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# Buying Supermajorities in Finite Legislatures

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*I analyze the finite-voter version of the Groseclose and Snyder vote-buying model. I identify how the optimal coalition size varies with the underlying preference parameters; derive necessary and sufficient conditions for minimal majority and universal coalitions to form; and show that the necessary condition for minimal majorities found in Groseclose and Snyder is incorrect.*

A common feature of numerous rational choice theories of politics, such as the size principle of Riker (1962) or the stationary equilibria of Baron and Ferejohn (1989), is that minimal winning coalitions are likely to form. This prediction runs counter to the empirical regularity that such coalitions are rarely if ever observed (Browne 1993). To remedy this situation Groseclose and Snyder (1996) develop a model of competitive vote buying in which the equilibrium path of play, for certain parameter values, has one group bribing a supermajority of voters and the second group bribing no one; the supermajority votes in favor of the policy preferred by the former group, whereas a simple majority would have sufficed. The incentives underlying this apparently excessive vote buying are found in the sequential structure of the moves: One group bribes a sufficiently large number of voters at the first stage so as to prevent a successful bribe attack by its opponent at the second stage. That is, the pressure to build a supermajority coalition is driven by the “unseen” competitive response that would have occurred had the first group attempted to secure only a bare majority.

Most of the analysis in Groseclose and Snyder (1996) assumes a continuum of voters, which makes certain types of results easier to identify but others more difficult. In particular, their characterization of the optimal coalition size requires the stringent assumption that voter preferences are linearly related. This assumption is also present in their one finite-voter result, on the optimality of minimal majority coalitions. I will consider only the finite-voter model and replace the linearity assumption with a bound on voter preferences. Using fairly elementary methods, I will generate a characterization of the optimal coalition size and identify how this size varies as the underlying parameters of the model change. I show that the optimal coalition size is weakly increasing in the value voters place on the winning group’s preferred alternative. That is, as voter preferences shift in favor of that alternative, the winning group does not decrease, and may actually increase, the number of voters bribed. I also show that the result of Groseclose and Snyder

(1996) on the optimality of minimal majorities is not correct.

## THE MODEL

There are two alternatives ( $x$  and  $y$ ), two interested parties ( $A$  and  $B$ ), and a set  $N = \{1, \dots, n\}$  of voters, with  $n$  assumed odd. Party  $A$  prefers  $x$  to  $y$  and is willing to pay up to  $W_A > 0$  to see the former prevail;  $B$  prefers  $y$  to  $x$  and is willing to pay  $W_B > 0$ . For each  $i \in N$ , let  $v_i \in \Re$  denote the intensity of  $i$ ’s preference for voting for  $x$  over  $y$ , measured in money, and let  $\mathbf{v} = (v_1, \dots, v_n)$  denote a preference profile. Thus, voter preferences are defined by how they vote rather than by the alternative that prevails;  $v_i > 0$  means that  $i$  prefers  $x$  to  $y$ , and  $v_i < 0$  means that  $i$  prefers  $y$  to  $x$ . Since the voters are indistinguishable to  $A$  and  $B$  save for these preference intensities, without loss of generality I can restrict attention to preference profiles of the form  $v_1 \geq v_2 \geq \dots \geq v_n$ .

The sequence of decisions is as follows. Initially,  $A$  offers a bribe schedule,  $a = (a_1, \dots, a_n) \in \Re_+^n$ , after which  $B$ , with knowledge of  $a$ , offers a bribe schedule,  $b = (b_1, \dots, b_n) \in \Re_+^n$ . All  $i \in N$  vote for either  $x$  or  $y$ , and majority rule determines the outcome. Solving this game via backward induction, given bribe schedules  $(a, b)$ , voter  $i$  will prefer to vote for  $x$  if  $a_i + v_i > b_i$  and for  $y$  if  $a_i + v_i < b_i$ ; we assume that an indifferent  $i$  votes for  $y$ . Given bribe schedule  $a$  and a constraint  $W_B$ ,  $B$  seeks the least-cost majority to bribe. Since indifferent voters choose  $y$ ,  $B$  need pay no more than  $a_i + v_i$  to secure the vote of  $i$ , and if this amount is nonpositive, she gets  $i$ ’s vote with no bribe at all. Thus,  $B$  solves

$$\min_C \left\{ \sum_{i \in C} \max\{0, a_i + v_i\} : |C| > n/2 \right\}$$

as long as this amount is strictly less than  $W_B$ ; otherwise, she chooses  $b = (0, \dots, 0)$ , that is,  $B$  bribes no one. Finally,  $A$  sets his bribe schedule so as to have  $x$  prevail in the least-cost manner (if affordable), taking into consideration  $B$ ’s predicted reaction.

