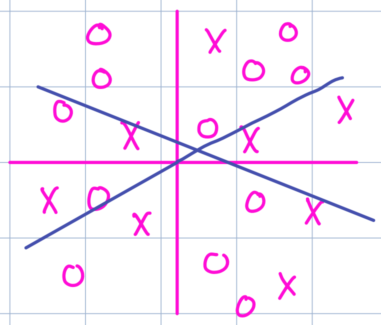
lab 8

Primal form: $\min \frac{1}{2}||w||^2$ subject to $\underbrace{(w^T x_i + b)\, y_i \geq 1}_{\text{constraint "penelty term"}}$ $\forall i$

weights

i = every point



$$\begin{cases} 0 & \text{when } (x^T x_i + b) y_i \geq 1 \\ \infty & \text{otherwise} \end{cases} = \max_{a_i \geq 0} a_i (1 - (w^T x_i + b)) y_i$$

Pull out the max

penalty for all points: $\min \left(\frac{1}{2}||w||^2 + \sum_{i=1}^{n} \max_{a_i \geq 0} a_i (1 - (w^T x_i + b)) y_i \right)$

$$\max_{a_i \geq 0} \min_{w,b} \left(\frac{1}{2}||w||^2 + \sum_{i=1}^{n} (1 - w^T x_i + b) y_i \right)$$

$$\max_{a_i \geq 0} \min_{w,b} \left(J(w, b ; a_i) \right)$$

- Minimize this function to find the dual form
  - ↳ taking the derivitive of this
  - ↳ find a term for w

task 1: minimize = take dirivtive and set it equal to zero

$$\max_{a_i \geq 0} \min_{w,b} \left(\frac{1}{2} ||w||^2 + \sum_{i=1}^{n} a_i (1 - w^T x_i + b) y_i \right)$$

$$\frac{\partial J}{\partial w} = \frac{1}{\cancel{2}} \cancel{2} ||w|| + \sum_{i=1}^{n} a_i x_i y_i$$

$$\frac{\partial J}{\partial b} = -\sum_{i=1}^{n} a_i y_i = 0$$

$$\frac{\partial J}{\partial w} = w + \sum_{i=1}^{n} a_i x_i y_i = 0 \qquad w = \sum_{i=1}^{n} a_i x_i y_i$$

$$J(w, b; a_i) = \frac{1}{2}\|w\|^2 + \sum_i a_i (1 - (w^T x_i + b) y_i)$$

$$= \frac{1}{2}\|w\|^2 + \sum_i a_i - \sum_i a_i y_i (w \cdot x_i) + \underbrace{\sum_i a_i y_i b}_{0}$$

$$= \frac{1}{2}\left\|\sum a_i x_i y_i\right\|^2 + \sum_i a_i - \sum_i a_i y_i \left(\sum_j a_j x_j y_j \cdot x_i\right)$$

dot product

$$= -\frac{1}{2}\sum_{ij} a_i a_j y_i y_j (x_i \cdot x_j) + \sum_i a_i$$

<u>this is the dual form</u>

$$L = \max_{a_i \geq 0} \left\{ \sum_i a_i - \frac{1}{2}\sum_{ij} a_i a_j y_i y_j (x_i \cdot x_j) \right\}$$

$$x_i \cdot x_j \rightarrow \phi x_i \cdot \phi x_j$$