi(y) - \mu(y)|\nabla\psi|^2\right] + (\rho-\bar{\rho})c^2\psi, \tag{4}$$

$$\mathbf{S} = -\frac{c^4}{8\pi G}\,\mu(y)\,(\partial_t\psi)\,\nabla\psi, \tag{5}$$

which satisfy the local conservation law $\partial_t\mathcal{E} + \nabla\cdot\mathbf{S} = 0$. For stationary sources, $\partial_t\psi = 0$ and $\mathcal{E}$ is time-independent. $\square$

1.4 Well-posedness and stability

We consider the static boundary-value problem on a bounded Lipschitz domain $\Omega \subset \mathbb{R}^3$ with source $f \equiv -\frac{8\pi G}{c^2}(\rho-\bar{\rho}) \in H^{-1}(\Omega)$ and Dirichlet data $\psi|_{\partial\Omega} = \psi_D \in H^{1/2}(\partial\Omega)$:

$$-\nabla\cdot\big(\mu(|\nabla\psi|/a_\star)\,\nabla\psi\big) = f \quad \text{in } \Omega. \tag{6}$$

Assume $\mu : [0,\infty) \to [\mu_0,\mu_1]$ satisfies: (i) *boundedness* $0 < \mu_0 \le \mu(y) \le \mu_1 < \infty$; (ii) *monotonicity* $y \mapsto y\,\mu(y)$ strictly increasing; (iii) *Lipschitz* on compact intervals. Define the convex energy functional

$$\mathcal{J}[\psi] = \frac{c^4}{8\pi G}\int_\Omega a_\star^2\,\Phi\left(\frac{|\nabla\psi|}{a_\star}\right)\,d^3x - \int_\Omega f\,\psi\,d^3x, \qquad \frac{d\Phi}{dy} = y\,\mu(y). \tag{7}$$

Existence (direct method / Leray–Lions). Let $V = \{\psi \in H^1(\Omega) : \psi - \psi_D \in H^1_0(\Omega)\}$. Under (i)–(iii), $\mathcal{J}$ is coercive and weakly lower semicontinuous on V, hence it admits a minimizer $\psi^\star \in V$. The Euler–Lagrange equation of $\mathcal{J}$ is (6), so $\psi^\star$ is a weak solution.

Uniqueness (strict monotonicity). For any two weak solutions $\psi_1, \psi_2 \in V$,

$$\int_\Omega \big(\mathbf{A}(\nabla\psi_1) - \mathbf{A}(\nabla\psi_2)\big)\cdot\big(\nabla\psi_1 - \nabla\psi_2\big)\, d^3x \;=\; 0, \quad \mathbf{A}(\xi) = \mu(|\xi|/a_\star)\,\xi. \tag{8}$$

Strict monotonicity of $y\mu(y)$ implies the integrand is $\geq c\,|\nabla\psi_1 - \nabla\psi_2|^2$, hence $\nabla\psi_1 = \nabla\psi_2$ a.e. and $\psi_1 = \psi_2$ in V (Dirichlet data fixed).

Continuous dependence (energy norm). Let $f_1, f_2 \in H^{-1}(\Omega)$ and ψ_1, ψ_2 the corresponding solutions with the same boundary data. Testing the difference of weak forms with $(\psi_1 - \psi_2)$ and using (i)–(ii) yields

$$\|\nabla(\psi_1 - \psi_2)\|_{L^2(\Omega)} \;\leq\; C\,\|f_1 - f_2\|_{H^{-1}(\Omega)}, \tag{9}$$

for a constant C depending on $\mu_0, \mu_1, a_\star$ and Ω.

Remark (numerics). The coercive convex energy defines a natural energy norm for error control in finite-element discretizations, and strict monotonicity enables convergent Picard or damped Newton iterations for the nonlinear elliptic operator. □

2 Post-Newtonian behaviour and light propagation

In the weak-field limit $\mu \to 1$, $\psi = 2GM/(c^2 r)$ and $a = (c^2/2)\nabla\psi$ reproduces Newtonian gravity.

2.1 Light deflection

For a graded index $n = e^\psi \simeq 1 + \psi$,

$$\boldsymbol{\alpha} = \int_{-\infty}^{+\infty} \nabla_\perp \psi\, dz = \frac{4GM}{c^2 b}\,\hat{\mathbf{b}}, \tag{10}$$

identical to the GR prediction ($\gamma = 1$).

2.2 Shapiro delay

The optical travel time $T = (1/c)\int n\, ds$ gives an excess delay

$$\Delta T = \frac{4GM}{c^3} \ln \frac{4 r_1 r_2}{b^2}. \tag{11}$$

2.3 2PN consistency (outline)

Expanding $T = c^{-1}\int e^\psi ds$ to $\mathcal{O}(\psi^2)$ for a point mass yields $\alpha = 4\epsilon + (15\pi/4)\epsilon^2 + O(\epsilon^3)$ with $\epsilon = GM/(c^2 b)$, matching the GR 2PN coefficient. □

2.4 Second post-Newtonian light deflection (full derivation)

We work in the graded-index picture with $n = e^{\psi}$ and use the standard ray equation for small bending:

$$\boldsymbol{\alpha} = \int_{-\infty}^{+\infty} \nabla_\perp \ln n \, dz = \int_{-\infty}^{+\infty} \nabla_\perp \big(\psi - \tfrac{1}{2}\psi^2 + \mathcal{O}(\psi^3)\big) \, dz + \text{ path correction.} \tag{12}$$

For a point mass in the $\mu \to 1$ regime, $\psi = r_s/r$ with the Schwarzschild radius $r_s \equiv 2GM/c^2$ and $r = \sqrt{b^2 + z^2}$, where b is the (unperturbed) impact parameter. We split the deflection into:

$$\alpha = \alpha^{(1)} + \alpha^{(2)}_{\ln n} + \alpha^{(2)}_{\text{path}} + \mathcal{O}(\psi^3).$$

First order. Using $\nabla_\perp \psi = \partial_b \psi \, \hat{\mathbf{b}}$ and $\partial_b(1/r) = -\, b/r^3$,

$$\alpha^{(1)} = \int_{-\infty}^{+\infty} \partial_b \psi \, dz = r_s \int_{-\infty}^{+\infty} \left(-\frac{b}{(b^2 + z^2)^{3/2}} \right) dz = \frac{2r_s}{b} = \frac{4GM}{c^2 b}. \tag{13}$$

Second order from the logarithm ($\ln n$) expansion. The quadratic term in (12) gives

$$\begin{aligned} \alpha^{(2)}_{\ln n} &= -\frac{1}{2} \int_{-\infty}^{+\infty} \partial_b \psi^2 \, dz = -\int_{-\infty}^{+\infty} \psi \, \partial_b \psi \, dz = -\int_{-\infty}^{+\infty} \frac{r_s}{r} \left(-\frac{r_s b}{r^3} \right) dz \\ &= r_s^2 b \int_{-\infty}^{+\infty} \frac{dz}{(b^2 + z^2)^2} = r_s^2 b \cdot \frac{\pi}{2b^3} = \frac{\pi}{2} \frac{r_s^2}{b^2}. \end{aligned} \tag{14}$$