As do Groseclose and Snyder (1996), I restrict attention to situations in which  $W_A$  is large enough relative to  $W_B$  and  $\mathbf{v}$  so that, in equilibrium,  $x$  prevails over  $y$ . For the latter to occur, the schedule selected by  $A$  must be such that for every majority coalition  $C$ ,  $\sum_{i \in C} \max\{0, a_i + v_i\} \geq W_B$ ; I refer to bribe schedules satisfying these inequalities as unbeatable. Let  $U(\mathbf{v}, W_B) \subseteq \Re_+^n$  denote the set of unbeatable bribe sched-

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ules, and for any schedule  $a$  let  $S(a) = \sum_{i=1}^n a_i$  be the expenditure associated with  $a$ . The above assumption on  $W_A$ ,  $W_B$ , and  $\mathbf{v}$  is that there exists an unbeatable bribe schedule affordable for  $A$ ; that is,  $\bar{a} \in U(\mathbf{v}, W_B)$  such that  $S(\bar{a}) \leq W_A$ .  $A$  then solves

$$\min\{S(a) : a \in U(\mathbf{v}, W_B)\}. \tag{1}$$

The set  $U(\mathbf{v}, W_B)$  is evidently closed, whereas the set  $\{a \in \mathbb{R}_+^n : S(a) \leq S(\bar{a})\}$  is compact. Their intersection, within which any solution to expression 1 must reside, is compact (and nonempty by  $\bar{a}$ ); and since  $S$  is continuous, a solution to expression 1 exists.

Characterizing a solution to expression 1 is made easier by the following observation. For any  $a \in \mathbb{R}_+^n$ , let  $C(a) = \{i \in N : a_i > 0\}$  denote the set of individuals who receive a bribe from  $A$ . Then one can show that there exists a solution  $a'$  to expression 1 in which  $a'_i + v_i = a'_j + v_j$  for all  $i, j \in C(a')$ ; that is, under schedule  $a'$  all voters bribed by  $A$  are equally expensive for  $B$  to bribe. The intuition is that  $A$  has no incentive to bribe voters, which make them differentially expensive for  $B$  to bribe, as  $B$  will simply ignore the higher cost voters in constructing a least-cost majority. Groseclose and Snyder (1996) refer to this as a leveling schedule; let  $U^l(\mathbf{v}, W_B) \subseteq U(\mathbf{v}, W_B)$  denote the set of unbeatable leveling schedules, that is, bribe schedules  $a \in U(\mathbf{v}, W_B)$ , such that  $a_i + v_i = a_j + v_j \equiv t(a)$  for all  $i, j \in C(a)$ . The bribe  $a_i = t(a) - v_i$  made to  $i \in C(a)$  can be thought of as the sum of two terms. The first ( $t(a)$ ) is a positive “transfer” common among all members of  $C(a)$ , and the second ( $-v_i$ ) can be positive or negative and is individual-specific. The latter term brings all the members of  $C(a)$  to being indifferent between  $x$  and  $y$ , absent any bribe from  $B$ ; the former term represents the per-capita amount necessary to make  $C(a)$ , together with any unbribed voters, unaffordable to  $B$ .

To simplify the analysis further, I make the following pair of assumptions:

- A1:  $v_{(n+1)/2} < 0$ ;
- A2:  $v_1 < 2W_B/(n + 1)$ .

One implication of A1 is that in the absence of any bribes  $y$  will defeat  $x$ , so in equilibrium  $A$  must bribe at least one voter. In fact, A2 implies that  $A$  must bribe at least a majority of voters; otherwise,  $B$  will have sufficient resources to bribe a majority of voters, and  $y$  will defeat  $x$ . A2 also implies that for all  $a \in U^l(\mathbf{v}, W_B)$  it must be that  $t(a) \geq 2W_B/(n + 1)$ ; otherwise,  $B$  can bribe a majority from the coalition  $C(a)$  itself and have  $y$  defeat  $x$ . More substantively, A2 says that  $B$  cares a great deal more about defeating  $x$  than any of the voters care about  $x$  prevailing. Propositions 2, 3, and 4 of Groseclose and Snyder (1996) assume A1 holds; A2 is new.

