1 Discrete univariate with finite support

1.1 Zipf–Mandelbrot law

Params.: $N \in \{1,2,3\ldots\}$ (integer), $q \in [0;\infty)$ (real), $s > 0$ (real); $\mathcal{W}(X)$: $k \in \{1,2,\ldots,N\}$; $\mathbb{E}[X]$: $\frac{H_{N,q,s-1}}{H_{N,q,s}} - q$;
f_x: $\frac{1/(k+q)^s}{H_{N,q,s}}$ F_x: $\frac{H_{k,q,s}}{H_{N,q,s}}$

1.2 Poisson binomial distribution

Params.: $\mathbf{p} \in [0,1]^n$ — success probabilities for each of the n trials; $\mathcal{W}(X)$: $k \in 0,\ldots,n$; $\mathbb{E}[X]$: $\sum_{i=1}^{n} p_i$; $Var[X]$: $\sigma^2 = \sum_{i=1}^{n}(1-p_i)p_i$;
f_x: $\sum_{A\in F_k}\prod_{i\in A} p_i \prod_{j\in A^c}(1-p_j)$ F_x: $\sum_{l=0}^{k}\sum_{A\in F_l}\prod_{i\in A} p_i \prod_{j\in A^c}(1-p_j)$

1.3 Rademacher distribution

$\mathcal{W}(X)$: $k \in \{-1,1\}$; $\mathbb{E}[X]$: 0; $Var[X]$: 1;
f_x:

$$f(k) = \begin{cases} 1/2 & \text{if } k=-1,\\ 1/2 & \text{if } k=+1,\\ 0 & \text{otherwise.}\end{cases}$$

F_x:

$$F(k) = \begin{cases} 0, & k<-1\\ 1/2, & -1\le k<1\\ 1, & k\ge 1\end{cases}$$

1.4 Bernoulli distribution

Params.: $0 \le p \le 1$, $q = 1-p$; $\mathcal{W}(X)$: $k \in \{0,1\}$; $\mathbb{E}[X]$: p; $Var[X]$: $p(1-p) = pq$;
f_x:

$$\begin{cases} q=1-p & \text{if } k=0\\ p & \text{if } k=1\end{cases}$$

F_x:

$$\begin{cases} 0 & \text{if } k<0\\ 1-p & \text{if } 0\le k<1\\ 1 & \text{if } k\ge 1\end{cases}$$

1.5 Beta-binomial distribution

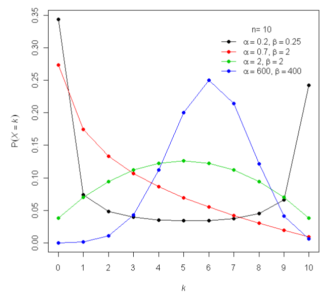

Abbildung 1: Probability mass function for the beta-binomial distribution

Params.: $n \in \mathbf{N}_0$ — number of trials, $\alpha > 0$ (real), $\beta > 0$ (real); $\mathcal{W}(X)$: $k \in 0,\ldots,n$; $\mathbb{E}[X]$: $\frac{n\alpha}{\alpha+\beta}$; $Var[X]$: $\frac{n\alpha\beta(\alpha+\beta+n)}{(\alpha+\beta)^2(\alpha+\beta+1)}$;
f_x: $\binom{n}{k}\frac{\mathrm{B}(k+\alpha,n-k+\beta)}{\mathrm{B}(\alpha,\beta)}$ F_x: $\begin{cases} 0, & k<0\\ \binom{n}{k}\frac{\mathrm{B}(k+\alpha,n-k+\beta)}{\mathrm{B}(\alpha,\beta)}{}_3F_2(\boldsymbol{a},\boldsymbol{b},k), & 0\le k<n\\ 1, & k\ge n\end{cases}$

, , where ¡big¿ ${}_3F_2(\mathbf{a},\mathbf{b},\mathrm{k})$¡/big¿ is the generalized hypergeometric function, ¡small¿ ${}_3F_2(1,-k,n-k+\beta;n-k-1,1-k-\alpha;1)$ ¡/small¿

1.6 Zipf's law

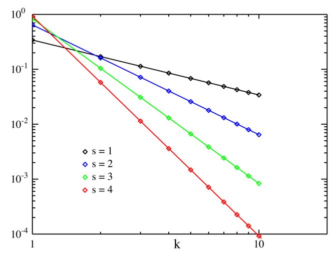

Abbildung 2: Plot of the Zipf PMF for $N = 10$

Params.: $s \ge 0$ (real), $N \in \{1,2,3\ldots\}$ (integer); $\mathcal{W}(X)$: $k \in \{1,2,\ldots,N\}$; $\mathbb{E}[X]$: $\frac{H_{N,s-1}}{H_{N,s}}$; $Var[X]$: $\frac{H_{N,s-2}}{H_{N,s}} - \frac{H^2_{N,s-1}}{H^2_{N,s}}$;
f_x: $\frac{1/k^s}{H_{N,s}}$ where $H_{N,s}$ is the Nth generalized harmonic number F_x: $\frac{H_{k,s}}{H_{N,s}}$

1.7 Binomial distribution

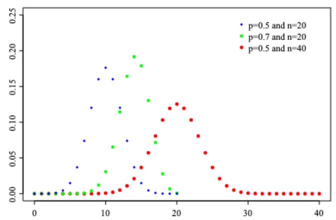

Abbildung 3: Probability mass function for the binomial distribution

Params.: $n \in \{0,1,2,\ldots\}$ – number of trials, $p \in [0,1]$ – success probability for each trial, $q = 1-p$; **Not.**: $B(n,p)$; $\mathcal{W}(X)$: $k \in \{0,1,\ldots,n\}$ – number of successes; $\mathbb{E}[X]$: np; $Var[X]$: npq;
f_x: $\binom{n}{k}p^k q^{n-k}$ F_x: $I_q(n-k,1+k)$

