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CSE 803: Computer Vision
Homework 3
October, 27

1 RANSAC

1.1 Fitting a Line

1.1.1

For fitting $y = mx + b$, we need to sample **two** points in an iteration to comute a putative model. Because, that is a equation of a straight line, and to form a straight line, we need two points.

1.1.2

Given,

$$\begin{aligned} \text{outlier ratio, } r &= 0.1 \\ \text{number of points, } s &= 2 \end{aligned}$$

So, the probability that a putative model fails to get a desired model:

$$1 - (1 - r)^s = 19\%$$

1.1.3

Given,

$$\begin{aligned} \text{outlier ratio, } r &= 0.1 \\ \text{number of points, } s &= 2 \\ \text{success rate, } c &= 0.95 \\ \text{Number of trials, } N &= ? \end{aligned}$$

So, the probability that a putative model fails to get a desired model:

$$\begin{aligned} 1 - (1 - (1 - r)^s)^N &= 0.95 \\ \implies N &= 1.83 \approx 2 \end{aligned}$$

1.2 Fitting Transformations

1.2.1

The degrees of freedom of M is 4 and at least 4 samples are required to find M. Because, no conditions are mentioned regarding M and M has $2x2 = 4$ parameters/elements.

1.2.2

$$\boldsymbol{y = Mx}$$

$$\begin{pmatrix} y_1 \\ \vdots \\ y_N \end{pmatrix} = \begin{pmatrix} -m_{00} & m_{01} & \dots \\ \vdots & \vdots & \vdots \\ -m_{N0} & \dots & \dots \end{pmatrix} \times \begin{pmatrix} x_1 \\ \vdots \\ x_N \end{pmatrix}$$

Let,

$$b = \begin{pmatrix} y_1 \\ \vdots \\ y_N \end{pmatrix}$$

$$\boldsymbol{A} = \begin{pmatrix} x_1 \\ \vdots \\ x_N \end{pmatrix}^T$$

Then,

$$b_{predict} = \boldsymbol{A}m$$

So, the fitting problem can be written in:

$$\begin{aligned} &\underset{m}{\operatorname{argmin}} \, ||\boldsymbol{b_{predict}} - \boldsymbol{b}||^2 \\ \implies &\underset{m}{\operatorname{argmin}} \, ||\boldsymbol{Am} - \boldsymbol{b}||^2 \end{aligned}$$

1.2.3

$$S = \begin{pmatrix} 2.04098127 & -3.0696843 \\ -1.01644528 & 0.94335775 \end{pmatrix}$$

$$t = \begin{bmatrix} 1.87154926 \\ -3.05145812 \end{bmatrix}$$

1.2.4

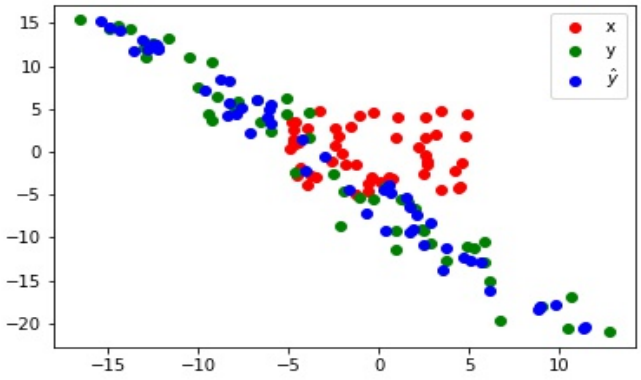

Figure 1: Plot of $x, y, \hat{y}$

1.2.5

Based on S and t, the transformation shrink (0.94) and scale up (0.94) the points. The diagonal elements 2.04 and 0.94 can be regarded as close to S_x and S_y in scaling transformation. Also, that transformation aligns the points as like they are negatively correlated (see other two diagonals). T denotes the intersection with axes of the points trend.

Yes, the fitting result is quite good. y and $\hat{y}$ almost overlaps for all data points.

