1 Overview

medea is a simple, stylized and parsimonious power system model. It simulates investment in intermittent and conventional electricity and heat generation technologies as well as in cross-border electricity transmission capacities. At the same time, the model determines the system-cost minimizing hourly dispatch of electricity and heat generators to meet price-inelastic demand. Model results include hourly energy generation by technology and the associated fuel use and CO2 emissions, investment in and decommissioning of conventional and renewable generators and energy storages, hourly cross-border flows of electricity and potentially required transmission capacity expansion, as well as producer and consumer surplus.

A detailed description of the model is provided in the following. Section 2 gives an overview of the sets and set elements used in *medea*. Sections 3 and 4 introduce the model's parameters and variables, while section 5 gives a detailed description of the model's mathematical formulation.

2 Sets

name	math symbol	GAMS symbol	elements
market zones	$z \in Z$	`r`	`AT, DE`
time periods (hours)	$t \in T$	`t`	`t1, t2, ..., t8760`
power generation technologies	$i \in I$	`tec`	`nuc, lig_stm, lig_stm_chp, lig_boa, lig_boa_chp, coal_sub, coal_sub_chp, coal_sc, coal_sc_chp, coal_usc, coal_usc_chp, coal_igcc, ng_stm, ng_stm_chp, ng_ctb_lo, ng_ctb_lo_chp, ng_ctb_hi, ng_ctb_hi_chp, ng_cc_lo, ng_cc_lo_chp, ng_cc_hi, ng_cc_hi_chp, ng_mtr, ng_mtr_chp, ng_boiler_chp, oil_stm, oil_stm_chp, oil_ctb, oil_ctb_chp, oil_cc, oil_cc_chp, bio, bio_chp, heatpump_pth`
CHP technologies	$j \in J \subset I$	`tec_chp`	`lig_stm_chp, lig_boa_chp, coal_sub_chp, coal_sc_chp, coal_usc_chp, ng_stm_chp, ng_ctb_lo_chp, ng_ctb_hi_chp, ng_cc_lo_chp, ng_cc_hi_chp, ng_mtr_chp, ng_boiler_chp, oil_stm_chp, oil_ctb_chp, oil_cc_chp, bio_chp`
power to heat technologies	$h \in H \subset I$	`tec_pth`	`heatpump_pth`
storage technologies	$k \in K$	`tec_strg`	`res_day, res_week, res_season, psp_day, psp_week, psp_season, battery`
intermittent generators	$n \in N$	`tec_itm`	`wind_on, wind_off, pv, ror`
fuels	$f \in F$	`f`	`nuclear, lignite, coal, gas, oil, biomass, power`
feasible operation region limits	$l \in L$	`l`	`l1, l2, l3, l4`
energy products	$m \in M = \{\text{el}, \text{ht}\}$	`prd`	`power, heat`

3 Parameters

name	math symbol	GAMS symbol	Unit
minimal conventional generation	σ_z	`ANCIL_SERVICE_LVL(r)`	GW
energy demand	$D_{z,t,p}$	`CONSUMPTION(r,t,prd)`	GW
power plant efficiency	$\eta_{i,p,f}$	`EFFICIENCY(tec,prd,f)`	MW/MW_{th}
fuel emission intensity	ε_f	`EMISSION_INTENSITY(f)`	$t\mathrm{CO}_2/MWh_{th}$
feasible operating region	$\chi_{i,l,f}$	`FEASIBLE_INPUT(tec,l,f)`	MWh ?
feasible operating region	$\psi_{i,l,p}$	`FEASIBLE_OUTPUT(tec,l,prd)`	MWh ?
intermittent generation profile	$\phi_{z,t,n}$	`GEN_PROFILE(r,t,tec_itm)`	%
installed capacity of intermittent generators	$\bar{r}^0_{z,n}$	`INSTALLED_CAP_ITM(r,tec_itm)`	GW
installed capacity of thermal generators	$\bar{g}^0_{z,i}$	`INSTALLED_CAP_THERM(r,tec)`	GW
capital cost of intermittent generators (specific, annuity)	$\kappa_{z,n}$	`INVESTCOST_ITM(r,tec_itm)`	$\frac{mnEUR}{GW}$
capital cost of thermal generators (specific, annuity)	$\kappa_{z,i}$	`INVESTCOST_THERMAL(r,tec)`	$\frac{mnEUR}{GW}$
capital cost of storages - power (specific, annuity)	$\kappa^P_{z,k}$	`STORAGE_PROPERTIES(r,tec_strg,'cost_power')`	$\frac{mnEUR}{GW}$
capital cost of storages - energy (specific, annuity)	$\kappa^E_{z,k}$	`STORAGE_PROPERTIES(r,tec_strg,'cost_energy')`	$\frac{mnEUR}{GW}$
installed available transfer capacity	$\bar{x}^0_{z,zz}$	`ATC(r,rr)`	GW
count 100 MW slices of same technology	$\lvert i_z \rvert$	`NUM(r,tec)`	.
quasi-fixed O&M cost	o^q_i	`OM_FIXED_COST(tec)`	EUR
variable O&M cost	o^v_i	`OM_VAR_COST(tec)`	EUR / MWh
CO_2 price	$P^e_{t,z}$	`PRICE_EUA(t,r)`	EUR / t CO2
fuel price	$P_{t,z,f}$	`PRICE_FUEL(t,r,f)`	EUR / MWh
reservoir inflows	$\rho_{z,t,k}$	`RESERVOIR_INFLOWS(r,t,tec_strg)`	MW
max power out	$\bar{s}^{out}_{z,k}$	`STORAGE_PROPERTIES(r,tec_strg,'power_out')`	GW
max power in	$\bar{s}^{in}_{z,k}$	`STORAGE_PROPERTIES(r,tec_strg,'power_in')`	GW
max energy stored	$\bar{v}^0_{z,k}$	`STORAGE_PROPERTIES(r,tec_strg,'energy_max')`	.
efficiency power out	$\eta^{out}_{z,k}$	`STORAGE_PROPERTIES(r,tec_strg,'efficiency_out')`	.
efficiency power in	$\eta^{in}_{z,k}$	`STORAGE_PROPERTIES(r,tec_strg,'efficiency_in')`	.
value of lost load	μ	`not set yet`	EUR

