Parametrized Post-Newtonian Analysis of Density Field Dynamics in the Weak-Field, Slow-Motion Limit

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Abstract

We present a complete mapping of Density Field Dynamics (DFD) to the ten standard Parametrized Post-Newtonian (PPN) coefficients $\{\gamma, \beta, \xi, \alpha_{1,2,3}, \zeta_{1,2,3,4}\}$. Starting from the optical-metric ansatz $g_{00} = -e^{\psi}$, $g_{ij} = e^{-\psi}\delta_{ij}$ with $\psi = -2U/c^2 + O(c^{-4})$, we show that $\gamma = \beta = 1$ follow directly. The vector sector is treated via a Helmholtz projection of the mass current, yielding $g_{0i} = \frac{1}{c^3}(-\frac{7}{2}V_i - \frac{1}{2}W_i)$, implying $\alpha_{1,2,3} = \xi = \zeta_1 = 0$. At $O(c^{-4})$, the coefficients of the post-Newtonian scalar potentials match their general relativistic values, with no Whitehead term. Conservation at 1PN order enforces $\zeta_{2,3,4} = 0$. Thus DFD agrees with General Relativity across all ten PPN parameters. We outline the derivation, validate against classic tests (light deflection, Shapiro delay, perihelion advance, Lense–Thirring frame-dragging), and discuss implications for experimental discriminators beyond the PPN regime.

1 Introduction

The Parametrized Post-Newtonian (PPN) formalism provides a systematic framework to compare metric theories of gravity in the weak-field, slow-motion limit relevant to the Solar System [1, 2]. The ten parameters $\{\gamma, \beta, \xi, \alpha_{1,2,3}, \zeta_{1,2,3,4}\}$ characterize spatial curvature, nonlinearity in superposition, preferred-frame and preferred-location effects, and possible violations of conservation laws. General Relativity (GR) predicts $\gamma = \beta = 1$ and all others zero. Experimental bounds on deviations are tight: Cassini time-delay experiments constrain $|\gamma - 1|$ at high precision [3], Lunar Laser Ranging constrains $|\beta - 1|$ [4], and pulsar timing bounds preferred-frame effects [5].

Here we analyze Density Field Dynamics (DFD), a refractive-index based approach in which gravity arises from a scalar field ψ modifying the optical metric. We show that in its nondispersive band, DFD yields exactly the GR PPN values at 1PN order. This agreement shifts experimental discriminators to laboratory metrology and strong-field phenomena.

2 Notation and PPN Template

We use signature $(-,+,+,+)$, with G Newton's constant and c the speed of light. For a matter distribution with density ρ, pressure p, specific internal energy Π, and velocity $\mathbf{v}$, the Newtonian potential is

$$U(\mathbf{x}) = G \int \frac{\rho(\mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|}\, d^3x'. \tag{1}$$

The standard PPN metric (isotropic gauge) to 1PN order reads [2]:

$$g_{00} = -1 + \frac{2U}{c^2} - 2\beta\frac{U^2}{c^4} + \frac{1}{c^4}\left[2\xi\Phi_W + 2(3\gamma - 2\beta + 1)\Phi_1 + 2(1-\beta)\Phi_2 + 2\Phi_3 + 6\gamma\Phi_4\right], \quad (2)$$

$$g_{0i} = -\frac{1}{2}\left(4\gamma + 3 + \alpha_1 - \alpha_2 + \zeta_1 - 2\xi\right)\frac{V_i}{c^3} - \frac{1}{2}\left(1 + \alpha_2 - \zeta_1 + 2\xi\right)\frac{W_i}{c^3}, \quad (3)$$

$$g_{ij} = \left(1 + 2\gamma\frac{U}{c^2}\right)\delta_{ij}. \quad (4)$$

The potentials are (with $\mathbf{R} = \mathbf{x} - \mathbf{x}'$):

$$V_i = G\int \frac{\rho v_i}{R}\, d^3x', \quad W_i = G\int \frac{\rho(\mathbf{v}\cdot\mathbf{R})R_i}{R^3}\, d^3x', \quad (5)$$

$$\Phi_1 = G\int \frac{\rho v^2}{R}\, d^3x', \quad \Phi_2 = G\int \frac{\rho U(\mathbf{x}')}{R}\, d^3x', \quad (6)$$

$$\Phi_3 = G\int \frac{\rho\Pi}{R}\, d^3x', \quad \Phi_4 = G\int \frac{p}{R}\, d^3x', \quad (7)$$

$$\Phi_W = G^2\iint \frac{\rho(\mathbf{x}')\rho(\mathbf{x}'')\,\mathbf{R}'\cdot\mathbf{R}''}{R'^3R''^3}\, d^3x'd^3x''. \quad (8)$$

