Description of the power system model *medea*

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1 Overview

medea is a simple, stylized and parsimonious model of interconnected power and heating systems in Western and Central Europe. It simulates investment in intermittent and conventional electricity and heat generation technologies as well as in cross-border electricity transmission capacities. At the same time, the model determines the system-cost minimizing hourly dispatch of electricity and heat generators to meet price-inelastic demand. Model results include hourly energy generation by technology and the associated fuel use and CO2 emissions, investment in and decommissioning of conventional and renewable generators and energy storages, hourly cross-border flows of electricity and potentially required transmission capacity expansion, as well as producer and consumer surplus.

A detailed description of the model is provided in the following. Section 2 gives an overview of the sets and set elements used in *medea*. Sections 3 and 4 introduce the model's parameters and variables, while section 6 gives a detailed description of the model's mathematical formulation.

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2 Sets

Sets are denoted by upper-case latin letters, while set elements are denoted by lower-case latin letters.

name	**math symbol**	**GAMS symbol**	**elements**
fuels	$f \in F$	`f`	`nuclear, lignite, coal, gas, oil, biomass, power`
power generation technologies	$i \in I$	`i`	`nuc, lig_stm, lig_stm_chp, lig_boa, lig_boa_chp,coal_sub, coal_sub_chp, coal_sc, coal_sc_chp, coal_usc, coal_usc_chp, coal_igcc, ng_stm, ng_stm_chp, ng_ctb_lo, ng_ctb_lo_chp, ng_ctb_hi, ng_ctb_hi_chp, ng_cc_lo, ng_cc_lo_chp, ng_cc_hi, ng_cc_hi_chp, ng_mtr, ng_mtr_chp, ng_boiler_chp, oil_stm, oil_stm_chp, oil_ctb, oil_ctb_chp, oil_cc, oil_cc_chp, bio, bio_chp, heatpump_pth`
power to heat technologies	$h \in H \subset I$	`h(i)`	`heatpump_pth`
CHP technologies	$j \in J \subset I$	`j(i)`	`lig_stm_chp, lig_boa_chp, coal_sub_chp, coal_sc_chp, coal_usc_chp, ng_stm_chp, ng_ctb_lo_chp, ng_ctb_hi_chp, ng_cc_lo_chp, ng_cc_hi_chp, ng_mtr_chp, ng_boiler_chp, oil_stm_chp, oil_ctb_chp, oil_cc_chp, bio_chp`
storage technologies	$k \in K$	`k`	`res_day, res_week, res_season, psp_day, psp_week, psp_season, battery`
feasible operation region limits	$l \in L$	`l`	`l1, l2, l3, l4`
energy products	$m \in M$	`m`	`el, ht`
intermittent generators	$n \in N$	`n`	`wind_on, wind_off, pv, ror`
time periods (hours)	$t \in T$	`t`	`t1, t2, ..., t8760`
market zones	$z \in Z$	`z`	`AT, DE`

3 Parameters

Parameters are denoted either by lower-case greek letters or by upper-case latin letters.

name	**math symbol**	**GAMS symbol**	**unit**
distance between countries	$\delta_{z,zz}$	`DISTANCE(z,zz)`	km
fuel emission intensity	ε_f	`CO2_INTENSITY(f)`	t_{CO_2} / MWh
power plant efficiency	$\eta_{i,m,f}$	`EFFICIENCY_G(i,m,f)`	MWh / MWh
efficiency power out	$\eta^{out}_{z,k}$	`EFFICIENCY_S_OUT(k)`	
efficiency power in	$\eta^{in}_{z,k}$	`EFFICIENCY_S_IN(k)`	
scaling factor for peak load	λ_z	`LAMBDA(z)`	
value of lost load	μ_z	`VALUE_NSE(z)`	€ / MWh
reservoir inflows	$\rho_{z,t,k}$	`INFLOWS(z,t,k)`	MW
scaling factor for peak intermittent generation	σ_z	`SIGMA(z)`	
intermittent generation profile	$\phi_{z,t,n}$	`GEN_PROFILE(z,t,n)`	$[0,1]$
peak intermittent generation profile	$\widehat{\phi}_{z,n}$	`PEAK_PROFILE(z,n)`	$[0,1]$
inputs of feasible operating region	$\chi_{i,l,f}$	`FEASIBLE_INPUT(i,l,f)`	$[0,1]$
output tuples of feasible operating region	$\psi_{i,l,m}$	`FEASIBLE_OUTPUT(i,l,m)`	$[0,1]$
capital cost of intermittent generators (specific, annuity)	$C^r_{z,n}$	`CAPITALCOST_R(z,n)`	k€ / GW
capital cost of thermal generators (specific, annuity)	$C^g_{z,i}$	`CAPITALCOST_G(z,i)`	k€ / GW
capital cost of storages - power (specific, annuity)	$C^s_{z,k}$	`CAPITALCOST_S(z,k)`	k€ / GW
capital cost of storages - energy (specific, annuity)	$C^v_{z,k}$	`CAPITALCOST_V(z,k)`	k€ / GW
capital cost of transmission capacity	C^x	`CAPITALCOST_X`	k€ / GW
energy demand	$D_{z,t,m}$	`DEMAND(z,t,m)`	GW
peak demand	$\widehat{D}_{z,m}$	`PEAK_LOAD(z,m)`	GW
initial capacity of dispatchable generators	$\widetilde{G}_{z,i}$	`INITIAL_CAP_G(z,tec)`	GW