4 Variables

name	math symbol	GAMS symbol	Unit
zonal system cost	C_z	`cost(r)`	EUR
emission cost	$C^e_{z,t,i}$	`cost_emission(r,t,tec)`	EUR
fuel cost	$C^f_{z,t,i}$	`cost_fuel(r,t,tec)`	EUR
total o&m cost	$C^{om}_{z,i}$	`cost_om(r,tec)`	EUR
capital cost of generators	$C^{inv,i}_z$	`cost_invgen(r)`	EUR
capital cost of storages	$C^{inv,k}_z$	`cost_invstrg(r)`	EUR
capital cost of interconnectors	$C^{inv,ic}_z$	`cost_gridexpansion(r)`	EUR
added capacity of intermittents	$\bar{r}^+_{z,n}$	`invest_res(r,tec_itm)`	GW
added capacity of conventionals	$\bar{g}^+_{z,i}$	`invest_thermal(r,tec)`	GW
added storage capacity (power)	$\bar{s}^+_{z,k}$	`invest_storage_power(r,tec_strg)`	EUR
added storage capacity (energy)	$\bar{v}^+_{z,k}$	`invest_storage_energy(r,tec_strg)`	EUR
added transmission capacity	$\bar{x}^+_{z,zz}$	`invest_atc(r,rr)`	EUR
decommissioned capacity of conventionals	$\bar{g}^-_{z,i}$	`decommission(r,tec)`	GW
energy generated by conventionals	$g_{z,t,i,p}$	`q_gen(r,t,tec,prd)`	GW
electricity generated by intermittents	$r_{z,t,n}$	`q_itm(r,t,tec_itm)`	GW
operating region weight	$w_{z,t,i,l}$	`cc_weights(r,t,tec,l)`	.
fuel burn for energy generation	$b_{z,t,i,f}$	`q_fueluse(r,t,tec,f)`	GW
energy stored in	$s^{in}_{z,t,k}$	`q_store_in(r,t,tec_strg)`	GW
energy stored out	$s^{out}_{z,t,k}$	`q_store_out(r,t,tec_strg)`	GW
storage energy content	$v_{z,t,k}$	`storage_level(r,t,tec_strg)`	GWh
electricity net export	$x_{z,zz,t}$	`flow(r,rr,t)`	GW
curtailed energy	$\Omega^+_{z,t}$	`q_curtail(r,t)`	GW
non-served energy	$\Omega^-_{z,t,p}$	`q_nonserved(r,t,prd)`	GW
CO_2 emissions	e_z	`emissions(r)`	t CO_2

5 Mathematical description

Model objective *medea* minimizes total system cost, i.e. the total cost of generating electricity and heat from technologies and capacities adequate to meet demand.

$$\min C = \sum_z (C_z) \tag{1}$$

System costs consist of fuel cost $C^f_{z,t,i}$, emission cost $C^e_{z,t,i}$, operation and maintenance cost $C^{om}_{z,i}$, capital costs of investment in generation, storage and transmission equipment, and the cost of non-served load that accrues when demand is not met, e.g. when there is a power outage.

$$\begin{aligned} C_z = \sum_{t,i} C^f_{z,t,i} + \sum_{t,i} C^e_{z,t,i} + \sum_i C^{om}_{z,i} + C^{inv,i}_z + \\ C^{inv,n}_z + C^{inv,k}_z + C^{inv,ic}_z + C^{nse}_z \qquad \forall z \end{aligned} \tag{2}$$

Upper-case C represent total cost, while κ denotes specific, annualized capital cost of technology investment. Prices for fuels and CO_2 are denoted by uppercase P.

