Decision Tree

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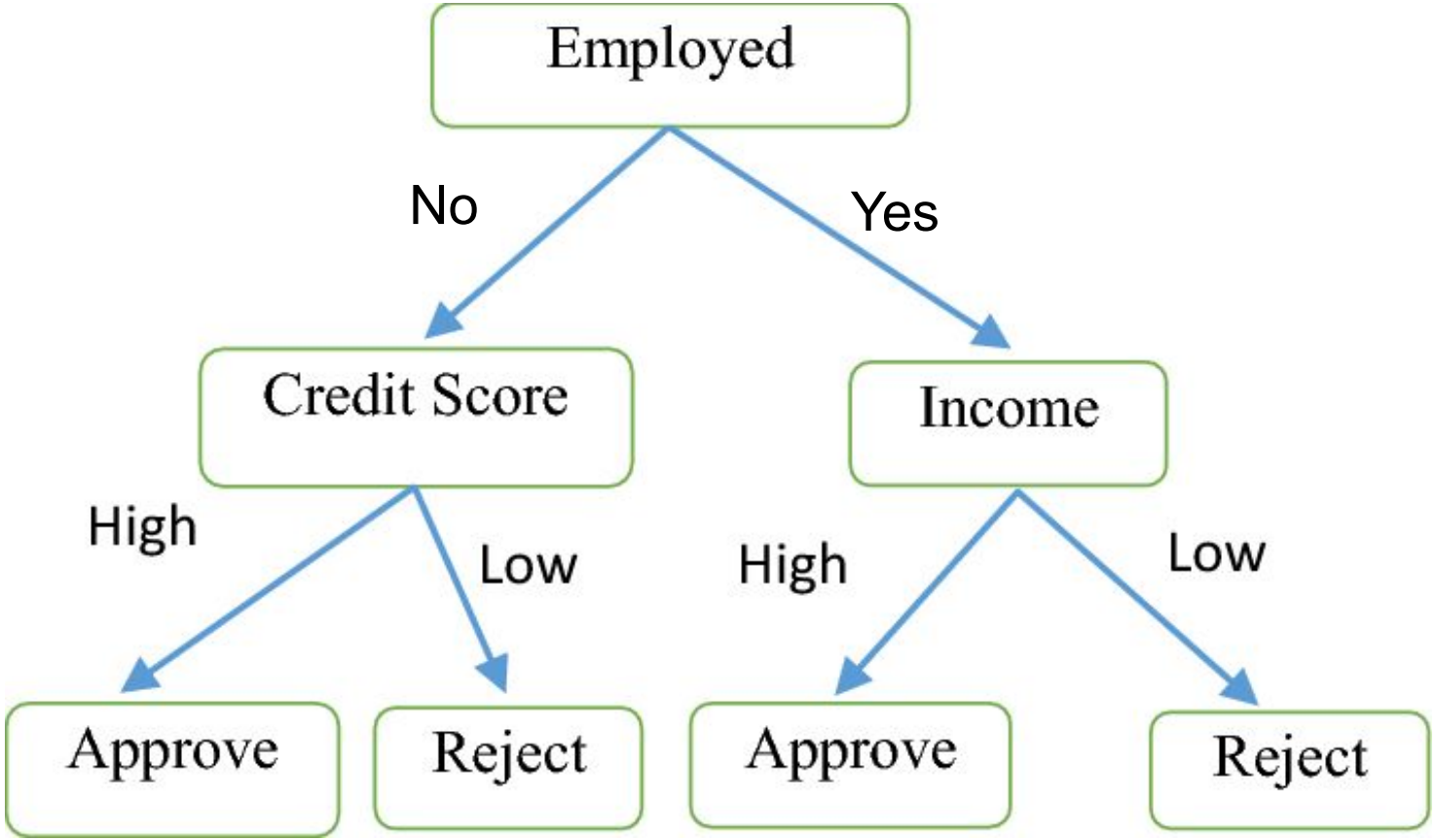
