Buying Supermajorities

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Introduction

- Minimal winning coalitions a key prediction or essential assumption in political economy
  - Minimum number of votes necessary to win
  - Present in almost all formal models of coalition formation, vote buying, and logrolling
- Intuition: If coalition builder must pay for each member, s/he never pays more than smallest number required to win

Motivation

- Empirically, oversized coalitions seem to be at least as prevalent as minimal winning coalitions
  - Divisions on legislative roll calls rarely 50-50
  - "Effectiveness" of majority party strictly increasing in size
  - "Defections" of majority party nonincreasing in size
- Potential explanations
  - Uncertainty (chance a legislator's vote fails)
  - Legislation norm of "universalism" (legislators prefer to support all distributive projects proposed each session)
  - Cost of "ideological diversity"

Implication

Non-minimal coalitions may actually be *cheaper* than minimal winning coalitions to maintain in a sequential vote-buying setting

- Second vote buyer bribes minimum number of members
- If first buyer bribes more than minimal winning coalition, he can decrease bribe paid per member, keeping constant the amount the second buyer must pay
- If savings from decreasing bribes greater than costs of bribing another legislator, then supermajority is cheaper

House vote on NAFTA

- Clinton and Republican House leaders said to have traded favors for votes, but final vote was 234-200
- 16 votes larger than minimum winning coalition
- Significant opposition from certain Democrats could have attempted to buy votes, but costs much higher for invading a supermajority coalition
- Pro-NAFTA leaders may have convinced opposition to concede the issue and spend resources elsewhere

Two assumptions

- There are two competing vote buyers instead of one
- Vote buyers move sequentially
  - Convenient (pure-strategy equilibria typically would not exist under simultaneous game)
  - More realistic interpretation with actual coalition building
  - Sequential model sensible under dynamic context and problem of *maintaining* a winning coalition

The Model

- A legislature is to decide by majority rule between the status quo s and a new policy x, $s, x \in \mathbb{R}$
- Legislator i has reservation price $v(i) = u_i(x) - u_i(s)$
    - Measured in money
    - Rank legislators so that $v(i)$ is a nonincreasing function
- Two vote buyers A and B, where WLOG, $x \succsim_A s, s \succsim_B x$
    - $WTP_A : W_A = U_A(x) - U_A(s)$
    - $WTP_B : W_B = U_B(s) - U_B(x)$
    - $a(i)$: A's offer to i, $b(i)$: B's offer to i

The Sequence and Dominant Voting Strategy

- t=1: A reveals and offers $a(\cdot)$
- t=2: B perfectly informed about $a(\cdot)$, counters with $b(\cdot)$
- Legislators take bribes as given and votes for the alternative with greater payoff
  - Only preferences over the vote, not the outcome
  - Dominant voting strategy once bribe offers known
  - Assume unbribed legislators indifferent between x and s vote for status quo

Example 1

- Seven legislators, $v(i) = 0$ for all legislators, $W_A \gg 0$
- Since B moves second and attacks the weakest part of A's coalition, A offers a to all legislators he bribes
- For A to win, he must spend enough so that B needs to spend more than W_B

Example 1

- Let $m+4 \geq 0$ be the size of the coalition A bribes
- B bribes at most $m+1$ members, spends at most W_B
  - $m=0$: B wins by paying $a+\epsilon$ to 1 member
    - $a \geq W_B$, A pays $4W_B$
  - $m=1$: B wins by paying $a+\epsilon$ to 2 members
    - $a \geq \frac{W_B}{2}$, A pays $\frac{5}{2}W_B$
  - $m=2$: B wins by paying $a+\epsilon$ to 3 members
    - $a \geq \frac{W_B}{3}$, A pays $2W_B$
  - $m=2$: B wins by paying $a+\epsilon$ to 4 members
    - $a \geq \frac{W_B}{4}$, A pays $\frac{7}{4}W_B$
- SPNE: $a(i) = \frac{W_B}{4}\forall i$, $b(i) = 0\forall i$

Example 1

Comment 1

Suppose the number of legislators n is odd, all legislators are initially indifferent between x and s, and $W_A \geq \frac{2nW_B}{n+1}$. Then, in equilibrium, A bribes all legislators, with $a(i) = \frac{2W_B}{n+1}$ for all i.

