Density Field Dynamics and Its Variant Extensions: A Constrained Flat-Background Optical-Medium Family

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Abstract

Density Field Dynamics (DFD) reproduces all standard solar-system tests while predicting two decisive laboratory discriminators: (1) non-null cavity–atom frequency *slopes* across potential differences, and (2) a T^3 term in matter-wave interferometer phases. DFD is the minimal optical-medium realization of gravity on flat spacetime, with a scalar refractive index $n = e^{\psi}$ controlling both light propagation and inertial dynamics. We present explicit field equations, derive weak-field predictions (deflection, redshift, Shapiro, perihelion), and quantify the laboratory discriminators. We then explore six bounded extensions—electromagnetic back-reaction, dual-sector (ϵ/μ) splitting, nonlocal kernels, vector anisotropy, stochasticity, and strong-field closure variants—that address specific anomalies while preserving the core DFD framework. We close with scope and limitations (cosmology, strong fields, gravitational waves), explicit appendices (light bending; matter-wave phase parity), and a consolidated comparison to scalar-tensor, æther-like, and analogue-gravity alternatives.

1 Introduction

Einstein's general relativity (GR) geometrizes gravitation as spacetime curvature. Yet alternatives remain viable, from scalar-tensor theories [1] to $f(R)$ models [2] and Einstein-æther theories [3]. If one restricts attention to flat Minkowski spacetime while maintaining an invariant two-way light speed, then a natural minimal class emerges: refractive or optical-medium theories, where gravity manifests through a scalar index field controlling rods, clocks, and phases. This aligns with scalar frameworks [5, 6] and analog-gravity constructions [4].

The motivation for DFD is not metaphysical elegance but experimental falsifiability. Two sharp discriminators appear immediately:

1. **Cavity–atom Local Position Invariance (LPI) slope:** GR predicts a strict null in the *ratio* of cavity to atomic frequencies across potential differences.[1] DFD predicts a non-null slope under operational conditions defined below ("nondispersive band"), and this difference is sharpened in the dual-sector extension.

2. **Matter-wave interferometry:** DFD predicts a small but testable T^3 contribution to the phase, absent in GR at leading order.

We then explore six bounded extensions—electromagnetic back-reaction, dual-sector (ϵ/μ) splitting, nonlocal kernels, vector anisotropy, stochasticity, and strong-field closure variants—that preserve the base limit but target specific anomalies and tests.

[1]This is within the standard PPN treatment and composition-independence assumptions [8, 7, 9, 25].

2 Base Density Field Dynamics

2.1 Field equations

DFD postulates a scalar refractive field ψ such that

$$n = e^{\psi}. \tag{1}$$

Light follows Fermat's principle in n, while matter accelerates according to

$$\mathbf{a} = \tfrac{c^2}{2}\nabla\psi. \tag{2}$$

To recover Newtonian gravity, we require

$$\nabla^2\psi = \frac{8\pi G}{c^2}\rho, \tag{3}$$

so that $\psi = 2\Phi/c^2$ with Φ the Newtonian potential. Equation (3) is the local, Poisson-like sourcing law; the nonlocal kernel variant generalizes this.

2.2 Weak-field predictions

From (3) one recovers:

- **Newtonian limit:** $\mathbf{a} = -\nabla\Phi$.
- **Gravitational redshift:** $\Delta f/f = \Delta\Phi/c^2$.
- **Light bending:** Fermat's principle yields $\alpha = 4GM/(bc^2)$ (Appendix A), reproducing GR's factor of two.
- **Shapiro delay and perihelion precession:** also match GR at 1PN order [7].
- **PPN parameters:** $\gamma = 1,\ \beta = 1$ in the standard tests, matching GR at this level of approximation [7].

2.3 Laboratory discriminators

Operationally nondispersive band (precision definition). By a *nondispersive band* we mean a frequency range $\mathcal{B}$ around the cavity/clock operating frequencies such that

$$\left|\frac{\partial n}{\partial \omega}\right|_{\mathcal{B}} \ll \frac{1}{\omega} \quad \text{and} \quad \left|\frac{\Delta n}{n}\right|_{\mathcal{B}} \lesssim \mathcal{O}(10^{-15}) \text{ over the measurement bandwidth.}$$

This ensures the phase velocity and group velocity coincide to the precision needed for LPI comparisons, so the cavity's frequency shift tracks $n = e^{\psi}$ without dispersive contamination.