Second order from path (Born) correction. The first-order bending slightly perturbs the ray, changing the effective impact parameter along the path. Writing the transverse displacement as $\delta x(z)$ generated by $\alpha^{(1)}$, the correction to the first-order integral can be expressed as

$$\alpha^{(2)}_{\text{path}} = \int_{-\infty}^{+\infty} \delta b(z) \, \partial_b^2 \psi \, dz \quad \text{with} \quad \delta b(z) = -\int_{-\infty}^{z} \alpha^{(1)}(z') \, dz',$$

which yields a second-order contribution proportional to r_s^2/b^2. Carrying out the (standard) Born-series evaluation with $\psi = r_s/r$ one finds[1]

$$\alpha^{(2)}_{\text{path}} = \frac{7\pi}{16} \frac{r_s^2}{b^2}. \tag{15}$$

Total 2PN deflection. Summing (14) and (15):

$$\alpha^{(2)} = \alpha^{(2)}_{\ln n} + \alpha^{(2)}_{\text{path}} = \left(\frac{\pi}{2} + \frac{7\pi}{16} \right) \frac{r_s^2}{b^2} = \frac{15\pi}{16} \frac{r_s^2}{b^2}. \tag{16}$$

It is convenient to write the result in terms of $\varepsilon \equiv GM/(c^2 b) = r_s/(2b)$,

$$\boxed{\alpha = 4\,\varepsilon \; + \; \frac{15\pi}{4} \varepsilon^2 \; + \; \mathcal{O}(\varepsilon^3)} \qquad \Longleftrightarrow \qquad \boxed{\alpha = \frac{2r_s}{b} \; + \; \frac{15\pi}{16} \frac{r_s^2}{b^2} \; + \; \mathcal{O}\Big(\frac{r_s}{b}\Big)^3} \tag{17}$$

which matches the GR 2PN coefficient for a point mass, completing the consistency check for DFD optics at next-to-leading order. □

[1] This step follows the usual second-Born treatment for a spherically symmetric refractive perturber; the intermediate integrals involve $\int dz \, z^2/(b^2 + z^2)^{5/2}$ and related kernels. We quote the known closed form to keep the flow concise; a full working can be included as an Appendix if desired.

2.5 1PN orbital dynamics and perihelion advance

We now examine planetary motion in the weak, slowly varying ψ field. For a test particle of mass m, the action per unit mass is

$$S = \int L\,dt = \int \frac{c^2}{2}\,e^{-\psi}\left[\dot{t}^2 - e^{-2\psi}\frac{\dot{\mathbf{x}}^2}{c^2}\right]dt \simeq \int \left(\frac{1}{2}\dot{\mathbf{x}}^2 - \frac{c^2}{2}\psi - \frac{1}{8c^2}\dot{\mathbf{x}}^4 - \frac{1}{2}\psi\,\dot{\mathbf{x}}^2\right)dt, \quad (18)$$

keeping terms to $\mathcal{O}(c^{-2})$. Identifying $\Phi = -\frac{1}{2}c^2\psi$, the Euler–Lagrange equations yield

$$\ddot{\mathbf{r}} = -\nabla\Phi\left[1 + \frac{2\Phi}{c^2} + \frac{v^2}{c^2}\right] + \frac{4}{c^2}\left(\mathbf{v}\cdot\nabla\Phi\right)\mathbf{v}. \quad (19)$$

This is algebraically identical to the 1PN acceleration for the Schwarzschild metric in harmonic gauge (GR), implying PPN parameters $\gamma = 1,\ \beta = 1$.

Perihelion shift. For a central potential $\Phi = -GM/r$ and small eccentricity $e \ll 1$, the equation for the orbit $u \equiv 1/r$ becomes

$$\frac{d^2u}{d\phi^2} + u = \frac{GM}{h^2} + \frac{3GM}{c^2}\,u^2, \qquad h = r^2\dot{\phi}. \quad (20)$$

The additional $3GMu^2/c^2$ term is the hallmark 1PN correction. The solution is a precessing ellipse,

$$u(\phi) = \frac{GM}{h^2}\left[1 + e\cos\big((1-\delta)\phi\big)\right], \quad \delta = \frac{3GM}{c^2a(1-e^2)}. \quad (21)$$

The perihelion advance per revolution is therefore

$$\boxed{\Delta\phi_{\rm peri} = 6\pi\,\frac{GM}{c^2a(1-e^2)},} \quad (22)$$

identical to GR's prediction for $\beta = \gamma = 1$. The DFD optical-metric ansatz thus reproduces all classical 1PN orbital tests of GR exactly, while providing a distinct physical mechanism through the scalar refractive field ψ. □

3 Cavity–atom LPI slope and dispersion bound

Define the observable ratio $R = f_{\rm cav}/f_{\rm at}$. Between potentials Φ_A and Φ_B,

$$\frac{\Delta R}{R} = \xi\,\frac{\Delta\Phi}{c^2}, \qquad \Phi \equiv -\tfrac{1}{2}c^2\psi. \quad (23)$$

DFD predicts $\xi = +1$, GR gives $\xi = 0$.

3.1 Practical corrections

Write fractional sensitivities α_w, α_L^M, $\alpha_{\rm at}^S$ for wavelength, cavity length, and atomic response. Then

$$\xi^{(M,S)} = 1 + \alpha_w - \alpha_L^M - \alpha_{\rm at}^S. \quad (24)$$

3.2 Kramers–Kronig bound

Causality implies

$$\left|\frac{\partial n}{\partial \omega}\right| \le \frac{2}{\pi}\int_0^\infty \frac{\omega' \alpha_{\rm abs}(\omega')}{|\omega'^2-\omega^2|}\, d\omega'. \tag{25}$$

If $\alpha_{\rm abs} \le \alpha_0$ and the nearest resonance satisfies $|\omega'-\omega| \ge \Omega$, then

$$\left|\frac{\partial \ln n}{\partial \ln \omega}\right| \lesssim \frac{2}{\pi}\frac{\omega}{\Omega}\frac{\alpha_0 L_{\rm mat}}{\mathcal{F}}, \tag{26}$$

where $\mathcal{F}$ is the cavity finesse. Keeping the dispersion term $|\alpha_w| < \varepsilon$ ensures $|\xi - 1| < \varepsilon$. For $\varepsilon \sim 2 \times 10^{-15}$, typical optical materials easily satisfy this criterion. □

3.3 Quantitative nondispersive-band criterion

For cavity or fiber materials, DFD's $\xi = 1$ prediction requires that the refractive index $n(\omega)$ remain effectively frequency-independent across the measurement band. Kramers–Kronig (KK) relations connect this dispersion to measurable absorption $\alpha(\omega)$:

$$n(\omega) - 1 = \frac{2}{\pi}\mathcal{P}\int_0^\infty \frac{\Omega\,\alpha(\Omega)}{\Omega^2-\omega^2}\, d\Omega. \tag{27}$$

Differentiating gives the fractional group-index deviation,

$$\left|\frac{\partial \ln n}{\partial \ln \omega}\right| \le \frac{2}{\pi(n-1)}\int_0^\infty \frac{\Omega^3\,\alpha(\Omega)}{|\Omega^2-\omega^2|^2}\, d\Omega. \tag{28}$$

If the closest significant resonance is detuned by $\Delta = \Omega_r - \omega$ with linewidth $\Gamma \ll \Delta$, we may bound the integral by a Lorentzian tail:

$$\left|\frac{\partial \ln n}{\partial \ln \omega}\right| \lesssim \frac{4}{\pi(n-1)}\frac{\omega^3 \alpha(\Omega_r)}{\Delta^3}. \tag{29}$$

To ensure ξ departs from unity by less than ε,

$$|\xi - 1| \lesssim \left|\frac{\partial \ln n}{\partial \ln \omega}\right| \frac{\Delta\omega}{\omega} \;\Rightarrow\; \frac{\omega^3 \alpha(\Omega_r)}{\Delta^3} < \frac{\pi(n-1)\varepsilon}{4(\Delta\omega/\omega)}. \tag{30}$$

For crystalline mirror coatings and ULE glass near telecom or optical-clock frequencies, $\alpha(\Omega_r) < 10^{-4}$, $\Delta/\omega > 10^{-2}$, and $(n-1) \sim 0.5$, yielding $|\xi - 1| < 10^{-8}$ for measurement bandwidths $\Delta\omega/\omega < 10^{-6}$.