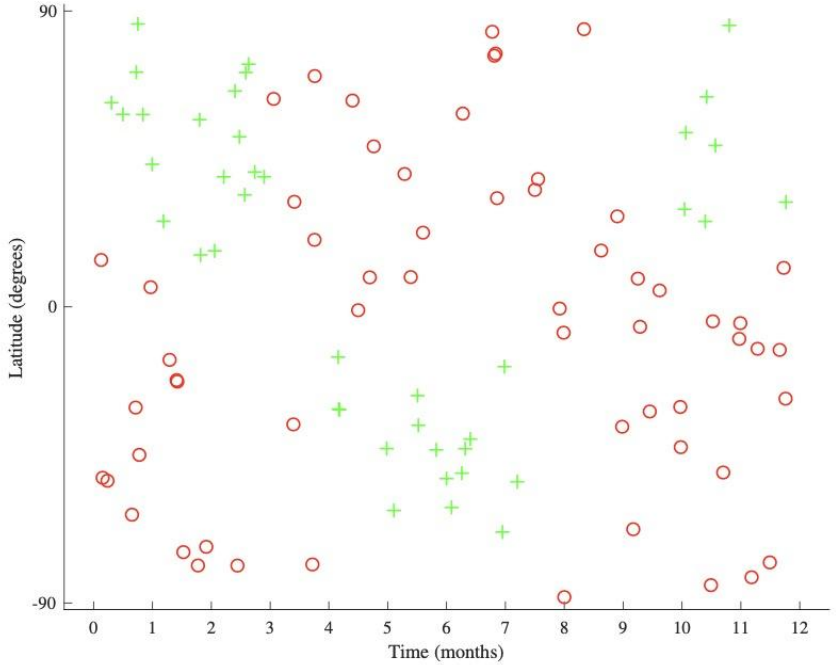
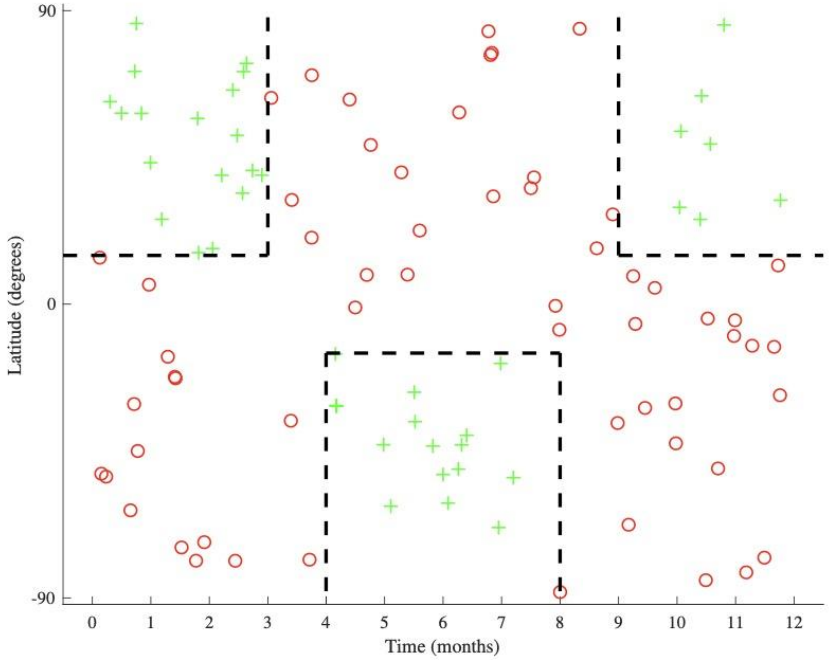

