The dynamic behavior of squash balls

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(Received 26 February 2010; accepted 23 September 2010)

The behavior of a squash ball constitutes an excellent case study of the dynamic behavior of rubbery materials. It is shown that the complex viscoelastic behavior of rubber can be investigated using simple drop bounce tests and compression tests. The drop tests show that the coefficient of restitution increases as the ball temperature increases. The compression tests show that as the speed of compression increases or as the ball temperature decreases, the compressive force and the energy loss both increase. These effects are due to the viscoelastic nature of the rubber and are an excellent example of the time-temperature equivalence of polymers. Compression tests were performed on balls with small holes at the base to separate the effects of the internal air pressure from the material deformation. It was found that the internal air pressure contributed about one-third to the compressive force, but contributed little to energy loss. This behavior shows that the rubber material dominates the rebound behavior and that the normal warming up process at the start of a squash game is important to raise the temperature of the rubber rather than to increase the internal air pressure.
[DOI: 10.1119/1.3531971]

I. INTRODUCTION

The bouncing of a ball is a familiar phenomenon that involves complex dynamics and materials behavior. Changes in the physics of the bounce can significantly change the nature of the activity and the chance of injury in many sports. The study of ball bouncing is amenable to simple techniques, is relevant to many students with interests in sports, and involves a range of physics principles that can be explored across a wide educational spectrum.

The behavior of squash balls presents an especially interesting case due to the nature of their materials and construction. Some balls rely mainly on internal air or gas pressure to provide the dynamic behavior, with tennis balls being the best example. There are good physics lessons here, with balls kept cool before use to minimize gas diffusion, balls being changed after a certain number of games as the internal pressure drops, and novel nanocomposite layers in the wall of Wilson double-core balls to reduce diffusion.[1] The behavior of solid balls is determined solely by the ball's material, with examples being baseball, cricket, and field hockey balls. Squash balls are intermediate because they are hollow, but have thick walls and are not internally pressurized. Their behavior is therefore partly controlled by the wall material and partly by the air inside. The rubber used for squash balls also produces interesting changes of the dynamic behavior with temperature, which is the reason that squash balls need to be warmed up before a game starts. It is also possible to use different grades of squash ball (indicated by colored spots), which give different levels of bounce and can be matched to the skill and fitness level of the players.

The aim of this paper is to show how the viscoelastic and temperature dependent nature of rubber can be demonstrated using simple experiments. The viscoelastic behavior of rubber has applications in tire technology, earthquake protection bearings, vibration damping, and seals (the Challenger Space Shuttle disaster was caused by such behavior). It is a subject in which the structure-property relations of a material can be related from the molecular level (how polymer chain segments move) to the macrolevel (the mechanical properties) and then to real-world situations. Despite its importance, it is a difficult subject to convey to students. The following study presents an interesting, amenable, and alternative way of demonstrating the principles of viscoelastic behavior.

II. BACKGROUND

The most important feature of squash balls is that they have low rebound resilience. The resilience of a material can be thought of as its ability to absorb energy elastically on loading and then to release that energy when the material is unloaded. When a squash ball makes contact with racket strings, a wall, or the floor of a court, some of its energy is stored elastically in the rubber, some in the racket strings, and some in the increased internal air pressure. Some energy will be lost as sound, but more of the energy becomes internal thermal energy in the ball itself. This energy has two effects—the air inside the ball becomes pressurized and the rubber compound from which the ball is made becomes more resilient. As a result, the ball bounces higher. The playing temperature of the squash ball is usually around 45 °C, which is achieved after the ball has been warmed up by the players.[2] This temperature is where equilibrium is reached, and the thermal energy lost to the strings, walls, floor, and air equals the energy gained from deformation.

The rebound resilience is defined as the ratio of the energy remaining in the ball after an impact to the energy before impact. It is related to the coefficient of restitution (COR), which is the ratio of speeds before and after impact. Although the rebound resilience is often easier to measure, the COR is more directly relevant to a squash game because it governs the speed of a bounce away from a wall and the floor. A perfectly elastic collision has COR = 1 and a rebound resilience of 100%. In this case, the ball bounces and returns to the height at which it was dropped. A perfectly inelastic collision has COR = 0, for example, a spherical lump of soft putty. No balls are perfectly elastic, although hard rubber "superballs," solid metal, and glass marbles bouncing on a rigid and elastic surface come close.