1.8 Discrete uniform distribution

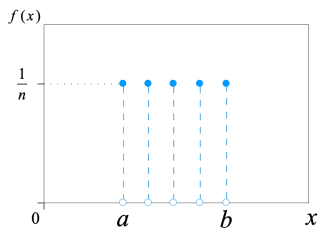

Abbildung 4: Discrete uniform probability mass function for $n = 5$

Params.: a, b integers with $b \ge a$, $n = b-a+1$; **Not.**: $\mathcal{U}\{a,b\}$ or $\mathrm{unif}\{a,b\}$; $\mathcal{W}(X)$: $k \in \{a,a+1,\ldots,b-1,b\}$; $\mathbb{E}[X]$: $\frac{a+b}{2}$; $Var[X]$: $\frac{(b-a+1)^2-1}{12}$;
f_x: $\frac{1}{n}$ F_x: $\frac{\lfloor k\rfloor - a+1}{n}$

2 Discrete univariate with infinite support

2.1 Beta negative binomial distribution

Params.: $\alpha > 0$ shape (real), $\beta > 0$ shape (real), $r > 0$ — number of failures until the experiment is stopped (integer but can be extended to real); $\mathcal{W}(X)$: $k \in 0, 1, 2, 3, \ldots$; $\mathbb{E}[X]$:

$$\begin{cases} \frac{r\beta}{\alpha-1} & \text{if } \alpha>1\\ \infty & \text{otherwise}\end{cases}$$

; $Var[X]$:

$$\begin{cases} \frac{r(\alpha+r-1)\beta(\alpha+\beta-1)}{(\alpha-2)(\alpha-1)^2} & \text{if } \alpha>2\\ \infty & \text{otherwise}\end{cases}$$

;
f_x: $\frac{\Gamma(r+k)}{k!\,\Gamma(r)}\frac{\mathrm{B}(\alpha+r,\beta+k)}{\mathrm{B}(\alpha,\beta)}$

2.2 Flory–Schulz distribution

Params.: 0 ¡ a ¡ 1 (real); $\mathcal{W}(X)$: $k \in 1, 2, 3, \ldots$; $\mathbb{E}[X]$: $\frac{2}{a} - 1$; $Var[X]$: $\frac{2-2a}{a^2}$;
f_x: $a^2k(1-a)^{k-1}$ F_x: $1-(1-a)^k(1+ak)$

2.3 Gauss–Kuzmin distribution

Params.: (none); $\mathcal{W}(X)$: $k \in \{1,2,\ldots\}$; $\mathbb{E}[X]$: $+\infty$; $Var[X]$: $+\infty$;
f_x: $-\log_2\left[1-\frac{1}{(k+1)^2}\right]$ F_x: $1-\log_2\left(\frac{k+2}{k+1}\right)$

2.4 Zeta distribution

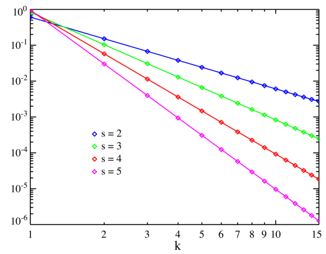

Abbildung 5: Plot of the Zeta PMF

Params.: $s \in (1,\infty)$; $\mathcal{W}(X)$: $k \in \{1,2,\ldots\}$; $\mathbb{E}[X]$: $\frac{\zeta(s-1)}{\zeta(s)}$ for $s > 2$; $Var[X]$: $\frac{\zeta(s)\zeta(s-2)-\zeta(s-1)^2}{\zeta(s)^2}$ for $s > 3$;
f_x: $\frac{1/k^s}{\zeta(s)}$ F_x: $\frac{H_{k,s}}{\zeta(s)}$

2.5 Logarithmic distribution

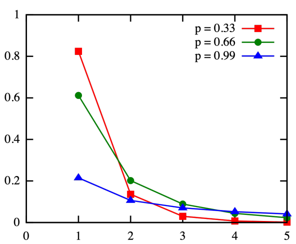

Abbildung 6: Plot of the logarithmic PMF

Params.: $0 < p < 1$; $\mathcal{W}(X)$: $k \in \{1,2,3,\ldots\}$; $\mathbb{E}[X]$: $\frac{-1}{\ln(1-p)}\frac{p}{1-p}$; $Var[X]$: $-\frac{p^2+p\ln(1-p)}{(1-p)^2(\ln(1-p))^2}$;
f_x: $\frac{-1}{\ln(1-p)}\frac{p^k}{k}$ F_x: $1+\frac{\mathrm{B}(p;k+1,0)}{\ln(1-p)}$

2.6 Yule–Simon distribution

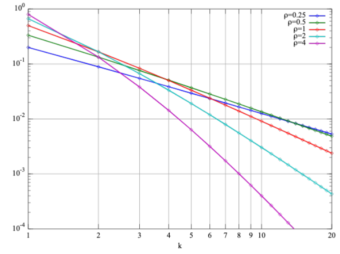

Abbildung 7: Plot of the Yule–Simon PMF

Params.: $\rho > 0$ shape (real); $\mathcal{W}(X)$: $k \in \{1,2,\ldots\}$; $\mathbb{E}[X]$: $\frac{\rho}{\rho-1}$ for $\rho > 1$; $Var[X]$: $\frac{\rho^2}{(\rho-1)^2(\rho-2)}$ for $\rho > 2$;
f_x: $\rho\,\mathrm{B}(k,\rho+1)$ F_x: $1-k\,\mathrm{B}(k,\rho+1)$

2.7 Skellam distribution

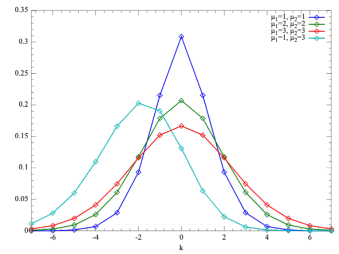

Abbildung 8: Examples of the probability mass function for the Skellam distribution.