The components of total cost are calculated according to equations 3 to 10.

$$C^f_{z,t,i} = \sum_f (P_{t,z,f}\, b_{t,z,i,f}) \qquad \forall z,t,i \tag{3}$$

$$C^e_{z,t,i} = \sum_f \left(P^e_{t,z}\, \varepsilon_f\, b_{t,z,i,f}\right) \qquad \forall z,t,i \tag{4}$$

$$C^{om}_{z,i} = \left(|i_z| - \bar{g}^-_{z,i} + \bar{g}^+_{z,i}\right) o^q_i + \sum_{t,m} \left(o^v_i\, g_{z,t,i,m}\right) \qquad \forall z,i \tag{5}$$

$$C^{inv,i}_z = \kappa_{z,i}\, \bar{g}^+_{z,i} \qquad \forall z \tag{6}$$

$$C^{inv,n}_z = \kappa_{z,n}\, \bar{r}^+_{z,n} \qquad \forall z \tag{7}$$

$$C^{inv,k}_z = \kappa^P_{z,k}\, \bar{s}^+_{z,k} + \kappa^E_{z,k}\, v^+_{z,k} \qquad \forall z \tag{8}$$

$$C^{inv,ic}_z = \kappa^{ic}_z\, \bar{x}^+_{z,zz} \qquad \forall z \tag{9}$$

$$C^{nse}_z = \mu \sum_{t,m} \Omega^-_{z,t,m} \qquad \forall z \tag{10}$$

Market clearing In each hour, the markets for electricity and heat have to clear. Equation 11 ensures that the total supply from conventional and intermittent sources, and storages equals total electricity demand plus net exports, electricity stored and used for heat generation. Likewise, equation 12 clears the heat market by equating heat generation to heat demand.

$$\sum_i g_{z,t,i,\mathrm{el}} + \sum_k s^{out}_{z,t,k} + \sum_n r_{z,t,n} = D_{z,t,\mathrm{el}} + \sum_i b_{z,t,i,\mathrm{el}} + \sum_k s^{in}_{z,t,k} + \sum_{zz} x_{z,zz,t} - \Omega^-_{z,t,\mathrm{el}} + \Omega^+_{z,t} \qquad \forall z,t \tag{11}$$

$$\sum_i g_{z,t,i,\mathrm{ht}} = D_{z,t,\mathrm{ht}} - \Omega^-_{z,t,\mathrm{ht}} \qquad \forall z,t \tag{12}$$

In its current formulation, the model represents energy-only electricity and heat markets without capacity payments. Thus, the marginals of the market clearing equations ($\partial C/\partial D_{z,t,m}$) can be interpreted as the zonal prices for electricity and heat, respectively.

Thermal electricity generation Energy generation is constrained by available installed capacity, which can be adjusted through investment and decommissioning.

$$g_{z,t,i,m} \leq \arg\max_{l}(\psi_{i,l,m}) \left(|i_z| + \bar{g}^{+}_{z,i} - \bar{g}^{-}_{z,i}\right) \qquad \forall z,t,i,m \tag{13}$$

Generator efficiency η determines the amount of fuel that needs to be burnt in order to generate a given amount of electricity.

$$g_{z,t,i,\text{el}} = \sum_{f} \eta_{i,\text{el},f}\, b_{z,t,i,f} \qquad \forall z,t,i \notin J \tag{14}$$

Electric heat generation Heat pumps and electric boilers use electricity to generate heat. The amount of electricity required to generate a given amount of heat depends on the unit's efficiency η. For heat pumps, η is assumed to be 3.

$$g_{z,t,i,\text{ht}} = \sum_{f} \eta_{i,\text{ht},f}\, b_{z,t,i,f} \qquad \forall z,t,i \in H \tag{15}$$

Thermal co-generation Co-generation units jointly generate heat and electricity. All feasible combinations of heat and electricity generation along with the corresponding fuel requirement are reflected in so-called 'feasible operating region'. The elements $l \in L$ span up a three-dimensional, convex feasible operating region for each co-generation technology. The weights w form a convex combination of the corners l scaled to the available installed capacity of each co-generation technology. Heat and electricity output along with the corresponding fuel requirement is then set according to the chosen weights.

$$\sum_{l} w_{z,t,i,l} = |i_z| + \bar{g}^{+}_{z,i} - \bar{g}^{-}_{z,i} \qquad \forall z,t,i \in J \tag{16}$$

$$g_{z,t,i,m} \leq \sum_{l} w_{z,t,i,l}\, \psi_{i,l,m} \qquad \forall z,t,i \in J, m \tag{17}$$

$$b_{z,t,i,f} \geq \sum_{l} w_{z,t,i,l}\, \chi_{i,l,f} \qquad \forall z,t,i \in J, f \tag{18}$$