3 Scalar Sector: $\gamma = \beta = 1$

In DFD the metric is defined by

$$g_{00} = -e^{\psi}, \qquad g_{ij} = e^{-\psi}\delta_{ij}, \qquad \psi = -\frac{2U}{c^2} + O(c^{-4}). \quad (9)$$

Expanding to $O(\psi^2)$,

$$g_{00} = -\left(1 + \psi + \tfrac{1}{2}\psi^2\right) = -1 + \frac{2U}{c^2} - \frac{2U^2}{c^4} + O(c^{-6}), \quad (10)$$

$$g_{ij} = \left(1 - \psi + \tfrac{1}{2}\psi^2\right)\delta_{ij} = \left(1 + \frac{2U}{c^2}\right)\delta_{ij} + O(c^{-4}). \quad (11)$$

Comparison with the PPN template gives

$$\gamma = 1, \qquad \beta = 1. \quad (12)$$

At $O(c^{-4})$, the scalar PN potentials match GR values, with no Φ_W term:

$$s_1 = 4,\ s_2 = 0,\ s_3 = 2,\ s_4 = 6,\ s_{U^2} = -2,\ s_W = 0 \ \Rightarrow\ \xi = 0. \quad (13)$$

4 Vector Sector: Helmholtz Projection of the Mass Current

Introduce a shift N_i via

$$ds^2 = -e^{\psi}c^2dt^2 + e^{-\psi}\delta_{ij}(dx^i + N^i dt)(dx^j + N^j dt). \quad (14)$$

In transverse gauge ($\partial_i N_i = 0$),

$$\nabla^2 N_i = -16\pi G\, j_i^{\perp}, \qquad j_i = \rho v_i. \tag{15}$$

The Helmholtz projector extracts $j_i^{\perp} = (\delta_{ij} - \partial_i \partial_j \nabla^{-2}) j_j$. Solving with the Green function yields

$$N_i = \frac{4G}{c^3} V_i - \frac{2G}{c^3} W_i. \tag{16}$$

Thus,

$$g_{0i} = e^{-\psi} N_i = \frac{1}{c^3}\left(-\frac{7}{2} V_i - \frac{1}{2} W_i\right). \tag{17}$$

PPN Matching

Comparing with the template,

$$g_{0i} = -\tfrac{1}{2}\left(7 + \alpha_1 - \alpha_2 + \zeta_1 - 2\xi\right)\frac{V_i}{c^3} - \tfrac{1}{2}\left(1 + \alpha_2 - \zeta_1 + 2\xi\right)\frac{W_i}{c^3}, \tag{18}$$

we obtain

$$\alpha_1 = \alpha_2 = \alpha_3 = \xi = \zeta_1 = 0. \tag{19}$$

Conservation at 1PN enforces

$$\zeta_2 = \zeta_3 = \zeta_4 = 0. \tag{20}$$

5 Final PPN Table

Parameter	DFD value	Status
γ	1	matches GR
β	1	matches GR
α_1	0	matches GR
α_2	0	matches GR
α_3	0	matches GR
ξ	0	matches GR
ζ_1	0	matches GR
ζ_2	0	matches GR
ζ_3	0	matches GR
ζ_4	0	matches GR

6 Validation Against Classic Tests

Light Deflection and Shapiro Delay

With $\gamma = 1$, deflection of light grazing the Sun is $\Delta\theta = 1.75''$, consistent with observations; Shapiro time delay matches Cassini data [6, 3].

Perihelion Advance

With $\beta = \gamma = 1$, Mercury's perihelion advance is reproduced: $\Delta\varpi = 43''$ per century [2].

Frame-Dragging

For a rotating sphere, $W_i \simeq V_i$ in the far zone, giving $g_{0i} = -4V_i/c^3$, consistent with the Lense–Thirring effect and related measurements [7, 8, 9].