Params.: $\mu_1 \ge 0$, $\mu_2 \ge 0$; $\mathcal{W}(X)$: $\{\ldots,-2,-1,0,1,2,\ldots\}$; $\mathbb{E}[X]$: $\mu_1-\mu_2$; $Var[X]$: $\mu_1+\mu_2$;
f_x:

$$e^{-(\mu_1+\mu_2)}\left(\frac{\mu_1}{\mu_2}\right)^{k/2} I_k(2\sqrt{\mu_1\mu_2})$$

2.8 Poisson distribution

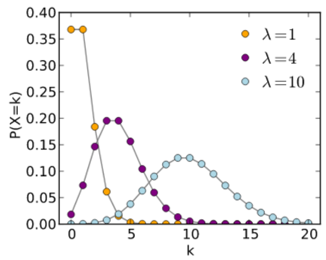

Abbildung 9: 325px

Params.: $\lambda \in (0,\infty)$ (rate); **Not.**: $\mathrm{Pois}(\lambda)$; $\mathcal{W}(X)$: $k \in \mathbb{N}_0$ (Natural numbers starting from 0); $\mathbb{E}[X]$: λ; $Var[X]$: λ;

f_x: $\frac{\lambda^k e^{-\lambda}}{k!}$ F_x: $\frac{\Gamma(\lfloor k+1\rfloor,\lambda)}{\lfloor k\rfloor!}$, or $e^{-\lambda}\sum_{i=0}^{\lfloor k\rfloor}\frac{\lambda^i}{i!}$, or $Q(\lfloor k+1\rfloor,\lambda)$ (for $k\geq 0$, where $\Gamma(x,y)$ is the upper incomplete gamma function, $\lfloor k\rfloor$ is the floor function, and Q is the regularized gamma function)

3 Continuous univariate supported on a bounded interval

3.1 Noncentral beta distribution

Params.: ¿ 0 shape (real), ¿ 0 shape (real), ¿= 0 noncentrality (real); **Not.**: Beta(, ,); $\mathcal{W}(X)$: $x\in[0;1]$; $\mathbb{E}[X]$: (type I) $e^{-\frac{\lambda}{2}}\frac{\Gamma(\alpha+1)}{\Gamma(\alpha)}\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha+\beta+1)}{}_2F_2\left(\alpha+\beta,\alpha+1;\alpha,\alpha+\beta+1;\frac{\lambda}{2}\right)$ (see Confluent hypergeometric function); $Var[X]$: (type I) $e^{-\frac{\lambda}{2}}\frac{\Gamma(\alpha+2)}{\Gamma(\alpha)}\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha+\beta+2}$ μ^2 where μ is the mean. (see Confluent hypergeometric function); f_x: (type I) $\sum_{j=0}^{\infty}e^{-\lambda/2}\frac{\left(\frac{\lambda}{2}\right)^j}{j!}\frac{x^{\alpha+j-1}(1-x)^{\beta-1}}{\mathrm{B}(\alpha+j,\beta)}$ F_x: (type I) $\sum_{j=0}^{\infty}e^{-\lambda/2}$

3.2 Beta rectangular distribution

Params.: $\alpha>0$ shape (real), $\beta>0$ shape (real) , $0<\theta<1$ mixture parameter; $\mathcal{W}(X)$: $x\in(a,b)$; $\mathbb{E}[X]$:

$$a+(b-a)\left(\frac{\theta\alpha}{\alpha+\beta}+\frac{1-\theta}{2}\right)$$

; $Var[X]$:

$$(b-a)^2\left(\frac{\theta\,\alpha(\alpha+1)}{k(k+1)}+\frac{1-\theta}{3}-\frac{\left(k+\theta(\alpha-\beta)\right)^2}{4k^2}\right)$$

where $k=\alpha+\beta$;
f_x:

$$\begin{cases}\frac{\theta\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\frac{(x-a)^{\alpha-1}(b-x)^{\beta-1}}{(b-a)^{\alpha+\beta+1}}+\frac{1-\theta}{b-a} & \text{for } x\in[a,b]\\ 0 & \text{otherwise}\end{cases}$$

F_x:

$$\begin{cases}0 & \text{for } x\leq a\\ \theta I_z(\alpha,\beta)+\frac{(1-\theta)(x-a)}{b-a} & \text{for } x\in[a,b]\\ 1 & \text{for } x\geq b\end{cases}$$

where $z=(x-a)/(b-a)$

3.3 U-quadratic distribution

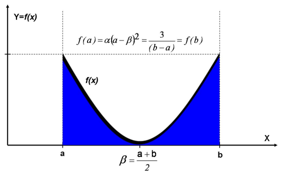

Abbildung 10: Plot of the U-Quadratic Density Function

Params.: a : $a\in(-\infty,\infty)$, b : $b\in(a,\infty)$, *or*, α : $\alpha\in(0,\infty)$, β : $\beta\in(-\infty,\infty)$,; $\mathcal{W}(X)$: $x\in[a,b]$; $\mathbb{E}[X]$: $\frac{a+b}{2}$; $Var[X]$: $\frac{3}{20}(b-a)^2$;
f_x: $\alpha\,(x-\beta)^2$ F_x: $\frac{\alpha}{3}\left((x-\beta)^3+(\beta-a)^3\right)$