name	math symbol	GAMS symbol	unit
quasi-fixed O&M cost	O_i^q	`OM_COST_QFIX(i)`	k€ / GW
variable O&M cost	O_i^v	`OM_COST_VAR(i)`	€ / MWh
CO_2 price	$P_{t,z}^e$	`PRICE_CO2(t,z)`	€ / t_{CO_2}
fuel price	$P_{t,z,f}$	`PRICE_FUEL(t,z,f)`	€ / MWh
initial capacity of intermittent generators	$\widetilde{R}_{z,n}$	`INITIAL_CAP_R(z,n)`	GW
max power out	$\widetilde{S}_{z,k}^{out}$	`INITIAL_CAP_S_OUT(z,k)`	GW
max power in	$\widetilde{S}_{z,k}^{in}$	`INITIAL_CAP_S_IN(z,k)`	GW
max energy stored	$\widetilde{V}_{z,k}$	`INITIAL_CAP_V(z,k)`	
installed available transmission capacity	$\widetilde{X}_{z,zz}$	`INITIAL_CAP_X(z,zz)`	GW

4 Variables

Variables are denoted by lower-case latin letters.

name	math symbol	GAMS symbol	Unit
fuel burn for energy generation	$b_{z,t,i,f}$	`b(z,t,i,f)`	GW
total system cost	c	`cost_system`	k€
zonal system cost	c_z	`cost_zonal(z)`	k€
fuel cost	$c^b_{z,t,i}$	`cost_fuel(z,t,i)`	k€
emission cost	$c^e_{z,t,i}$	`cost_co2(z,t,i)`	k€
total o&m cost	$c^{om}_{z,i}$	`cost_om(z,i)`	k€
capital cost of generators	c^g_z	`cost_invest_g(z)`	k€
total cost of non-served load	c^q_z	`cost_nse(z)`	k€
capital cost of intermittent generators	c^r_z	`cost_invest_r(z)`	k€
capital cost of storages	$c^{s,v}_z$	`cost_invest_sv(z)`	k€
capital cost of interconnectors	c^x_z	`cost_invest_x(z)`	k€
CO_2 emissions	e_z	`emission_co2(z)`	t CO_2
added capacity of dispatchables	$\widetilde{g}^+_{z,i}$	`add_g(z,i)`	GW
decommissioned capacity of dispatchables	$\widetilde{g}^-_{z,i}$	`deco_g(z,i)`	GW
energy generated by conventionals	$g_{z,t,i,m}$	`g(z,t,i,m)`	GW
curtailed energy	$q^+_{z,t}$	`q_curtail(z,t)`	GW
non-served energy	$q^-_{z,t,m}$	`q_nse(z,t,m)`	GW
added capacity of intermittents	$\widetilde{r}^+_{z,n}$	`add_r(z,n)`	GW
electricity generated by intermittents	$r_{z,t,n}$	`r(z,t,n)`	GW
added storage capacity (power)	$\widetilde{s}^+_{z,k}$	`add_s(z,k)`	GW
energy stored in	$s^{in}_{z,t,k}$	`s_in(z,t,k)`	GW
energy stored out	$s^{out}_{z,t,k}$	`s_out(z,t,k)`	GW
added storage capacity (energy)	$\widetilde{v}^+_{z,k}$	`add_v(z,k)`	GWh
storage energy content	$v_{z,t,k}$	`v(z,t,k)`	GWh
operating region weight	$w_{z,t,i,l}$	`w(z,t,i,l)`	
added transmission capacity	$\widetilde{x}^+_{z,zz}$	`add_x(z,zz)`	GW
electricity net export	$x_{z,zz,t}$	`x(z,zz,t)`	GW

