Wavelet Coherence

Open this Example

The continuous wavelet transform (CWT) allows you to analyze the temporal evolution of the frequency content of a given signal or time series. The application of the CWT to two time series and the cross examination of the two decompositions can reveal localized similarities in time and scale. Areas in the time-frequency plane where two time series exhibit common power or consistent phase behavior indicate a relationship between the signals.

For jointly stationary time series, the cross spectrum and associated coherence function based on the Fourier transform are key tools for detecting common behavior in frequency. In the general nonstationary case, wavelet-based counterparts can be defined to provide time-localized alternatives. The `wcoher.m` function provides these wavelet-based estimators. The purpose of this example is to understand how to use `wcoher.m` and interpret the results.

In all of the following examples, complex-valued wavelets are used. When a complex wavelet is used, the CWT, Cx(a,b), of a real-valued time series, x, is a complex-valued function of the scale parameter a and the location parameter b.

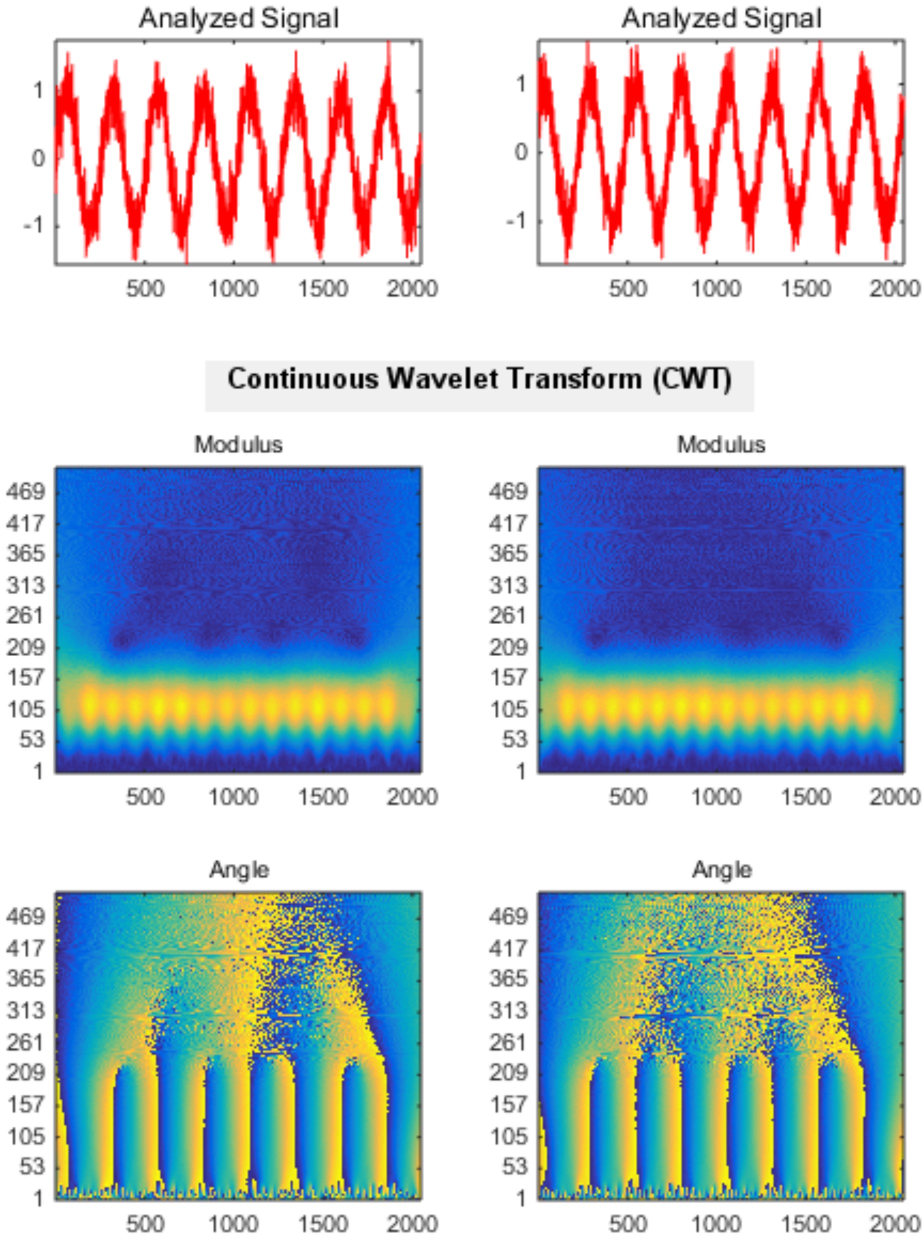
On this page…

Example 1: Two Sine Waves in Gaussian Noise

The first example introduces the different graphical representations provided by the `wcoher.m` function. The example also highlights the usefulness of the phase information obtained from using complex-valued wavelets.

Consider two sine functions on the interval [0,1] using 2048 points. Both sine functions have a frequency of 8 Hz. One of the functions has an initial phase offset of pi/4 radians. Both sine functions are corrupted by additive zero-mean Gaussian noise with a variance of 0.5. Consider the CWT of the two signals (denoted by x and y) using the cgau3 complex wavelet for integer scales from 1 to 512.

```
wname  = 'cgau2';
scales = 1:512;
ntw = 21;
t  = linspace(0,1,2048);
x = sin(16*pi*t)+0.25*randn(size(t));
y = sin(16*pi*t+pi/4)+0.25*randn(size(t));
wcoher(x,y,scales,wname,'ntw',ntw,'plot','cwt');
```



The common period of the signals at scale 128 is clearly detected in the moduli of the individual wavelet spectra. Using

```
Freq = scal2frq(128,'cgau3',1/2048);
```

