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Ejercicio 1

a) $H(N,C,V) = \sum_{\forall_N}\sum_{\forall_C}\sum_{\forall_V} p(N,C,V) \log_2 \frac{1}{p(N,C,V)} \approx H(N,C,V{=}a) + H(N,C,V{=}e) + H(N,C,V{=}i)$

$1{,}3125 + 1{,}625 + 1{,}3125 = \boxed{4{,}25 \text{ bits}}$

V = a Tenemos la tabla de $P(N,C,V{=}a)$

$H(N,C,V{=}a) = \sum_{\forall_N}\sum_{\forall_C} p(N,C,V{=}a) \cdot \log_2\left(\frac{1}{p(N,C,V{=}a)}\right)$

$\log_2\left(\frac{1}{p(N,C,V=a)}\right)$	1	2	3
b	3	0	0
d	4	6	6
g	5	4	6
l	0	0	0
m	0	0	0

$H(N,C,V{=}a) = \frac{3}{8} + \frac{4}{16} + \frac{6}{64} + \frac{6}{64} + \frac{5}{32} + \frac{4}{16} + \frac{6}{64} = \frac{42}{32} \approx 1{,}3125$ bits

V = e $H(N,C,V{=}e) = \sum\sum p(N,C,V{=}e) \log_2\left(\frac{1}{p(N,C,V{=}e)}\right)$

$\log_2\left(\frac{1}{p(N,C,V=e)}\right)$	1	2	3
b	3	0	0
d	4	6	6
g	0	0	0
l	5	5	5
m	0	6	4

$H(N,C,V{=}e) = \frac{3}{8} + \frac{4}{16} + \frac{6}{64} + \frac{6}{64} + \frac{5}{32} + \frac{5}{32} + \frac{5}{32} + \frac{6}{64} + \frac{4}{16} = \frac{52}{32} = 1{,}625$ bits

V = i

$\log_2\left(\frac{1}{p(N,C,V=i)}\right)$	1	2	3
b	0	0	0
d	0	0	0
g	5	4	4
l	5	5	5
m	[illegible]	[illegible]	4

$H(N,C,V{=}i) = \frac{5}{32} + \frac{4}{16} + \frac{4}{16} + 3\left(\frac{5}{32}\right) + \frac{6}{64} + \frac{4}{16} = 1{,}3125$ bits

Se ve en la table

b)

$(1/8 + 1/8 + 0)$

$p(n,c)$	1	2	3
b	2/8	0	0
d	2/16	2/64	2/64
f	2/32	2/16	2/64
l	2/32	2/32	2/32
m	0	2/64	2/16

$(1/8 + 1/16 + 1/32 + 0 + 0)$

$p(n,v)$	1	2	3
a	7/32	5/64	1/32
e	7/32	1/16	7/64
i	1/16	7/64	7/64

$(1/16 + 1/64 + 1/64)$

$p(c,v)$	b	d	f	l	m
a	1/8	3/32	7/64	0	0
e	1/8	3/32	0	3/32	5/64
i	0	0	7/64	3/32	5/64

c) • $H(N,C) = \sum_{\forall n} \sum_{\forall c} p(n,c) \log_2\left(\frac{1}{p(n,c)}\right)$

$H(N,C) = \frac{2}{4} + \frac{3}{8} + \frac{5}{32} + \frac{5}{32} + \frac{4}{16} + \frac{3}{8} + \frac{5}{32} + 3\cdot\left(\frac{4}{16}\right) + \frac{5}{32} + \frac{3}{8} = \frac{104}{32} = \boxed{3{,}25 \text{ bits}}$

$\log_2\left(\frac{1}{p(n,c)}\right)$	1	2	3
b	2	0	0
d	3	5	5
f	4	3	5
l	4	4	4
m	0	5	3

①

• $H(N,V) = \sum_{\forall n} \sum_{\forall v} p(n,v) \log_2\left(\frac{1}{p(n,v)}\right)$