Example 2

- Seven legislators, $v(i) = -1$ for all legislators, $W_A \gg 0$, $W_B = 3$
- A again offers a to all legislators he bribes
- Let $m + 4 \geq 0$ be the size of the coalition A bribes
  - $m = 0$: A sets $a = 1 + W_B = 4$ to win, pays 16
  - $m = 1$: A sets $a = 1 + W_B/2 = 5/2$ to win, pays $25/2$
  - $m = 2$: A sets $a = 1 + W_B/3 = 2$ to win, pays 12
  - $m = 3$: A sets $a = 1 + W_B/4 = 7/3$ to win, pays $49/4$
- Optimal strategy is to bribe six legislators

Relaxing Model Assumptions

- Generalizable to finite periods of vote buying
- Defender of status quo need not to be arbitrarily given last-mover advantage
  - In game where vote buyers prefer sequence, status quo prefers to never initiate and wants to move last
- We assume legislators do not have preferences over which policy wins but only over how they vote
  - Such preferences only matter if a legislator is pivotal
- Legislators receive cash transfers as bribes, but may be more natural to treat them as benefits written in the bill
  - Legislators no longer indifferent about bribes to other legislators if they must be tax-funded

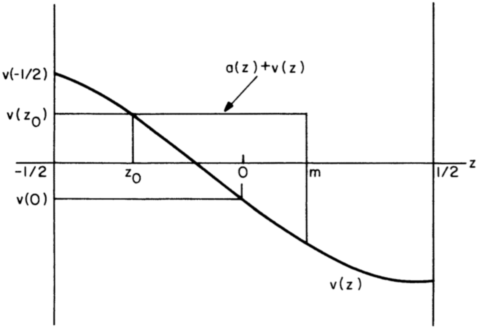
Model with Continuum of Legislators

- Set of legislators indexed $\sim U\left[-\frac{1}{2}, \frac{1}{2}\right]$
  - Median voter at zero
- Let $v(z)$ be the reservation-price function
- Strategies of A and B are functions $a(\cdot)$ and $b(\cdot)$ on $\left[-\frac{1}{2}, \frac{1}{2}\right]$
- Focus on cases in which $W_A \gg 0$
- Let $m + \frac{1}{2}$ be the fraction of legislators, both bribed, and unbribed, who vote for x
  - m the "excess" size of A's coalition relative to a minimal winning coalition
  - B must bribe at least m members of A's coalition

Leveling Strategy

- A's bribe offer function $a(\cdot)$ a *leveling strategy* if there is a legislator z_0, such that $v(z) + a(z) = v(z_0)$ for all bribed legislators $(z : a(z) > 0)$
  - A leaves B with a level field of legislators from which to choose when deciding whom to bribe
- Whenever there are equilibria in which x wins, there is always one in which A plays a leveling strategy

FIGURE 1. Payments by *A* under a Leveling Strategy



Note: Each legislator bribed by *A* costs the same for *B* to buy, $v(z) + a(z) = v(z_0)$.

Example 1

Proposition 1

Suppose $a^*(\cdot)$ and $b^*(\cdot)$ constitute an equilibrium in which x wins. Then, exactly one of the following cases holds:

1. if $v(0) > 0$ and $W_B \leq \int_0^{v^{-1}(0)} v(z)dz$, then $a^*(z) = 0$ for all z;
2. if $v(0) > 0$ and $\int_0^{v^{-1}(0)} v(z)dz < W_B < v(0)v^{-1}(0)$, then $a^*(z)$ satisfies $a^*(z) = 0$ for $z \notin [0, v^{-1}(0)]$, $a^*(z) \leq v(0) - v(z)$ for all $z \in [0, v^{-1}(0)]$, and $\int_0^{v^{-1}(0)}[v(z) + a^*(z)]dz = W_B$;
3. if $v(0) \leq 0$ or $W_B \geq v(0)v^{-1}(0)$, then $a^*(\cdot)$ is a leveling strategy, with $a^*(z) = W_B/m - v(z)$ for all z such that $a^*(z) > 0$, where m satisfies $m > max\{0, v^{-1}(0)\}$ and $W_B/m > v(0)$.