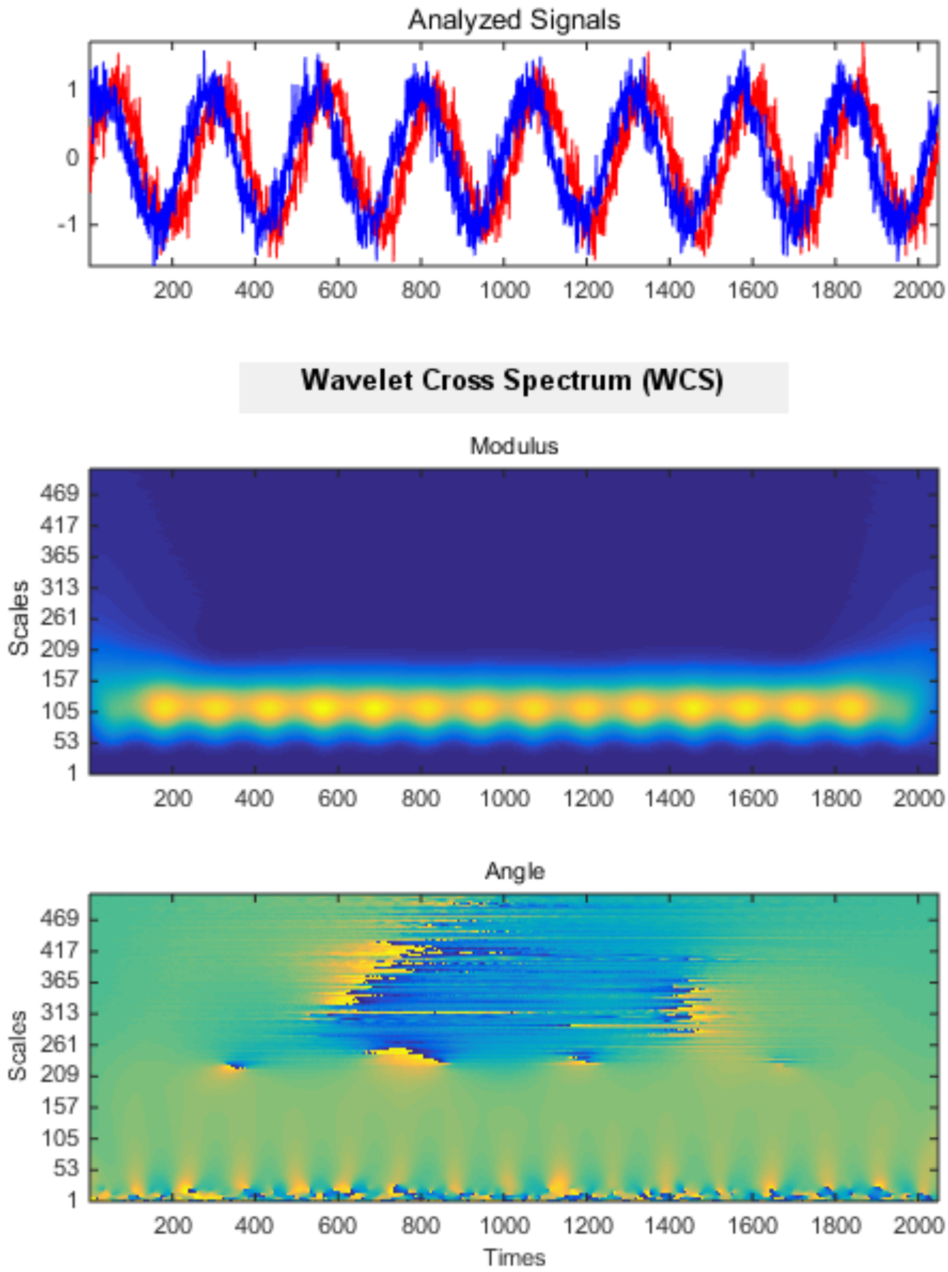
note that this corresponds to a frequency of 8 Hz, which is equivalent to (1/8)*2048 = 256 samples per period with the given sampling frequency.

The wavelet spectrum, defined for each signal, is characterized by the modulus and the phase of the CWT obtained using the complex-valued wavelet. Denote the individual wavelet spectra as Cx(a,b) and Cy(a,b). The two decompositions are exactly the same, up to a translation, since the CWT is translation-invariant. To examine the relationship between the two signals in the time-scale plane, consider the wavelet cross spectrum Cxy(a,b), which is defined as

$$Cxy(a,b) = \overline{Cx(a,b)}Cy(a,b)$$

where $\bar{z}$ denotes the complex conjugate of z. A smoothed version of this function is depicted in the following figure.

```
wcoher(x,y,scales,wname,'ntw',ntw,'plot','wcs');
```



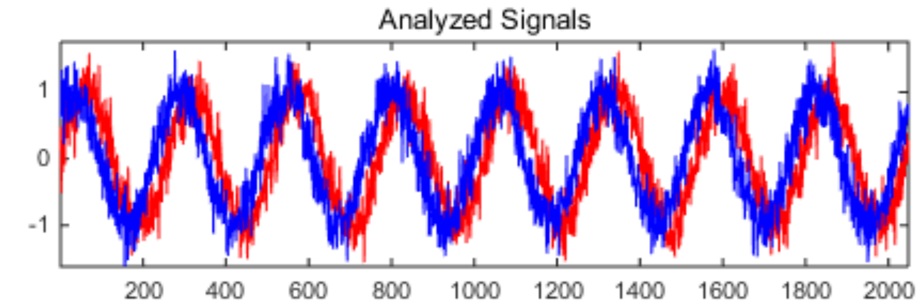
The magnitude of the wavelet cross spectrum can be interpreted as the absolute value of the local covariance between the two time series in the time-scale plane. In this example, this nonnormalized quantity highlights the fact that both signals have a significant contribution around scale 128 at all positions.

