Mathematical modelling

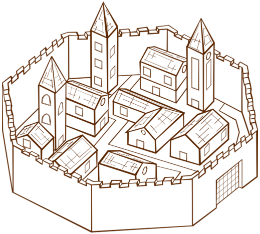
Opportunities and Limitations

Lecture 2

UNIVERSITY OF BERGEN

Assumptions

"All models are wrong, but some are useful" - George Box



N - population size
$S(t)$ - susceptible
$I(t)$ - infective

Population is closed: $S(t) + I(t) = N$

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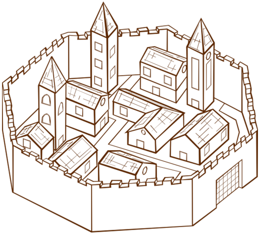
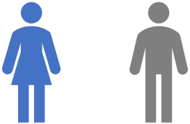


N - population size
$S(t)$ - susceptible
$I(t)$ - infective

Population is closed: $S(t) + I(t) = N$

- If you meet one person, what is the chance that they are infective?



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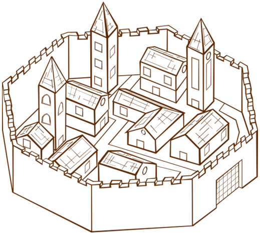

N - population size
$S(t)$ - susceptible
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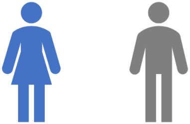
Population is closed: $S(t) + I(t) = N$

- If you meet one person, what is the chance that they are infective?



$$\frac{I}{N-1} \approx \frac{I}{N}$$

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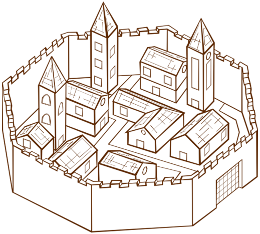
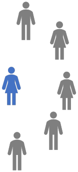


N - population size
$S(t)$ - susceptible
$I(t)$ - infective

Population is closed: $S(t) + I(t) = N$

- If you meet five people, what is the chance at least one of them was infective?



$$\approx 5\frac{I}{N}$$

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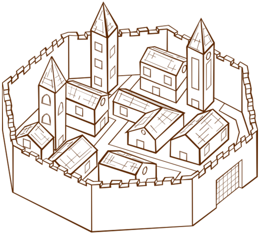
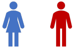


N - population size
$S(t)$ - susceptible
$I(t)$ - infective

Population is closed: $S(t) + I(t) = N$

- If you met an infective, what is the chance of getting ill?



depends on the virus,...
(transmissibility)

Model I

$$S \xrightarrow{S\,\beta\,\frac{I}{N}} I$$

$$\beta = \underbrace{\text{contact rate}}_{e.g.5\ ppl\ per\ day} \times \underbrace{\text{transmissibility}}_{e.g.\ 25\%}\ \text{(a.k.a. }\textbf{effective contact rate}\text{)}$$

SI-model

$$\begin{cases} \dfrac{dS}{dt} = -\beta S\,\dfrac{I}{N} \\[2ex] \dfrac{dI}{dt} = \beta S\,\dfrac{I}{N} \end{cases}, \qquad t \in [0, T_{end}]$$

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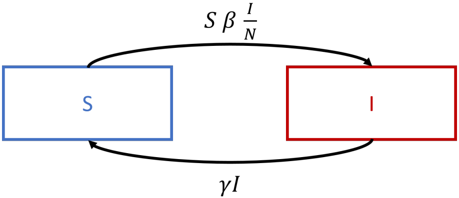
Model II



$$\beta = \underbrace{\text{contact rate}}_{e.g.\,5\text{ ppl per day}} \times \underbrace{\text{transmissibility}}_{e.g.\,25\%}$$ (a.k.a. **effective contact rate**)

$$\gamma = 1/\ (\#\text{ days})$$ (a.k.a. **recovery rate**)

SIS-model

$$\begin{cases} \dfrac{dS}{dt} = -\beta S\,\dfrac{I}{N} + \gamma I \\[2ex] \dfrac{dI}{dt} = \beta S\,\dfrac{I}{N} - \gamma I \end{cases}, \qquad t \in [0, T_{end}]$$

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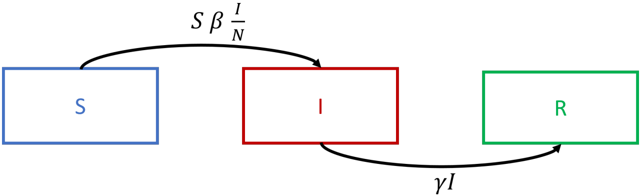
Model III



$\beta = \underbrace{\text{contact rate}}_{e.g. 5\ ppl\ per\ day} \times \underbrace{\text{transmissibility}}_{e.g.\ 25\%}$ (a.k.a. **effective contact rate**)

$\gamma = 1/$ (# days) (a.k.a. **recovery rate**)

SIR-model

$$\begin{cases} \frac{dS}{dt} = -\beta S \frac{I}{N} \\ \frac{dI}{dt} = \beta S \frac{I}{N} - \gamma I \\ \frac{dR}{dt} = \gamma I \end{cases}, \qquad t \in [0, T_{end}]$$

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