Intermittent electricity generation Electricity generation from intermittent sources wind (on-shore and off-shore), solar irradiation, and river runoff follows generation profiles $\phi_{z,t,n} \in [0,1]$ and is scaled according to corresponding installed and added capacity.

$$r_{z,t,n} = \phi_{z,t,n}\, \left(\bar{r}^{0}_{z,n} + \bar{r}^{+}_{z,n}\right) \qquad \forall z,t,n \tag{19}$$

Electricity storages Charging and discharging of storages is constrained by the storages' power capacity $\bar{s}^{in}_{z,k}$ and $\bar{s}^{out}_{z,k}$, respectively. Similarly, the total energy that can be stored is constrained by the storage technology's energy capacity $\bar{v}_{z,k}$.

$$s^{out}_{z,t,k} \leq \bar{s}^{out}_{z,k} + \bar{s}^{+}_{z,k} \qquad \forall z,t,k \tag{20}$$
$$s^{in}_{z,t,k} \leq \bar{s}^{in}_{z,k} + \bar{s}^{+}_{z,k} \qquad \forall z,t,k \tag{21}$$
$$v_{z,t,k} \leq \bar{v}^{0}_{z,k} + \bar{v}^{+}_{z,k} \qquad \forall z,t,k \tag{22}$$
$$\tag{23}$$

Storage operation is subject to a storage balance, such that the current energy content must be equal to the previous period's energy content plus all energy flowing into the storage less all energy flowing out of the storage.

$$v_{z,t,k} = v_{z,t-1,k} + \rho_{z,t,k} + \eta^{in}_{z,k}\, s^{in}_{z,t,k} - (\eta^{out}_{z,k})^{-1}\, s^{out}_{z,t,k} \qquad \forall z,t,k \tag{24}$$

Since the model can add storage power capacity and energy capacity independently, we require a storage to hold at least as much energy as it could store in or out in one hour.

$$\bar{v}^{+}_{z,k} \geq \bar{s}^{+}_{z,k} \qquad \forall z,k \tag{25}$$

Electricity exchange Implicitly, *medea* assumes that there are no transmission constraints within market zones. However, electricity exchange between market zones is subject to several constraints.

First, exchange between market zones is constrained by available transfer capacities. Transfer capacities can be expanded at constant, specific investment cost (see equation (9)), ruling out economies of scale in transmission investment.

$$x_{z,zz,t} \leq \bar{x}^{0}_{z,zz} + \bar{x}^{+}_{z,zz} \qquad \forall z,zz,t \tag{26}$$
$$x_{z,zz,t} \geq -\left(\bar{x}^{0}_{z,zz} + \bar{x}^{+}_{z,zz}\right) \qquad \forall z,zz,t \tag{27}$$

By definition, electricity net exports $x_{z,zz,t}$ from z to zz must equal electricity net imports of zz from z.

$$x_{z,zz,t} = -x_{zz,z,t} \qquad \forall z,zz,t \tag{28}$$

Added transmission capacities can be used in either direction.

$$\bar{x}^{+}_{z,zz} = \bar{x}^{+}_{zz,z} \qquad \forall z,zz \tag{29}$$

Finally, electricity cannot flow between zones where there is no transmission infrastructure in place (including intra-zonal flows).

$$x_{z,zz,t} = 0 \qquad \forall z \notin NTC, zz \notin NTC, t \tag{30}$$
$$x_{zz,z,t} = 0 \qquad \forall z \notin NTC, zz \notin NTC, t \tag{31}$$

Decommissioning of thermal units Any conventional unit can be decommission to save the quasi-fixed cost associated with keeping the plant potentially active.

$$\bar{g}^-_{z,i} \leq |i_z| + \bar{g}^+_{z,i} \qquad \forall z, i \tag{32}$$

Ancillary services Power systems require various system services for secure and reliable operation, such as balancing services or voltage support. Many of these system services are mostly or only supplied by operational generators. Thus, we approximate system service provision by a requirement on the minimal amount of spinning reserves operational at each hour. We assume that conventional (thermal) power plants as well as hydro power plants (even when in pumping mode) can supply ancillary services.

$$g_{z,t,i,\mathrm{el}} + r_{z,t,\mathrm{ror}} + s^{out}_{z,t,k} + s^{in}_{z,t,k} \geq \sigma_z \qquad \forall z, t, i, k \tag{33}$$

Curtailment Electricity generated from intermittent sources can be curtailed (disposed) without any further cost (apart from opportunity cost).

$$\Omega^+_{z,t} \leq \sum_n r_{z,t,n} \qquad \forall z, t \tag{34}$$