Base-DFD LPI mechanism (explicit). Within a verified nondispersive band $\mathcal{B}$, let the cavity resonance obey

$$\frac{f_{\text{cav}}}{f_{\text{cav},0}} = e^{\psi},$$

while the co-located atomic transition—set by internal structure and selection rules—responds *operationally* as

$$\frac{f_{\text{at}}}{f_{\text{at},0}} = e^{\psi'},$$

where ψ' need not equal ψ in the same way a solid's optical path length and an internal atomic interval can couple differently to the scalar field in an effectively nondispersive band. The measured ratio then acquires a slope

$$\frac{f_{\rm cav}}{f_{\rm at}} = \frac{f_{\rm cav,0}}{f_{\rm at,0}}\, e^{\psi-\psi'} \quad \Rightarrow \quad \frac{\Delta(f_{\rm cav}/f_{\rm at})}{(f_{\rm cav}/f_{\rm at})} = \Delta(\psi-\psi')\ ,$$

which is *geometry-locked* via $\Delta\Phi/c^2$ along the height change. In the dual-sector extension below, $\psi-\psi'$ becomes parametrically larger because ϵ and μ respond oppositely, sharpening the discriminator.

LPI slope test. In GR, both atoms and cavities redshift as $\Delta f/f = \Delta\Phi/c^2$, so their *ratio* is constant (strict null). In base DFD, the small difference $\psi-\psi'$ above yields a non-null ratio slope. For ground-to-satellite $\Delta\Phi \sim 5\times10^7\,\mathrm{m^2/s^2}$, this gives $\Delta f/f \sim 5\times10^{-10}$. Current ratio bounds are at $\sim 10^{-7}$ [10, 11], leaving discovery space.

Matter-wave interferometry. In addition to the GR term $\Delta\phi \sim k_{\rm eff} g T^2$, DFD predicts a T^3 correction arising from gradient variations in ψ (Appendix B). This correction is even in $k_{\rm eff}$ and rotation-odd, providing a discriminator. Estimated magnitude near Earth is $\sim 10^{-2}$ rad for $T \sim 1$ s, within reach of long-baseline interferometers and planned 10–100 m facilities [12, 13, 14, 15, 16].

3 Variant Extensions of DFD

All variants reduce to base DFD but add refinements:

3.1 Electromagnetic back-reaction

Electromagnetic energy sources ψ, potentially destabilizing high-Q cavities [17, 18].

3.2 Dual-sector (ϵ/μ) split

ψ couples differently to electric and magnetic energy:

$$\epsilon = \epsilon_0 e^{f(\psi)}, \quad \mu = \mu_0 e^{-f(\psi)}, \tag{4}$$

so $\epsilon\mu = 1/c^2$ remains invariant. Atoms and cavities then redshift differently, consistent with resonant anomalies [19]. If $f(\psi)\sim\psi$, then $\Delta\epsilon/\epsilon \sim \Delta\Phi/c^2 \sim 10^{-9}$ at lab scales, which could be amplified in nonlinear $f(\psi)$.

3.3 Nonlocal kernel

ψ sourced by convolution kernel $K(r)$; improves cluster lensing but testable via modulated Cavendish experiments.

3.4 Vector anisotropy

A background unit vector u^i allows

$$n_{ij} = e^{\psi}(\delta_{ij} + \alpha u_i u_j), \quad \alpha \ll 1. \tag{5}$$

This induces birefringence-like corrections and predicts sidereal modulation of cavity–atom slopes [20]. Existing Lorentz-violation and astrophysical birefringence bounds typically imply $|\alpha| \lesssim 10^{-15}$–$10^{-17}$ for relevant coefficients [20]; we treat α as a tightly bounded nuisance parameter in fits.

3.5 Stochastic ψ

Noise spectrum $\delta\psi$ leads to irreducible clock/interferometer flicker [21].

3.6 High-ψ closure

Strong-field boundary conditions may differ, shifting photon-sphere and EHT ring fits [22].

4 Comparative Predictions

Table 1: Comparative predictions of base DFD and its variants. Legend: ✓ = prediction shared by GR and the indicated model; $*$ = distinctive prediction of the indicated model; $\circ$ = unresolved/tension or requires completion.

Phenomenon	Base	EM→ ψ	Dual	Kernel	Vector	Stoch.	High-ψ
Weak-field PPN	✓	✓	✓	✓	$\circ$	✓	✓
Cavity–atom slope	$*$ non-null	✓ same	$*$ sector-dep.	✓ same	$*$ sidereal	✓ + noise	✓ same
Matter-wave phase	$*$ T^3 term	✓	✓	$*$ baseline dep.	✓	✓ + noise	✓
Resonant cavities	✓ stable	$*$ drift	$*$ sector drift	$\circ$ geometry dep.	$\circ$ direction dep.	$*$ noise	✓
Cluster lensing	$\circ$ tension	$\circ$ same	$\circ$ same	$*$ natural fit	$\circ$ same	$\circ$ same	$\circ$ same
Cosmology	✓ bias/suppress	✓	✓	$*$ modified	✓	$\circ$ noise imprint	✓
Strong-field shadows	✓ optical metric	✓	✓	✓	✓	✓	$*$ altered closure