3.4 Continuous Bernoulli distribution

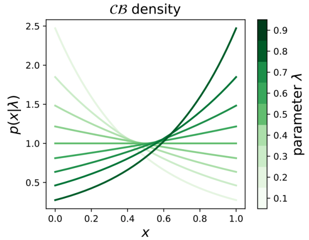

Abbildung 11: Probability density function of the continuous Bernoulli distribution

Params.: $\lambda\in(0,1)$; **Not.**: $\mathcal{CB}(\lambda)$; $\mathcal{W}(X)$: $x\in[0,1]$; $\mathbb{E}[X]$: $\mathrm{E}[X]=\begin{cases}\frac{\lambda}{2\lambda-1}+\frac{1}{2\tanh^{-1}(1-2\lambda)} & \text{if } \lambda\neq\frac{1}{2}\\ \frac{1}{2} & \text{otherwise}\end{cases}$; $Var[X]$: $\mathrm{var}[X]=\begin{cases}\frac{(1-\lambda)\lambda}{(1-2\lambda)^2}+\frac{1}{(2\tanh^{-1}(1-2\lambda))^2} & \text{if } \lambda\neq\frac{1}{2}\\ \frac{1}{12} & \text{otherwise}\end{cases}$;
f_x: $C(\lambda)\lambda^x(1-\lambda)^{1-x}$, where $C(\lambda)=\begin{cases}\frac{2\tanh^{-1}(1-2\lambda)}{1-2\lambda} & \text{if } \lambda\neq\frac{1}{2}\\ 2 & \text{otherwise}\end{cases}$ F_x: $\begin{cases}\frac{\lambda^x(1-\lambda)^{1-x}+\lambda-1}{2\lambda-1} & \text{if } \lambda\neq\frac{1}{2}\\ x & \text{otherwise}\end{cases}$

3.5 Triangular distribution

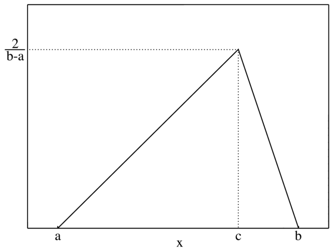

Abbildung 12: Plot of the Triangular PMF

Params.: a : $a\in(-\infty,\infty)$, b : $a<b$, c : $a\leq c\leq b$; $\mathcal{W}(X)$: $a\leq x\leq b$; $\mathbb{E}[X]$: $\frac{a+b+c}{3}$; $Var[X]$: $\frac{a^2+b^2+c^2-ab-ac-bc}{18}$;
f_x:

$$\begin{cases}0 & \text{for } x<a,\\ \frac{2(x-a)}{(b-a)(c-a)} & \text{for } a\leq x<c,\\ \frac{2}{b-a} & \text{for } x=c,\\ \frac{2(b-x)}{(b-a)(b-c)} & \text{for } c<x\leq b,\\ 0 & \text{for } b<x.\end{cases}$$

F_x:

$$\begin{cases}0 & \text{for } x\leq a,\\ \frac{(x-a)^2}{(b-a)(c-a)} & \text{for } a<x\leq c,\\ 1-\frac{(b-x)^2}{(b-a)(b-c)} & \text{for } c<x<b,\\ 1 & \text{for } b\leq x.\end{cases}$$

3.6 Arcsine distribution

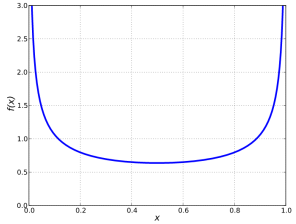

Abbildung 13: Probability density function for the arcsine distribution

Params.: none; $\mathcal{W}(X)$: $x\in[0,1]$; $\mathbb{E}[X]$: $\frac{1}{2}$; $Var[X]$: $\frac{1}{8}$;
f_x: $f(x)=\frac{1}{\pi\sqrt{x(1-x)}}$ F_x: $F(x)=\frac{2}{\pi}\arcsin\left(\sqrt{x}\right)$

3.7 Raised cosine distribution

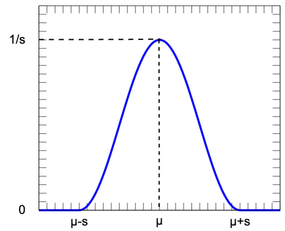

Abbildung 14: Plot of the raised cosine PDF

Params.: μ (real), $s>0$ (real); $\mathcal{W}(X)$: $x\in[\mu-s,\mu+s]$; $\mathbb{E}[X]$: μ; $Var[X]$: $s^2\left(\frac{1}{3}-\frac{2}{\pi^2}\right)$;
f_x:

$$\frac{1}{2s}\left[1+\cos\left(\frac{x-\mu}{s}\pi\right)\right]=\frac{1}{s}\operatorname{hvc}\left(\frac{x-\mu}{s}\pi\right)$$

F_x:

$$\frac{1}{2}\left[1+\frac{x-\mu}{s}+\frac{1}{\pi}\sin\left(\frac{x-\mu}{s}\pi\right)\right]$$

3.8 Balding–Nichols model

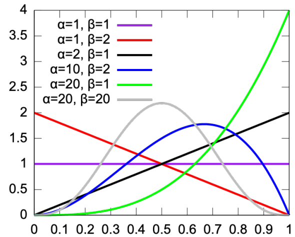

Abbildung 15: 352px

Params.: $0<F<1$ (real), $0<p<1$ (real), For ease of notation, let, $\alpha=\frac{1-F}{F}p$, and , $\beta=\frac{1-F}{F}(1-p)$; $\mathcal{W}(X)$: $x\in(0;1)$; $\mathbb{E}[X]$: p; $Var[X]$: $Fp(1-p)$;
f_x: $\frac{x^{\alpha-1}(1-x)^{\beta-1}}{\mathrm{B}(\alpha,\beta)}$ F_x: $I_x(\alpha,\beta)$

