ProMCDA: A Python package for Probabilistic Multi-Criteria Decision Analysis

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Summary

Multi-Criteria Decision Analysis (MCDA) is a formal process to assist decision makers (DMs) in structuring their decision problems and to provide them with tools and methods leading to recommendations on the decisions at stake (Roy (1996)). The recommendations are based on a comprehensive identification of the alternatives considered and the selection of criteria/subcriteria/etc. to evaluate them, which are aggregated taking into account the preferences of the DMs (Bouyssou et al. (2006)). In the literature, there is a wide range of MCDA methods used to integrate information and either classify alternatives into preference classes or rank them from best to worst (Cinelli et al. (2022)). In the context of ranking and benchmarking alternatives across complex concepts, composite indicators (CIs) are the most widely used synthetic measures (Greco et al. (2019)). Indeed, they have been applied, for example, in the context of environmental quality (Oţoiu and Grădinaru (2018)), resilience of energy supply (Gasser et al. (2020)), sustainability (Volkart et al. (2016)), global competitiveness (Klaus Schwab (2018)), etc. However, the uncertainty of the criteria, the effect of tuning the weights relative to them, and the choice of methods (normalization/aggregation) to construct CIs, have been shown to influence the final ranking of alternatives (e.g. Cinelli et al. (2020)).

The `ProMCDA` Python module proposed here allows a DM to explore the sensitivity and robustness of the CIs results in a user-friendly way. In other words, it allows the user to assess either the sensitivity related to the choice of normalization and/or aggregation method, but also to account for uncertainty in the criteria and weights.

Statement of need

There are already dedicated tools for estimating CIs in the literature. In *R*, there is an existing package called `COINr`, which allows the user to develop CIs by including all common operations, from criteria selection, data treatment, normalization and aggregation, and sensitivity analysis (Becker et al. (2022)). There are also other packages in R, such as `compind`, that focus on weighting and aggregation (Fusco, Vidoli, and Sahoo (2018)). In *MATLAB*, there are some packages dedicated to specific parts of CI development, such as the `CIAO` tool (Lindén et al. (2021)). The Python module `Decisi-o-Rama` (Chacon-Hurtado and Scholten (2021)) focuses on the implementation of the Multi-Attribute Utility Theory (MAUT) to normalize criteria, considering a hierarchical criteria structure and uncertain criteria, and to aggregate the results using different aggregation methods. Finally, the web tool called `MCDA Index Tool` allows sensitivity analysis based on different combinations of normalization functions and aggregation methods (MCDA Index Tool).

`ProMCDA` is a Python module for performing CIs MCDA considering a full probabilistic approach. The tool provides sensitivity and robustness analysis of the ranking results. The sensitivity of the MCDA scores is caused by the different pairs of normalization/aggregation functions (Cinelli et al. (2020)) that can be used in the evaluation process. The uncertainty is instead caused by either the variability associated with the criteria values (Stewart and Durbach (2016)) or the randomness that may be associated with their weights (Lahdelma, Hokkanen, and Salminen (1998)). `ProMCDA` is unique in combining all these different sources of variability and providing a systematic analysis.

The tool is designed to be used by both researchers and practitioners in operations research. The approach has a wide range of potential applications, ranging from sustainability to healthcare and risk assessment, to name but a few. `ProMCDA` has been developed as a core methodology for the development of a decision support system for forest management (FutureForest). However, the tool is generic and can be used in any other domain involving multi-criteria decision-making.

Overview

`ProMCDA` is a module consisting of a set of functions that allow CIs to be constructed considering the uncertainty associated with the criteria, the weights and the combination of normalization/aggregation methods. The evaluation process behind `ProMCDA` is based on two main steps of data manipulation:

- data normalisation, to work with data values on the same scale;
- data aggregation, to estimate a single composite indicator from all criteria.