$H(N,V) = 2\times\left[\frac{7}{32}\cdot(2{,}1926)\right] + \frac{5}{64}\cdot 3{,}6780 + \frac{5}{32} + \left(\frac{4}{16}\right)\cdot 2 + \left(\frac{7}{64}\cdot 3{,}192\right)\times 3$

$\rightarrow \boxed{2{,}9504 \text{ bits}}$

$\log_2\left(\frac{1}{p(n,v)}\right)$	1	2	3
a	2,1926	3,6780	5
e	2,1926	4	3,1926
i	4	3,1926	3,1926

• $H(C,V) = \sum_{\forall c} \sum_{\forall v} p(c,v) \log_2\left(\frac{1}{p(c,v)}\right)$

$H(C,V) = 2\times\left[\frac{3}{8}\right] + 4\times\left(\frac{3}{32}\times 3{,}415\right) + 2\times\left[\frac{7}{64}\cdot 3{,}1926\right] + 2\cdot\left[\frac{5}{64}\cdot 3{,}678\right] = \boxed{3{,}303 \text{ bits}}$

$\log_2\left(\frac{1}{p(c,v)}\right)$	b	d	f	l	m
a	3	3,415	3,1926	0	0
e	3	3,415	0	3,415	3,678
i	0	0	3,1926	3,415	3,678

d) $p(n) = \sum_{\forall v} p(n,v)$ $\begin{cases} p(n=1) = 1/2 \rightarrow \frac{7}{32} + \frac{7}{32} + \frac{1}{16} \\ p(n=2) = 1/4 \rightarrow \frac{5}{64} + \frac{1}{16} + \frac{7}{64} \\ p(n=3) = \frac{1}{4} \rightarrow \frac{1}{32} + \frac{7}{64} + \frac{7}{64} \end{cases}$ $\rightarrow 1$

$p(c) = \sum_{\forall v} p(c,v) = \begin{cases} p(c=b) = 1/4 \\ p(c=d) = 3/16 \\ p(c=g) = 7/32 \\ p(c=l) = 3/16 \\ p(c=m) = 5/32 \end{cases}$ $\rightarrow 1$

$p(v) = \sum_{\forall n} p(n,v) = \begin{cases} p(v=a) = 21/64 \\ p(v=e) = 25/64 \\ p(v=i) = 9/32 \end{cases}$ $\rightarrow 1$

e) $H(N) = \sum_{\forall n} p(n) \log_2 \frac{1}{p(n)} \rightarrow H(N) = \frac{1}{2} + \frac{1}{4} \cdot 2 + \frac{1}{4} \cdot 2 = 1{,}5$ bits

$H(C) = \sum_{\forall c} p(c) \log_2 \frac{1}{p(c)} \rightarrow H(C) = \frac{2}{4} + 2\left(\frac{3}{16} \cdot 2{,}415\right) + \left(\frac{7}{32} \cdot 2{,}1926\right) + \left(\frac{5}{32} \cdot 2{,}678\right)$

$\hookrightarrow = 2{,}303$ bits

$H(V) = \sum_{\forall v} p(v) \log_2 \frac{1}{pv} \Rightarrow H(V) = \frac{21}{64} \cdot 1{,}6076 + \frac{25}{64} \cdot 1{,}3561 + \frac{9}{32} \cdot 1{,}8301$

$\hookrightarrow 1{,}5719$ bits

f) $p(n, v/c) = \frac{p(n,c,v)}{p(c)} \sim$

$\frac{p(1,b,a)}{p(b)}$

v \ n	1	2	3
a	1/2	0	0
e	1/2	0	0
i	0	0	0

$P(c=b) = 1/4$

v \ n	1	2	3
a	1/3	1/12	1/12
e	1/3	1/12	1/12
i	0	0	0

$P(c=d) = 3/16$

v \ n	1	2	3
a	1/7	2/7	1/14
e	0	0	0
i	1/7	2/7	1/14

$P(c=g) = \frac{7}{32}$

v \ n	1	2	3
a	0	0	0
e	1/6	1/6	1/6
i	1/6	1/6	1/6

$P(c=l) = 3/16$

	1	2	3
a	0	0	0
e	0	1/10	2/5
i	0	1/10	2/5

$P(c=m) = \frac{5}{..}$

g) $H(N, V/C) = H(N, V, C) - H(C) = 4{,}25 - 2{,}303 = \boxed{1{,}947 \text{ bits}}$