FIGURE 2. Proposition 1, Case 1: Area I > W_B by Assumption, So Total Payments by A = Zero

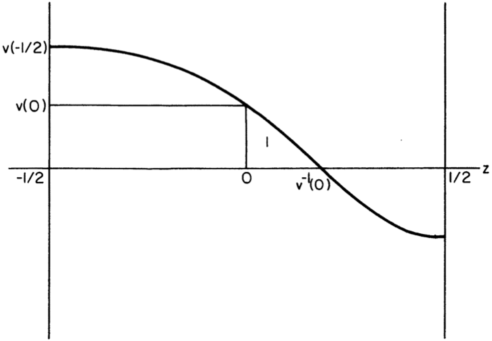
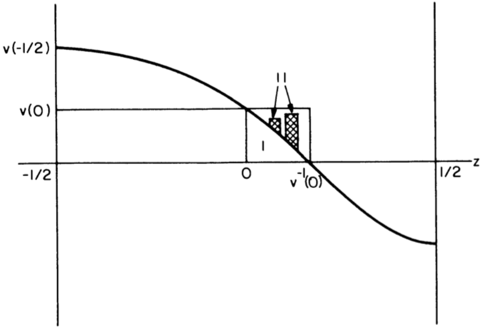


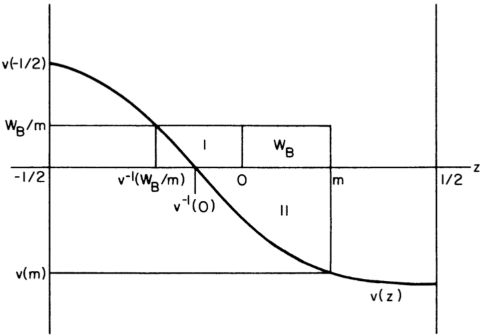
FIGURE 3. Proposition 1, Case 2: Area II Represents Payments by *A*. At the Optimum, I + II = W_B



Subcase of Case 3

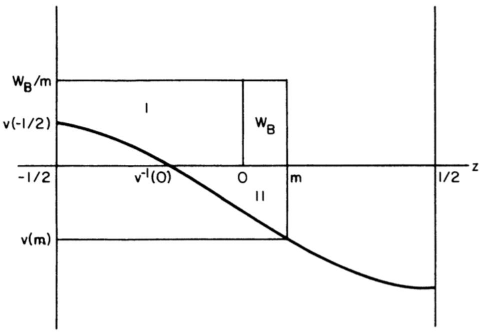
- One may typically imagine bribes taking place when majority of legislature initially opposed to the vote buyer ($v(0) \leq 0$)
  - Two conditions: $v\left(-\frac{1}{2}\right) \geq \frac{W_B}{m}$, and $v\left(-\frac{1}{2}\right) < \frac{W_B}{m}$
    - $\frac{W_B}{m}$ the minimum amount B must pay to buy the vote of a member of A's coalition
- A coalition is *flooded* if A bribes every member of his coalition

FIGURE 4. Payments by *A* When *A* Constructs a Nonflooded Coalition



Note: Total payments = I + II + W_B.

FIGURE 5. Payments by *A* When *A* Constructs a Flooded Coalition



Note: Total payments = I + II + W_B.