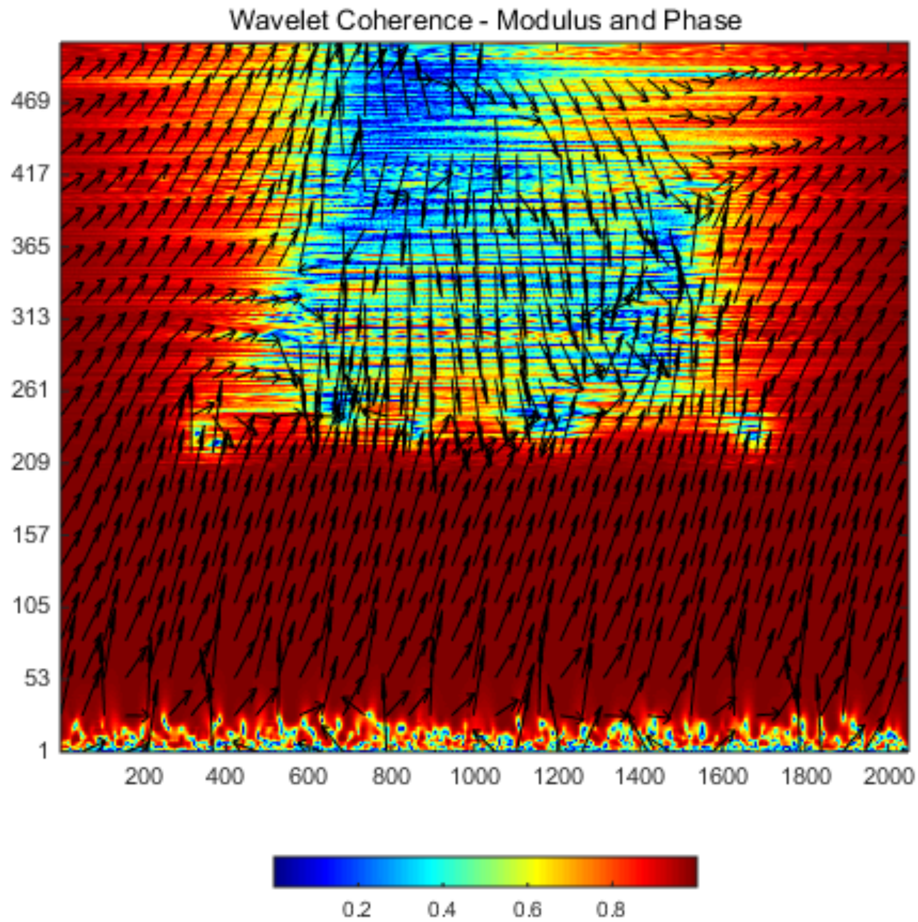
The next figure displays the wavelet coherence and is the most important. The empirical wavelet coherence for x and y is defined as the ratio:

$$S(Cxy(a,b))/\sqrt{S(|Cx(a,b)|^2)}\sqrt{S(|Cy(a,b)|^2)}$$

where S stands for a smoothing operator in time and scale. The wavelet coherence can be interpreted as the local squared correlation coefficient in the time-scale plane.

```
wcoher(x,y,scales,wname,'ntw',ntw,'plot');
```





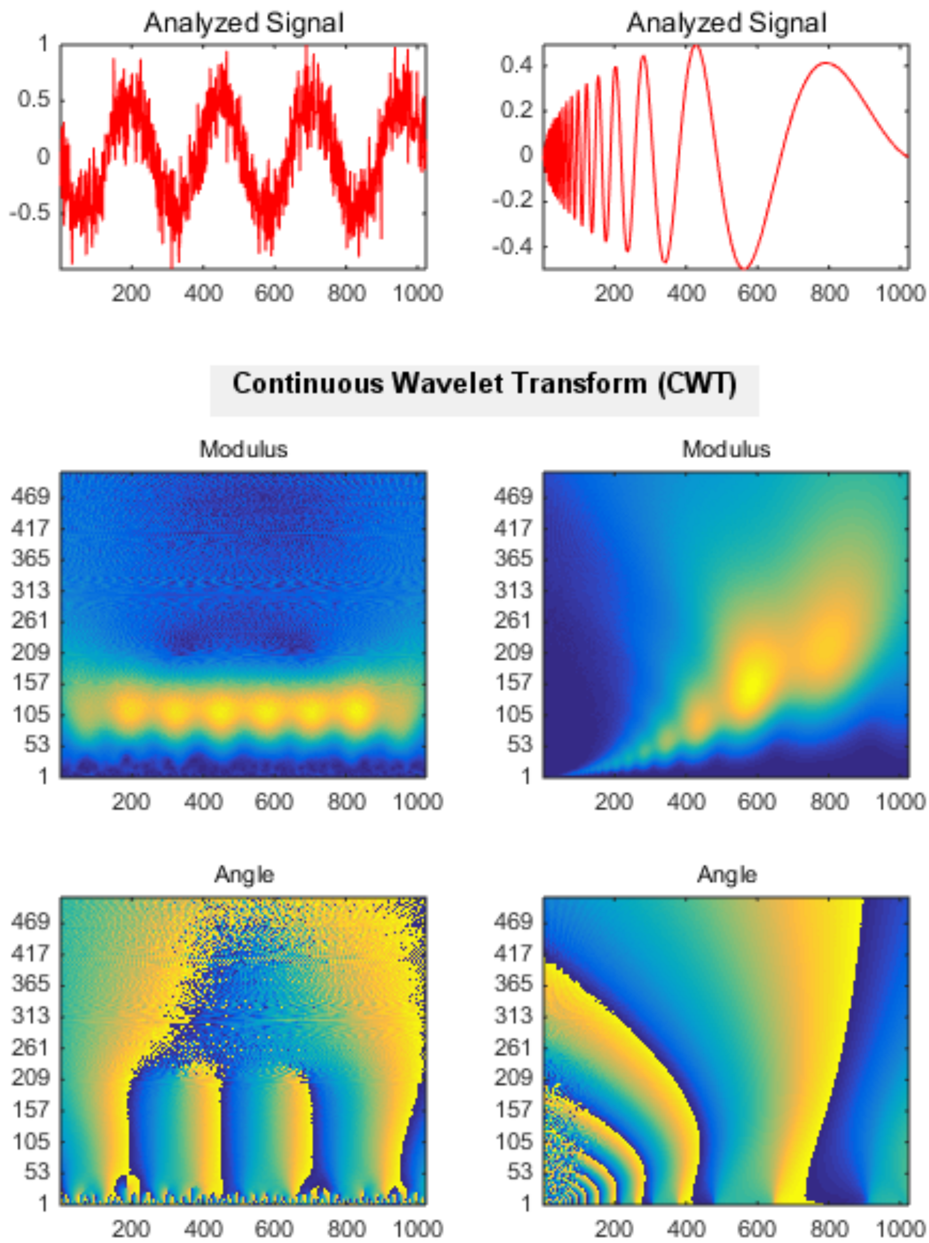
The arrows in the figure represent the relative phase between the two signals as a function of scale and position. The relative phase information is obtained from the smoothed estimate of the wavelet cross spectrum, S(Cxy(a,b)). The plot of the relative phases is superimposed on the wavelet coherence. The relative phase information produces a local measure of the delay between the two time series. Note that for scales around 128, the direction of the arrows captures the relative phase difference between the two signals of pi/4 radians.