Employed
No
Yes
Credit Score
Income
High
Low
High
Low
Approve
Reject
Approve
Reject

Is it possible to ski?

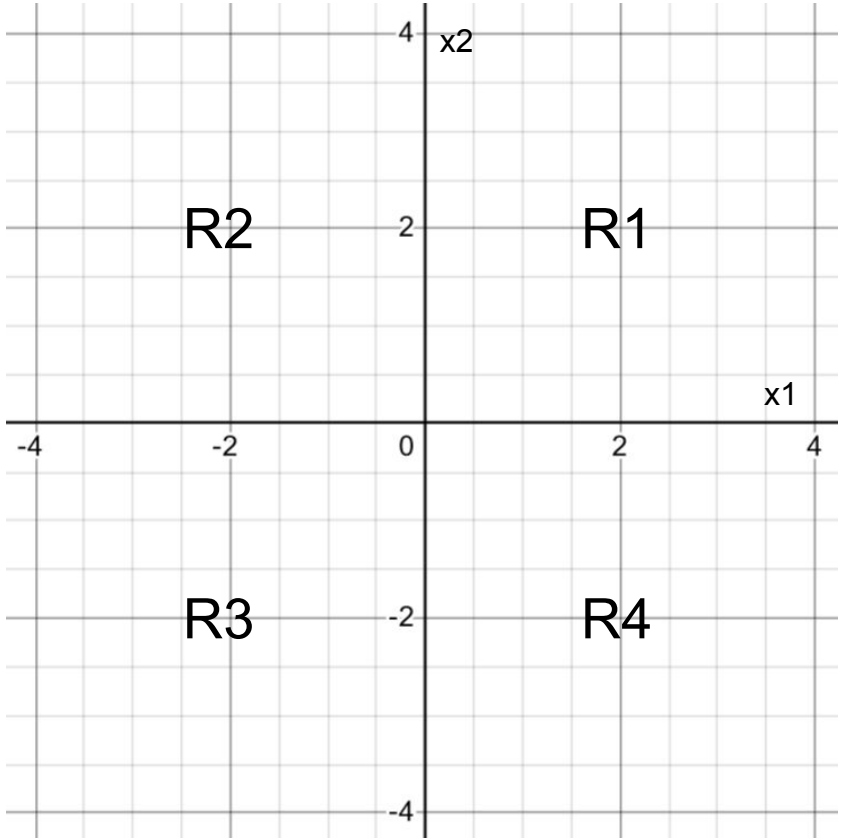




Partition the input domain into disjoint regions R_i such that the union of these regions recovers the entire input domain.

$$\mathcal{X} = \bigcup_{i=0}^{n} R_i$$

$$\text{s.t.} \qquad R_i \cap R_j = \emptyset \ \text{ for } \ i \neq j$$

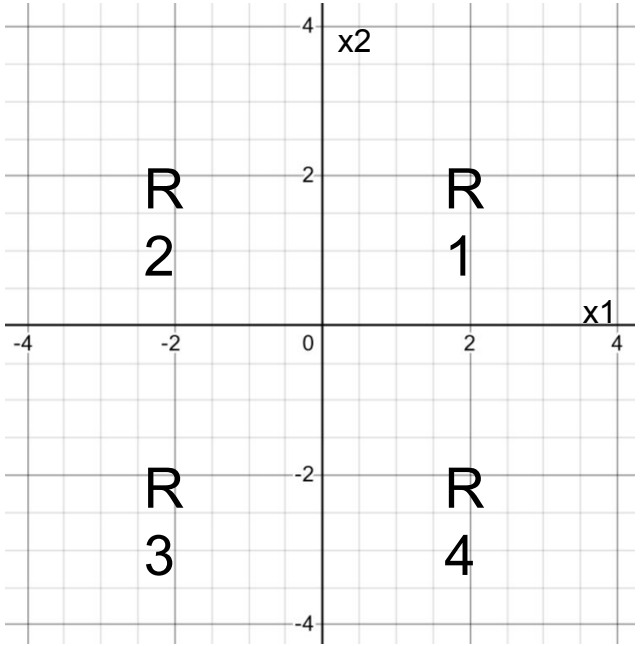


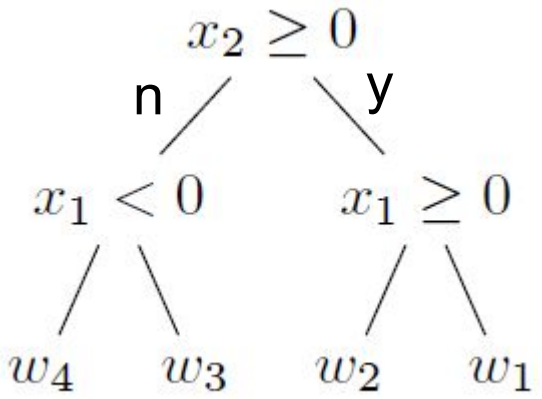
Input domain: $\mathbb{R}^2$

$$R_1 = \{x : x_1 \geq 0, x_2 \geq 0\},$$
$$R_2 = \{x : x_1 < 0, x_2 \geq 0\},$$
$$R_3 = \{x : x_1 < 0, x_2 < 0\},$$
$$R_4 = \{x : x_1 \geq 0, x_2 < 0\},$$

