

What is a Decision Tree?

A tree-like model that predicts a class (or value) by asking a sequence of questions about the attributes.

- Each internal node → asks about an attribute
 - Each branch → represents an outcome of that test
 - Each leaf → represents a class label or decision
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Hunt's Algorithm (How a Decision Tree Learns)

It's a recursive algorithm that keeps splitting the dataset until each subset is as "pure" as possible (meaning it mostly contains one class).

Algorithm Steps:

1. Start with all the data at the top of the tree.
 2. Check:
 - a. If every record in this group has the same class → stop! That becomes a leaf with that class.
 - b. If there are no records left → make it a leaf that predicts the majority class.
 3. Otherwise:
 - a. Pick the best feature to ask about — the one that separates the classes most clearly.
 - b. Split the data into smaller groups based on that feature's values.
 - c. Repeat the same process (go back to step 2) for each smaller group.
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What Do We Mean by "Best Split"?

When building a decision tree, we want to choose the question (feature split) that best separates the data into purer groups — groups that mostly contain only one class.

So a good split makes each branch have data points that are mostly of a single class (like mostly "Yes" or mostly "No"). A bad split leaves branches that are still very mixed (lots of both "Yes" and "No").

Types of Splits: two main kinds

1. Binary split: at a split, only split to 2 groups

Binary splits are common because they keep the tree simpler.

2. Multi-Way Split

Splits the data into more than two groups at once.

Gini Index

The Gini index measures how impure or mixed a node is.

It tells us how often a randomly chosen sample from that node would be misclassified if we labeled it randomly according to the class distribution.

- Gini = 0 → pure node (only one class)
- Gini = high → mixed node

$$Gini(t) = 1 - \sum_j p(j|t)^2$$

$p(j|t)$ = proportion of samples in node t that belong to class j.

Suppose a node has:

- 8 total samples
- 1 “No” and 7 “Yes”

Then $p(\text{yes}) = \frac{7}{8}$, $p(\text{no}) = \frac{1}{8}$

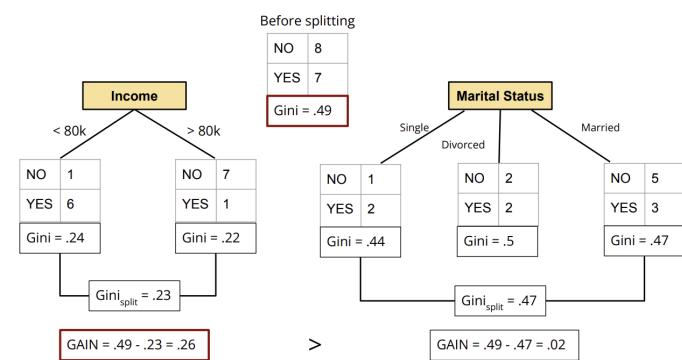
$$\text{Gini} = 1 - (\frac{1}{8})^2 - (\frac{7}{8})^2 = 1 - (49/64 + 1/64) = 1 - 50/64 = 14/64 = 0.22$$

When you split a node into several child nodes, the total impurity is the weighted average of their Gini indices:

$$Gini_{split} = \sum_t \frac{n_t}{n} Gini(t)$$

Goal: pick the attribute/split with the lowest Gini_split (i.e., the highest purity).

Example of deciding what to split on:



Limitations of Decision Trees

1. Overfitting

- Trees can easily become too complex, splitting until every leaf perfectly fits the training data.
- This makes the model perform poorly on new data.
- Example: the tree keeps creating small rules just to fit outliers.

2. Unstable

- Small changes in the data can lead to a completely different tree structure.

3. Bias toward attributes with many values

- A feature with many unique values (like “Customer ID”) can create many small splits — even if it doesn’t truly help classification.

4. Hard to capture smooth or linear boundaries

- Trees split data into boxes — they can’t easily represent gradual changes like a line or curve.

Solutions:

- Early termination
 - Stop at some specified depth
 - Stop if size of node is below some threshold
 - Stop if gini does not improv
- Pruning
 - Create fully grown tree then trim

Extensions

1. Entropy: Measures information disorder — used in Information Gain

2. Misclassification error: Simpler measure that looks only at the most common class in a node.

- It’s less sensitive than Gini or Entropy — so it’s used less often when growing the tree, but sometimes used for pruning (simplifying the tree).
- Ex: If 70% “Yes”, 30% “No” error = $1 - 0.7 = 0.3$