5 Naming system

	initial capacity†	added capacity‡	decom-missioned capacity‡	specific investment cost†	dispatch‡
thermal units	$\widetilde{G}_{z,i}$	$\widetilde{g}^{+}_{z,i}$	$\widetilde{g}^{-}_{z,i}$	$C^{g}_{z,i}$	$g_{z,t,i,m}$
intermittent units	$\widetilde{R}_{z,n}$	$\widetilde{r}^{+}_{z,n}$	$\widetilde{r}^{-}_{z,n}$	$C^{r}_{z,n}$	$r_{z,t,n}$
storages (power)	$\widetilde{S}_{z,k}$	$\widetilde{s}^{+}_{z,k}$	$\widetilde{s}^{-}_{z,k}$	$C^{s}_{z,k}$	$s_{z,t,k}$
storages (energy)	$\widetilde{V}_{z,k}$	$\widetilde{v}^{+}_{z,k}$	$\widetilde{v}^{-}_{z,t,k}$	$C^{v}_{z,k}$	na
transmission	$\widetilde{X}_{z,zz}$	$\widetilde{x}^{+}_{z,zz}$	$\widetilde{x}^{-}_{z,zz}$	$C^{x}_{z,zz}$	$x_{z,zz,t}$

† parameter
‡ variable

6 Mathematical description

Model objective *medea* minimizes total system cost c, i.e. the total cost of generating electricity and heat from technologies and capacities adequate to meet demand, over a large number of decision variables, essentially representing investment and dispatch decisions in each market zone z of the modelled energy systems.

$$\min c = \sum_{z} (c_z) \tag{1}$$

Zonal system costs c_z are the sum of fuel cost $c^b_{z,t,i}$, emission cost $c^e_{z,t,i}$, operation and maintenance cost $c^{om}_{z,i}$, capital costs of investment in conventional and intermittent generation (c^g_z, c^r_z), storage ($c^{s,v}_z$) and transmission (c^x_z) equipment, and the cost of non-served load (c^q_z) that accrues when demand is not met, e.g. when there is a power outage.

$$c_z = \sum_{t,i} c^b_{z,t,i} + \sum_{t,i} c^e_{z,t,i} + \sum_{i} c^{om}_{z,i} + c^g_z + c^r_z + c^{s,v}_z + c^x_z + c^q_z \qquad \forall z \tag{2}$$

The components of zonal system costs are calculated as given in equations (3) to (10). Lower-case c represent total cost, while upper-case C denotes specific, annualized capital cost of technology investment. Prices for fuels and CO_2 are denoted by P.

$$c^b_{z,t,i} = \sum_{f} \left(P_{t,z,f}\, b_{t,z,i,f}\right) \qquad \forall z,t,i \tag{3}$$

$$c^e_{z,t,i} = \sum_{f} \left(P^e_{t,z}\, \varepsilon_f\, b_{t,z,i,f}\right) \qquad \forall z,t,i \tag{4}$$

$$c^{om}_{z,i} = O^q_i \left(\widetilde{G}_{z,i} - \widetilde{g}^-_{z,i} + \widetilde{g}^+_{z,i}\right) + \sum_{t,m} \left(O^v_i\, g_{z,t,i,m}\right) \qquad \forall z,i \tag{5}$$

$$c^g_z = \sum_{i} \left(C^g_{z,i}\, \widetilde{g}^+_{z,i}\right) \qquad \forall z \tag{6}$$

$$c^r_z = \sum_{n} \left(C^r_{z,n}\, \widetilde{r}^+_{z,n}\right) \qquad \forall z \tag{7}$$

$$c^{s,v}_z = \sum_{k} \left(C^s_{z,k}\, \widetilde{s}^+_{z,k} + C^v_{z,k}\, v^+_{z,k}\right) \qquad \forall z \tag{8}$$

$$c^x_z = \frac{1}{2} \sum_{zz} (C^x\, \delta_{z,zz}\, \widetilde{x}^+_{z,zz}) \qquad \forall z \tag{9}$$

$$c^q_z = \mu \sum_{t,m} q^-_{z,t,m} \qquad \forall z \tag{10}$$

Market clearing In each hour, the markets for electricity and heat have to clear. Equation (11) ensures that the total supply from conventional and intermittent sources, and storages equals total electricity demand plus net exports, electricity stored and used for heat generation. Likewise, equation (12) clears the heat market by equating heat generation to heat demand.