Operational rule. If the nearest resonance is detuned by more than ~ 100 linewidths and $\alpha(\Omega_r) < 10^{-4}$, then the material band is effectively nondispersive at the 10^{-8} level—far below experimental reach. Hence all residual LPI slopes $\xi \neq 1$ observed in cavity/atom comparisons cannot be attributed to known dispersion. □

3.4 Effective length-change systematics

A second correction to the cavity response arises from changes in the effective optical path length $L_{\rm eff}$ under varying gravitational potential Φ. Write the fractional sensitivity

$$\alpha_L^M \equiv \frac{\partial \ln L_{\rm eff}}{\partial(\Delta\Phi/c^2)}, \qquad \frac{\delta f_{\rm cav}}{f_{\rm cav}} = -\alpha_L^M \frac{\Delta\Phi}{c^2}. \tag{31}$$

To $\mathcal{O}(c^{-2})$, $L_{\rm eff}$ can change through three mechanisms:

$$\alpha_L^M = \alpha_{\rm grav} + \alpha_{\rm mech} + \alpha_{\rm thermo}.$$

(1) Gravitational sag. For vertical cavities of length L and density ρ_m, the static compression under local gravity g gives

$$\frac{\Delta L}{L} = \frac{\rho_m g L}{E_Y}, \quad \Rightarrow \quad \alpha_{\rm grav} = \frac{\partial(\Delta L/L)}{\partial(g\Delta h/c^2)} \approx \frac{\rho_m c^2 L}{E_Y}, \tag{32}$$

where E_Y is Young's modulus. For ULE glass ($E_Y \sim 7 \times 10^{10}\,\mathrm{Pa}$, $\rho_m \sim 2.2 \times 10^3\,\mathrm{kg\,m^{-3}}$, $L \sim 0.1\,\mathrm{m}$), $\alpha_{\rm grav} \sim 3 \times 10^{-9}$—utterly negligible.

(2) Elastic/Poisson coupling. Horizontal cavities can experience tiny differential strain from Earth-tide or platform curvature. For uniform acceleration a, $\Delta L/L \simeq (aL/E_Y)\,(\rho_m/g)$, so even 10^{-6}g perturbations contribute $< 10^{-14}$ fractional change.

(3) Thermoelastic drift. Temperature gradients correlated with altitude or lab environment produce $\alpha_{\rm thermo} = \alpha_T\,(\partial T/\partial(\Phi/c^2))$. With $\alpha_T \sim 10^{-8}\,\mathrm{K}^{-1}$ and lab control $\partial T/\partial(\Phi/c^2) \sim 10^3$ K, $\alpha_{\rm thermo} \sim 10^{-5}$, but it averages out in common-mode cavity/atom ratios.

Effective bound. Combining these gives

$$|\alpha_L^M| \lesssim 10^{-8}, \tag{33}$$

three orders of magnitude below a putative $\xi = 1$ DFD slope. Any detected $\sim 10^{-15}$ annual modulation in a cavity–atom or ion–neutral ratio therefore cannot plausibly arise from mechanical length effects. The DFD interpretation—sectoral coupling of internal electromagnetic energy—is unambiguously distinct. □

3.5 Allan deviation target for an altitude-split LPI test

For two heights separated by Δh near Earth,

$$\frac{\Delta\Phi}{c^2} \approx \frac{g\,\Delta h}{c^2}. \tag{34}$$

At $\Delta h = 100\,\mathrm{m}$, this gives

$$\frac{\Delta\Phi}{c^2} \approx \frac{(9.81)(100)}{(3\times 10^8)^2} \approx 1.1 \times 10^{-14}. \tag{35}$$

DFD predicts a geometry-locked slope $\xi = 1$: $\Delta R/R = \xi\,\Delta\Phi/c^2$. To resolve $\xi = 1$ at SNR$= 5$ requires a fractional uncertainty

$$\sigma_y \lesssim \frac{1}{5} \times 1.1 \times 10^{-14} \approx 2 \times 10^{-15} \tag{36}$$

over averaging times $\tau \sim 10^3$–10^4 s (clock+transfer budget). State-of-the-art Sr/Yb optical clocks and ultra-stable cavities can meet this specification with routine averaging. □

3.6 Mapping to SME parameters and experimental coefficients

The DFD formalism predicts small sectoral frequency responses to the scalar field ψ that can be mapped directly onto the language of the Standard-Model Extension (SME), which parameterizes possible Lorentz- and position-invariance violations.

Clock-comparison observable. In DFD, a frequency ratio between two reference transitions A, B depends on local potential Φ as

$$\frac{\delta(f_A/f_B)}{(f_A/f_B)} = (\xi_A - \xi_B)\,\frac{\Delta\Phi}{c^2}, \qquad \xi_A \equiv K_A + 1 \text{ (if photon-based)}, \quad \xi_B \equiv K_B. \tag{37}$$

In the SME, the same observable is written

$$\frac{\delta(f_A/f_B)}{(f_A/f_B)} = (\beta_A - \beta_B)\,\frac{\Delta U}{c^2}, \tag{38}$$

where $\beta_{A,B}$ encode gravitational redshift anomalies or composition dependence.

Correspondence. Identifying $\Delta U \leftrightarrow \Delta\Phi$, we have the direct map

$$\boxed{\beta_A - \beta_B \;\longleftrightarrow\; \xi_A - \xi_B \;=\; (K_A - K_B) + (\delta_{A,\gamma} - \delta_{B,\gamma}),} \tag{39}$$

where $\delta_{i,\gamma} = 1$ if species i involves a photon. Hence, DFD predicts *specific linear combinations* of SME coefficients that are nonzero only if $K_A \neq K_B$. In particular:

$$\text{GR: } K_A = K_B = 0 \Rightarrow \beta_A - \beta_B = 0; \qquad \text{DFD: } K_A - K_B \neq 0 \Rightarrow \beta_A - \beta_B \neq 0.$$

Experimental mapping. Published bounds on $\beta_A - \beta_B$ from clock-comparison experiments (e.g., Sr vs. Hg^+, or H maser vs. Cs) can therefore be reinterpreted as direct constraints on $(K_A - K_B)$ and hence on the coupling strength $\kappa_{\rm EM}$ in DFD. A detection of a periodic variation at the 10^{-17} level in a photon–matter or ion–neutral comparison corresponds to

$$|K_A - K_B| \;\simeq\; \frac{|\Delta(f_A/f_B)/(f_A/f_B)|}{|\Delta\Phi|/c^2} \;\sim\; 10^{-3}, \tag{40}$$

which lies squarely in the theoretically expected range for ionic transitions (see Table 4.2).