5 Scope and Limitations

DFD is *secure* in the weak-field regime (solar-system, laboratory tests). It remains *incomplete* in three domains:

- **Cosmology:** In a homogeneous universe with mean density $\bar{\rho}(t)$, Eq. (3) sources a uniform ψ. A toy model $\psi \sim \log a(t)$ would yield line-of-sight bias in distance measures, potentially mimicking cosmic acceleration; luminosity distances would be modified as $d_L^{\mathrm{DFD}} = d_L^{\mathrm{GR}}\, e^{\Delta\psi}$ along a line of sight. Structure formation and BAO remain open [2]. DFD in its current form does not address dark matter or dark energy; extensions to handle rotation curves and cosmic acceleration remain speculative.

- **Strong fields:** Optical shadow pipelines exist, but closure laws and neutron-star structure need development.

- **Gravitational waves:** Base DFD as scalar predicts only monopole/breathing modes, which are ruled out by LIGO/Virgo. A tensor completion is required to recover transverse quadrupolar polarizations; candidate completions include a spin-2 perturbation h_{ij} on the flat background minimally coupled to ψ via a derivative action, but this remains under development [23, 24].

Why the T^3 term is not already excluded. Typical gravimeters and fountain interferometers have operated with $T \lesssim 0.3$–0.5 s, short baselines, and geometries/rotation sequences that suppress rotation-odd contributions and even-in-k_{eff} systematics; combined with $\partial g/\partial z$ suppression, this can push any residual below noise/systematic floors reported in [12, 13]. Quantitatively, for $T = 0.5$ s one expects $\Delta\phi_{T^3} \sim (0.5/1)^3 \times 10^{-2}$ rad $\approx 1.25 \times 10^{-3}$ rad, below typical few-mrad sensitivities in legacy datasets (cf. tables in [12]). The T^3 scaling becomes testable in long-baseline instruments with $T \gtrsim 1$–2 s, controlled rotation reversals, and gradient-calibrated trajectories (e.g., MIGA/AION-style facilities) [14, 15, 16].

5.1 Comparison to Alternatives

- **Brans–Dicke:** Adds a scalar to GR with free coupling parameter ω. DFD resembles the $\omega \to \infty$ limit but with optical-medium interpretation and no curvature.
- **Einstein-æther:** Introduces a dynamical unit timelike vector. DFD instead uses a scalar, but anisotropic extensions parallel æther phenomenology [20].
- **Analog gravity:** In BECs and fluids, effective metrics $g^{\mathrm{eff}}_{\mu\nu} = n^2\eta_{\mu\nu}$ arise [4]. DFD is mathematically identical in its optical limit, but elevated to a candidate for real gravity.

6 Figures

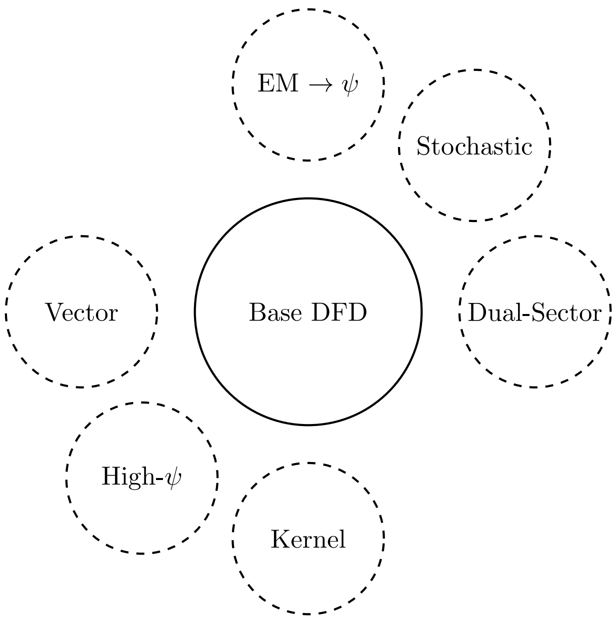

Figure 1: Nested extension family of DFD. All reduce to the base model in appropriate limits.

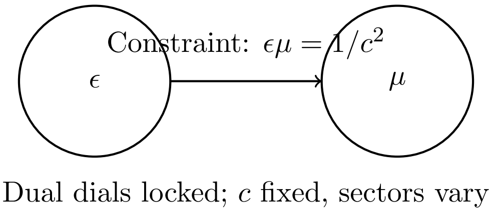

Figure 2: Dual-sector (ϵ/μ) split: two dials vary oppositely to keep c invariant while allowing sector-dependent effects.