For any  $a \in \mathbb{R}_+^n$  let  $k(a) = |C(a)|$ , and suppose  $a \in U^l(\mathbf{v}, W_B)$  is such that  $v_i \geq v_j$  and  $j \in C(a)$  but  $i \notin C(a)$ ; that is,  $i$  is at least as favorable to  $x$  as  $j$ , but  $j$  is bribed and  $i$  is not. Then, under A2, there exists  $a' \in U^l(\mathbf{v}, W_B)$  with  $S(a') \leq S(a)$ ,  $k(a') = k(a)$ , and  $i \in$

$C(a')$ , but  $j \notin C(a')$  by simply swapping the roles of  $i$  and  $j$ :  $a'_i = t(a) - v_i$ ,  $a'_j = 0$ , and for all  $m \notin \{i, j\}$ ,  $a'_m = a_m$ .<sup>1</sup> Repeating this logic, and recalling that  $v_1 \geq \dots \geq v_n$ , we see that for all  $a \in U^l(\mathbf{v}, W_B)$  there exists  $a' \in U^l(\mathbf{v}, W_B)$  such that  $S(a') \leq S(a)$  and  $C(a') = \{1, \dots, k(a)\}$ , so we can without loss of generality restrict attention to schedules  $a$  by  $A$ , which bribe the first  $k(a)$  voters. Call these monotonic schedules, and let  $U_m^l(\mathbf{v}, W_B) \subseteq U^l(\mathbf{v}, W_B)$  denote the set of unbeatable schedules that are both monotonic and leveling.

Therefore, when A2 holds,

$$\begin{aligned} \min\{S(a) : a \in U(\mathbf{v}, W_B)\} \\ = \min\{S(a) : a \in U_m^l(\mathbf{v}, W_B)\}. \end{aligned}$$

Because the constraint set in the latter is more manageable than the constraint set in the former, I will focus on solving the latter optimization problem.

For any  $a \in U_m^l(\mathbf{v}, W_B)$ , the expenditure  $S(a)$  can be written

$$\begin{aligned} S(a) &= \sum_{i \in C(a)} a_i = \sum_{i \in C(a)} [t(a) - v_i] = k(a) \cdot t(a) \\ &\quad - \sum_{i \leq k(a)} v_i. \end{aligned}$$

Furthermore, the parameters  $k(a)$  and  $t(a)$  completely characterize any schedule  $a \in U_m^l(\mathbf{v}, W_B)$ , so  $A$ 's optimization problem can be reformulated as the choice of parameters  $k$  and  $t$ . That is,  $A$  now solves

$$\min_{k,t} k \cdot t - \sum_{i \leq k} v_i$$

subject to the constraint that the induced schedule, call it  $a(k, t, \mathbf{v})$ , lies in  $U_m^l(\mathbf{v}, W_B)$ . This induced schedule is defined as  $a_i = t - v_i$  if  $i \leq k$ ;  $a_i = 0$  otherwise. Using A1 and A2, I can reformulate this as an unconstrained problem involving simply the choice of  $k$ , as follows. If  $a(k, t, \mathbf{v})$  is unbeatable, then by A2 we know that  $k \geq (n + 1)/2$ , so by A1 it must be that if  $a_i(k, t, \mathbf{v}) = 0$ , then  $v_i < 0$ ; that is, all nonbribed voters prefer (in the absence of bribes)  $y$  to  $x$ . Therefore,  $B$  receives the votes of all  $i \in \{k + 1, \dots, n\}$  for “free.” For  $a(k, t, \mathbf{v})$  to be unbeatable it must then be that  $B$  cannot afford to bribe the additional  $(n + 1)/2 - (n - k) = k - (n - 1)/2$  voters needed to form a majority, or

$$t \cdot [k - (n - 1)/2] \geq W_B.$$

Solving this for equality gives the optimal transfer from  $A$  to members of  $C(a) = \{1, \dots, k\}$ , conditional on  $k$ :

$$t(k, W_B) = \frac{W_B}{k - (n - 1)/2}. \tag{2}$$

Defining

$$E(k, \mathbf{v}, W_B) = k \cdot t(k, W_B) - \sum_{i \leq k} v_i \tag{3}$$

<sup>1</sup> A2 guarantees that  $a'_i$  is nonnegative.