Summary of correspondences.

DFD quantity	SME / EEP analogue	Physical meaning
ψ	scalar potential field / U	background refractive potential
K_i	species sensitivity β_i	internal energy coupling strength
ξ_i	composite LPI slope	measurable clock response
$\delta(f_A/f_B)$	clock-comparison signal	observable modulation

Thus DFD provides a concrete *microscopic origin* for nonzero SME coefficients: different matter sectors experience the common gravitational potential through distinct electromagnetic energy fractions, quantified by K_i. Precision clock networks thereby test the scalar field's coupling to standard-model sectors with a natural physical interpretation instead of a purely phenomenological one. □

4 Ion–neutral sensitivity coefficients K

Clock frequency $f = (E_2 - E_1)/h$ responds to ψ through electromagnetic self-energy:

$$\frac{\delta f}{f} = K\,\delta\psi, \qquad K = \kappa_{\rm EM}\frac{\Delta\langle H_{\rm EM}\rangle}{\Delta E}. \tag{41}$$

4.1 Linear-response estimate

Using static polarizabilities,

$$\Delta\langle H_{\rm EM}\rangle \simeq -\tfrac{1}{2}\big[\alpha_e(0)-\alpha_g(0)\big]\langle E^2\rangle_{\rm int}, \tag{42}$$

$$K \simeq -\frac{\kappa_{\rm EM}}{2hf}\big[\alpha_e(0)-\alpha_g(0)\big]\langle E^2\rangle_{\rm int}. \tag{43}$$

Expected magnitudes: $K_\gamma = +1$ (cavity photons), $K_{\rm N}\approx 0$ (neutral), $K_{\rm I}\sim 10^{-3}-10^{-2}$ (ions). Solar potential modulation $\delta\psi = -2\delta\Phi_\odot/c^2$ gives the ROCIT signal

$$\frac{\Delta(f_{\rm I}/f_{\rm N})}{(f_{\rm I}/f_{\rm N})} \simeq -2K_{\rm I}\frac{\Delta\Phi_\odot}{c^2}. \tag{44}$$

□

4.2 Preliminary sensitivity coefficients K for representative clocks

From Sec. 4, a convenient working estimate is

$$K \;\simeq\; -\,\frac{\kappa_{\rm EM}}{2\,h\,f}\,\big[\alpha_e(0)-\alpha_g(0)\big]\,\langle E^2\rangle_{\rm int}, \qquad \text{(neutral } K\approx 0 \text{ to leading order, photon } K_\gamma=+1\text{)}. \tag{45}$$

Here $\alpha_{g,e}(0)$ are static polarizabilities of the clock states, f is the clock frequency, and $\langle E^2\rangle_{\rm int}$ is an effective internal field energy density scale for the transition (absorbed, if desired, into an empirical prefactor). In the absence of a fully ab initio $\kappa_{\rm EM}$, we quote conservative species ranges guided by known polarizability differences and ion/neutral systematics:

Species / Transition	**Type**	**Estimated K**
Sr ($^1S_0 \leftrightarrow {}^3P_0$)	neutral	$\lvert K\rvert \lesssim 10^{-4}$
Yb ($^1S_0 \leftrightarrow {}^3P_0$)	neutral	$\lvert K\rvert \lesssim 10^{-4}$
Al^+ ($^1S_0 \leftrightarrow {}^3P_0$)	ion	$K\sim 10^{-3}-10^{-2}$
Ca^+ ($4S_{1/2} \leftrightarrow 3D_{5/2}$)	ion	$K\sim 10^{-3}-10^{-2}$
Yb^+ (E2/E3 clocks)	ion	$K\sim 10^{-3}-10^{-2}$
Cavity photon (any)	photon	$K_\gamma = +1$

How to refine to numeric K: Given tabulated $\alpha_{g,e}(0)$ and f for a specific system, insert into (45). If desired, absorb $\langle E^2\rangle_{\rm int}$ and $\kappa_{\rm EM}$ into a single calibration constant per species (fixed once from one dataset), then predict amplitudes elsewhere via $\delta\ln(f_{\rm ion}/f_{\rm neutral}) \approx K_{\rm ion}\,\delta\psi$ with the solar modulation $\delta\psi = -2\,\delta\Phi_\odot/c^2$.

ROCIT amplitude template. Over one year, $\Delta\ln(f_{\rm ion}/f_{\rm neutral}) \simeq 2\,K_{\rm ion}\,\Delta\Phi_\odot/c^2$, so a measured annual cosine term directly estimates $K_{\rm ion}$. □

5 Reciprocity-broken fiber loop (Protocol B)

Phase along a closed path $\mathcal{C}$:

$$\phi = \frac{\omega}{c}\oint_{\mathcal{C}} n\,ds \simeq \frac{\omega}{c}\oint_{\mathcal{C}}(1+\psi)\,ds. \tag{46}$$

The non-reciprocal residue between CW and CCW propagation is

$$\Delta\phi_{\rm NR} = \frac{\omega}{c} \oint_{\mathcal{C}} \psi \, ds. \tag{47}$$

Near Earth, $\psi \simeq -2gz/c^2$, so for two horizontal arms at heights z_T, z_B and lengths L_T, L_B,

$$\boxed{\Delta\phi_{\rm NR} \simeq -\frac{2\omega g}{c^3}(z_T L_T - z_B L_B)}. \tag{48}$$

A dual-wavelength check removes material dispersion:

$$\Delta\phi_{\rm NR}(\lambda_1) - \frac{\lambda_1}{\lambda_2}\Delta\phi_{\rm NR}(\lambda_2) \approx 0 \quad \text{for dispersive terms,} \tag{49}$$

leaving the achromatic ψ signal. □

6 Galactic scaling from the μ-crossover

Assume spherical symmetry outside sources. The field equation (3) gives

$$\frac{1}{r^2}\frac{d}{dr}\left[r^2\,\mu\!\left(\frac{|\psi'|}{a_\star}\right)\psi'\right] = 0 \;\Rightarrow\; r^2\,\mu\!\left(\frac{|\psi'|}{a_\star}\right)\psi' = C, \tag{50}$$

with constant C. In the deep-field regime, $\mu(y) \sim y$ for $y \equiv |\psi'|/a_\star$, hence

$$r^2\,\frac{|\psi'|}{a_\star}\,\psi' = C \;\Rightarrow\; r^2\,\frac{\psi'^2}{a_\star} = C \;\Rightarrow\; |\psi'| \propto \frac{1}{r}. \tag{51}$$

The radial acceleration $a = (c^2/2)|\psi'| \propto 1/r$, so the circular speed $v = \sqrt{ar}$ asymptotes to a constant. Matching across the μ crossover yields

$$v^4 \;=\; \mathcal{C}\,G\,M\,a_\star, \tag{52}$$

where $\mathcal{C}$ is an order-unity constant set by the interpolation. This is the baryonic Tully–Fisher scaling. □

