MTH 493/593 COMPUTATIONAL COMMUTATIVE ALGEBRA SPRING 2020

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Schedule: TR 4:00-5:50 pm

Credit Hours: MTH 493 is 3 credits, MTH 593 is 4 credits

What is Computational Commutative Algebra? Polynomial functions and polynomial equations are very common in mathematics. One of the objectives of commutative algebra is to study the properties of systems of polynomial equations and their solutions. The computational aspect comes from developing algorithms that allow one to answer many problems about systems of polynomial equations using computers. Problems of interest include:

- Finding solutions symbolically (as opposed to approximating solutions numerically);
- Eliminating variables (find Cartesian equations for a rational parametric curve/surface);
- Ideal membership problem (can a polynomial be written as a linear combination of a given set of polynomials?).

Course objectives. In order to address the questions above, we will explore the algebra of multivariable polynomials and develop the fundamental concept of a Gröbner basis, a special collection of polynomials with particularly convenient properties from a computational perspective. In order to find Gröbner bases, we will study monomial orders, monomial ideals, and the Buchberger algorithm. Participants will also be introduced to the basics of algebraic geometry, namely the study of geometric objects defined by algebraic equations. Further topics will be explored in student projects.

Prerequisites. For 493: A grade of C or better in MTH 288 and MTH 358, or permission of the instructor. For 593: MTH 514. For both: A willingness to work with abstract mathematics and proofs. Familiarity with basic computer programming is helpful.

Differences between 493 and 593. Students enrolled in 593 will cover more material and will receive additional homework problems. Moreover, 593 students will be asked to present some of their work in class.

Textbook. (tentatively) Cox, Little, O'Shea, *Ideals, Varieties, and Algorithms*, 3rd edition, Springer.