1.2.6

The homography matrix $\boldsymbol{H}$ for all cases:

```
For case 0 [[-5.84619486e-01 -9.38188546e-04  7.85709946e-02]
 [-1.48861804e-03 -3.61934740e-01  4.27826653e-01]
 [-2.62615122e-05 -2.09196954e-05 -5.81387269e-01]]
For case 1 [[ 3.68911272e-01  8.92258703e-04 -3.93043451e-01]
 [ 2.75059839e-05  5.96020476e-01 -2.41643577e-02]
 [ 2.17658979e-07  1.34347784e-05  5.94639553e-01]]
For case 2 [[-2.53193239e-01 -3.99057800e-03  5.69038181e-01]
 [-5.45228988e-03 -2.53238879e-01  6.28011972e-01]
 [-1.21872179e-04 -1.08513265e-04 -3.91808736e-01]]
For case 3 [[ 4.29709571e-12  5.77350269e-01 -3.07598061e-10]
 [ 5.77350269e-01  1.15147653e-12 -9.97500158e-11]
 [ 5.31784754e-14  2.87491195e-14  5.77350269e-01]]
For case 4 [[-7.91063985e-03  2.28491079e-13  6.01208628e-01]
 [-5.35800683e-14 -7.91063985e-03  7.98974625e-01]
 [ 2.89392053e-15 -1.06881468e-15  7.91063985e-03]]
For case 5 [[ 1.38048111e-01  2.19005570e-01 -6.17163607e-01]
 [ 2.20297434e-01  1.36451811e-01 -6.05518007e-01]
 [ 7.60197607e-05  4.81267787e-05  3.43927668e-01]]
For case 6 [[ 1.34941272e-01  7.96016480e-03 -1.11241055e-02]
 [-1.37954817e-02  1.51370758e-01  9.63220032e-01]
 [-5.34941898e-05 -1.11646326e-05  1.75235524e-01]]
For case 7 [[-1.44485744e-01  8.35368093e-03 -7.39389853e-01]
 [ 8.19996491e-03 -1.69535496e-01 -6.11590373e-01]
 [ 7.82502801e-05 -1.85804532e-05 -1.71768495e-01]]
```