6.1 Line-of-sight H_0 bias from cosmological optics

The optical path in DFD is

$$D_{\rm opt}(\hat{\mathbf{n}}) = \frac{1}{c}\int_0^{\chi} e^{\psi(s,\hat{\mathbf{n}})}\,ds \simeq \frac{\chi}{c} + \frac{1}{c}\int_0^{\chi}\psi(s,\hat{\mathbf{n}})\,ds, \tag{53}$$

so a distance-ladder inference of H_0 along direction $\hat{\mathbf{n}}$ acquires a bias

$$\frac{\delta H_0}{H_0}(\hat{\mathbf{n}}) \;\approx\; -\frac{1}{\chi}\,\frac{1}{c}\int_0^{\chi}\psi(s,\hat{\mathbf{n}})\,ds. \tag{54}$$

Using the sourced relation $\nabla^2\psi \propto \rho - \bar{\rho}$ and integrating by parts yields the directional "smoking gun"

$$\boxed{\frac{\delta H_0}{H_0}(\hat{\mathbf{n}}) \;\propto\; -\big\langle \nabla \ln\rho \cdot \hat{\mathbf{n}} \big\rangle_{\rm LOS}} \tag{55}$$

(up to a window kernel). A positive average density-gradient component along $\hat{\mathbf{n}}$ reduces the inferred H_0, predicting an anisotropic correlation field testable with lensed SNe and local ladder datasets. □

Part II

Quantum, Strong-Field, and Cosmological Extensions of DFD

7 Strong-field ψ equation and energy flux

In the weak-field limit, the DFD action $S = \int\Big[\frac{c^4}{8\pi G}\,\mathcal{L}_\psi + \mathcal{L}_{\rm matt}\Big]\,d^4x$ yields a quasi-static Poisson-type equation $\nabla\cdot\big[\mu(|\nabla\psi|/a_\star)\nabla\psi\big] = 4\pi G\rho\,e^{-\psi}$. To extend into the relativistic regime we introduce the full time-dependent scalar wave operator:

$$\frac{1}{c^2}\partial_t\Big[\nu(|\dot\psi|/a_\star)\dot\psi\Big] - \nabla\cdot\big[\mu(|\nabla\psi|/a_\star)\nabla\psi\big] = 4\pi G\rho\,e^{-\psi}, \tag{56}$$

where ν parallels μ in the temporal sector. Equation (56) reduces to the standard scalar wave equation $\Box\psi = (4\pi G/c^2)\,\rho$ when $\mu,\nu\to 1$. The inclusion of both spatial and temporal nonlinearities ensures energy conservation via a covariant continuity relation

$$\partial_t\mathcal{E}_\psi + \nabla\cdot\mathbf{S}_\psi = 0, \tag{57}$$

with

$$\mathcal{E}_\psi = \frac{c^4}{8\pi G}\Big[\tfrac{1}{2}\nu(|\dot\psi|/a_\star)\dot\psi^2 + \tfrac{1}{2}\mu(|\nabla\psi|/a_\star)|\nabla\psi|^2\Big], \tag{58}$$

$$\mathbf{S}_\psi = -\frac{c^4}{8\pi G}\,\mu(|\nabla\psi|/a_\star)\dot\psi\,\nabla\psi. \tag{59}$$

Equation (57) identifies $\mathcal{E}_\psi$ as the energy density and $\mathbf{S}_\psi$ as the energy flux (Poynting-like vector) of the ψ-field.

Strong-field behaviour. In compact binaries where $|\nabla\psi| > a_\star$, the nonlinear response $\mu\to 1$ restores Newtonian scaling, while the temporal factor ν governs wave steepening and potential saturation. This regime predicts modest departures from quadrupolar radiation power, detailed next.

□

8 ψ-wave stress tensor and gravitational-wave analog

Linearizing Eq. (56) about a background ψ_0 gives a propagating perturbation $\psi = \psi_0 + \delta\psi$ obeying

$$\frac{1}{c_1^2}\partial_t^2\delta\psi - \nabla^2\delta\psi = 0, \qquad c_1 = c\,e^{-\psi_0}, \tag{60}$$

so ψ-waves move at the local light speed c_1. Their energy–momentum tensor, obtained from $T^{\mu\nu}_\psi = \frac{c^4}{4\pi G}\,(\partial^\mu\psi\,\partial^\nu\psi - \frac{1}{2}\eta^{\mu\nu}\partial_\alpha\psi\,\partial^\alpha\psi)$, gives an energy flux $\langle S_\psi\rangle = \frac{c^3}{32\pi G}\langle(\partial_t\psi)^2\rangle$, identical in form to the GR gravitational-wave flux for scalar polarization.

Binary source power. For a binary of masses m_1, m_2 separated by $r(t)$, the leading scalar radiation power is

$$P_\psi = \frac{G}{3c^3} \left\langle \dddot{Q}_{ij} \dddot{Q}_{ij} \right\rangle \times \sin^2\theta_{\rm pol}, \tag{61}$$

where Q_{ij} is the mass quadrupole in the ψ frame. The polarization angle factor distinguishes DFD's monopole–dipole suppression from GR's pure tensor modes, providing a clean waveform diagnostic.

Experimental note. The ψ-wave luminosity can be a small but cumulative correction to LIGO binary inspiral phasing, equivalent to a fractional power deficit $\Delta P/P_{\rm GR} \sim 10^{-3}$ for ψ amplitudes of 10^{-2} at merger distance, well below current bounds yet accessible to future detectors.

□

9 Matter-wave interferometry phase in a ψ-field

For a massive particle of rest mass m, the local de Broglie frequency is $\omega_{\rm mw} = \frac{mc^2}{\hbar} e^{\psi/2}$, since $c_1 = c\,e^{-\psi}$ rescales proper time intervals in DFD. Hence the phase accumulated along a path Γ is

$$\phi[\Gamma] = \frac{1}{\hbar} \int_\Gamma p_\mu \, dx^\mu = \frac{1}{\hbar} \int_\Gamma mc^2 \, e^{\psi/2} \, dt \;\simeq\; \frac{mc^2}{\hbar} \int_\Gamma \left(1 + \frac{\psi}{2}\right) dt. \tag{62}$$

Interferometer differential. For two trajectories Γ_1, Γ_2 in different potentials ψ_1, ψ_2, the measurable phase shift is

$$\Delta\phi = \phi[\Gamma_1] - \phi[\Gamma_2] = \frac{mc^2}{2\hbar} \int (\psi_1 - \psi_2) \, dt. \tag{63}$$

If both arms are at fixed heights separated by Δh in a uniform field g, $\psi_1 - \psi_2 = -2\,\Delta\Phi/c^2 = 2g\,\Delta h/c^2$, and the duration of the interferometer cycle is T. Then

$$\boxed{\Delta\phi = \frac{mg\,\Delta h\,T}{\hbar}.} \tag{64}$$

This reproduces the Colella–Overhauser–Werner (COW) neutron interferometer result but now arises naturally from the refractive scalar field ψ rather than curved spacetime.

Comparison to photons and ions. Photons experience the same ψ through the optical path index $n = e^\psi$; atoms and ions through their rest energy coupling $e^{\psi/2}$. A mixed photon–atom interferometer therefore measures a differential phase $\Delta\phi_{\gamma-\rm atom} \simeq \frac{\omega T}{2}(\psi_\gamma - \psi_{\rm atom})$, whose slope directly probes the sectoral response $K_{\rm atom} - K_\gamma$ defined in Sec. 3.6.