Now consider an example involving transient behavior and a more subtle relationship between two time series.

Example 2: Sine and Doppler Signal

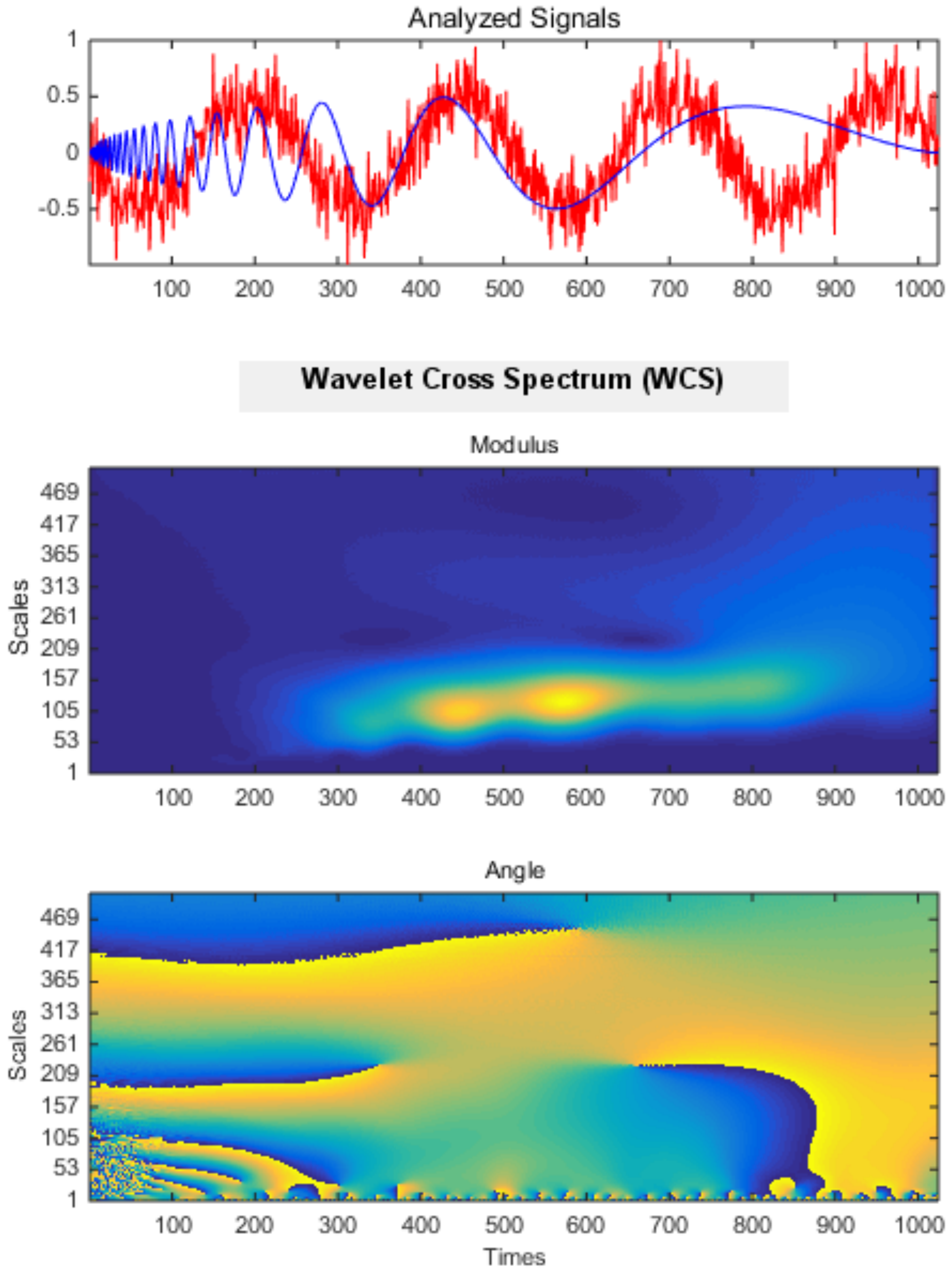
A 4-Hz sine wave with additive Gaussian noise is sampled on a grid of 1024 points over the interval [0,1]. The second time series is a Doppler signal with decreasing frequency over time. Consider the CWT of the two signals (denoted by x and y) using the cgau3 complex wavelet for integer scales from 1 to 512.

```
wname  = 'cgau2';
scales = 1:512;
ntw = 21;
t = linspace(0,1,1024);
x = -sin(8*pi*t) + 0.4*randn(1,1024);
x = x/max(abs(x));
y = wnoise('doppler',10);
wcoher(x,y,scales,wname,'ntw',ntw,'plot','cwt');
```



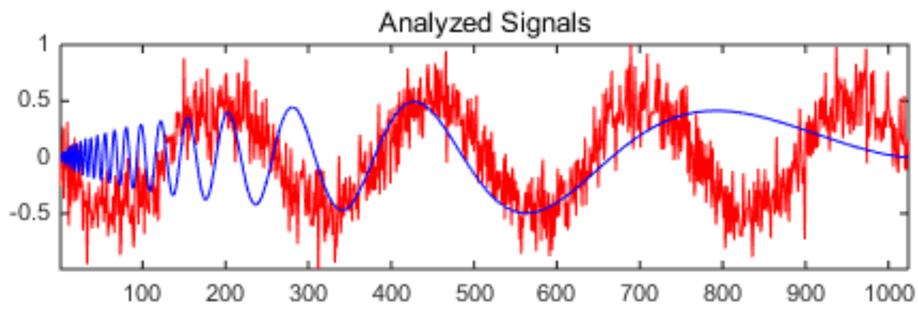
The analysis of the sine function on the left exhibits the scale associated with the period (which is equal to 1024/8 = 128). The analysis of the Doppler signal highlights a typical time-scale picture illustrating the decreasing frequency (increasing scale) as a function of time. The wavelet cross spectrum is shown in the following figure.