Figure 2: $\boldsymbol{H}$ for all cases

1.2.7

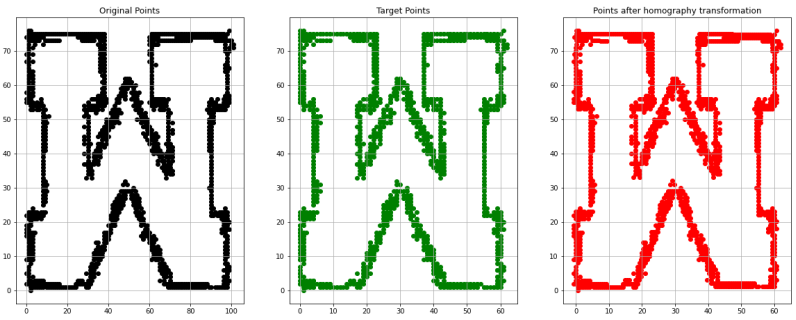

Figure 3: Visualization for case 0

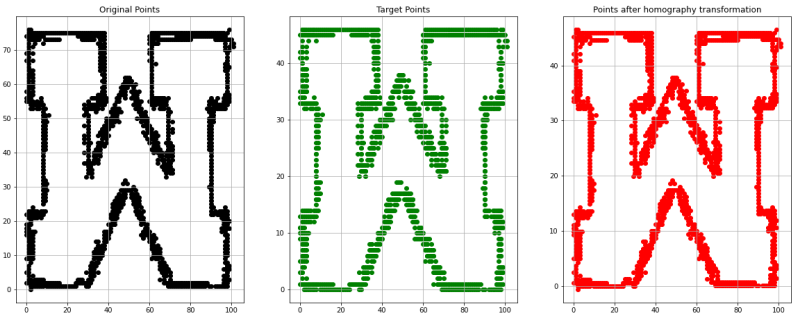

Figure 4: Visualization for case 1

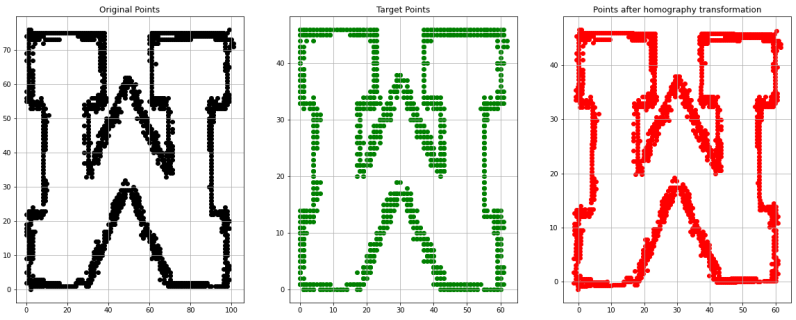

Figure 5: Visualization for case 2

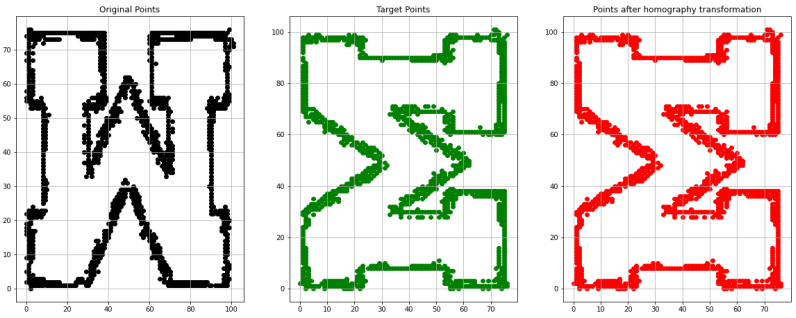

Figure 6: Visualization for case 3

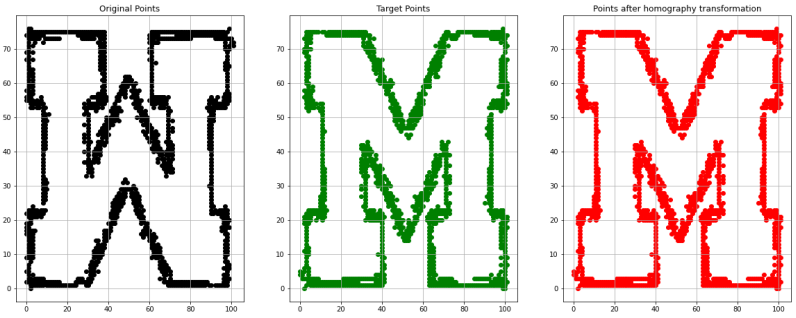

Figure 7: Visualization for case 4

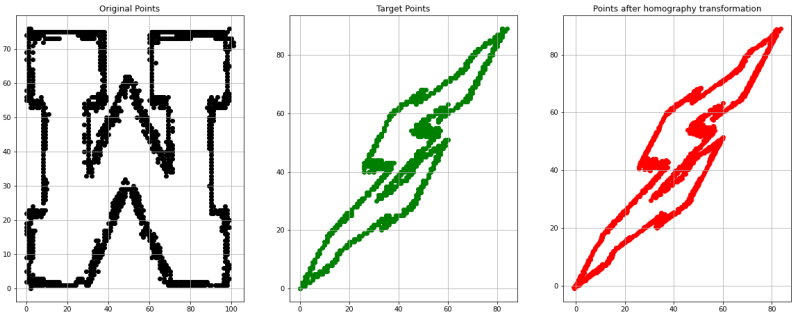

Figure 8: Visualization for case 5

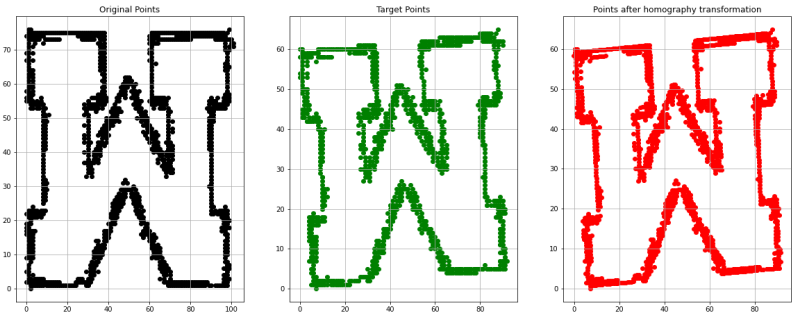

Figure 9: Visualization for case 6

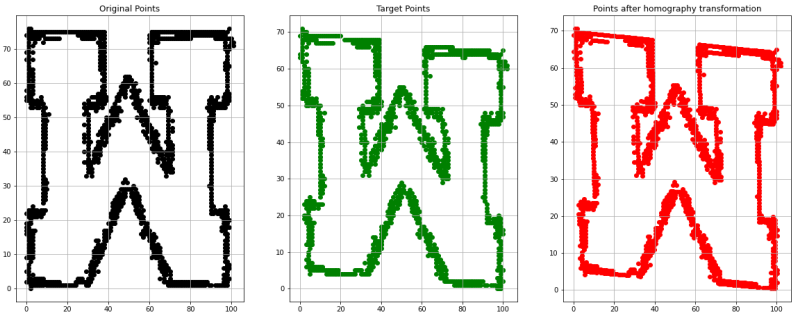

Figure 10: Visualization for case 7

1.2.8

Yes, all of these transformations make senses.