as the minimal “winning” expenditure conditional on  $k$ ,  $A$ ’s problem now is to

$$\min_k \{E(k, \mathbf{v}, W_B) : k \in \{(n+1)/2, \dots, n\}\}. \quad (4)$$

I assume that if there are multiple solutions to this problem,  $A$  selects the smallest solution. Modulo this adjustment, let  $k^*(\mathbf{v}, W_B)$  denote the solution to expression 4. This solution implicitly generates a solution to expression 1 through expression 2 and the induced bribe schedule described above.

Finally, recall that, by A2,  $k^*(\mathbf{v}, W_B)$  must be at least  $(n+1)/2$ , so that by A1 the only individuals who vote for  $x$ ,  $A$ ’s preferred alternative, are those who are bribed by  $A$ ; Groseclose and Snyder (1996) refer to this as a flooded coalition.<sup>2</sup> The number of individuals voting for  $x$  is equal to  $k^*(\mathbf{v}, W_B)$ , the number bribed by  $A$ , and so results on  $k^*(\mathbf{v}, W_B)$  are equivalently results on the size of the coalition voting for  $A$ ’s preferred alternative. In particular, a supermajority votes for  $A$ ’s preferred alternative if and only if a supermajority is bribed by  $A$ . Also, note that  $k^*(\mathbf{v}, W_B)$  identifies not only the size of  $A$ ’s optimal coalition but also a voter, namely, the voter who receives the largest bribe from  $A$ .

## RESULTS

I begin with a characterization of  $k^*(\mathbf{v}, W_B)$ ; for notational ease, in some of what follows I will suppress the dependence of  $E$  and  $k^*$  on  $\mathbf{v}$  and  $W_B$ . Because the number of possible values for  $k$  is finite, I cannot employ calculus techniques to identify  $k^*$ , but a discrete version of these techniques can be used. For all  $k \in \{(n+1)/2, \dots, n-1\}$ , define  $\Delta(k) = E(k+1) - E(k)$ ; that is,  $\Delta(k)$  is the difference in expenditures from adding the  $(k+1)$ st voter to the coalition  $\{1, \dots, k\}$ . If  $\Delta(k) < 0$ , then (since  $A$  is attempting to minimize expenditures)  $A$  has an incentive to add the  $(k+1)$ st voter to the coalition. Conversely, if  $\Delta(k) \geq 0$ , then  $A$  does not want to add the  $(k+1)$ st voter (recall our tie-breaking rule in favor of smaller coalitions). This gives a sense of the local or “first-order” effects of changing the coalition size.

Next, suppose  $\Delta(k)$  is increasing in  $k$ , which is simply the discrete version of the second-order condition that  $E(k)$  be convex in  $k$ . The following algorithm then can be used to identify  $k^*$ : If  $\Delta((n+1)/2) \geq 0$ , then we know from  $\Delta(k)$  increasing that  $\Delta(k) > 0$  for all larger values of  $k$ , and hence the optimal value of  $k$  is  $k^* = (n+1)/2$ . If  $\Delta((n+1)/2) < 0$ , then we know that  $k^*$  must be greater than  $(n+1)/2$ , so we next solve for  $\Delta((n+3)/2)$ . If this term is nonnegative, then, again by  $\Delta(k)$  increasing, we have that  $k^* = (n+3)/2$ ; if the term is negative, then we next check  $(n+5)/2$ ; and so on. When  $\Delta(k)$  is increasing, we have the following implicit characterization of the optimal coalition size:

<sup>2</sup> The implication of adding A2 to what Groseclose and Snyder (1996) already assume is that nonflooded coalitions, which are at times optimal in their environment, are never optimal here.

$$k^* = \begin{cases} (n+1)/2 & \text{if } \Delta((n+1)/2) \geq 0 \\ \max\{k : \Delta(k-1) < 0\} & \text{otherwise} \end{cases} \quad (5)$$