`ProMCDA` receives all the necessary input information via a configuration file in JSON format (for more details see the README). The alternatives are represented in an input matrix (in CSV file format) as rows and described by the different values of the criteria in the columns. The sensitivity analysis is performed by comparing the different scores associated with the alternatives, which are obtained by using different combinations of normalization and aggregation functions. `ProMCDA` implements 4 different normalization and 4 different aggregation functions, as described in Table 1 and Table 2 respectively. However, the user can decide to run `ProMCDA` with a specific pair of normalization and aggregation functions, and thus switching off the sensitivity analysis.

Table 1: Normalization functions used in `ProMCDA`.

Normalization methods		Formula	Description	Comments
Linear scale	**Min-max**	$N_{ia} = \frac{x_{ic} - \min(x_i)}{\max(x_i) - \min(x_i)}$	It applies a linear transformation to rescale the data in a specified range (typically 0-1).	Most common normalization method used. The order and proximity of the data points are maintained. Outliers can have a significant impact on the transformation. Loss of information: it compresses the range of the original data.
	Standardization (z-score)	$N_{ia} = \frac{x_{ia} - x_{ia=\bar{a}}}{\sigma_{ia=\bar{a}}}$	It applies a linear transformation with mean of 0 and standard deviation of 1.	The order and proximity of the data points are maintained. The standardized data is not bounded. High values have a great impact on the result, which is desirable if the wish is to reward exceptional behaviour. It preserves the shape and distribution of the original data. Loss of information: none.
Ratio scale	**Target**	$N_{ia} = \frac{x_{ia}}{\max\,(x_i)}$	It normalizes the upper limit to 1.	The order and proximity of the data points are maintained. No fixed range. Sensitive to outliers. It can be useful when the maximum value is of particular interest or importance in the analysis. Loss of information: it can reduce the relative differences between values, potentially compressing the data.
Ordinal	**Rank**	$N_{ia} = rank(x_{ia})$	The data points are ranked based on their relative values.	The order and proximity of the data points are maintained. It does not impose a fixed range on the transformed data. It eliminates magnitude differences. It can be useful when the exact values are not important, but rather the relative positions or comparisons between values. Robust to outliers.

Legend

N_{ia}: the normalized value of indicator *i* for alternative *a.*
x_{ia}: the value of indicator i for alternative *a.*
$x_{ia=\bar{a}}$the average value of indicator *i* across all alternatives.
$\sigma_{ia=\bar{a}}$: the standard deviation of indicator *i* across all alternatives.
$min(x_i)$: the minimum value of indicator *i* across all alternatives.
$max(x_i)$: the maximum value of indicator *i* across all alternatives.

Table 2: Aggregation functions used in `ProMCDA`.The sum of the weights is normalized to 1 as in Langhans, Reichert, and Schuwirth (2014).

Aggregation methods	Formula	Level of compensation	Comments
Additive (weighted arithmetic mean)	$score_a = \sum_{i=1}^{n} N_{ia}\, w_i$	Full	Most common aggregation method used. It is a linear combination. It amplifies the effect of the higher values. Commonly used in situations where variables are considered equally important.
Geometric (weighted geometric mean)	$score_a = \prod_{i=1}^{n} {N_{ia}}_i^{\,w_i}$	Partial	The indicators values should be larger than 0. It is a non-linear combination. The impact of each variable's value is not proportional to its magnitude, and the relative contribution of each variable depends on the other variables involved. It amplifies the impact of variables with small values. The method is commonly used in situations where the interaction or joint effect of variables is of interest.
Harmonic	$score_a = \frac{\sum_{i=1}^{n} w_i}{\sum_{i=1}^{n} \frac{w_i}{N_{ia}}}$	Partial (less than Geometric)	The indicators values should strictly be larger than 0. It is a non-linear combination. The impact of each value is not proportional to its magnitude, and the relative contribution of each variable depends on the other variables involved. Insensitivity to extreme values. It is primarily used in situations where smaller values are considered more important or when dealing with ratios or rates.
Minimum	$N_{ia} = min(N_{1a}, N_{2a}, \ldots, N_{na})$	None	The worst performing indicator equals the final score. Suitable if the DM is interested in an assessment driven by the worst performing indicator.
Legend $score_a$: the composite score for alternative *a*. n: the number of indicators. w_i: the weight of indicator *i*. N_{ia}: the normalized value of indicator *i* for alternative *a*.			