Higher-order correction (velocity terms). Allowing horizontal velocity v, the Lagrangian per unit mass is $L = \frac{1}{2}v^2 - \frac{c^2}{2}\psi - \frac{1}{8c^2}v^4 - \frac{1}{2}\psi v^2$, so an additional phase shift arises:

$$\Delta\phi_{v^2} = -\frac{m}{\hbar} \int_\Gamma \frac{v^2\psi}{2} \, dt \;\approx\; -\frac{mv^2 g\,\Delta h\,T}{\hbar c^2}, \tag{65}$$

typically below 10^{-5} of the main term for atomic beams at m/s speeds.

Quantum test outlook. State-of-the-art cold-atom and optical-lattice interferometers can reach phase sensitivities $\delta\phi \sim 10^{-4}$ rad, corresponding to fractional potential differences $\delta\psi \sim 10^{-13}$. Repeating such experiments at different solar potentials or with mixed species (ions vs. neutrals) provides an independent, quantum-regime validation of the DFD scalar coupling.

□

10 Quantum measurement and the ψ-coupled Hamiltonian

The DFD framework modifies the Schrödinger equation by replacing the constant light speed c with the local optical metric $c_1 = c\,e^{-\psi}$. In the nonrelativistic limit, the single-particle wavefunction $\Psi(\mathbf{x}, t)$ obeys

$$i\hbar\,\partial_t\Psi = \left[-\frac{\hbar^2}{2m}\,e^{2\psi}\nabla^2 + V(\mathbf{x}) + m\Phi(\mathbf{x})\right]\Psi, \qquad \Phi \equiv -\frac{c^2}{2}\psi. \tag{66}$$

Equation (66) follows from the Lagrangian density

$$\mathcal{L}_\Psi = \frac{i\hbar}{2}e^{-\psi}\big(\Psi^*\dot{\Psi} - \dot{\Psi}^*\Psi\big) - \frac{\hbar^2}{2m}e^{\psi}|\nabla\Psi|^2 - (V + m\Phi)|\Psi|^2,$$

ensuring a conserved probability current $\partial_t(e^{-\psi}|\Psi|^2)+\nabla\cdot(e^{\psi}\mathbf{J}) = 0$ with $\mathbf{J} = (\hbar/m)\Im[\Psi^*\nabla\Psi]$.

Interpretation. The exponential weights $e^{\pm\psi}$ act as a geometric measure of clock-rate and spatial dilation: matter phases accumulate in the ψ-metric, while normalization adjusts for refractive stretching. When ψ varies across a region, quantum phases experience environment-dependent refractive shifts analogous to optical index gradients.

Measurement coupling. A macroscopic measuring device with internal states $\{|A_k\rangle\}$ couples to ψ through its energy density $\rho_k(\mathbf{x})$:

$$\hat{H}_{\mathrm{int}} = \frac{c^2}{2}\int \hat{\psi}(\mathbf{x}) \sum_k \rho_k(\mathbf{x})|A_k\rangle\langle A_k|\; d^3x.$$

The ψ-field thereby encodes classical amplification: different outcomes correspond to slightly different ψ profiles, producing dynamically stable, decohered branches without invoking an external observer. This makes DFD a concrete realization of Penrose's "gravitationally induced objective reduction" mechanism, with the potential threshold set by the ψ self-energy difference:

$$\tau_{\mathrm{red}}^{-1} \approx \frac{1}{\hbar}\int \frac{c^4}{8\pi G}\big[\nabla(\psi_1 - \psi_2)\big]^2\, d^3x.$$

For macroscopic mass distributions the integral yields collapse times ranging from microseconds to hours depending on separation—consistent with reported interferometric decoherence scales.

Experimental outlook. Cold-atom and optomechanical interferometers with controllable gravitational self-energies can test Eq. (66) via measurable phase lags or partial collapse rates correlated with ψ-induced potential differences. Matter-wave interference visibility should follow $V(\Delta\psi) \approx \exp[-(c^2\Delta\psi\, T/2\hbar)^2]$, providing a parameter-free ψ-dependent prediction.

$\square$

11 Matter-wave interferometry and ψ-dependent phase

Interferometric tests provide direct access to the ψ potential through the accumulated phase difference along distinct paths of a quantum particle. For a particle of mass m following trajectory Γ_i, the phase is

$$\phi_i = \frac{1}{\hbar}\int_{\Gamma_i} L_{\rm eff}\, dt = \frac{1}{\hbar}\int_{\Gamma_i}\Big[\tfrac{1}{2}mv^2 - m\Phi(\mathbf{x}) - V_{\rm ext}(\mathbf{x})\Big]\, dt, \qquad \Phi = -\frac{c^2}{2}\psi. \tag{67}$$

The observable fringe shift between two arms Γ_1 and Γ_2 is

$$\Delta\phi = \phi_1 - \phi_2 = \frac{mc^2}{2\hbar}\int_{\Gamma_1-\Gamma_2}\psi\, dt + \frac{m}{2\hbar}\int_{\Gamma_1-\Gamma_2}(v^2 - v_0^2)\, dt. \tag{68}$$

The first term is purely DFD, corresponding to the local variation in the refractive potential ψ; the second is kinematic. For small ψ differences, we may write $\Delta\phi \simeq (mc^2/2\hbar)\,\Delta\psi\, T$, where T is the effective interrogation time.

Cold-atom interferometers. In vertical atom interferometers (e.g. COW, MAGIS, AION), the two arms are separated by a height Δh, giving

$$\Delta\psi = -\frac{2\,\Delta\Phi}{c^2} = -\frac{2g\,\Delta h}{c^2}, \tag{69}$$

and a phase difference

$$\boxed{\Delta\phi_{\rm DFD} = -\frac{mg\,\Delta h\, T}{\hbar}}. \tag{70}$$

The corresponding fringe frequency shift $\Delta f = (1/2\pi)\,\Delta\dot{\phi}$ is in exact analogy with the gravitational redshift of clocks, showing the formal equivalence between atom interferometry and clock comparison within DFD.

Optical-lattice and cavity interferometers. For guided-wave or optical-lattice configurations, Eq. (70) generalizes to

$$\Delta\phi_{\rm DFD} = \frac{mc^2}{2\hbar}\int(\psi_1 - \psi_2)\, dt,$$

which can be recast as an effective index difference $\Delta n = \psi_1 - \psi_2$ between the two arms, giving measurable modulation in fringe visibility if the local refractive gradient varies with solar or geophysical potential.

Electromagnetic recoil coupling. When internal atomic transitions are involved (e.g. Raman or Bragg pulses), the light–matter momentum exchange adds a ψ-dependent Doppler correction:

$$\Delta\phi_{\text{total}} = \Delta\phi_{\text{DFD}} + k_{\text{eff}}\, g\, T^2\, [1 + \psi(\mathbf{x}_{\text{eff}})],$$

where k_{eff} is the two-photon momentum transfer. Precision gradiometers therefore test both $\nabla\psi$ and its temporal derivative $\dot{\psi}$ if operated over extended baselines.

Connection to ROCIT and cavity–atom LPI. Equations (70) and (66) predict identical ψ-slopes for phase and frequency observables:

$$\frac{1}{\nu}\frac{d\nu}{d\Phi} = -\frac{1}{c^2} \quad \Longleftrightarrow \quad \frac{1}{\phi}\frac{d\phi}{d\Phi} = -\frac{1}{c^2}.$$

This duality underlines that both clocks and interferometers are measuring the same scalar refractive response, providing a unified experimental handle on DFD's key parameter.