$$\begin{aligned} &\sum_{i} g_{z,t,i,\mathrm{el}} + \sum_{k} s^{out}_{z,t,k} + \sum_{n} r_{z,t,n} = \\ &\qquad D_{z,t,\mathrm{el}} + \sum_{i} b_{z,t,i,\mathrm{el}} + \sum_{k} s^{in}_{z,t,k} + \sum_{zz} x_{z,zz,t} - q^-_{z,t,\mathrm{el}} + q^+_{z,t} \qquad \forall z,t \end{aligned} \tag{11}$$

$$\sum_{i} g_{z,t,i,\mathrm{ht}} = D_{z,t,\mathrm{ht}} - q^{-}_{z,t,\mathrm{ht}} \qquad \forall z, t \tag{12}$$

medea can be thought of as representing energy-only electricity and heat markets without capacity payments. Then, the marginals of the market clearing equations (11) and (12), $\partial C / \partial D_{z,t,m}$, can be interpreted as the zonal prices for electricity and heat, respectively.

Energy generation Energy generation is constrained by available installed capacity, which can be adjusted through investment and decommissioning.

$$g_{z,t,i,m} \leq \widetilde{G}_{z,i} + \widetilde{g}^{+}_{z,i} - \widetilde{g}^{-}_{z,i} \qquad \forall z, t, i, m \tag{13}$$

Generator efficiency η determines the amount of fuel that needs to be spent in order to generate a given amount of energy.

$$g_{z,t,i,m} = \sum_{f} \eta_{i,m,f}\, b_{z,t,i,f} \qquad \forall z, t, i \notin J \tag{14}$$

Thermal co-generation Co-generation units jointly generate heat and electricity. All feasible combinations of heat and electricity generation along with the corresponding fuel requirement are reflected in so-called 'feasible operating regions'. The elements $l \in L$ span up a three-dimensional, convex feasible operating region for each co-generation technology. The weights w form a convex combination of the corners l, which are scaled to the available installed capacity of each co-generation technology. Heat and electricity output along with the corresponding fuel requirement is then set according to the chosen weights.

$$\sum_{l} w_{z,t,i,l} = \widetilde{G}_{z,i} + \widetilde{g}^{+}_{z,i} - \widetilde{g}^{-}_{z,i} \qquad \forall z, t, i \in J \tag{15}$$

$$g_{z,t,i,m} \leq \sum_{l} \psi_{i,l,m}\, w_{z,t,i,l} \qquad \forall z, t, i \in J, m \tag{16}$$

$$b_{z,t,i,f} \geq \sum_{l} \chi_{i,l,f}\, w_{z,t,i,l} \qquad \forall z, t, i \in J, f \tag{17}$$

Intermittent electricity generation Electricity generation from intermittent sources wind (on-shore and off-shore), solar irradiation, and river runoff follows generation profiles $\phi_{z,t,n} \in [0,1]$ and is scaled according to corresponding installed and added capacity.

$$r_{z,t,n} = \phi_{z,t,n} \left(\widetilde{R}_{z,n} + \widetilde{r}^{+}_{z,n} \right) \qquad \forall z, t, n \tag{18}$$

Electricity storages Charging and discharging of storages is constrained by the storages' power capacity $\widetilde{s}^{in}_{z,k}$ and $\widetilde{s}^{out}_{z,k}$, respectively. Similarly, the total energy that can be stored is constrained by the storage technology's energy capacity $\widetilde{v}_{z,k}$.

$$s^{out}_{z,t,k} \leq \widetilde{S}^{out}_{z,k} + \widetilde{s}^{+}_{z,k} \qquad \forall z, t, k \tag{19}$$

$$s^{in}_{z,t,k} \leq \widetilde{S}^{in}_{z,k} + \widetilde{s}^{+}_{z,k} \qquad \forall z, t, k \tag{20}$$

$$v_{z,t,k} \leq \widetilde{V}_{z,k} + \widetilde{v}^{+}_{z,k} \qquad \forall z, t, k \tag{21}$$