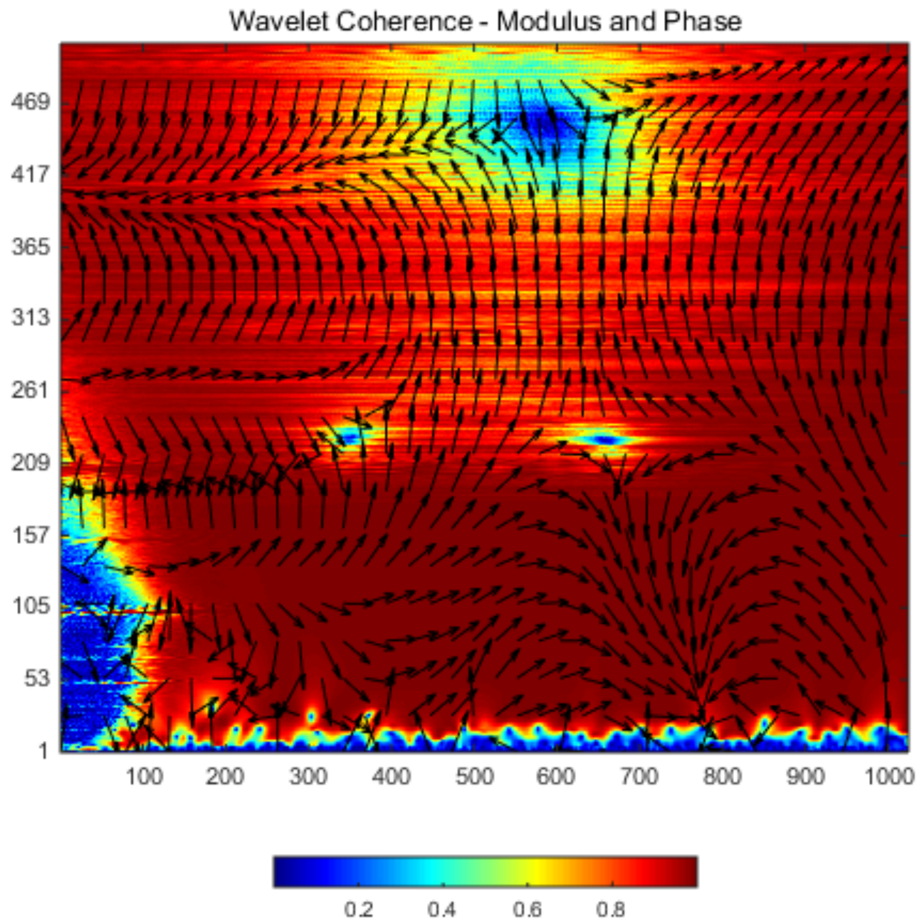
```
wcoher(x,y,scales,wname,'ntw',ntw,'nsw',1,'plot','wcs');
```



The magnitude is the more instructive and shows the similarity of the local frequency behavior of the two time series in the time-scale plane. Both signals have a similar contribution around scale 128 over the interval [300, 700]. This is consistent with the behavior observed by visual inspection of the time-domain plot. Additional interesting information is discernible in the wavelet coherence.

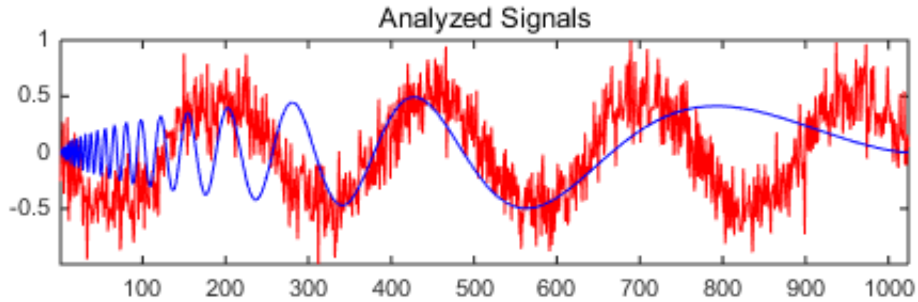
```
wcoher(x,y,scales,wname,'ntw',ntw,'plot');
```

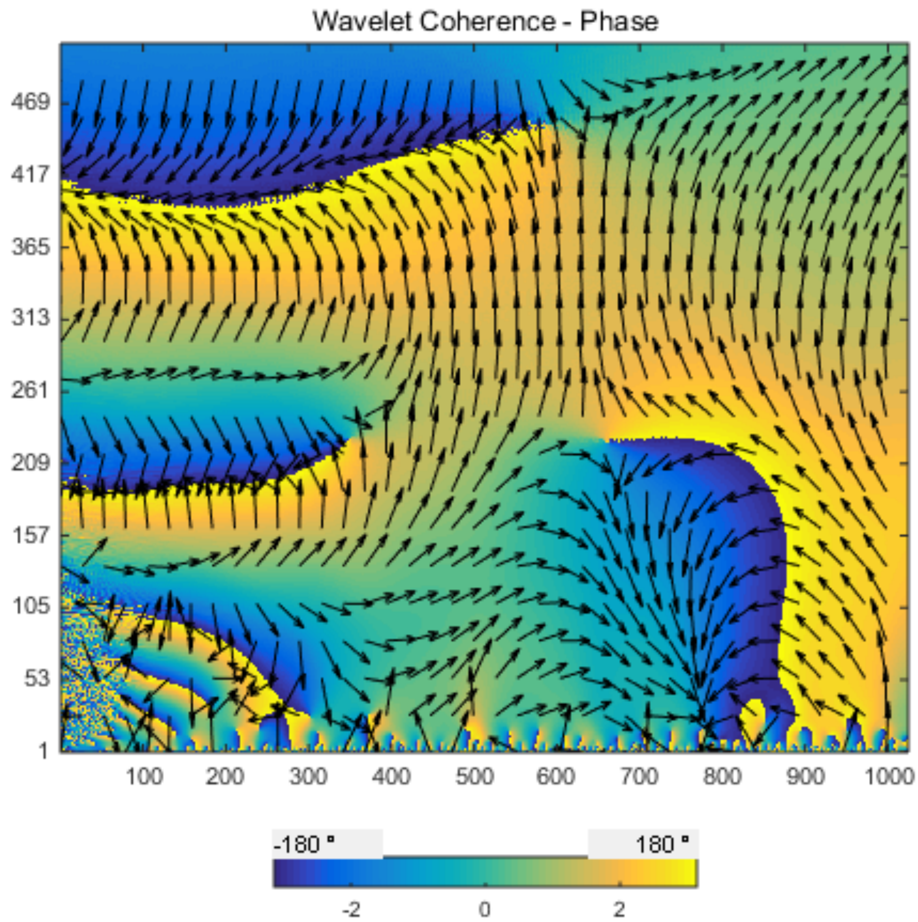




The phase information can be interpreted by locating different regions of the time-scale plane and highlighting some coherent behaviors. Some transient minor contributions to the variability of the time series occur at small scales at the beginning of the Doppler signal, which exhibits rapid oscillations. The behavior is not coherent and the phase changes very quickly. However, at positions greater than 150 and scales greater than 130, numerous coherent regions can be easily detected, delimited by the stability of the phase information. Because phase information is so useful in determining coherent behavior, another representation tool is available for focusing on the phase. The phase information is coded both by the arrow, or vector, orientation and by the background color. The background color is associated with a mapping onto the interval [-pi,pi].

```
wcoher(x,y,scales,wname,'ntw',ntw,'plot','wcoh');
```