Balls used for all sports exhibit some energy loss due to damping. Several models have been developed for the behavior of balls, which most commonly use combinations of masses, springs, and dampers to account for energy losses.[3–5] These models can account reasonably well for the change in behavior with speed,[3,6] but have not been used to deal with the effects of changes in temperature.

The most common ways of assessing the properties of balls are to measure the coefficient of restitution and the force required to compress the ball. Measurements of the coefficient of restitution are most commonly made by dropping balls onto hard rigid surfaces and measuring the height of rebound.[7,8] This height can then be converted into the speed ratio. Drop tests only give fairly low impact speeds, which means that measurements at higher speeds require more advanced measurement systems such as high speed video photography.[6] The measurement of the coefficient of restitution is relevant to the impact speed seen during play, although in squash, a range of speeds are seen, from the high speed rebound of a hard shot into the wall compared to a gentle drop-shot against the wall and then onto the court floor.[9]

The other common method is to determine the force required to compress the ball. The compression behavior is important because it determines the forces and time scales of impact. For instance, a more compressible ball will have a slower impact and lower peak forces, which usually means lower injury potential. However, more compressible baseballs have a greater chance of chest injury due to the impact, which produces resonant vibrations in the chest cavity and vital organs.[11] A study of lacrosse balls found that more compressible balls had a greater chance of passing through face guards.[10] Although squash balls have less potential for injury, there is major concern over impacts near the eye socket. For this reason, the World Squash Federation specifies a maximum compression stiffness.[12] Compression tests are done at much lower speeds than that found in an impact.

By continually measuring the force-displacement behavior, the hysteresis of the ball (percentage energy lost during compression and unloading) can be measured, which can then be related to the dynamic energy losses measured by the coefficient of restitution.[7,8] Ideally, the speed of compression should match the impact speed, but this ideal is difficult to achieve. Scarton *et al.*[11] developed a dynamic hardness measure from impact force measurements and claimed that this measure relates more to injury potential.

The aim of our investigation was to examine the mechanics of two types of squash balls using methods amenable to most educational institutions. To explore the effects of time scale and temperature on the energy lost during deformation of a ball, two test methods were used. Drop tests were used to study the effects of increasing temperature on the coefficient of restitution. We also measured the force required to compress the squash balls as a function of speed and temperature.

Previous research suggests that the elasticity of the rubber is the determining feature of bounce height rather than the pressure of the air in the ball.[13] This suggestion was investigated by compressing balls with small holes to allow free movement of air in and out of the ball. These results allow us to make comparisons of ball stiffness and rebound resilience with the World Squash Federation specifications.[12]

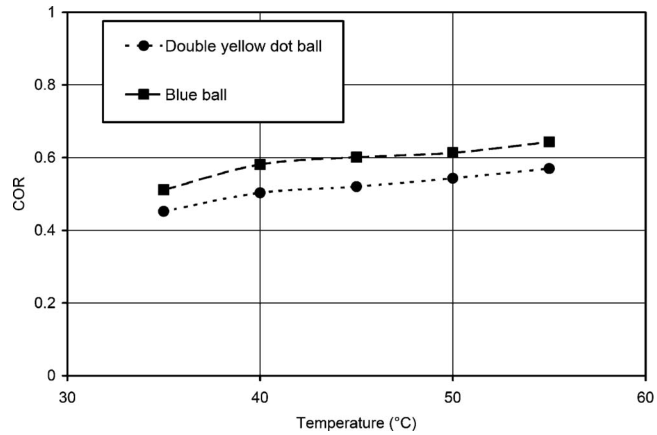

Fig. 1. The temperature dependence of the coefficient of restitution for two types of squash balls.

III. MEASUREMENT OF COEFFICIENT OF RESTITUTION

We used Dunlop international competition double yellow dot balls and Dunlop Max blue balls. The double yellow dot ball is used for competitions and has a low resilience. The Max blue ball has a diameter of 44.8 mm, which is 12% larger than the standard size of 40 mm, has a higher resilience and a 40% longer time between vertical bounces than the double yellow dot, and is designed more for beginners.