Like, in Case 0, Case 1, Case 2, the transformations look like same to the original input, but shading/points directions give a perspective representation for different shadings. Case 3 gives a 90 degree rotational perspective. Case 4 gives a 180 degree or flip perspective. Case 5 gives a shear perspective. Case 6 and Case 7 gives a perspective from upper-left and upper right viewpoints. So, all the transformations make sense.

2 Image Stitching

2.1

Figure 11: Left

Figure 12: Right

2.2

Here, SIFT is used as it looks like SIFT features are more appropriate than SURF feature points (according to given images)

Figure 13: Left image with SIFT feature points

Figure 14: Left image with SIFT feature points

2.3

I have experimented with normalized correlation and Euclidean distance. Here, I choose Euclidian distance of the descriptors. Before applying euclidian distance, all the descriptors are normalized (zero mean and unit standard deviation).

2.4

I have experimented with the selection of putative matches. Here, I choose the ratio test described in lecture (nearest-neighbor to second-nearest-neighbor ratio test). I have set the ratio as 0.5 and got the putative matches.

Among 5317 descriptors from the left image and 4195 descriptors from the right image, the number of putative matches based on 0.5 ration test is 721.

2.5

Number of inliers for the best fit: 49.
Average residual for the inliers: 5.638504690936416.

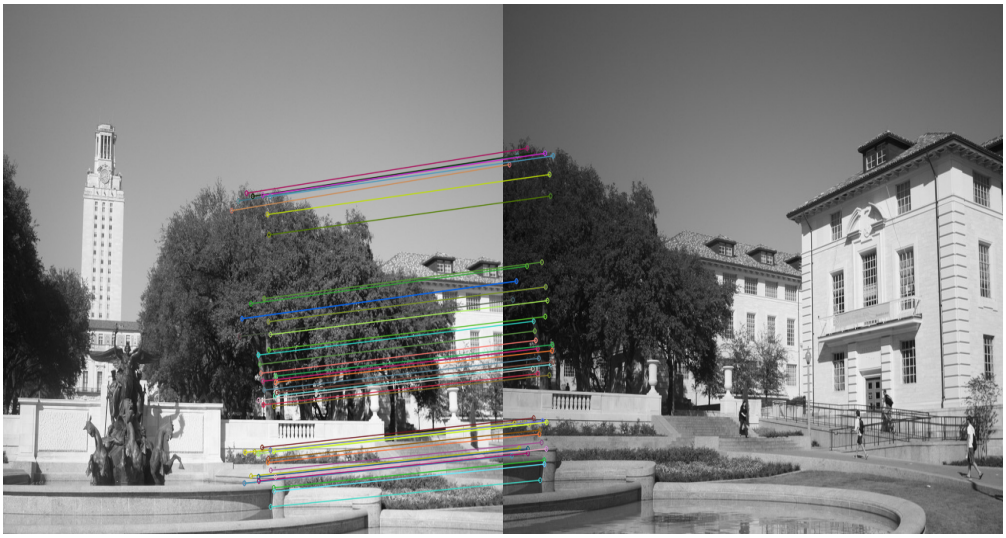

Figure 15: Inlier matches

2.6

I have Warped one image into another (see code, section 4 in the function).

Figure 16: Warped Image

2.7

Composite image with simple averaging pixel values.

Figure 17: Composite/Stitched Image

2.8

Number of inliers for the best fit: 45.
Average residual for the inliers: 10.424224.