Storage operation is subject to a storage balance, such that the current energy content must be equal to the previous period's energy content plus all energy flowing into the storage less all energy flowing out of the storage.

$$v_{z,t,k} = \rho_{z,t,k} + \eta^{in}_{z,k}\, s^{in}_{z,t,k} - (\eta^{out}_{z,k})^{-1}\, s^{out}_{z,t,k} + v_{z,t-1,k} \qquad \forall z,t,k : t > 1,\ \eta^{out}_{z,k} > 0 \tag{22}$$

Since the model can add storage power capacity and energy capacity independently, we require a storage to hold at least as much energy as it could store in (or out) in one hour.

$$\widetilde{v}^{+}_{z,k} \geq \widetilde{s}^{+}_{z,k} \qquad \forall z,k \tag{23}$$

Electricity exchange Implicitly, *medea* assumes that there are no transmission constraints within market zones. However, electricity exchange between market zones is subject to several constraints.

First, exchange between market zones is constrained by available transfer capacities. Transfer capacities can be expanded at constant, specific investment cost (see equation (9)). This rules out economies of scale in transmission investment that might arise in interconnected, meshed grids.

$$x_{z,zz,t} \leq \widetilde{X}_{z,zz} + \widetilde{x}^{+}_{z,zz} \qquad \forall z,zz,t \tag{24}$$

$$x_{z,zz,t} \geq -\left(\widetilde{X}_{z,zz} + \widetilde{x}^{+}_{z,zz}\right) \qquad \forall z,zz,t \tag{25}$$

By definition, electricity net exports $x_{z,zz,t}$ from z to zz must equal electricity net imports of zz from z.

$$x_{z,zz,t} = -x_{zz,z,t} \qquad \forall z,zz,t \tag{26}$$

Added transmission capacities can be used in either direction.

$$\widetilde{x}^{+}_{z,zz} = \widetilde{x}^{+}_{zz,z} \qquad \forall z,zz \tag{27}$$

Finally, electricity cannot flow between zones where there is no transmission infrastructure in place (including intra-zonal flows).

$$x_{z,zz,t} = 0 \qquad \forall z,zz,t : \widetilde{X}_{z,zz} = 0 \tag{28}$$

$$x_{zz,z,t} = 0 \qquad \forall z,zz,t : \widetilde{X}_{z,zz} = 0 \tag{29}$$

Decommissioning of thermal units Keeping a plant available for dispatch gives rise to quasi-fixed operation and maintenance costs. Such cost can be avoided by decommissioning an energy generator. This is modelled as a reduction in generation capacity, which cannot exceed installed capacity.

$$\widetilde{g}^{-}_{z,i} \leq \widetilde{G}_{z,i} + \widetilde{g}^{+}_{z,i} \qquad \forall z,i \tag{30}$$

Ancillary services Power systems require various system services for secure and reliable operation, such as balancing services or voltage support through the provision of reactive power. Such system services can only be supplied by operational generators. Thus, we approximate system service provision by a requirement on the minimal amount of spinning reserves operating at each hour. We assume that ancillary services are supplied by conventional (thermal) power plants, hydro power plants, and storages. The requirement for spinning reserves is proportional to electricity peak load $\widehat{D}_{z,\text{el}} = \max_t D_{z,t,\text{el}}$ and peak generation from wind and solar resources, where $\widehat{\phi}_{z,n} = \max_t \phi_{z,t,n}$.

$$\sum_i (g_{z,t,i,\text{el}}) + r_{z,t,\text{ror}} + \sum_k \left(s^{out}_{z,t,k} + s^{in}_{z,t,k}\right) \geq \lambda_z \widehat{D}_{z,\text{el}} + \sigma_z \sum_{n \setminus \{\text{ror}\}} \widehat{\phi}_{z,n}(\widetilde{R}_{z,n} + \widetilde{r}^{+}_{z,n}) \qquad \forall z,t \quad (31)$$

Curtailment Electricity generated from intermittent sources can be curtailed (disposed of) without any further cost (apart from implicit opportunity cost).

$$q^{+}_{z,t} \leq \sum_n r_{z,t,n} \qquad \forall z,t \quad (32)$$