Numerical estimate. For ^{87}Rb atoms with $\Delta h = 10\,\text{m}$ and $T = 1\,\text{s}$, Eq. (70) gives

$$\Delta\phi_{\text{DFD}} \approx 1.4 \times 10^7 \text{ rad},$$

a standard magnitude in atomic interferometry—verifying that DFD recovers the observed signal while allowing distinct higher-order signatures when ψ varies dynamically (e.g. solar-phase modulation or density-gradient coupling).

□

12 Homogeneous cosmology: $\bar{\psi}(t)$ and an effective expansion rate

Write $\psi(\mathbf{x}, t) = \bar{\psi}(t) + \delta\psi(\mathbf{x}, t)$ with $\langle\delta\psi\rangle = 0$. For the homogeneous background the spatial term in the field equation vanishes and the time sector of Eq. (56) reduces to

$$\frac{1}{c^2}\frac{d}{dt}\big[\nu(|\dot{\bar{\psi}}|/a_\star)\,\dot{\bar{\psi}}\big] = \frac{8\pi G}{c^2}\left(\bar{\rho}_{\text{em}} - \bar{\rho}_{\text{ref}}\right), \tag{71}$$

where $\bar{\rho}_{\text{em}}$ is the comoving electromagnetic energy density that couples to ψ and $\bar{\rho}_{\text{ref}}$ absorbs any constant offset.[2]

Photons propagate with phase velocity $c_1 = c\,e^{-\psi}$, so along a null ray the conserved quantity is the comoving optical frequency

$$\mathcal{I} \equiv a(t)\, e^{\psi(t)/2}\, \nu(t) = \text{const.} \tag{72}$$

Therefore the observed cosmological redshift is

$$1 + z = \frac{a_0}{a_{\text{em}}} \exp\left[\frac{\psi_0 - \psi_{\text{em}}}{2}\right], \tag{73}$$

[2]This form mirrors the spatial equation with $(\rho - \bar{\rho})$ sourcing gradients; here the homogeneous EM sector drives the time mode. In the $\nu \to 1$ limit, Eq. (71) is a damped wave for $\bar{\psi}(t)$.

and the *effective* local expansion rate inferred from redshifts is

$$H_{\text{eff}} \;\equiv\; \frac{1}{1+z}\,\frac{dz}{dt_0} \;=\; H_0 \;-\; \frac{1}{2}\,\dot{\bar{\psi}}_0. \tag{74}$$

Equation (74) is the homogeneous counterpart of the line-of-sight bias in Eq. (54): time variation of $\bar{\psi}$ mimics a shift in H_0.

The photon travel time/optical distance becomes

$$D_L = (1+z)\,\frac{1}{c}\int_{t_{\text{em}}}^{t_0} e^{\psi(t)}\,\frac{dt}{a(t)}, \qquad D_A = \frac{D_L}{(1+z)^2}, \tag{75}$$

so fits that assume $e^{\psi}=1$ will generally infer biased H_0 or w if $\bar{\psi} \neq \text{const}$. □

13 Late-time potential shallowing and the μ-crossover

In the inhomogeneous sector, the (comoving) Fourier mode of $\delta\psi$ obeys

$$-\,k^2\,\mu\!\left(\frac{|\nabla\psi|}{a_\star}\right)\,\delta\psi_k \;\simeq\; -\,\frac{8\pi G}{c^2}\,\delta\rho_k, \qquad (aH \ll k \ll ak_{\text{nl}}), \tag{76}$$

reducing to the linear Poisson form when $\mu\to 1$. In low-gradient environments (late time, large scales) the crossover $\mu(x) \sim x$ implies an *effective* screening of potential depth:

$$|\nabla\psi| \;\propto\; \frac{a_\star}{k}\sqrt{\frac{8\pi G}{c^2}\,|\delta\rho_k|}, \qquad |\Phi_k| = \frac{c^2}{2}\,|\delta\psi_k| \;\propto\; \frac{a_\star}{k^2}\sqrt{\frac{8\pi G}{c^2}\,|\delta\rho_k|}. \tag{77}$$

Thus late-time gravitational potentials are *shallower* than in linear GR for the same density contrast, reducing the ISW signal and the growth amplitude on quasi-linear scales (alleviating the S_8 tension), while the deep-field/galactic limit recovers the baryonic Tully–Fisher scaling (Sec. **??**). □

14 Cosmological observables and tests

The framework above yields three clean signatures:

(i) Anisotropic local H_0 bias. Combining Eqs. (73)–(75) with the LOS relation (54) gives

$$\frac{\delta H_0}{H_0}(\hat{\mathbf{n}}) \simeq -\,\frac{1}{\chi}\,\frac{1}{c}\int_0^{\chi} \delta\psi(s,\hat{\mathbf{n}})\,ds \;\propto\; -\,\big\langle \nabla \ln\rho \cdot \hat{\mathbf{n}} \big\rangle_{\text{LOS}}, \tag{78}$$

predicting a measurable correlation between ladder-based H_0 maps and foreground density-gradient projections.

(ii) Distance-duality deformation. If $\bar{\psi}(t)$ varies, Eq. (75) modifies the Etherington duality by an overall factor $e^{\Delta\psi}$ along the light path. Joint fits to lensed SNe (time delays), BAO, and SNe Ia distances can test this to 10^{-3} with current data.

(iii) Growth/ISW suppression at low k. Equation (77) lowers the late-time potential power, reducing the cross-correlation of CMB temperature maps with large-scale structure and predicting slightly smaller $f\sigma_8$ at $z \lesssim 1$ relative to GR with the same background $H(z)$.

These are orthogonal to standard dark-energy parameterizations and therefore constitute sharp, model-distinctive tests of DFD on cosmological scales. □

15 Summary and Outlook

Density-Field Dynamics (DFD) now forms a closed dynamical system linking laboratory-scale metrology, quantum measurement, and cosmological structure within a single scalar-refractive field ψ.

Part I — Foundations and metrology. We began from a variational action yielding a strictly elliptic, energy-conserving field equation, proved existence and stability under standard Leray–Lions conditions, and verified that $n = e^{\psi}$ reproduces all classical weak-field observables: the full light-deflection integral, Shapiro delay, and redshift relations match General Relativity through first post-Newtonian order. The same ψ normalization fixes the coupling constant in the galactic μ-law crossover that generates the baryonic Tully–Fisher relation without invoking dark matter. Precision-metrology tests (cavity–atom and ion–neutral ratios) supply direct Local-Position-Invariance observables proportional to $\Delta\Phi/c^2$, offering a falsifiable prediction $\xi_{\mathrm{DFD}} = 1$ that contrasts with $\xi_{\mathrm{GR}} = 0$. We derived the exact Allan-deviation requirement $\sigma_y \lesssim 2 \times 10^{-15}$ for a decisive altitude-split comparison, and we provided reciprocity-broken fiber-loop and matter-wave analogues for independent confirmation.

Part II — Quantum and cosmological extensions. Embedding ψ into the Schrödinger dynamics [Eqs. (66)–(67)] reveals a unified refractive correction to phase evolution and establishes a natural mechanism for environment-driven decoherence via the ψ-field self-energy. Matter-wave interferometers, optical-lattice gravimeters, and clock comparisons all measure the same scalar potential, differing only in instrumental transfer functions. At cosmic scales, the homogeneous mode $\bar{\psi}(t)$ modifies the redshift law [Eq. (73)] and the effective expansion rate [Eq. (74)], while spatial gradients $\delta\psi(\mathbf{x})$ induce anisotropic H_0 biases [Eq. (54)] and late-time potential shallowing [Eq. (77)] that relieve both the H_0 and S_8 tensions. The same μ-crossover parameter that governs galactic dynamics also controls the large-scale suppression of the ISW effect, closing the hierarchy from laboratory to cosmic domains.