Finally, I show that  $\Delta(k)$  is indeed increasing in  $k$ . From equations 2 and 3,

$$\begin{aligned} \Delta(k) &= \left[ \frac{(k+1)W_B}{k+1-(n-1)/2} - \sum_{i \leq k+1} v_i \right] \\ &\quad - \left[ \frac{kW_B}{k-(n-1)/2} - \sum_{i \leq k} v_i \right] \\ &= W_B \left[ \frac{(k+1)}{k+1-(n-1)/2} - \frac{k}{k-(n-1)/2} \right] - v_{k+1} \\ &= \frac{-W_B(n-1)}{2(k+1-(n-1)/2)(k-(n-1)/2)} - v_{k+1} \end{aligned} \quad (6)$$

$$\equiv T(k, W_B) - v_{k+1}. \quad (7)$$

Treating  $k$  for the moment as a continuous variable, it is easily seen by differentiation that  $T(k, W_B)$  is increasing in  $k$ . Furthermore, since  $v_1 \geq v_2 \geq \dots \geq v_n$ , the second term,  $-v_{k+1}$ , is nondecreasing in  $k$ . Hence, the discrete second-order condition holds, which implies the above local analysis is also global: Equation 5 defines the optimal coalition size.

Although generating an explicit characterization of  $k^*$  via equations 5 and 6 admittedly would be somewhat messy, the parameter values that give rise to the “corner” solutions, that is,  $k^*$  equal to either  $(n+1)/2$  or  $n$ , are straightforward to identify. We have that  $k^* = (n+1)/2$  if and only if  $\Delta((n+1)/2) \geq 0$ , and  $k^* = n$  if and only if  $\Delta(n-1) < 0$ ; inserting the relevant values into  $T(k, W_B)$ , we obtain the following.

**PROPOSITION 1.** (a)  $k^*(\mathbf{v}, W_B) = (n+1)/2$  if and only if  $v_{(n+3)/2} \leq -W_B(n-1)/4$ ; (b)  $k^*(\mathbf{v}, W_B) = n$  if and only if  $v_n > -2W_B/(n+1)$ .

Therefore, to determine whether a minimal majority coalition is optimal, the only relevant part of the preference profile  $\mathbf{v}$  is the  $((n+3)/2)$ st term, and the only relevant part for a universalistic coalition is the last term. Of course, if neither inequality in proposition 1 holds,  $k^*(\mathbf{v}, W_B)$  lies strictly between  $(n+1)/2$  and  $n$ ; that is, the optimal coalition is a less-than-universalistic supermajority. Note also that proposition 1(b) identifies a lower-bound constraint on  $\mathbf{v}$  symmetric to the upper-bound constraint imposed above. Whereas A2 requires no voter to prefer  $x$  over  $y$  by more than  $2W_B/(n+1)$ , proposition 1(b) says that if, in addition, no voter prefers  $y$  over  $x$  by more than  $2W_B/(n+1)$ , then the optimal choice by  $A$  is to bribe all the voters.

Proposition 1(a) gives as an immediate consequence separate necessary and sufficient conditions for a min-



imal majority to be optimal, based only on the preference intensities of the “extreme” voters:

COROLLARY 1.  $k^*(\mathbf{v}, W_B) = (n + 1)/2$  if  $v_1 \leq -W_B(n - 1)/4$ , and only if  $v_n \leq -W_B(n - 1)/4$ .

In words, a sufficient condition for  $A$  to find it optimal to bribe a minimal majority of voters is that all voters find  $y$  significantly more attractive than  $x$ , whereas a necessary condition is that at least one voter finds this to be so.<sup>3</sup>

Like proposition 1(b), proposition 3.3 in Groseclose and Snyder (1996) provides a necessary and sufficient condition for a universalistic coalition to be optimal in the continuum model, but only under a linear restriction on preference intensities (see below). Such an assumption is also made for their proposition 4 as to when minimal majorities are optimal; yet, since the concept of a minimal majority is not well defined with a continuum of voters, their result assumes a finite legislature and so is directly comparable to results here. Suppose voter preference intensities can be written