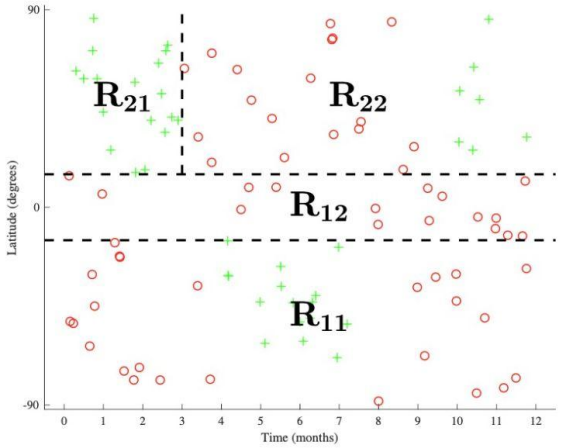
$$f(x) = \begin{cases} w_1 & \text{if } x \in R_1, \\ w_2 & \text{if } x \in R_2, \\ \vdots & \vdots \\ w_k & \text{if } x \in R_k. \end{cases}$$

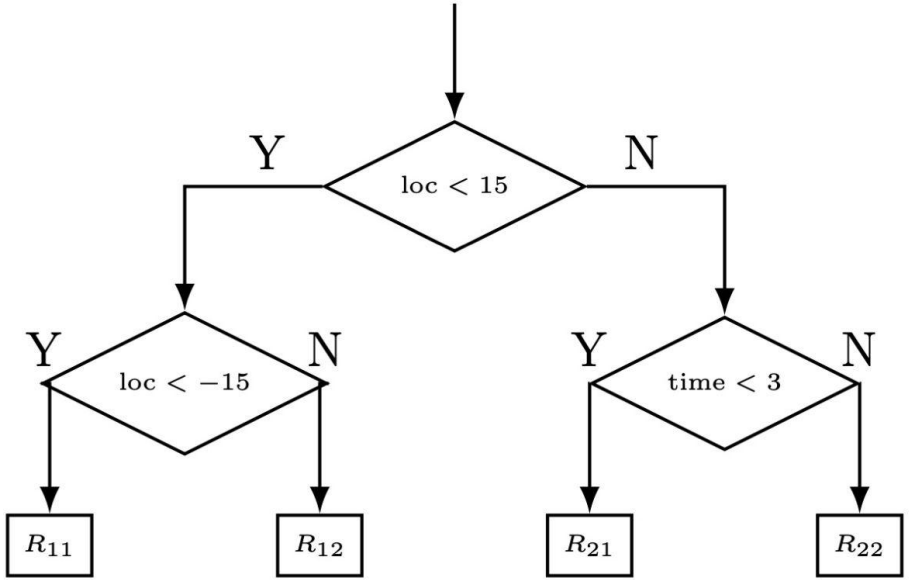
Mapping represented as trees





Mapping represented as trees

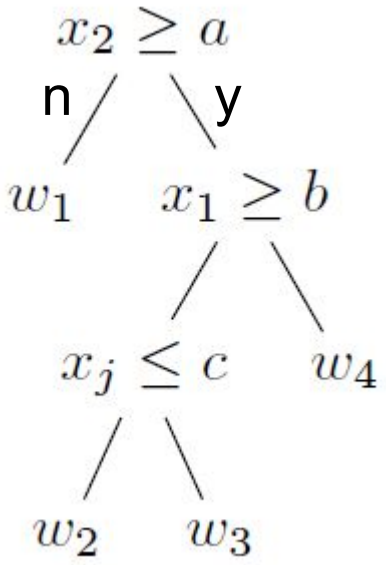




Exercise: Draw out the resulting regions in R^2

Given:

- d = 2
- a,b,c ∈ R
- j ∈ {1, 2}



To fit the model

$$\mathcal{L}(f) = \sum_{i=1}^{n} \ell\big(y^{(i)}, f(x^{(i)})\big) = \sum_{j=1}^{k} \sum_{x^{(i)} \in R_j} \ell(y^{(i)}, w_j).$$