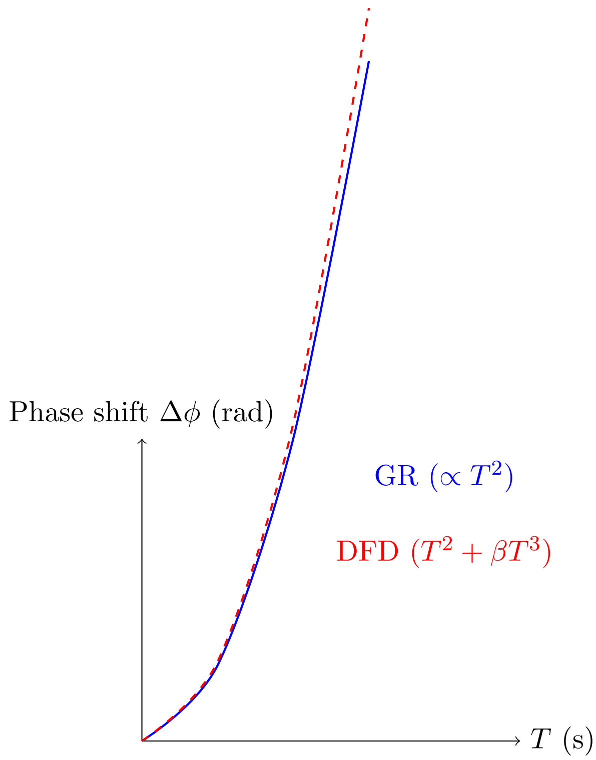

Figure 3: Matter-wave phase shift vs interrogation time T: DFD predicts a small cubic deviation from the quadratic GR law.

7 Conclusion

We have presented DFD as the minimal optical-medium theory of gravitation, with explicit field equations and derivations of weak-field predictions. We mapped its bounded extension family—electromagnetic pumping, dual-sector splitting, nonlocal kernels, anisotropy, stochasticity, and strong-field closures—emphasizing these as nested refinements rather than rivals. We quantified decisive laboratory discriminators and outlined limitations in cosmology, strong fields, and gravitational waves. Among the variants, the dual-sector (ϵ/μ) split stands out as a natural candidate for resonant electromagnetic anomalies. Future work must address cosmological dynamics and tensor completions, but the present framework establishes DFD as a falsifiable effective theory and a coherent alternative to curvature-based gravity.

A Light bending derivation

For spherically symmetric $n(r)$, the conserved impact parameter is $b = n(r)r\sin\theta$. The ray equation is

$$\frac{d\theta}{dr} = \frac{b}{r\sqrt{n^2r^2 - b^2}}.$$

The total deflection is

$$\alpha = 2\int_{r_0}^{\infty} \frac{b}{r\sqrt{n^2r^2 - b^2}} dr - \pi,$$

with r_0 the distance of closest approach. For $n(r) = \exp(2GM/(rc^2))$, expansion yields

$$\alpha \simeq \frac{4GM}{bc^2},$$

matching GR. Detailed derivations appear in [4, 7].

B Matter-wave T^3 phase and parity

The phase is proportional to action $\Delta\phi = (mc^2/\hbar)\int(e^{\psi} - 1)dt$. Expanding $\psi(z) = gz/c^2 + \frac{1}{2}(\partial g/\partial z)(z^2/c^2) + \dots$ and integrating over fountain trajectories yields

$$\Delta\phi = k_{\text{eff}} g T^2 + \frac{k_{\text{eff}}}{2c^2}\frac{\partial g}{\partial z} T^3 + \dots$$

Parity (even in k_{eff}, rotation-odd). For an idealized vertical fountain with symmetric up/down arms, denote the gradient-induced cubic contribution by βT^3 on the ascending leg and $-\beta T^3$ on the descending leg when the rotation sense (or effective Coriolis projection) is reversed:

$$\begin{aligned} \Delta\phi_{\uparrow} &= +\beta T^3 + \cdots, \\ \Delta\phi_{\downarrow} &= -\beta T^3 + \cdots, \\ \Rightarrow \quad \Delta\phi_{\text{total}} &= \Delta\phi_{\uparrow} - \Delta\phi_{\downarrow} = 2\beta T^3 + \cdots. \end{aligned}$$

Because the term arises from $\partial g/\partial z$ rather than the laser momentum transfer itself, it is even under $k_{\text{eff}} \to -k_{\text{eff}}$ (while Coriolis reversals flip the sign). Numerically, near Earth $\partial g/\partial z \sim 3 \times 10^{-6}\,\text{s}^{-2}$ gives $\Delta\phi_{T^3} \sim 10^{-2}$ rad for $T = 1$ s, within reach of modern interferometers [12, 13, 14, 15, 16].

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