3.9 Uniform distribution (continuous)

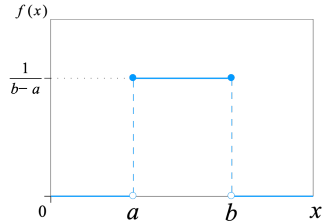

Abbildung 16: the maximum convention

Params.: $-\infty<a<b<\infty$; **Not.**: $\mathcal{U}(a,b)$ or $\mathrm{unif}(a,b)$; $\mathcal{W}(X)$ $x\in[a,b]$; $\mathbb{E}[X]$: $\frac{1}{2}(a+b)$; $Var[X]$: $\frac{1}{12}(b-a)^2$;
f_x:

$$\begin{cases}\frac{1}{b-a} & \text{for } x\in[a,b]\\ 0 & \text{otherwise}\end{cases}$$

F_x:

$$\begin{cases}0 & \text{for } x<a\\ \frac{x-a}{b-a} & \text{for } x\in[a,b]\\ 1 & \text{for } x>b\end{cases}$$

3.10 Kumaraswamy distribution

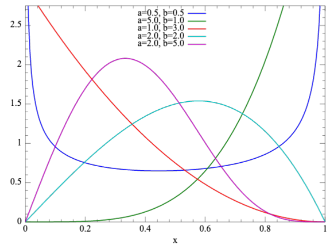

Abbildung 17: Probability density function

Params.: $a>0$ (real), $b>0$ (real); $\mathcal{W}(X)$: $x\in(0,1)$; $\mathbb{E}[X]$: $\frac{b\Gamma(1+\frac{1}{a})\Gamma(b)}{\Gamma(1+\frac{1}{a}+b)}$; $Var[X]$: (complicated-see text);
f_x: $abx^{a-1}(1-x^a)^{b-1}$ F_x: $1-(1-x^a)^b$

3.11 Irwin–Hall distribution

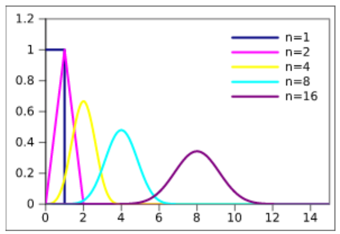

Abbildung 18: Probability mass function for the distribution

Params.: $n\in\mathbf{N}_0$; $\mathcal{W}(X)$: $x\in[0,n]$; $\mathbb{E}[X]$: $\frac{n}{2}$; $Var[X]$: $\frac{n}{12}$;
f_x: $\frac{1}{(n-1)!}\sum_{k=0}^{\lfloor x\rfloor}(-1)^k\binom{n}{k}(x-k)^{n-1}$ F_x: $\frac{1}{n!}\sum_{k=0}^{\lfloor x\rfloor}(-1)^k\binom{n}{k}(x-k)^n$

3.12 Wigner semicircle distribution

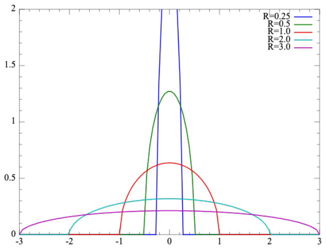

Abbildung 19: Plot of the Wigner semicircle PDF

Params.: $R > 0$ radius (real); $\mathcal{W}(X)$: $x \in [-R; +R]$; $\mathbb{E}[X]$: 0; $Var[X]$: $\frac{R^2}{4}$;
f_x: $\frac{2}{\pi R^2}\sqrt{R^2-x^2}$ F_x: $\frac{1}{2}+\frac{x\sqrt{R^2-x^2}}{\pi R^2}+\frac{\arcsin\left(\frac{x}{R}\right)}{\pi}$, for $-R \leq x \leq R$

3.13 Reciprocal distribution

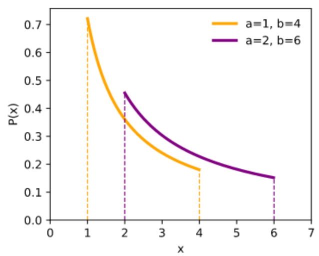

Abbildung 20: Probability density function

Params.: $0 < a < b, a, b \in \mathbb{R}$; $\mathcal{W}(X)$: $[a, b]$; $\mathbb{E}[X]$: $\frac{b-a}{\ln\frac{b}{a}}$; $Var[X]$: $\frac{b^2-a^2}{2\ln\frac{b}{a}}-\left(\frac{b-a}{\ln\frac{b}{a}}\right)^2$;
f_x: $\frac{1}{x\ln\frac{b}{a}}$ F_x: $\log_{\frac{b}{a}}\frac{x}{a}$

3.14 Beta distribution

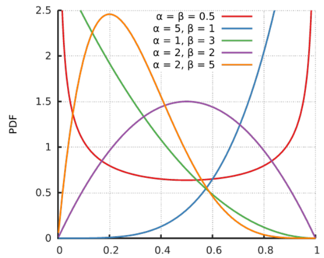

Abbildung 21: Probability density function for the Beta distribution

Params.: ¿ 0 shape (real), ¿ 0 shape (real); **Not.**: Beta(,); $\mathcal{W}(X)$: $x \in [0,1]$ or $x \in (0,1)$; $\mathbb{E}[X]$: $\mathrm{E}[X] = \frac{\alpha}{\alpha+\beta}$, $\mathrm{E}[\ln X] = \psi(\alpha)-\psi(\alpha+\beta)$, , $\mathrm{E}[X\ \ln X] = \frac{\alpha}{\alpha+\beta}\left[\psi(\alpha+1)-\psi(\alpha+\beta+1)\right]$, (see digamma function and see section: Geometric mean); $Var[X]$: $\mathrm{var}[X] = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$, $\mathrm{var}[\ln X] = \psi_1(\alpha)-\psi_1(\alpha+\beta)$, (see trigamma function and see section: Geometric variance);
f_x: $\frac{x^{\alpha-1}(1-x)^{\beta-1}}{\mathrm{B}(\alpha,\beta)}$, where $\mathrm{B}(\alpha,\beta) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$ and Γ is the Gamma function. F_x: $I_x(\alpha,\beta)$ (the regularized incomplete beta function)