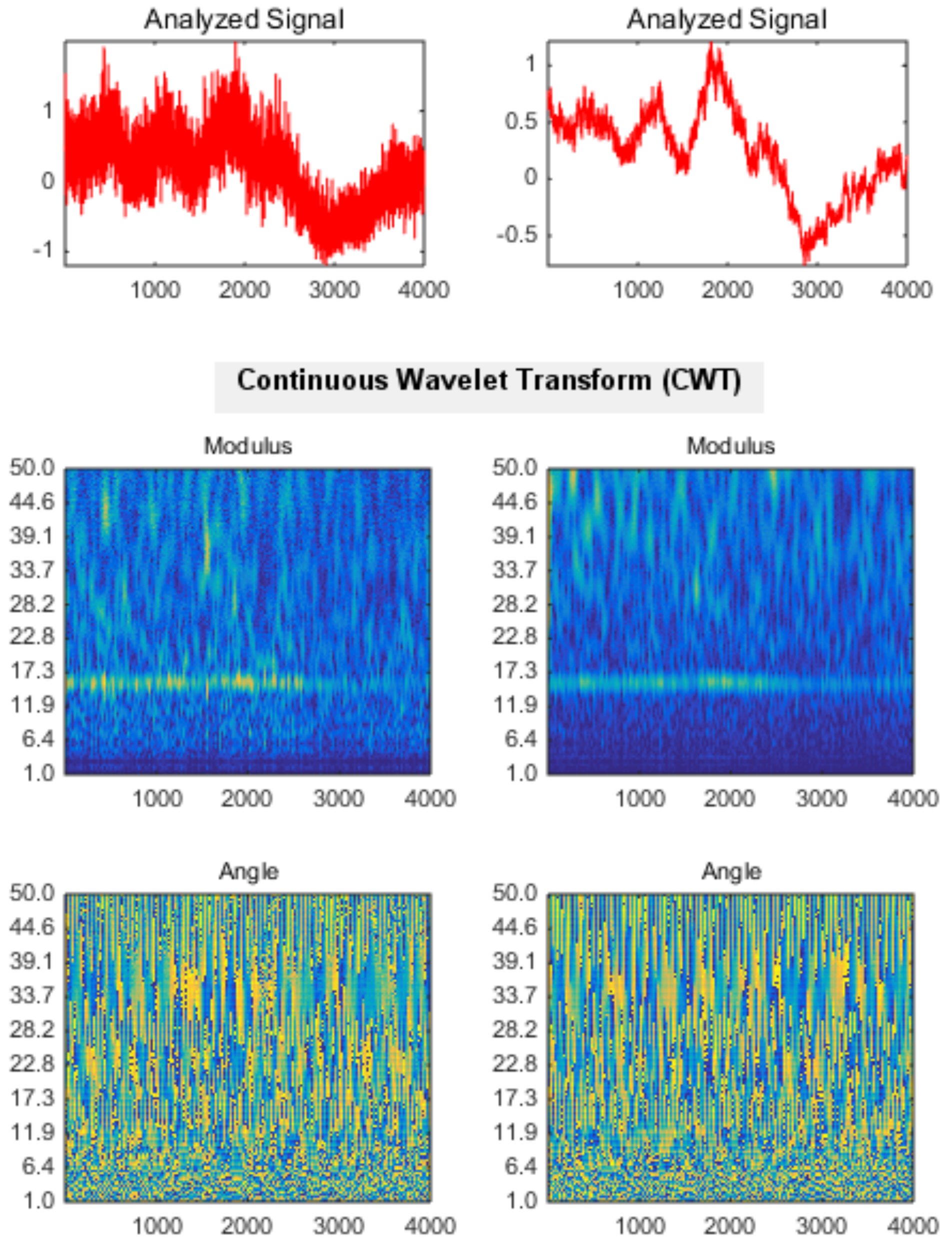


Example 3: Detection of System Anomaly Using Wavelet Coherence

The data analyzed in this example were generously provided by the oil company Total. The two signals, sensor1 and sensor2, were recorded at two different spatial locations. A quasi-periodic signal appearing at both sensors indicates a system anomaly. While it is possible to detect the oscillatory behavior by examining data recorded at an individual sensor, analyzing the joint time-scale variations in the sampled input at two spatially-separated sensors results in a reduction in the false positive rate. Prior studies using the complex Morlet wavelet show that the spurious signal evolves around scale 15. This example uses the complex Morlet wavelet cmor1-3 and the scales 1:0.1:50.

```
scales = 1:0.1:50;
wname  = 'cmor1-3';
```