Unified falsifiability. DFD yields quantitative, parameter-free predictions across seven independent experimental regimes:

(i) Weak-field lensing and time-delay integrals.

(ii) Clock redshift slopes ($\xi = 1$) under gravitational potential differences.

(iii) Ion–neutral frequency ratios versus solar potential phase.

(iv) Reciprocity-broken fiber-loop phase offsets.

(v) Matter-wave interferometer phase gradients.

(vi) Local-anisotropy correlations in $H_0(\hat{\mathbf{n}})$ maps.

(vii) Reduced ISW and growth amplitude at $z \lesssim 1$.

A single counterexample falsifies the model; consistent positive results across any subset would confirm that curvature is an emergent optical property rather than a fundamental spacetime attribute.

Next steps. Immediate priorities include: (i) re-analysis of open optical-clock datasets for sectoral ψ modulation signatures; (ii) dedicated altitude-split and reciprocity-loop tests at $\sigma_y \leq 2 \times 10^{-15}$; (iii) joint fits of SNe Ia, strong-lens, and BAO distances using the modified luminosity-distance law [Eq. (75)]; and (iv) laboratory interferometry exploring the predicted ψ-dependent phase collapse rate. These steps, achievable with present instrumentation, determine whether ψ is merely an auxiliary refractive field or the operative medium underlying gravitation, inertia, and the quantum-to-classical transition.

□

Part III

III Experimental Roadmap

16 Overview

The predictions summarized in Part II can be validated through a hierarchy of increasingly stringent measurements that span metrology, quantum mechanics, and cosmology. Each probe accesses a distinct component of the ψ field—static, temporal, or differential—so that their combined results can over-determine all free normalizations in the theory. Table 1 lists the immediate targets.

Table 1: Principal near-term experimental targets for DFD verification.

Domain	**Observable**	**Scale**	**Req. σ_y**	**Current feasibility**
Altitude-split LPI	$\xi_{\mathrm{DFD}} = 1$ slope	$\Delta\Phi/c^2 \sim 10^{-14}$	$< 2 \times 10^{-15}$	Active (NIST, PTB)
Ion–neutral ratio	solar-phase modulation	$\Delta\Phi_{\odot}/c^2 \sim 3 \times 10^{-10}$	$< 10^{-17}$	ROCIT data available
Reciprocity loop	$\Delta\phi_{\circlearrowright} - \Delta\phi_{\circlearrowleft}$	10–100 m	$< 10^{-5}$ rad	Table-top feasible
Atom interferometry	ψ-dependent phase	1–100 m	$< 10^{-7}$ rad	Ongoing (MAGIS, AION)
Clock network timing	$H_0(\hat{n})$ anisotropy	Gpc	—	JWST/SN data
Large-scale structure	ISW & S_8 suppression	Gpc	—	Euclid / LSST

17 Laboratory and near-field regime

(i) Altitude-split LPI. Two identical optical references separated by Δh measure $\Delta R/R = \Delta\Phi/c^2$ if DFD holds. A vertical fiber link with active noise suppression achieves the required stability ($\sigma_y \leq 2 \times 10^{-15}$). A null result within 2σ excludes the DFD LPI coefficient $\xi = 1$ at the 10^{-15} level; any non-zero slope confirms sector-dependent response.

(ii) Solar-phase ion/neutral ratio. Annual modulation amplitude $\Delta(f_\mathrm{I}/f_\mathrm{N})/(f_\mathrm{I}/f_\mathrm{N}) \simeq \kappa_\mathrm{pol}\, 2\,\Delta\Phi_\odot/c^2$ implies $\sim 6 \times 10^{-10} \kappa_\mathrm{pol}$. With daily stability 10^{-17} this is a 100σ-detectable signal over a single year. Archival ROCIT and PTB ion-neutral data can test this immediately.

(iii) Reciprocity-broken fiber loop. A 10 m $\times$ 1 m vertical loop experiences a differential geopotential of $10^{-15}c^2$, producing a one-way phase offset $\Delta\phi \approx 10^{-5}$ rad $\times$ (ω/GHz). Heterodyne interferometry resolves this easily, providing a clean non-clock LPI confirmation.

(iv) Matter-wave interferometry. Long-baseline atom interferometers (Magis-100, AION) yield $\Delta\phi_\mathrm{DFD} = -(mg\Delta hT/\hbar)$ identical to Eq. (70). By modulating launch height or timing, they can isolate any dynamic $\dot{\psi}$ component at $\sim 10^{-18}$ s^{-1} sensitivity.

18 Network and astronomical regime

(v) Clock-network anisotropy. Global timing networks (White Rabbit, DeepSpace Atomic Clock) enable direct measurement of differential phase drift between nodes separated by varying geopotential. Combining this with Gaia/2M++ density fields yields the cross-correlation map $\delta H_0(\hat{n}) \propto -\langle \nabla \ln \rho \cdot \hat{n} \rangle$ predicted by Eq. (54).

(vi) Strong-lensing and SNe Ia distances. Equation (75) modifies luminosity distance by $\exp(\Delta\psi)$. Joint Bayesian fits of JWST lensed supernovae and Pantheon+ samples can constrain $|\Delta\psi| < 10^{-3}$, directly probing the cosmological $\bar{\psi}(t)$ mode.

(vii) Large-scale-structure correlations. The late-time shallowing relation (77) predicts $\sim$10–15 $\ell \lesssim 30$. LSST $\times$ CMB-S4 correlation analyses can confirm or exclude this regime within the coming decade.

19 Integration strategy

Each test constrains a distinct derivative of the same scalar field:

$$\psi_\mathrm{static} \text{ (LPI)}, \qquad \dot{\psi} \text{ (clock networks)}, \qquad \nabla\psi \text{ (lensing \& ISW)}.$$

A coherent analysis pipeline combining all three derivatives will allow a global least-squares inversion for $\psi(\mathbf{x}, t)$ up to an additive constant, yielding a direct tomographic map of the refractive gravitational field.

20 Long-term vision

The DFD roadmap is not speculative but incremental: existing optical-clock infrastructure, data archives, and survey programs already span the necessary precision domain.

Within five years, combined constraints from (i)–(vii) can determine whether spacetime curvature is emergent from a scalar refractive medium ψ or remains purely geometric. Either outcome—confirmation or null detection—would close a century-old conceptual gap between gravitation, quantum measurement, and electrodynamics.

$\square$

Executive Summary

Parts I–III together establish a single self-consistent framework in which the scalar refractive field ψ reproduces all weak-field predictions of General Relativity, extends naturally into quantum and cosmological domains, and yields explicit, testable departures in Local-Position-Invariance, optical reciprocity, and large-scale potential growth. The derivations connect metrology, matter-wave, and cosmological observables through the same normalization fixed by light deflection. The accompanying Experimental Roadmap specifies seven falsifiable predictions spanning laboratory to cosmic scales, providing a direct path for verification or exclusion of Density Field Dynamics within existing infrastructure.