3.15 Logit-normal distribution

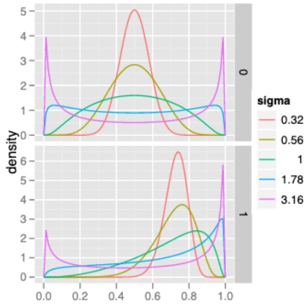

Abbildung 22: Plot of the Logitnormal PDF

Params.: ^{2} ¿ 0 — squared scale (real),, $\mu \in \mathbf{R}$ — location; **Not.**: $P(\mathcal{N}(\mu,\sigma^2))$; $\mathcal{W}(X)$: $x \in (0,1)$; $\mathbb{E}[X]$: no analytical solution; $Var[X]$: no analytical solution;
f_x: $\frac{1}{\sigma\sqrt{2\pi}}\,e^{-\frac{(\mathrm{logit}(x)-\mu)^2}{2\sigma^2}}\frac{1}{x(1-x)}$ F_x: $\frac{1}{2}\left[1+\mathrm{erf}\left(\frac{\mathrm{logit}(x)-\mu}{\sqrt{2\sigma^2}}\right)\right]$

3.16 Bates distribution

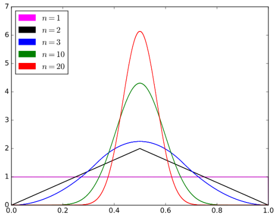

Abbildung 23: 325px

Params.: $-\infty < a < b < \infty$, $n \geq 1$ integer; $\mathcal{W}(X)$: $x \in [a, b]$; $\mathbb{E}[X]$: $\frac{1}{2}(a+b)$; $Var[X]$: $\frac{1}{12n}(b-a)^2$;
f_x: see below

3.17 ARGUS distribution

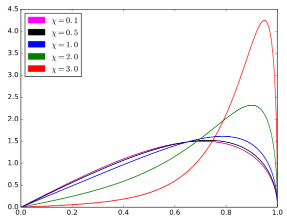

Abbildung 24: 325px

Params.: $c > 0$ cut-off (real), $\chi > 0$ curvature (real); $\mathcal{W}(X)$: $x \in (0, c)$; $\mathbb{E}[X]$: $\mu = c\sqrt{\pi/8}\ \frac{\chi e^{-\frac{\chi^2}{4}} I_1\left(\frac{\chi^2}{4}\right)}{\Psi(\chi)}$, , where I_1 is the Modified Bessel function of the first kind of order 1, and $\Psi(x)$ is given in the text.; $Var[X]$: $c^2\left(1-\frac{3}{\chi^2}+\frac{\chi\varphi(\chi)}{\Psi(\chi)}\right)-\mu^2$;
f_x: see text F_x: see text

4 Continuous univariate supported on a semi-infinite interval

4.1 Discrete Weibull distribution

Params.: $\alpha > 0$ scale , $\beta > 0$ shape; $\mathcal{W}(X)$: $x \in \{0, 1, 2, \ldots\}$;
f_x:

$$\exp\left[-\left(\frac{x}{\alpha}\right)^\beta\right]-\exp\left[-\left(\frac{x+1}{\alpha}\right)^\beta\right]$$

F_x: $1-\exp\left[-\left(\frac{x+1}{\alpha}\right)^\beta\right]$

4.2 Benktander type I distribution

Params.: $a > 0$ (real), $b > 0$ real; $\mathcal{W}(X)$: $x \geq 1$; $\mathbb{E}[X]$: $1+\frac{1}{a}$; $Var[X]$: $\frac{-\sqrt{b}+ae^{\frac{(a-1)^2}{4b}}\sqrt{\pi}\,\mathrm{erfc}\left(\frac{a-1}{2\sqrt{b}}\right)}{a^2\sqrt{b}}$;
f_x: $\left(\left[\left(1+\frac{2b\log x}{a}\right)(1+a+2b\log x)\right]-\frac{2b}{a}\right)x^{-(2+a+b\log x)}$ F_x: $1-(1$ -

4.3 Davis distribution

Params.: $b > 0$ scale, $n > 0$ shape, $\mu > 0$ location; $\mathcal{W}(X)$: $x > \mu$; $\mathbb{E}[X]$:

$$\begin{cases}\mu+\frac{b\zeta(n-1)}{(n-1)\zeta(n)} & \text{if } n > 2\\ \text{Indeterminate} & \text{otherwise}\end{cases}$$

; $Var[X]$:

$$\begin{cases}\frac{b^2\left(-(n-2)\zeta(n-1)^2+(n-1)\zeta(n-2)\zeta(n)\right)}{(n-2)(n-1)^2\zeta(n)^2} & \text{if } n > 3\\ \text{Indeterminate} & \text{otherwise}\end{cases}$$

;
f_x: $\frac{b^n(x-\mu)^{-1-n}}{\left(e^{\frac{b}{x-\mu}}-1\right)\Gamma(n)\zeta(n)}$, Where $\Gamma(n)$ is the Gamma function and $\zeta(n)$ is the Riemann zeta function