The coefficient of restitution of the squash balls was measured using a simple drop test. The collision was between the ball and a wooden squash court floor, creating a testing environment reflecting the nature of the game. The balls were dropped from the viewing balcony of a standard squash court onto the court below (a distance of 3.55 m). A Sony DCR-HC62E video camera operating at 50 frames/s on a rigid tripod was used to record the bounce of the squash ball against a background including a 1 m calibration rule. Six of each type of squash ball were tested at different temperatures achieved by immersion in an electrically powered thermostatic water bath set to various temperatures. The temperature of the ball on impact is somewhat lower due to cooling during the drop, but by dropping balls immediately after removal from the water bath, the cooling is small.

The coefficient of restitution was determined using the relation

$$\mathrm{COR} = \sqrt{\frac{h_2}{h_1}}, \tag{1}$$

where h_1 is the drop height and h_2 is the bounce height. The effects of air resistance were taken into account when calculating the coefficient of restitutions from rebound heights using a standard drag coefficient for a sphere and was shown to give about a 3% difference. Although this correction is not described here, such a calculation can be used to demonstrate the use of differential equations and can be solved either analytically or numerically.

The vertical speeds just before impact (about 8.2 m/s) and just after impact were measured from the video images. Although this method is not as precise, the results were in agreement with the coefficient of restitution calculated from rebound heights. Figure 1 shows the results, from which it can be seen that the coefficient of restitution is between 0.45

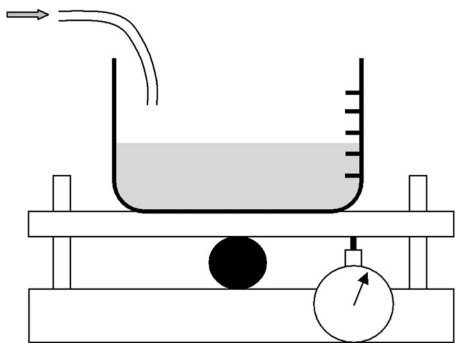

Fig. 2. Simple apparatus for measuring compression and recovery.

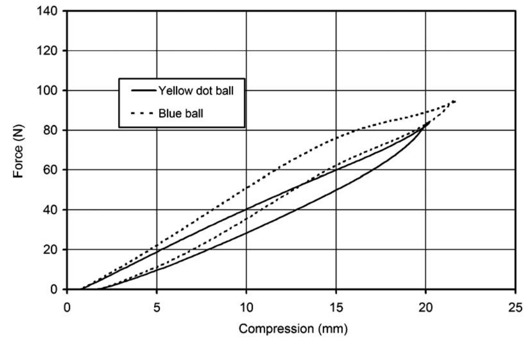

Fig. 4. Force/compression behavior at 100 mm/min for two types of squash balls.

and 0.65 (resilience between 20% and 35%), increases with increasing temperature, and is higher for the Max blue balls than for the double yellow dot balls. In a real squash game, the ball speeds could be considerably higher. The World Squash Federation specifications for the double yellow dot ball states that the resilience should be between 26% and 33%, as measured in a drop test at 45 °C. Our results give a value of 27.1% ± 0.5%, which is just inside the lower limit set by the World Squash Federation.

IV. COMPRESSION TESTS

Compression tests were performed using a Hounsfield 25 kN mechanical testing machine with balls compressed between parallel steel plates. The plates were moved together at a constant rate until the deformation reached 20 mm, at which the plates were moved apart at the same rate. The compression rates were 10, 30, 100, 300, and 1000 mm/min. The testing machine includes a load cell above the top compression plate. This cell comprises of a stiff bending beam, which is instrumented with strain gauges and allows the force to be recorded continuously during each test. Although this type of equipment is standard to materials testing laboratories, not all physics departments have access to this equipment, and a simpler compression apparatus can be constructed using a water tank for loading and a dial gauge, as shown in Fig. 2. Loading and unloading can be achieved rapidly by filling and emptying tubes, and a video camera can be used to record and then analyze a fast moving dial gauge and the liquid level. The compression to 20 mm was chosen as being half of the ball diameter. This compression is realistic because the energy of compression to 20 mm is later shown to be about 1 J, similar to the kinetic energy of the squash ball in the drop tests.