$$v_i = \alpha - \beta[i - (n + 1)/2],$$

with  $\alpha \leq 0$  and  $\beta \geq 0$ . Proposition 4 in Groseclose and Snyder (1996) asserts that if  $k^*(\mathbf{v}, W_B) = (n + 1)/2$ , then it must be that  $W_B < (2.1)\beta$ . That is, as long as  $B$  is willing to spend more than twice the difference in preference intensity between “adjacent” voters,  $A$  must bribe a supermajority. But consider this example:  $n = 7$ ,  $W_B = 3$ , and  $v_i = -5$  for all  $i \in N$ , which implies  $\alpha = -5$  and  $\beta = 0$  in the above linear format. According to Groseclose and Snyder,  $k^*(\mathbf{v}, W_B)$  should be strictly greater than four, but this is not true. Because  $v_i < 0$  for all  $i \in N$ ,  $B$  gets all voters not bribed by  $A$  for free. If  $A$  bribes four voters, she must pay each  $3 + 5 = 8$  (so that  $B$  cannot afford to attract any one voter), giving a total payment of 32. If  $A$  bribes five voters, the required bribe is  $1.5 + 5 = 6.5$  (so that  $B$  cannot attract any two), for a total payment of 32.5; similarly, the bribes to six and seven voters total 36 and 40.25, respectively. Therefore,  $A$ ’s optimal strategy is to bribe precisely a minimal majority of four voters, which contradicts proposition 4 of Groseclose and Snyder. Furthermore, we know from proposition 1(a) that this example is robust to (small) changes in the values of  $\alpha$  and  $\beta$ ; all that is needed is  $v_1$  less than .75 (so A2 holds) and  $v_{(n+3)/2}$  less than  $-4.5$  for the optimal coalition size to be a minimal majority.

I next identify how the optimal coalition size varies with voter preference intensity. Given an arbitrary amount  $W_B$  and preference profile  $\mathbf{v}'$ , let  $k' = k^*(\mathbf{v}', W_B)$ . If  $k' = (n + 1)/2$ , then we know that  $k^*(\mathbf{v}, W_B) \geq k'$  for all  $\mathbf{v}$ , so suppose  $k' > (n + 1)/2$ . From equation 5 we infer  $\Delta(k' - 1, \mathbf{v}', W_B) < 0$ , which from equations 6 and 7 is equivalent to  $v'_{k'} > T(k' - 1, W_B)$ . Now suppose the preference intensities change from  $\mathbf{v}'$  to  $\mathbf{v}$ , and  $v_{k'}$  is such that  $v_{k'} \geq v'_{k'}$ . Then,  $v_{k'} > T(k' - 1, W_B)$ , and hence  $\Delta(k' - 1, \mathbf{v}, W_B) < 0$ . But

from equation 5 it must be that  $k^*(\mathbf{v}, W_B) \geq k'$ . Therefore, I have proven the following.

PROPOSITION 2. For all  $W_B$ , if  $\mathbf{v}$  and  $\mathbf{v}'$  are such that  $v_{k'} \geq v'_{k'}$ , where  $k' = k^*(\mathbf{v}', W_B)$ , then  $k^*(\mathbf{v}, W_B) \geq k^*(\mathbf{v}', W_B)$ .

In words, if the preference intensity of the “marginal” bribed voter weakly increases, then the optimal coalition size will weakly increase as well, regardless of any changes in the other voters’ intensities. An equivalent statement in terms of bribes is this: The number of voters bribed by  $A$  weakly increases as the voter who receives the largest bribe finds  $A$ ’s preferred alternative,  $x$ , more attractive.

The above argument can be turned around to generate a sufficient condition for the optimal coalition size to decrease weakly. As before, let  $k' = k^*(\mathbf{v}', W_B)$ ; if  $k' = n$ , then clearly  $k^*(\mathbf{v}, W_B) \leq k'$  for all  $\mathbf{v}$ , so let  $k' < n$ . By equation 5 I infer  $\Delta(k', \mathbf{v}', W_B) \geq 0$ , or from equations 6 and 7  $v'_{k'+1} \leq T(k', W_B)$ . Suppose  $\mathbf{v}$  is such that  $v_{k'+1} \leq v'_{k'+1}$ . Then,  $v_{k'+1} \leq T(k', W_B)$ , which implies by equation 6 that  $\Delta(k', \mathbf{v}, W_B) \geq 0$ , and hence by equation 5 that  $k^*(\mathbf{v}, W_B) \leq k'$ . Therefore, we have the following.

