A Sharp, Testable Slope Prediction for a Sector-Resolved Cavity–Atom LPI Test

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September 10, 2025

Summary. This note records the precise, falsifiable slope prediction for a co-located cavity–atom redshift comparison across a vertical geopotential difference. Using the sector-resolved formalism defined in my LPI letter, the measurable ratio slope

$$\frac{\Delta R^{(M,S)}}{R^{(M,S)}} \equiv \xi^{(M,S)}\,\frac{\Delta\Phi}{c^2}, \qquad \xi^{(M,S)} = \alpha_w - \alpha_L^{(M)} - \alpha_{\rm at}^{(S)}, \tag{1}$$

is predicted in Density Field Dynamics (DFD) to be *nonzero* in a verified nondispersive optical band, with leading value

$$\xi^{(\rm ULE,Sr)} \simeq +1 \quad \text{(DFD, nondispersive band).}$$

Thus, for a height change Δh on Earth,

$$\boxed{\frac{\Delta R}{R} \;=\; [1+\varepsilon_{\rm disp}+\varepsilon_{\rm thermo}+\varepsilon_{\rm sag}+\varepsilon_{\rm at}]\,\frac{g\,\Delta h}{c^2}} \tag{2}$$

with $g \simeq 9.8\,\mathrm{m\,s^{-2}}$. Numerically,

$$\frac{g\,\Delta h}{c^2} = \begin{cases} 1.09\times10^{-14} & (\Delta h = 100\,\mathrm{m}),\\ 3.27\times10^{-15} & (\Delta h = 30\,\mathrm{m}),\\ 3.27\times10^{-14} & (\Delta h = 300\,\mathrm{m}).\end{cases}$$

General Relativity (GR) corresponds to $\xi = 0$ in Eq. (1), hence a strict null for the co-transported ratio.[1]

Definitions and identifiability

Following the sector basis of Ref. [1], the cavity and atomic fractional redshifts are

$$\left(\frac{\Delta f}{f}\right)^{(M)}_{\rm cav} = \left(\alpha_w - \alpha_L^{(M)}\right)\frac{\Delta\Phi}{c^2}, \tag{3}$$

$$\left(\frac{\Delta f}{f}\right)^{(S)}_{\rm at} = \alpha_{\rm at}^{(S)}\,\frac{\Delta\Phi}{c^2}, \tag{4}$$

[1]In the sector parameterization with GR normalization $\alpha_w = 1$, $\alpha_L^{(M)} = 0$, $\alpha_{\rm at}^{(S)} = 1$, GR implies $\xi = 0$. See Eq. (2) below and Ref. [1] for details.

and the four measured slopes across two cavity materials (ULE, Si) and two atomic species (Sr, Yb) identify

$$\delta_{\rm tot} \equiv \alpha_w - \alpha_L^{\rm ULE} - \alpha_{\rm at}^{\rm Sr} = \xi^{(\rm ULE,Sr)},$$
$$\delta_L \equiv \alpha_L^{\rm Si} - \alpha_L^{\rm ULE}, \qquad \delta_{\rm at} \equiv \alpha_{\rm at}^{\rm Yb} - \alpha_{\rm at}^{\rm Sr}, \tag{5}$$

via an over-determined $4 \to 3$ GLS solution with full covariance [1].

DFD leading-order prediction

In DFD's optical-metric sector, photons propagate with phase velocity $v_{\rm phase} = c/n = c\,e^{-\psi}$ (*nondispersive band*), so an evacuated cavity tracks $n = e^{\psi}$ to leading order, while the co-located atomic transition is leading-order ψ-insensitive in this sector. Therefore,

$$\alpha_w \to 1, \quad \alpha_L^{(M)} \to 0, \quad \alpha_{\rm at}^{(S)} \to 0 \;\Rightarrow\; \xi^{(M,S)} \to +1,$$

giving Eq. (2). This yields an *order*-$\Delta\Phi/c^2$ geometry-locked slope: $|\Delta R/R| \sim \Delta\Phi/c^2 \approx 1.1 \times 10^{-14}$ per 100 m on Earth.

Correction controls (as implemented in the protocol)

The LPI protocol specifies explicit controls so that any allowed deviations enter Eq. (2) only through small, bounded ε-terms:

- **Dispersion/thermo-optic bound** via dual-wavelength probing within the low-loss band, requiring $|\xi_{\lambda_1} - \xi_{\lambda_2}| < 0.1\,|\xi|_{\rm targ}$ (and $< 2\sigma_\Delta$), which caps $|\varepsilon_{\rm disp}|$ at $\lesssim 10\%$ of a per-slope target and $\lesssim 2\%$ in the GLS solution [2].
- **Elastic sag / orientation flip** modeling plus 180° flips at each height distinguish mechanical-length artifacts (sign-reversing) from genuine redshift (sign-preserving), bounding $|\varepsilon_{\rm sag}|$ at $\lesssim 10^{-16}$ per window [3].
- **Environmental thresholds / hardware swaps** (vibration, temperature, pressure, magnetic reversals; mirror and electronics swaps) encode residual configuration offsets in the covariance, further suppressing bias [4].

Numerical statement to be compared with data

For a vertical separation Δh measured geodetically (beyond $g\Delta h$),

$$\boxed{\left.\frac{\Delta R}{R}\right|_{\rm DFD,\,ULE/Sr} = (1 \pm 0.1_{\rm disp} \pm 0.02_{\rm GLS})\,\frac{g\,\Delta h}{c^2} \;+\; \mathcal{O}(10^{-16})\,,} \tag{6}$$

where the $\pm$ terms reflect the protocol's internal dispersion/GLS bounds when the dual-λ and stationarity criteria are satisfied [2, 4]. GR predicts zero for the same co-transported ratio.

Falsification

A result consistent with $\xi = 0$ at or below $|\Delta\Phi|/c^2$ after applying the above controls would falsify the *nondispersive-band* DFD prediction stated here.

Data/Code

Upon request, I will supply a minimal script computing Eq. (6) for arbitrary Δh and site geodesy.

Acknowledgments

I thank colleagues in precision metrology for guidance on geodesy, vibration immunity, and fieldable clocks/comb systems.

References

[1] G. Alcock, "Sector-Resolved Test of Local Position Invariance with Co-Located Cavity–Atom Frequency Ratios," (2025). Formalism, identifiability, and GR limit summarized in Eqs. (1)–(4); see especially the ratio-slope definition and δ-basis mapping.

[2] G. Alcock, *ibid.*, dual-λ dispersion/thermo-optic bound and acceptance criterion ($|\xi_{\lambda_1} - \xi_{\lambda_2}| < 0.1|\xi|_{\rm targ}$; $< 2\sigma_\Delta$).

[3] G. Alcock, *ibid.*, elastic-sag model and 180° orientation flips bounding mechanical artifacts.

[4] G. Alcock, *ibid.*, environmental thresholds, swaps, and the ratio Allan model used in GLS.