For each test speed, two balls of each type were each tested twice. In this way, each ball was tested twice at each speed and variability from ball to ball could also be assessed. The first test conducted on a ball gave a higher force than subsequent tests due to the well-known Mullins effect,[14] whereby weak bonds formed during manufacture are broken on the first deformation. Following the first compression, which was discounted, the results were reproducible, with standard deviations of the order of 5%. The Mullins effect was not significant for the drop tests because all balls were dropped several times before the actual measurements.

Typical force/compression curves are shown in Fig. 3 for the double yellow dot ball at various test speeds. The two faster tests shown are offset by 60 and 120 N to show the behavior clearly on one graph. Figure 3 shows that the force/compression behavior is not quite linear, with a slight upward curvature. The force on unloading is smaller, giving a hysteresis loop between loading and unloading. As the test speed increases, the forces increase and the area of the hysteresis loop also increases. Figure 4 shows the force/compression behavior for the double yellow dot balls and the Max blue balls, both at a test speed of 100 mm/min. The data show that the Max blue balls require a slightly higher force to achieve the same compression, not surprising given their larger size. There is little difference in the size of the hysteresis loops.

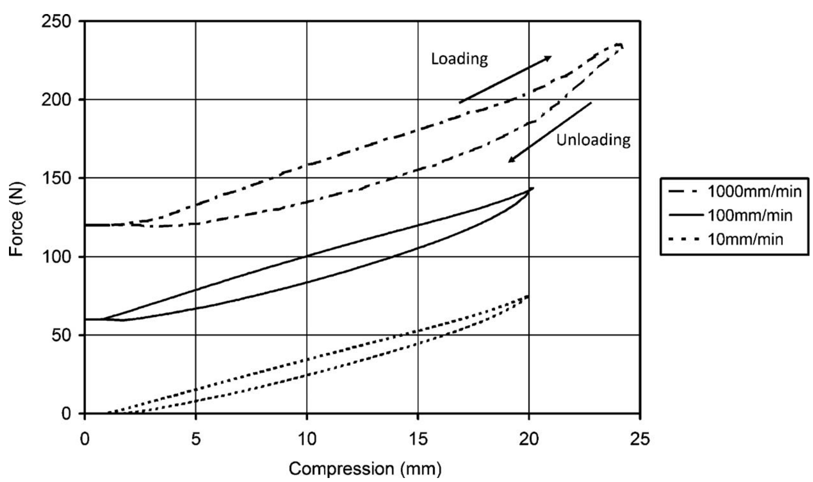

Fig. 3. Force/compression behavior of yellow dot balls at different compression rates.

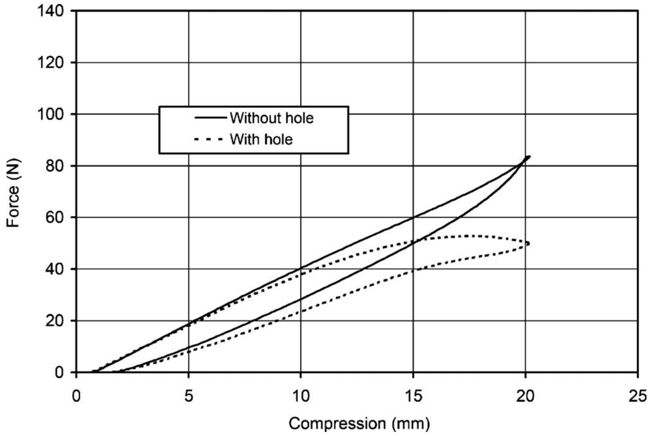

Fig. 5. Force/compression behavior at 100 mm/min for yellow dot balls with and without a hole.

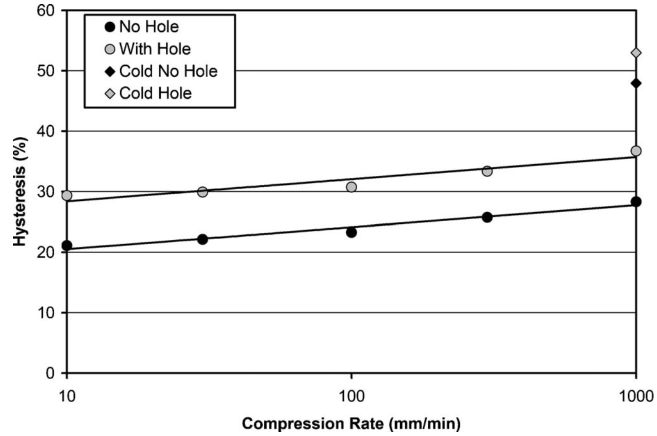

Fig. 7. Variation of hysteresis with rate and temperature for yellow dot balls.