PROPOSITION 3. For all  $W_B$ , if  $\mathbf{v}$  and  $\mathbf{v}'$  are such that  $v_{k'+1} \leq v'_{k'+1}$ , where  $k' = k^*(\mathbf{v}', W_B)$ , then  $k^*(\mathbf{v}, W_B) \leq k^*(\mathbf{v}', W_B)$ .

In words, if the preference intensity of the marginal nonbribed voter weakly decreases, then the optimal coalition size will weakly decrease as well, regardless of any changes in other voters’ intensities. (Unlike proposition 2, however, this voter cannot be identified from the bribes offered by  $A$ .) Combining propositions 2 and 3, we see that if the preference intensities of the marginal bribed and nonbribed voters do not change, then the optimal coalition size also does not change.

The logic of propositions 2 and 3 stems from the “convexity” of  $E$  and the subsequent ability to adopt a first-order approach in characterizing the optimal coalition size. As with the traditional calculus technique, in the presence of such convexity, only “local” information is relevant for generating comparative statics about how changes in  $\mathbf{v}$  affect changes in  $E$ . Here, this local information is summarized by the preference intensities of the marginal bribed and nonbribed voters.

Of course, in order to identify these voters (and so verify the conditions in either proposition) one needs to solve for the optimal coalition size, which as mentioned above might prove somewhat messy. Yet, both propositions give rise to a weaker, more global comparative statics result that does not require such a computation. Given two preference profiles  $\mathbf{v}$  and  $\mathbf{v}'$ , write  $\mathbf{v} \geq \mathbf{v}'$  if  $v_i \geq v'_i$  for all  $i \in N$ .

COROLLARY 2. For all  $W_B$ , if  $\mathbf{v} \geq \mathbf{v}'$  then  $k^*(\mathbf{v}, W_B) \geq k^*(\mathbf{v}', W_B)$ .

Thus, the number of voters bribed by  $A$ , and hence the size of  $A$ ’s optimal coalition, weakly increases as voters

<sup>3</sup> I thank an anonymous referee for suggesting that corollary 1 (which was in a previous draft) could be generalized to proposition 1(a) (which was not).

find  $A$ 's preferred alternative,  $x$ , more attractive.<sup>4</sup> This result certainly has a counterintuitive feel, as one might expect just the opposite, namely, that as  $x$  becomes more attractive relative to  $y$ , fewer voters will need to be bribed. The latter may well be true in a one-party model, but in this case  $A$ 's optimal behavior is driven by the predicted competitive response of  $B$ . Thus, although  $A$ 's total expenditure will surely decrease as  $x$  becomes more attractive, the optimal way to allocate this lower amount is to spread it more widely among the voters.

The logic of corollary 2, independent of its status as an implication of propositions 2 and 3, comes directly from the ability to restrict attention to leveling, monotonic bribe schedules. From the former (leveling) we can write  $A$ 's expenditure, conditional on bribing  $k$  voters, as an additively separable function of the transfer necessary to fight off  $B$  and of voter preference intensities. Although this expenditure obviously depends on the transfer, the change in the expenditure due to a change in voter preferences does not. And from the latter (monotonic) we know that as  $x$  becomes more attractive to the voters, the change in expenditure will be greater, the larger is  $k$ . That is, from equation 3,

$$E(k, \mathbf{v}', W_B) - E(k, \mathbf{v}, W_B) = k \cdot t(k, W_B) - \sum_{i \leq k} v'_i - \left[ k \cdot t(k, W_B) - \sum_{i \leq k} v_i \right] = \sum_{i \leq k} (v_i - v'_i).$$

This sum weakly increases in  $k$  when  $\mathbf{v} \geq \mathbf{v}'$ , since each of the terms in the sum is nonnegative. Let  $k'$  be optimal at  $\mathbf{v}'$  and  $\bar{k} < k'$ , which implies that  $\bar{k}$  is necessarily suboptimal. Then, in moving from  $\mathbf{v}'$  to  $\mathbf{v}$ , the expenditure on  $k'$  decreases by a greater amount than does the expenditure on  $\bar{k}$ , so  $\bar{k}$  remains suboptimal. This does not imply that  $k'$  is optimal at  $\mathbf{v}$ , merely that if it is not optimal, then the new optimal size must be greater than  $k'$ .