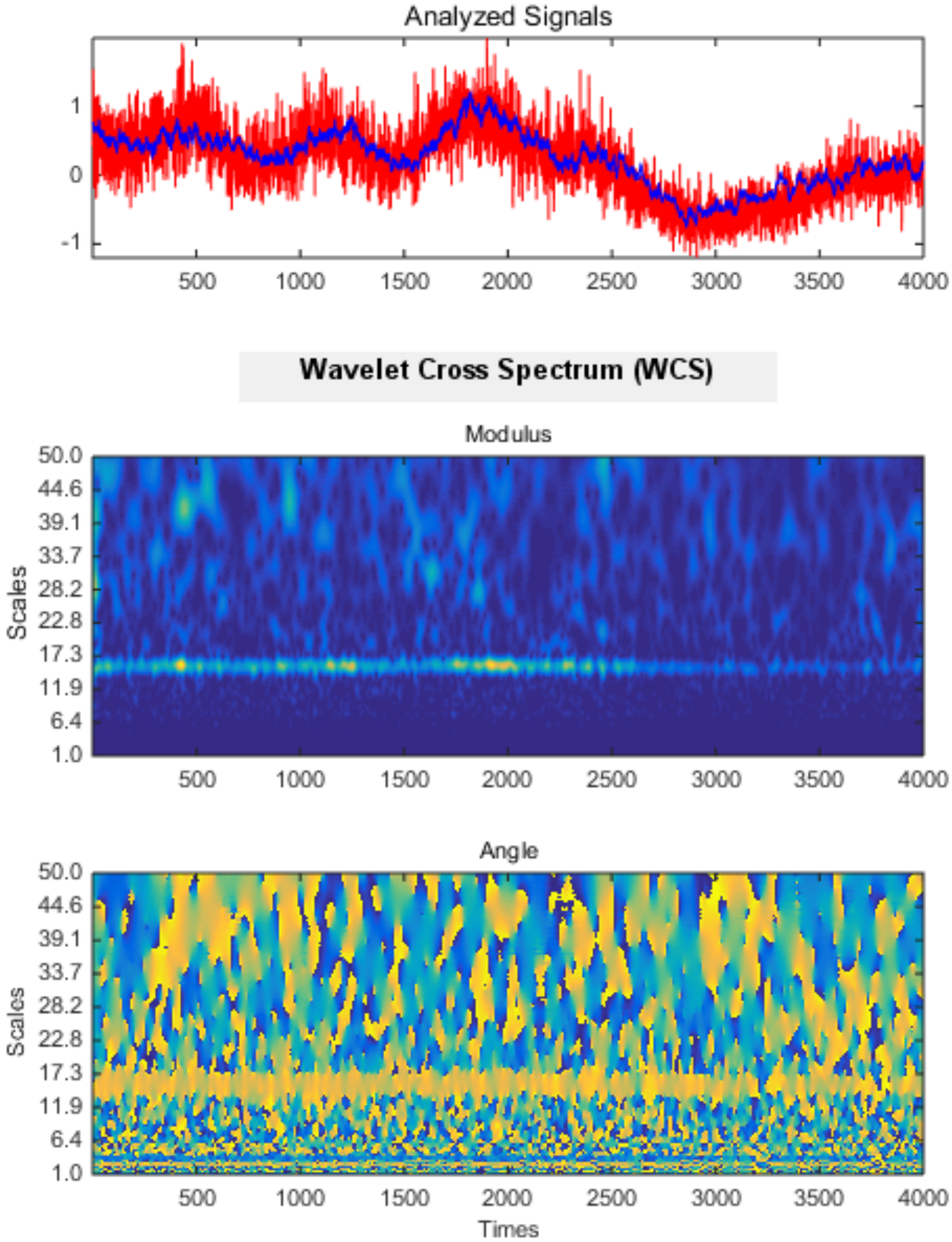
```
load sensor1; load sensor2
long  = 4000;
first = 5000;
last  = first + long-1;
indices = (first:last);
s1 = sensor1(indices);
s2 = sensor2(indices);
wcoher(s1,s2,scales,wname,'ntw',ntw,'plot','cwt');
```



The figure shows both CWT decompositions. The data from sensor1 is to the left, and the data from sensor2 to the right. The middle figure panels contain plots of the individual CWT moduli for sensor1 and sensor2. Both

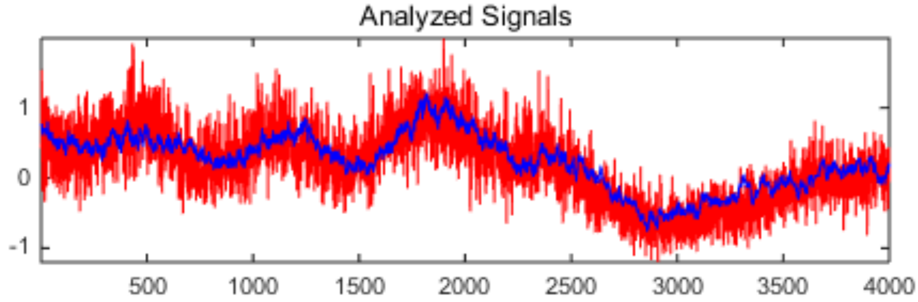
CWT moduli plots reveal strong components around scale 16 over approximately the first 2000 points. Both the scale and positions of the components show good agreement at the two spatially-separated sensors. The next step shows the wavelet cross spectrum.

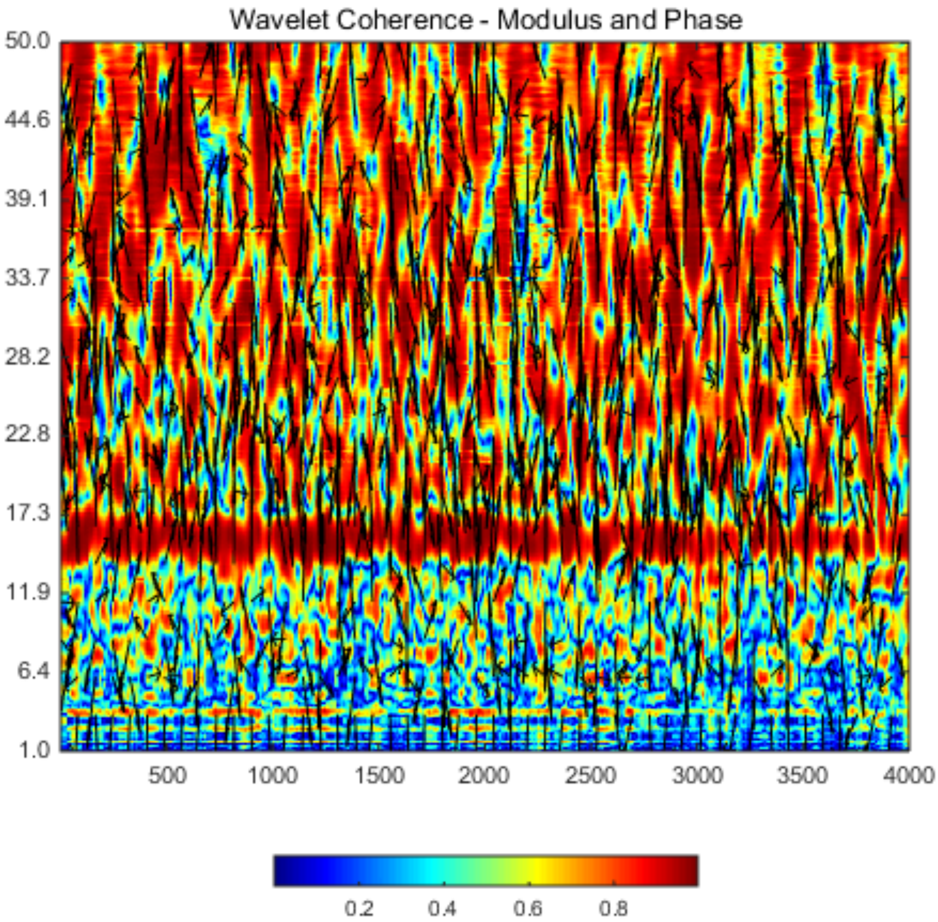
```
wcoher(s1,s2,scales,wname,'ntw',ntw,'plot','wcs');
```