Greedy approach to fit the model: Maximize the Gain Function

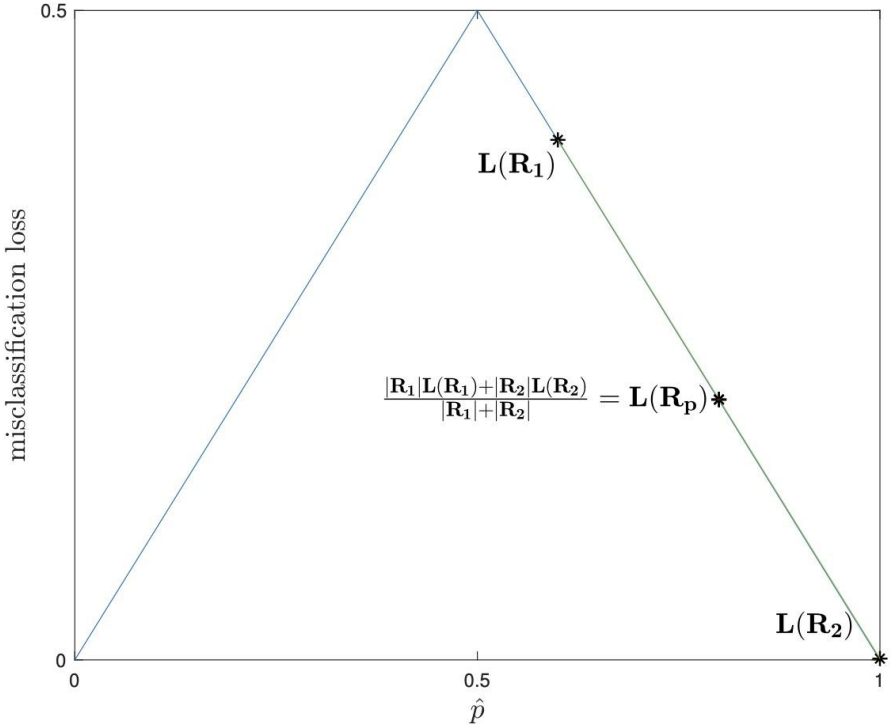
$$j, \theta = \arg\max_{j,\theta} G(j, \theta).$$

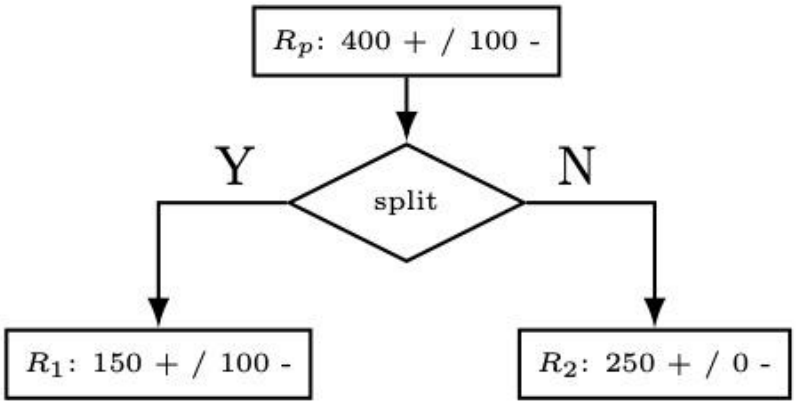
- Gain function: G(j, θ)
- j ∈ {1, . . . , d}
- θ ∈ R

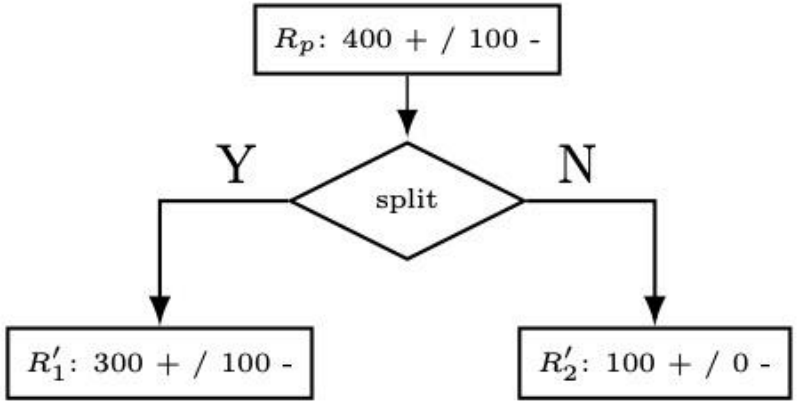
Maximizing gain by minimizing cost

$$G(j,\theta) = C(S) - \left[\frac{|L|}{|S|}C(L) + \frac{|R|}{|S|}C(R)\right]$$

$$L_{\text{misclass}}(R) = 1 - \max_{c}(\hat{p}_c)$$




R_p: 400 + / 100 -
Y
split
N
R_1: 150 + / 100 -
R_2: 250 + / 0 -


R_p: 400 + / 100 -
Y
split
N
R'_1: 300 + / 100 -
R'_2: 100 + / 0 -

Misclassification Loss Analysis

Parent Region:

- **Positive Examples (+):** 400
- **Negative Examples (-):** 100
- **Total Examples:** 500
- **Misclassification Loss:** $L_{\text{misclass}}(\text{Parent}) = 1 - \max(0.8, 0.2) = 0.2$

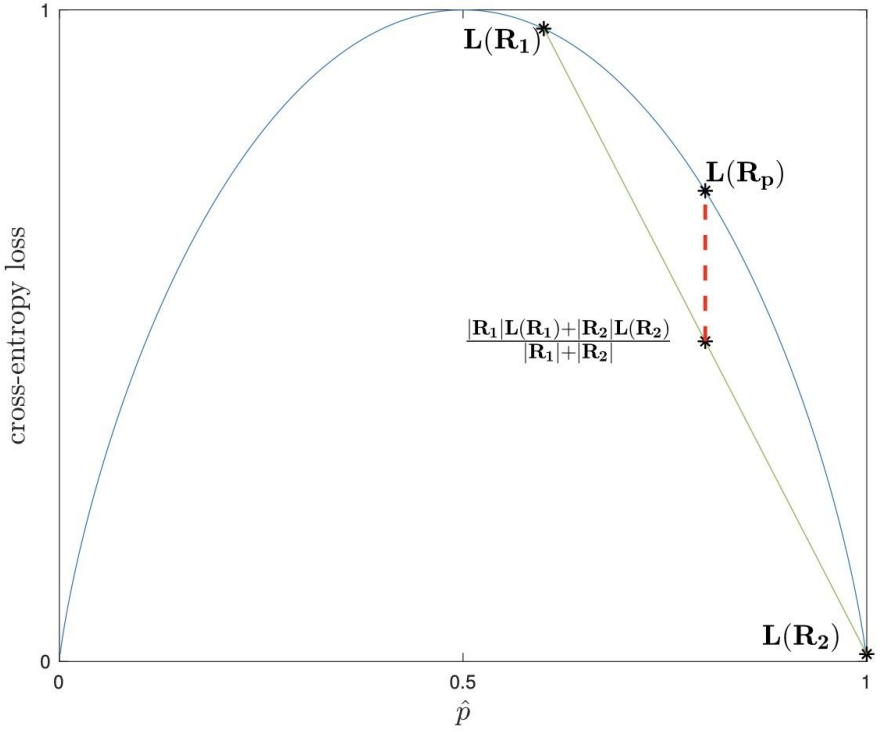
Split 1:

- **Child Region 1:** 150 (+), 100 (-)
  - **Misclassification Loss:** $L_{\text{misclass}}(\text{R1}) = 1 - \max(0.6, 0.4) = 0.4$
- **Child Region 2:** 250 (+), 0 (-)
  - **Misclassification Loss:** $L_{\text{misclass}}(\text{R2}) = 1 - \max(1, 0) = 0$
- **Total Misclassification Loss After Split 1:** $\frac{250}{500} \times 0.4 + \frac{250}{500} \times 0 = 0.2$

Split 2:

- **Child Region 1:** 300 (+), 100 (-)
  - **Misclassification Loss:** $L_{\text{misclass}}(\text{R1}) = 1 - \max(0.75, 0.25) = 0.25$
- **Child Region 2:** 100 (+), 0 (-)
  - **Misclassification Loss:** $L_{\text{misclass}}(\text{R2}) = 1 - \max(1, 0) = 0$
- **Total Misclassification Loss After Split 2:** $\frac{400}{500} \times 0.25 + \frac{100}{500} \times 0 = 0.2$

$$L_{cross}(R) = -\sum_c \hat{p}_c \log_2 \hat{p}_c$$



For regression:

$$\hat{y} = \frac{\sum_{i \in R} y_i}{|R|}$$

$$L_{\text{squared}}(R) = \frac{\sum_{i \in R} (y_i - \hat{y})^2}{|R|}$$

Regularization

Problems arising when we let the greedy algorithm described above run indefinitely:

- Overfitting to training data
- Poor generalization

Regularization methods

- Limiting the number of leaves (distinct regions) in the model.
- barring additional splits when there are too few datapoints in a node.
- not splitting if the gain described above is beneath some threshold.
- **Pruning**

Advantages and disadvantages

Advantages:

- Highly interpretable
- Pretty good fit relatively quickly, even on large datasets

Disadvantages:

- Lack of Additive Structure
- Poor generalization

Lack of Additive structure