Instructions: This homework is based on the background material necessary for the course, in terms of linear algebra and basic machine learning concepts. Please refer to the background slides for a helpful material.

- Filename: Submit solutions in PDF format titled written-assignment-4-msunetid.pdf. Submissions in other formats or with other filenames will not be graded. You can produce your file however you like, LATEX, Word or scan. Handwritten scans that are not legible will not be graded.
- **Submission:** Only homeworks uploaded to Google Classroom will be graded. Make sure to show all the steps of your derivations in order to receive full credit.
- **Integrity and Collaboration:** You are expected to work on the homeworks by yourself. You are not permitted to discuss them with anyone except the instructor. The homework that you hand in should be entirely your own work. You may be asked to demonstrate how you got any results that you report.
- Clarifications: If you have any question, please look at Google Classroom first. Other students may have encountered the same problem, and is solved already. If not, post your question there. We will respond as soon as possible.

1 Reversible Architectures [3pts]:

1.

$$x_2 = y_2 \circ \exp(-s)$$

$$x_1 = (y_1 - \mathcal{F}(y_2 \circ \exp(-s)) \circ \exp(-\mathcal{G}(y_2 \circ \exp(-s))))$$
(1)

2.

$$\frac{\partial \mathbf{y}}{\partial \mathbf{x}} = \begin{bmatrix} \operatorname{diag}(\exp(\mathcal{G}(\mathbf{x}_2))) & \operatorname{diag}(\mathcal{G}'(\mathbf{x}_2)) \circ \exp(\mathcal{G}(\mathbf{x}_2)) \circ \mathbf{x}_1 + \mathcal{F}'(\mathbf{x}_2) \\ \mathbf{0} & \operatorname{diag}(\exp(\mathbf{s})) \end{bmatrix}$$
(2)

3. $\prod_{i=1}^{d/2} \exp(\mathcal{G}(x_{2,i}) + s_i)$. This is not volume preserving.

2

- 1. use Bayes theorem
- 2.
- 3. $D_{KL}(q_i(z_i)||p_i(z_i)) = \frac{1}{2}(\mu_i^2 + \sigma_i^2 1) \log(\sigma_i)$
- 4. $\nabla_{\boldsymbol{\theta}} D_{KL}(q_i(z_i)||p_i(z_i)) = \begin{bmatrix} \mu_i \\ \sigma_i \frac{1}{\sigma_i} \end{bmatrix}$