Figure 18: Left image with SIFT feature points

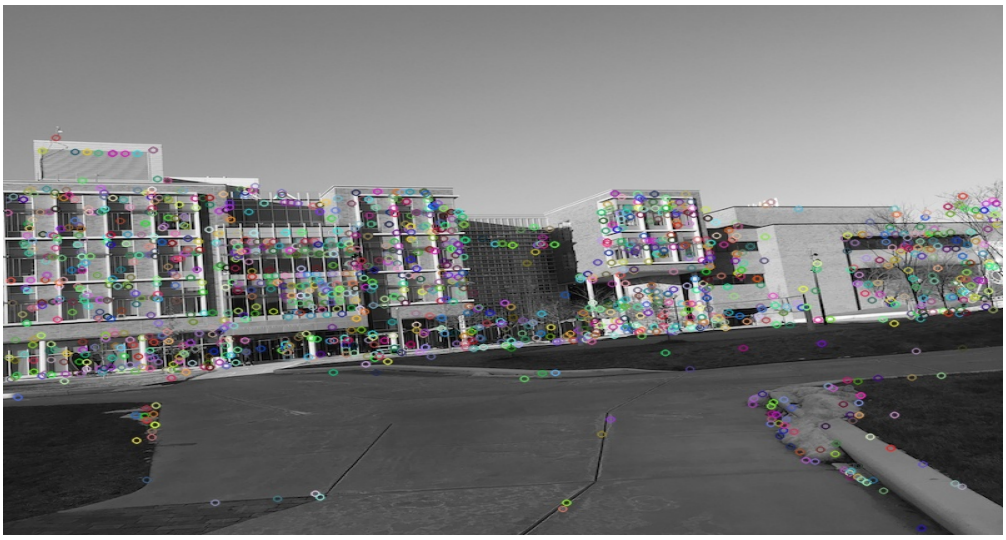

Figure 19: Right image with SIFT feature points

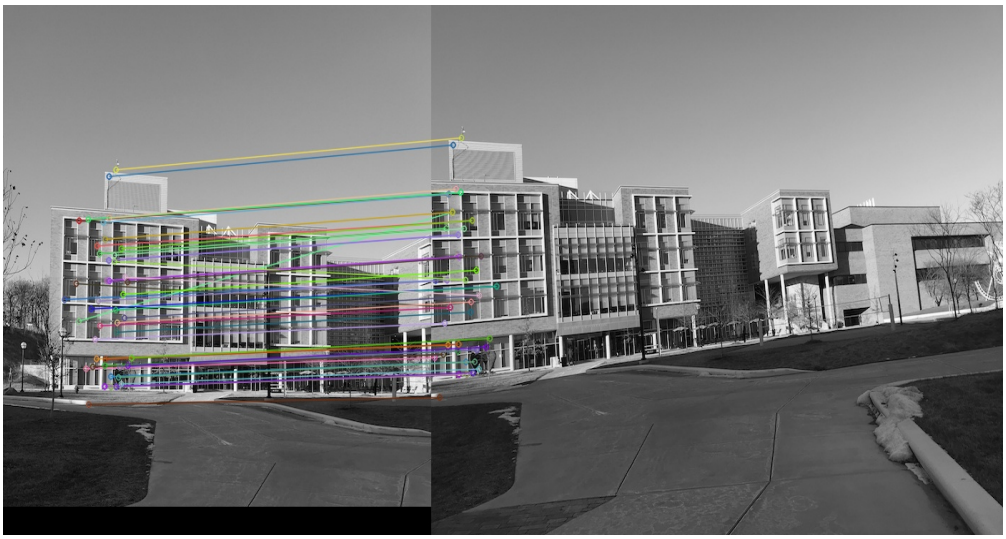

Figure 20: Matching inliers after RANSAC

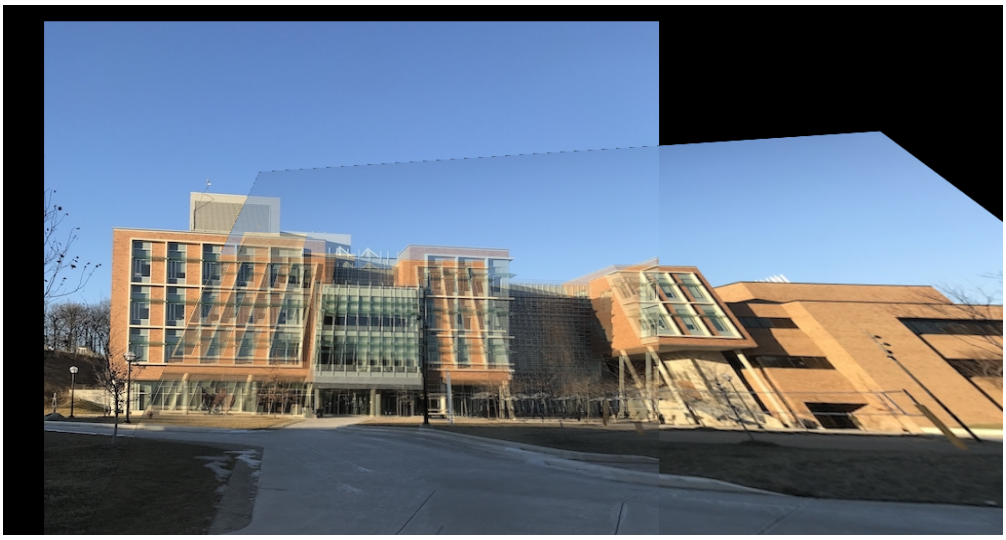

Figure 21: Composite/Stitched Image