There has long been considerable debate over the relative importance of the rubber and internal air for squash balls.[12] To investigate this importance, compression tests were conducted with squash balls with holes. One small hole of about 3 mm diameter was cut in each ball using a scalpel. The ball was then compressed as described, with the hole carefully positioned at the base of the test so that the effect of the hole on the rubber deformation is minimized. Because the purpose of the hole is to allow free passage of air in and out of the ball, the ball was placed on a steel disk that had a central hole linked to a groove on the base of the disk, thereby allowing free passage of air. The force/compression behavior of the double yellow dot balls with and without holes is shown in Fig. 5. The data show that the initial behavior is almost identical. Only when the compression has reached 15 mm does the hole make a significant difference, causing a decrease in the force. The area of the hysteresis loop is about the same size. Measurements of the area under the loading and unloading curves show that the energy lost by a ball with a hole is only slightly smaller than one without a hole. The force at 20 mm compression is 82 N without a hole, compared to 50 N with a hole, indicating that at this degree of compression about 61% of the compression force of a squash ball arises from rubber deformation, with about 39% arising from compression of the air inside the ball. More significantly, almost all of the energy lost during a compression comes from deformation of the rubber.

There is a widely held view that the purpose of warming up a squash ball is to achieve more bounce due to the internal air warming and the pressure increasing.[2] This interpretation is incorrect because it is the rubber that determines almost all of the energy loss, and it is the increase in the rubber temperature that gives a higher bounce.

To demonstrate the effects of deformation speed and temperature on the behavior of squash balls, compression tests were also conducted with cold balls. Although it is possible to use a low temperature chamber around the test machine, such facilities are not that common, and hence a simpler method was used of precooling the balls in a freezer, followed by immediate testing at high speed. Balls were returned to the freezer for at least 1 h between tests. Figure 6 shows the force/compression curves for the double yellow dot balls at 23 and 0 °C, both at a speed of 1000 mm/min. The data show that it takes larger forces to compress the cold ball, and there is also a significantly larger hysteresis loop, which would mean more energy lost during compression and so a lower bounce.

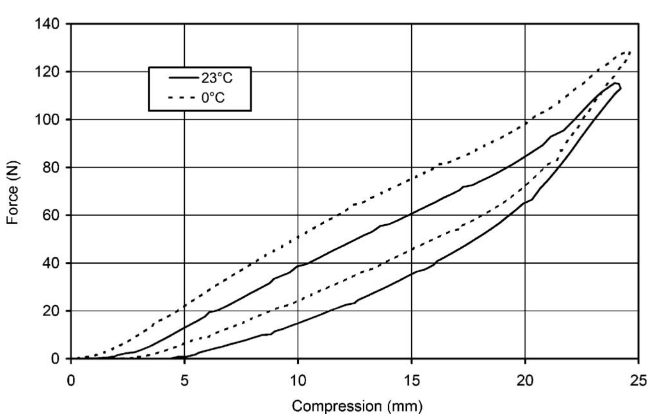

Fig. 6. Force/compression behavior at 1000 mm/min for yellow dot balls at 23 and 0 °C.

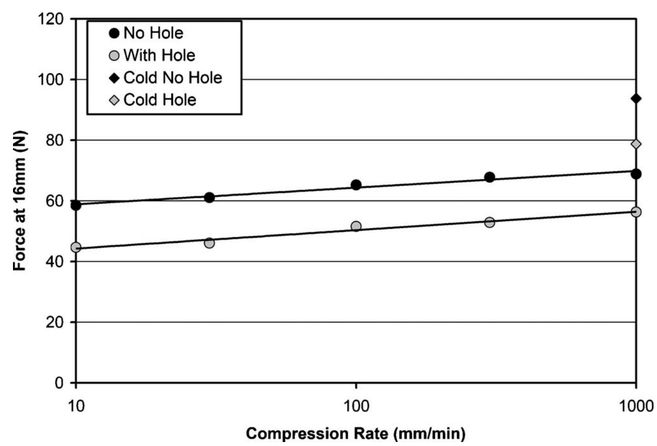

Fig. 8. The variation of force at 16 mm compression with rate and temperature for double yellow dot balls.