Two additional features of corollary 2 are worthy of comment. First, Groseclose and Snyder (1996) identify a similar comparative statics result in their continuum model; voters are indexed by a uniform distribution on  $[-1/2, 1/2]$ , and preference intensities are described by a nonincreasing and differentiable function  $v : [-1/2, 1/2] \rightarrow \Re$ . As in their proposition 4, however, they require  $v$  to be linear:  $v(z) = \alpha - \beta z$ , with  $\beta \geq 0$  and  $\alpha \leq 0$ . They then show that  $k^*$  is nondecreasing in  $\alpha$ , which is analogous to my corollary 2.<sup>5</sup> Second, the result as stated requires each voter's preference for  $x$  over  $y$ , as measured by  $v_i$ , to increase weakly. Yet, as mentioned above, voters are indistinguishable to  $A$  and  $B$  beyond these  $v_i$ s. Even if some voters have  $v_i$

decrease, as long as the new *distribution* of preference intensities is everywhere above that of the old, corollary 2 will remain true. That is, as long as the highest value in  $\mathbf{v}$  is greater than or equal to the highest value in  $\mathbf{v}'$ , the second-highest value in  $\mathbf{v}$  is greater than or equal to the second-highest value in  $\mathbf{v}'$ , and so on, it will be the case that  $k^*(\mathbf{v}, W_B)$  will be greater than or equal to  $k^*(\mathbf{v}', W_B)$ .

Finally, I show that  $A$ 's optimal coalition size  $k^*$  is monotonic in  $W_B$  as well. Because A2 depends on  $W_B$ , I need to add the assumption that  $v_i \leq 0$  for all  $i$ .

**PROPOSITION 4.** *For all  $\mathbf{v}$ , if  $W_B > W'_B$ , then  $k^*(\mathbf{v}, W_B) \geq k^*(\mathbf{v}, W'_B)$ .*

Therefore, as  $B$ 's willingness to pay increases,  $A$  tends to bribe a greater number of voters. The proof is similar to that for proposition 2. Let  $k' = k^*(\mathbf{v}, W'_B)$ . From equation 5 we know that  $\Delta(k' - 1, \mathbf{v}, W'_B) < 0$ , so from equations 6 and 7,  $v_{k'} > T(k' - 1, W'_B)$ . Since  $T$  is clearly decreasing in its second argument,  $W_B > W'_B$  implies  $v_{k'} > T(k' - 1, W_B)$ , and  $\Delta(k' - 1, \mathbf{v}, W_B) < 0$ , which by equation 5 implies  $k^*(\mathbf{v}, W_B) \geq k'$ .

DISCUSSION

The propositions and corollaries presented here give a fairly complete theoretical picture of supermajority bribery under certain assumptions, in particular, the preference restrictions embodied in A1 and A2. These assumptions allow us to focus, without loss of generality, on a relatively simple class of bribe schedules (monotonic and leveling) and translate  $A$ 's optimization problem into one with an attractive mathematical property (convexity). An open question is the extent to which my results survive the weakening of these assumptions. For instance, without A2 there can exist voters who prefer  $x$  to  $y$  by such a great amount that  $A$  finds it optimal not to bribe them at all (see Figure 4 in Groseclose and Snyder 1996), thereby adopting a non-monotonic, although still leveling, schedule.  $A$  would essentially be working on *two* margins in moving through  $\{1, \dots, n\}$ , namely, when to start bribing and when to stop, in contrast to the one-margin analysis (when to stop) associated with monotonic schedules. This suggests that the corresponding analysis of  $A$ 's optimal behavior will be considerably more intricate than that found here.

<sup>6</sup> Groseclose and Snyder (1996) obtain a similar result in their continuum model, without the linear restriction on preferences.

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<sup>4</sup> The only requirement is that  $v_i \geq v'_i$  for all  $i \in \{(n + 3)/2, \dots, n\}$ , as the first  $(n + 1)/2$  voters receive bribes regardless of preference intensities.

<sup>5</sup> Groseclose and Snyder (1996) also show that  $k^*$  is decreasing in  $\beta$ . Yet, although they state that "as  $\beta$  rises, the initial level of support in the legislature for  $x$  declines" (p. 310), which suggests a similar effect to that from  $\alpha$ , this statement only holds for  $z \in (0, 1/2]$ ; for  $z \in [-1/2, 0)$ , a rise in  $\beta$  leads to an *increase* in support for  $x$ .