h) $p(c, v/n) = \dfrac{p(n, c, v)}{p(n)}$

$\frac{P(1, b, a)}{P(n=1)}$

v \ c	b	d	f	l	m
a	1/4	1/8	1/16	0	0
e	1/4	1/8	0	1/16	0
i	0	0	1/16	1/16	0

$P(n=1) = 1/2$

v \ c	b	d	f	l	m
a	0	1/16	1/4	0	0
e	0	1/16	0	1/8	1/16
i	0	0	1/4	1/8	1/16

$P(n=2) = 1/4$

v \ c	b	d	f	l	m
a	0	1/16	1/16	0	0
e	0	1/16	0	1/8	1/4
i	0	0	1/16	1/8	1/4

$P(n=3) = 1/4$

j) $H(C, V/N) = H(N, C, V) - H(N) = 4{,}25 - 1{,}5 = \boxed{2{,}75 \text{ bits}}$

i) $H(C, V/N) = \sum_{\forall v} \sum_{\forall c} p(c, v/n=1) \cdot \log_2 \left(\frac{1}{p(c, v, n=1)} \right) = *$ $\hookrightarrow N=1$

$* \; 2 \cdot \left(\frac{1}{4} \cdot 2 \right) + 2 \cdot \left(\frac{1}{8} \cdot 3 \right) + 4 \cdot \left(\frac{1}{16} \cdot 4 \right) = \boxed{2{,}75 \text{ bits}}$

$-H(V, C) + \cancel{H(C)}$

k) $H(N/C, V) = H(N, V/C) - H(V/C) = H(N, V, C) - \cancel{H(C)} - H(V/C)$

$\hookrightarrow 4{,}25 - 3{,}303 = 0{,}947$ bits

$H(V/N, C) = H(N, V, C) - H(N, C) = 4{,}25 - 3{,}25 = 1$ bit

$H(C/N, V) = H(N, V, C) - H(N, V) = 4{,}25 - 2{,}9504 = 1{,}2996$ bits

l) $I(N; C) = H(N) - H(N/C) = H(N) - H(N, C) + H(C) = 1{,}5 - 3{,}25 + 2{,}303$

$\hookrightarrow 0{,}553$ bits

$I(C; V) = H(C) - H(C/V) = H(C) - H(C, V) + H(V) = 2{,}303 - 3{,}303 + 1{,}5719$

$\hookrightarrow 0{,}5719$ bits

$I(N; V) = H(N) - H(N/V) = H(N) - H(N, V) + H(V) = 1{,}5 - 2{,}9504 + 1{,}5719$

$\hookrightarrow 0{,}1215$ bits

m) $I(N; C/V) = H(N/V) - H(N/C,V) = H(N,V) - H(V) - H(N/C,V) =$

$= 2{,}9504 - 1{,}8719 - 0{,}947 = 0{,}4319$

$I(C; V/N) = H(C/N) - H(C/V,N) = H(C,N) - H(N) - H(C/V,N) =$

$= 3{,}25 - 1{,}5 - 1{,}2996 = 0{,}4504$ bits

$H(C/V,N) = H(N,V,C) - H(N,V) = H(C/N,V)$

$I(N; V/C) = H(N/C) - H(N/V,C) = H(N,C) - H(C) - H(N/V,C) =$

$= 3{,}25 - 2{,}303 - 0{,}947 = 0$ bits

n) Dependencia markoviana debe cumplir $I(x;y) \geq I(x;z)$

$I(N;V) = 0{,}1215 \not\geq I(N;C) = 0{,}553$ No hay

$I(N;C) \geq I(N;V)$ Hay dep. markov $N \to C \to V$

$I(C;V) = 0{,}57198 > I(C;N) = 0{,}553$ $C \to V \to N$