4.4 Benini distribution

Params.: $\alpha > 0$ shape (real), $\beta > 0$ shape (real), $\sigma > 0$ scale (real); $\mathcal{W}(X)$: $x > \sigma$; $\mathbb{E}[X]$: $\sigma+\frac{\sigma}{\sqrt{2\beta}}H_{-1}\left(\frac{-1+\alpha}{\sqrt{2\beta}}\right)$, where $H_n(x)$ is the **probabilists' Hermite polynomials"**; $Var[X]$: $\left(\sigma^2+\frac{2\sigma^2}{\sqrt{2\beta}}H_{-1}\left(\frac{-2+\alpha}{\sqrt{2\beta}}\right)\right)-\mu^2$;
f_x: $e^{-\alpha\log\frac{x}{\sigma}-\beta\left[\log\frac{x}{\sigma}\right]^2}\left(\frac{\alpha}{x}+\frac{2\beta\log\frac{x}{\sigma}}{x}\right)$ F_x: $1-e^{-\alpha\log\frac{x}{\sigma}-\beta\left[\log\frac{x}{\sigma}\right]^2}$

4.5 Type-2 Gumbel distribution

Params.: a (real), $b$shape (real); $\mathbb{E}[X]$: $b^{1/a}\Gamma(1-1/a)$; $Var[X]$: $b^{2/a}(\Gamma(1-1/a)-\Gamma(1-1/a)^2)$;
f_x: $abx^{-a-1}e^{-bx^{-a}}$ F_x: $e^{-bx^{-a}}$

4.6 Hypoexponential distribution

Params.: $\lambda_1,\ldots,\lambda_k > 0$ rates (real); $\mathcal{W}(X)$: $x \in [0;\infty)$; $\mathbb{E}[X]$: $\sum_{i=1}^k 1/\lambda_i$; $Var[X]$: $\sum_{i=1}^k 1/\lambda_i^2$;
f_x: Expressed as a phase-type distribution, $-\boldsymbol{\alpha}e^{x\Theta}\Theta\mathbf{1}$, Has no other simple form; see article for details F_x: Expressed as a phase-type distribution, $1-\boldsymbol{\alpha}e^{x\Theta}\mathbf{1}$

4.7 Phase-type distribution

Params.: S, $m\times m$ subgenerator matrix, $\boldsymbol{\alpha}$, probability row vector; $\mathcal{W}(X)$: $x \in [0;\infty)$; $\mathbb{E}[X]$: $-\boldsymbol{\alpha}S^{-1}\mathbf{1}$; $Var[X]$: $2\boldsymbol{\alpha}S^{-2}\mathbf{1}-(\boldsymbol{\alpha}S^{-1}\mathbf{1})^2$;
f_x: $\boldsymbol{\alpha}e^{xS}\boldsymbol{S}^0$, See article for details F_x: $1-\boldsymbol{\alpha}e^{xS}\mathbf{1}$

4.8 Log-logistic distribution

Params.: $\alpha > 0$ scale, $\beta > 0$ shape; $\mathcal{W}(X)$: $x \in [0,\infty)$; $\mathbb{E}[X]$: $\frac{\alpha\,\pi/\beta}{\sin(\pi/\beta)}$, if $\beta > 1$, else undefined; $Var[X]$: See main text;
f_x:

$$\frac{(\beta/\alpha)(x/\alpha)^{\beta-1}}{\left(1+(x/\alpha)^\beta\right)^2}$$

F_x: $\frac{1}{1+(x/\alpha)^{-\beta}}$

4.9 Log-Cauchy distribution

Params.: μ (real), $\sigma > 0$ (real); $\mathcal{W}(X)$: $x \in (0,+\infty)$; $\mathbb{E}[X]$: infinite; $Var[X]$: infinite;
f_x: $\frac{1}{x\pi}\left[\frac{\sigma}{(\ln x-\mu)^2+\sigma^2}\right]$, $x > 0$ F_x: $\frac{1}{\pi}\arctan\left(\frac{\ln x-\mu}{\sigma}\right)+\frac{1}{2}$, $x > 0$

4.10 Noncentral chi-squared distribution

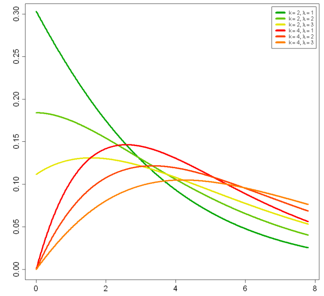

Abbildung 25: 325px

Params.: $k > 0$ degrees of freedom, $\lambda > 0$ non-centrality parameter; $\mathcal{W}(X)$: $x \in [0;+\infty)$; $\mathbb{E}[X]$: $k+\lambda$; $Var[X]$: $2(k+2\lambda)$;
f_x:

$$\frac{1}{2}e^{-(x+\lambda)/2}\left(\frac{x}{\lambda}\right)^{k/4-1/2}I_{k/2-1}(\sqrt{\lambda x})$$

F_x: $1-Q_{\frac{k}{2}}\left(\sqrt{\lambda},\sqrt{x}\right)$ with Marcum Q-function $Q_M(a,b)$

4.11 Dagum distribution

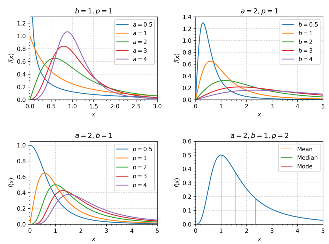

Abbildung 26: The pdf of the Dagum distribution for various parameter specifications.