Proposition 2

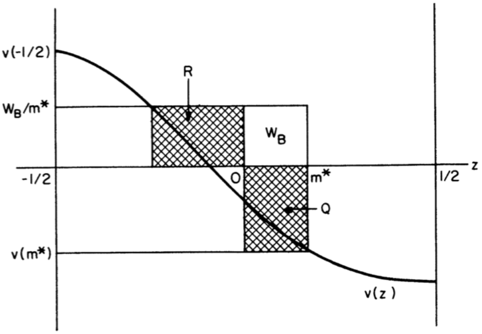
Suppose $v(\cdot)$ is nonincreasing and differentiable, and $v(0) \leq 0$. Then m^* is unique, and exactly one of the following holds:

1. *A* constructs a nonflooded, nonuniversalistic coalition, in which case $m^* \geq W_B/v(-1/2)$, $m^* < 1/2$, and m^* satisfies $-(W_B/m^*)v^{-1}(W_B/m^*) = -m^* v(m^*)$;
2. *A* constructs a flooded, nonuniversalistic coalition, in which case $m^* < W_B/v(-1/2)$, $m^* < 1/2$, and m^* satisfies $(W_B/m^*)(1/2) = -m^* v(m^*)$;
3. *A* constructs a universalistic coalition, in which case $m^* = 1/2$.

Proposition 2 Implications

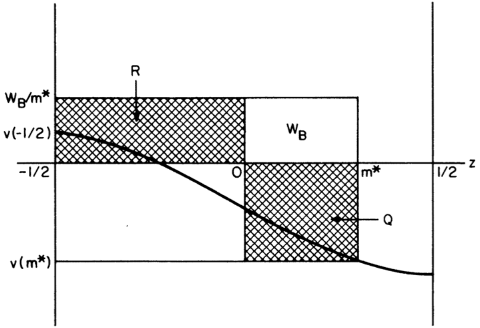
- At an interior m^*, two particular rectuangular areas must be equal
- In cases 1 and 2, $\frac{\delta m^*}{\delta W_B} > 0$
  - If $W_B = 0$, A faces no vote buying opposition and only bribes a minimal winning coalition

FIGURE 6. Payments by *A* When a Nonflooded Coalition Is Optimal



Note: At the optimum, $R = Q$.

FIGURE 7. Payments by *A* When Flooded Coalition Is Optimal



Note: At the optimum, $R = Q$.

Proposition 3

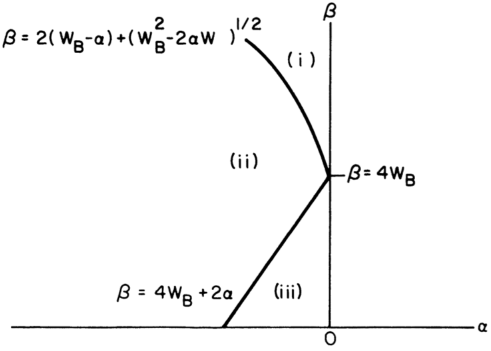
Suppose $v(z) = \alpha - \beta z$, with $\beta \geq 0$ and $\alpha \leq 0$. Then, the types of coalitions formed and m^* are characterized as follows.

1. A constructs a nonflooded, nonuniversalistic coalition iff $\beta \geq 2(W_B - \alpha) + (W_B^2 - 2\alpha W_B)^{1/2}$. In this case, $m^* = (W_B/\beta)^{1/2}$.
2. A constructs a flooded, nonuniversalistic coalition iff $4W_B + 2\alpha < \beta < 2(W_B - \alpha) + (W_B^2 - 2\alpha W_B)^{1/2}$. In this case, m^* solves $\beta(m^*)^2 = \alpha m^* + W_B/2m^*$.
3. A constructs a flooded, universalistic condition iff $\beta \leq 4W_B + 2\alpha$. In this case, $m^* = 1/2$.
4. A never constructs a nonflooded, universalistic coalition.

Proposition 3 Implications

- m^* is a continuous function of α, β, and W_B.
- m^* is differentiable except at the boundaries

FIGURE 8. Values of α and β Corresponding to Cases 1, 2, and 3 of Proposition 3



Proposition 4

Suppose $v(i) = \alpha - \beta[i - (n+1)/2]$, with $\beta \geq 0$ and $\alpha \leq 0$. If $a^*(\cdot)$ and $b^*(\cdot)$ consitute an equilibrium in which x wins, then $m^* = 0$ only if $W_B \leq [1/3 + (28/9)^{1/2}]\beta < (2.1)\beta$.

FIGURE 9. Payments by *A* with a Finite Legislature and Linear *v* Function