The modulus of the wavelet cross spectrum clearly shows a strong component around scale 16. The magnitude of the component increases and decreases over time, but is generally strong for approximately the first 2500 points of the signals. Finally, computing the wavelet coherence and superimposing the phase of the smoothed wavelet cross spectrum shows that data from the two sensors exhibit coherence near 1 and approximately constant relative phase at the scales of interest.

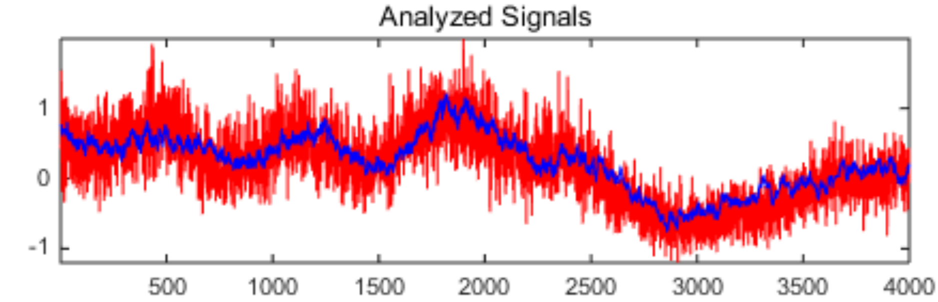
```
ntw = 51;    % N-point time window
nts = 10;    % N-point scale window
nat = 50;    % number of arrows in time
nas = 20;    % number of arrows in scale
asc = 0.75; % scale factor  for the arrows (see QUIVER).
wcoher(s1,s2,scales,wname,'nat',nat,'nas',nas, ...
    'ntw',ntw,'nts',nts,'asc',asc,'plot');
```

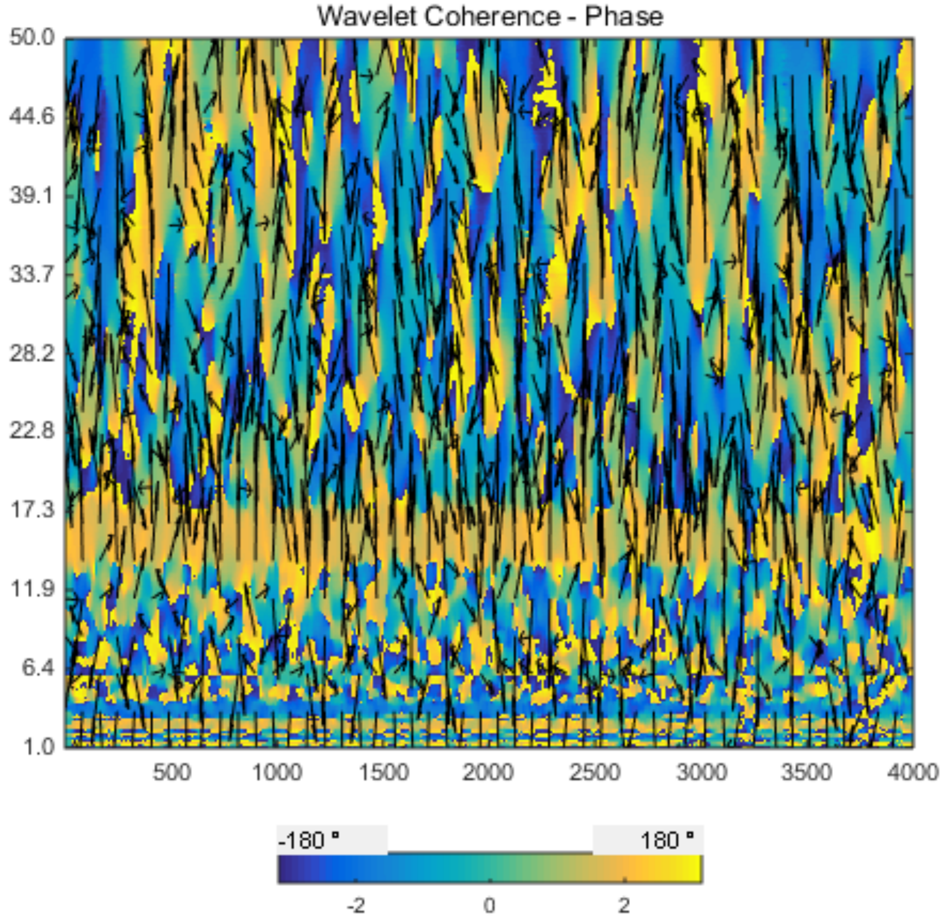




Coding the background color in the coherence plot for phase highlights the consistent relative phase behavior at the two sensors at the scales of interest.

```
wcoher(s1,s2,scales,wname,'nat',nat,'nas',nas, ...
    'ntw',ntw,'nts',nts,'asc',asc,'plot','wcoh');
```





Wavelet coherence analysis greatly facilitates the detection of the quasi-periodic component indicative of a system anomaly. The use of other wavelets (cgaus2 or cmor1.5) does not alter these conclusions, but the precise localization in scale of the critical phenomenon naturally changes depending on the analyzing wavelet.

Summary

This example has shown you how to use wavelet cross spectrum and wavelet coherence to reveal localized similarities between two time series in the time-scale plane and to interpret the results.