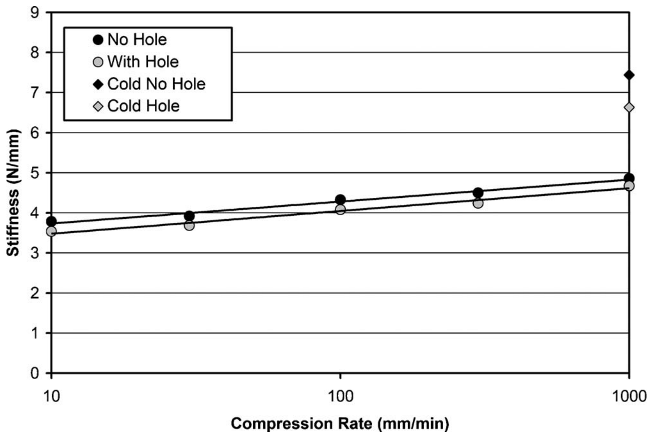

Fig. 9. The variation of stiffness with rate and temperature for double yellow dot balls.

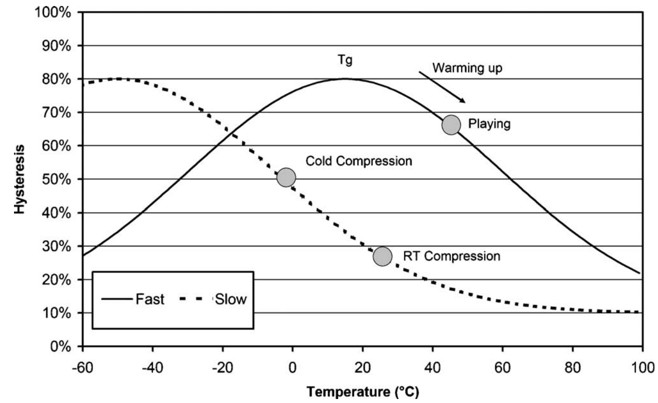

Fig. 10. The effects of temperature and rate on the hysteresis of rubber.

V. THE EFFECTS OF TEMPERATURE AND RATE ON COMPRESSION TESTS

The hysteresis was determined by measuring the area between the loading and the unloading curves and dividing by the area under the loading curve. Figure 7 shows the average hysteresis for the double yellow dot balls plotted against the loading rate, with the effects of holes and low temperature also shown. We see that the hysteresis increases slightly as the rate of compression increases and more significantly as the temperature decreases. The hysteresis values of the balls with holes are larger, reflecting the lower energy stored rather than any more energy lost. The Max blue balls exhibited a similar behavior.

Figure 8 shows the force at 16 mm deformation, which also increases with the loading rate and with lower temperature. The balls with holes have considerably lower values because the contribution from compressing the air is now absent. Figure 9 shows the stiffness of the balls as determined from the slope of the loading curve. The stiffness increases with loading rate and with lower temperature, but shows little effect of the holes, indicating that the compression of the air only becomes important with more compression.

VI. ENERGY LOSS

By measuring the hysteresis loops of the compression tests and comparing them with the rebound tests, it is possible to investigate the effects of time scale and temperature on the percentage of energy lost during deformation. The coefficient of restitution values were converted to hysteresis as

$$\text{Hysteresis}(\%) = (1 - \text{COR}^2)100. \tag{2}$$

The time scale of the impacts in the bounce tests was estimated by comparing the change in momentum (from the coefficient of restitution results) with a typical average force during impact taken to be 50 N from the compression tests. The average hysteresis values for each test condition are shown in Table I. We see that as the rate of deformation increases, the hysteresis increases, and as the temperature increases, the hysteresis decreases. This dependence is an excellent example of the time-temperature equivalence of the time-dependent (viscoelastic) mechanical behavior of polymers. The mechanical properties arise from the movement of polymer chains. More polymer chain movement occurs with slower compression rates (more time) or at higher temperatures (more mobile chains). This mechanism of deformation leads to the idea that slower compression tests at a low temperature are equivalent to faster compression rates at a higher temperature.

Table I. Hysteresis determined from bounce and compression tests. The last two columns are in percent.