The user can also decide to run ProMCDA with or without a robustness analysis. The robustness analysis is triggered by adding randomness to either the weights or the criteria. This means that either the weights or the criteria values are randomly sampled using a Monte Carlo method. In ProMCDA randomness is not allowed for both weights and criteria in order to make the results as transparent as possible. In fact, mixing uncertainty from both weights and criteria would lead to a lack of distinction between the effect of one or the other. Randomness in the weights can be applied to one weight at a time or to all weights at the same time. In the first case, the aim is to be able to analyse the effect of each individual criteria on the scores; in the second case, it is to have an overview of the uncertainty associated with all the weights. In both cases, by default, the weights are sampled from a uniform distribution [0-1]. On the other hand, if the user decides to analyse the robustness of the criteria, he/she has to provide the parameters defining the marginal distribution (i.e. a probability density function, pdf) that best describes the criteria, rather than the criteria values. This means that if a criterion is characterized by a pdf described by 2 parameters, two columns should be allocated in the input CSV file for it. In ProMCDA 4 different pdfs describing the criteria uncertainty are considered:

- *uniform*, which is described by 2 parameters, i.e., minimum and maximum
- *normal*, which is described by 2 parameters, i.e., mean and standard deviation
- *lognormal*, which is described by 2 parameters, i.e., log(mean) and log(standard deviation)

- *Poisson*, which is described by 1 parameter, i.e., the rate.

Once the pdf for each criterion is selected and the input parameters are in place in the input CSV file, `ProMCDA` randomly samples n-values of each criterion per alternative from the given pdf and assesses the score and ranking of the alternatives, by considering robustness at the criteria level. The number of samples is given in the configuration file by the user.

Once the pdfs for each criterion are selected and the input parameters are in the input CSV file, `ProMCDA` randomly samples n-values of each criterion per alternative from the given pdf to evaluate the score and ranking of the alternatives, taking into account robustness at the criteria level.

Finally, in all possible cases (i.e. a simple MCDA; MCDA with sensitivity analysis for the different normalization/aggregation functions used; MCDA with robustness investigation related either to randomness on the weights or on the indicators), `ProMCDA` will output a CSV file with the scores/average scores and their plots. For a quick overview of the functionality of `ProMCDA`, refer to Table 3. For more details, refer to the README.

Table 3: Overview on the functionalities of ProMCDA.

Possible usages of ProMCDA	Specifications	Notes
Simple MCDA No sensitivity nor robustness analysis is performed.	The specific pair normalization /aggregation to be used for the evaluation of the alternatives.	For a fully controlled MCDA.
Sensitivity analysis Focus is on the role of the normalization and aggregation functions.	All normalization and aggregation pairs are used for the evaluation of the alternatives.	Each pair normalization/aggregation will produce different scores for every alternative. **The sensitivity analysis can be associated with the robustness analysis due to the weights or the indicators.**
Robustness analysis of one weight at time Focus is on the role of one indicator and its relative weight at time.	One single weight at time is sampled from the uniform distribution [0,1].	This run can help investigate the importance of each indicator for the final scores. Average results are reported a number-of-indicator times. ***This robustness analysis cannot be used together with the robustness analysis associated with the indicators.***
Robustness analysis of all weights Focus is on the role of the weights.	All weights are sampled from the uniform distribution [0,1].	This run can help understanding the overall impact of the uncertainty due to the weights. ***This robustness analysis cannot be used together with the robustness analysis associated with the indicators.***
Robustness analysis of the indicators Focus is on the role of the uncertainty of the indicators.	All indicators, whose values are distributed as a non-exact pdf, are randomly sampled. *ProMCDA* needs N-values for each indicator per alternative to build N random input-matrices.	This run let the user analyse the impact of the uncertainty on the indicators for the final scores. ***This robustness analysis cannot be used together with the robustness analysis associated to the weights.***

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