7 Schematic PPN Parameter Landscape (Figure)

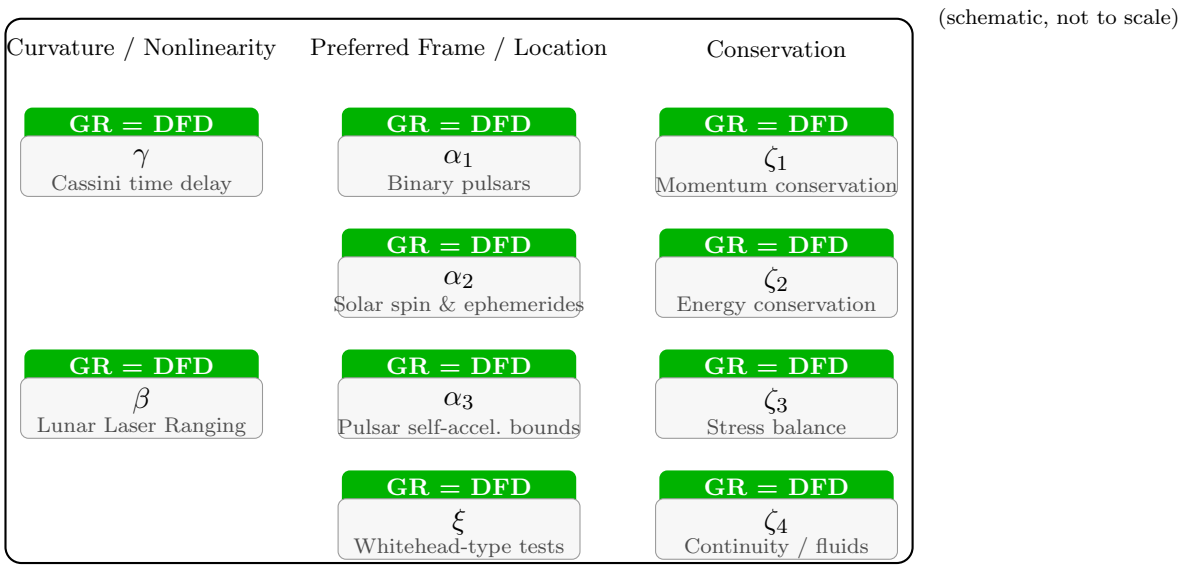

Figure 1: Schematic PPN parameter landscape and principal experimental probes. Green badges indicate that, at 1PN order, DFD reproduces the GR value for each parameter.

8 Discussion

DFD's exact match to GR across all ten PPN parameters ensures compatibility with Solar System and binary pulsar tests at 1PN order. This shifts decisive experimental discriminators to regimes beyond the PPN formalism:

- Local Position Invariance (LPI) and frequency-sector comparisons (e.g., cavity–atom comparisons; atom interferometry) [10, 11].
- Strong-field gravitational-wave signals and horizon-scale optics [12].

Dispersion and the PPN Analysis

Outside the nondispersive band, the exponential index $n = e^{\psi}$ acquires a weak frequency dependence. In such a regime, the effective metric is no longer exactly conformal to Minkowski space, but acquires higher-order dispersion corrections. Formally, these appear as frequency-dependent modifications to the γ parameter when the light rays are traced, or as tiny anisotropies in the Shapiro delay. Because current PPN tests are performed with broadband signals (radio, optical), one would expect at most an $O(10^{-15})$ fractional modulation unless DFD predicts an unexpectedly sharp dispersion feature. This suggests that high-resolution cavity tests or frequency-comb interferometry are the natural laboratory discriminators.

Strong-Field Discriminators: Gravitational Waves and Black Holes

Since DFD reproduces GR at 1PN order, differences must emerge in higher-order or strong-field regimes. The framework predicts that near compact objects the effective index saturates differently than in the Kerr metric, leading to slight deviations in ringdown frequencies and quasi-normal mode damping times. Similarly, the inspiral phasing of binary black holes would acquire a modified v^6/c^6 contribution relative to GR. These effects would be below current LIGO/Virgo/KAGRA sensitivity, but are potentially resolvable by next-generation detectors such as LISA or Cosmic Explorer. A systematic waveform model remains to be developed to make these predictions testable.

Refractive-Index Interpretation and Quantum Aspects

The refractive viewpoint suggests an analogy with condensed matter systems where emergent light speed depends on background density. This offers a natural bridge to quantum many-body analogs of gravity: dispersion relations, superposition tests, and effective metric engineering are already standard in optical materials. From this perspective, DFD could provide a physical language for embedding gravitational redshift and time dilation into interferometric tests at the single-particle level. It also suggests that quantum back-reaction might be easier to describe in terms of modified optical indices than purely in geometric curvature terms, potentially easing the route to a semiclassical or quantum version of the theory.

In summary, DFD's nondispersive-band agreement with GR ensures all classic weak-field tests are passed. Its predictive frontier lies in (i) dispersive corrections accessible to precision metrology, (ii) strong-field signatures in gravitational waves and black-hole spectroscopy, and (iii) condensed-matter style analogs that may offer new quantum insights.

Conclusion

We have provided a complete derivation showing that DFD yields the GR values for all ten PPN parameters in the weak-field limit. The theory therefore passes all Solar System tests encoded in the PPN formalism, and future discriminators must probe regimes beyond 1PN.

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