Conditions	Deformation time scale (ms)	Hysteresis of double yellow dot balls	Hysteresis of Max blue balls
Bounce at 35 °C	5.5	79.5	73.7
Bounce at 40 °C	6	74.7	66.1
Bounce at 45 °C	6	72.9	63.6
Bounce at 50 °C	6	70.5	62.1
Bounce at 55 °C	6	67.5	58.4
Compression 10 mm/min 23 °C	240,000	21.1	21.4
Compression 30 mm/min 23 °C	80,000	22.1	21.5
Compression 100 mm/min 23 °C	24,000	23.3	22.4
Compression 300 mm/min 23 °C	8,000	25.7	23.6
Compression 1000 mm/min 23 °C	2,400	28.3	25.0
Compression 1000 mm/min 0 °C	2,400	47.9	43.1

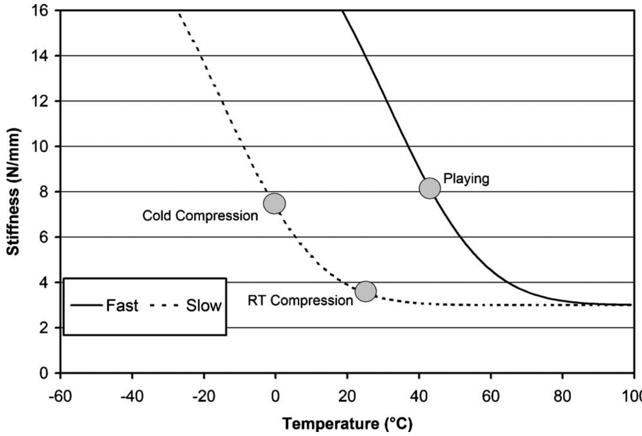

Fig. 11. The effects of temperature and rate on the stiffness of rubber.

The hysteresis of a polymer is related to its glass-transition temperature T_g, the region where its behavior changes from a hard rigid glassy material (below T_g) to a softer rubbery material (above T_g).[15] Near T_g the stiffness decreases rapidly with temperature, and the hysteresis increases.[15] This behavior occurs because some chain segments are flexible, while some are still more immobile; moving one past another causes energy losses.

Figure 10 shows a schematic of the effect of time scale and temperature on hysteresis for rubber, where T_g (defined as the peak of the curve) is below room temperature. The two curves represent different deformation rates. The faster deformation leads to a T_g that is significantly higher than for the slow rate. The behavior of squash ball rubber is indicated by the gray circles. The playing conditions at 45 °C and fast deformation rates give a large hysteresis. The effects of warming are indicated by the arrow, where increasing temperature leads to a decrease in hysteresis (more bounce). The much slower compression tests are also shown, which explains why the hysteresis is less for the compression tests than for the bounce tests. A more detailed analysis of the results shows that the hysteresis values from the rebound tests are higher than those from the compression tests, even accounting for the effects of loading rate and temperature. The explanation comes from the differences between the two types of tests. In the compression test, the ball is compressed equally on both sides between steel plates at a constant rate. In the rebound test compression is on just one side, onto a slightly compliant wooden floor, and the rate of compression varies through the bounce as the ball slows down and then rebounds. The equivalent curves for the blue balls would be shifted slightly to the left because the different rubber compound has a slightly lower value of T_g.

Figure 10 also shows that if compression tests are to be used to predict the playing behavior of a squash ball, it is better to perform the tests cold as the behavior will be closer to the behavior of a ball during play. This point is more clearly illustrated in Fig. 11, which shows the ball stiffness plotted against temperature for two deformation rates. The stiffness drops dramatically with increasing temperature near T_g and does so at a higher temperature for the faster rate. If the compression tests are used to determine the injury potential of a squash ball, then compression at low temperatures would give a better indication of the stiffness under playing conditions than compression at room temperature, as is currently specified by the World Squash Federation.[13]

VII. COMMENTS

The use of much slower compression tests needs to be treated with caution for predicting squash ball behavior. If such tests are to be used, it is better to use balls cooled to a lower temperature. The use of lower temperatures means the properties are closer to those of much faster rate tests at higher temperatures as found under playing conditions. This understanding suggests that the specifications of the World Squash Federation regarding compression should be modified.

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