Params.: $p > 0$ shape , $a > 0$ shape , $b > 0$ scale; $\mathcal{W}(X)$: $x > 0$; $\mathbb{E}[X]$:

$$\begin{cases}-\frac{b}{a}\frac{\Gamma\left(-\frac{1}{a}\right)\Gamma\left(\frac{1}{a}+p\right)}{\Gamma(p)} & \text{if } a > 1\\ \text{Indeterminate} & \text{otherwise}\end{cases}$$

; $Var[X]$:

$$\begin{cases} -\frac{b^2}{a^2}\left(2a\frac{\Gamma\left(-\frac{2}{a}\right)\Gamma\left(\frac{2}{a}+p\right)}{\Gamma(p)}+\left(\frac{\Gamma\left(-\frac{1}{a}\right)\Gamma\left(\frac{1}{a}+p\right)}{\Gamma(p)}\right)^2\right) & \text{if } a>2 \\ \text{Indeterminate} & \text{otherwise} \end{cases}$$

;

f_x: $\frac{ap}{x}\left(\frac{(\frac{x}{b})^{ap}}{((\frac{x}{b})^a+1)^{p+1}}\right)$ F_x: $\left(1+\left(\frac{x}{b}\right)^{-a}\right)^{-p}$

4.12 Inverse-chi-squared distribution

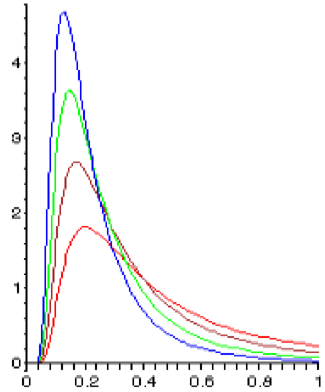

Abbildung 27:

Params.: $\nu>0$; $\mathcal{W}(X)$: $x\in(0,\infty)$; $\mathbb{E}[X]$: $\frac{1}{\nu-2}$ for $\nu>2$; $Var[X]$: $\frac{2}{(\nu-2)^2(\nu-4)}$ for $\nu>4$; f_x: $\frac{2^{-\nu/2}}{\Gamma(\nu/2)}x^{-\nu/2-1}e^{-1/(2x)}$ F_x:

$$\Gamma\left(\frac{\nu}{2},\frac{1}{2x}\right)\Big/\Gamma\left(\frac{\nu}{2}\right)$$

4.13 Generalized gamma distribution

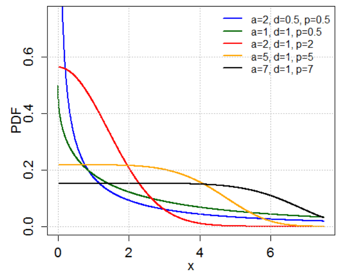

Abbildung 28: Gen Gamma PDF plot

Params.: $a>0$ (scale), $d>0, p>0$; $\mathcal{W}(X)$: $x\in(0,\infty)$; $\mathbb{E}[X]$: $a\frac{\Gamma((d+1)/p)}{\Gamma(d/p)}$; $Var[X]$: $a^2\left(\frac{\Gamma((d+2)/p)}{\Gamma(d/p)}-\left(\frac{\Gamma((d+1)/p)}{\Gamma(d/p)}\right)^2\right)$; f_x: $\frac{p/a^d}{\Gamma(d/p)}x^{d-1}e^{-(x/a)^p}$ F_x: $\frac{\gamma(d/p,(x/a)^p)}{\Gamma(d/p)}$

4.14 Rice distribution

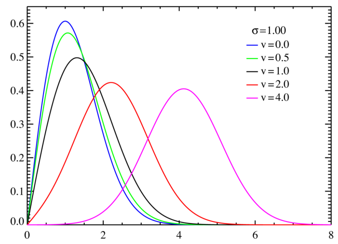

Abbildung 29: Rice probability density functions = 1.0

Params.: $\nu\geq 0$, distance between the reference point and the center of the bivariate distribution,, $\sigma\geq 0$, spread; $\mathcal{W}(X)$: $x\in[0,\infty)$; $\mathbb{E}[X]$: $\sigma\sqrt{\pi/2}\,L_{1/2}(-\nu^2/2\sigma^2)$; $Var[X]$: $2\sigma^2+\nu^2-\frac{\pi\sigma^2}{2}L^2_{1/}$ f_x:

$$\frac{x}{\sigma^2}\exp\left(\frac{-(x^2+\nu^2)}{2\sigma^2}\right)I_0\left(\frac{x\nu}{\sigma^2}\right)$$

F_x: $1-Q_1\left(\frac{\nu}{\sigma},\frac{x}{\sigma}\right)$ where Q_1 is the Marcum Q-function

4.15 Scaled inverse chi-squared distribution

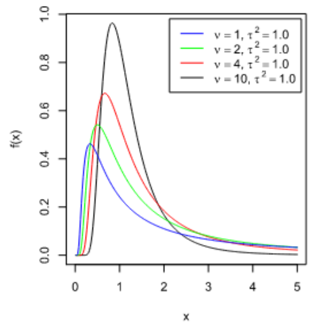

Abbildung 30: 250px

Params.: $\nu>0$, $\tau^2>0$; $\mathcal{W}(X)$: $x\in(0,\infty)$; $\mathbb{E}[X]$: $\frac{\nu\tau^2}{\nu-2}$ for $\nu>2$; $Var[X]$: $\frac{2\nu^2\tau^4}{(\nu-2)^2(\nu-4)}$ for $\nu>4$; f_x:

$$\frac{(\tau^2\nu/2)^{\nu/2}}{\Gamma(\nu/2)}\frac{\exp\left[\frac{-\nu\tau^2}{2x}\right]}{x^{1+\nu/2}}$$

F_x:

$$\Gamma\left(\frac{\nu}{2},\frac{\tau^2\nu}{2x}\right)\Big/\Gamma\left(\frac{\nu}{2}\right)$$

4.16 Beta prime distribution

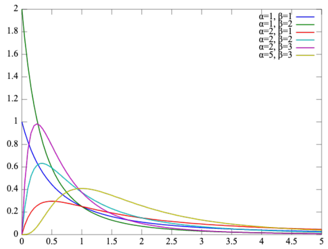

Abbildung 31: 325px

Params.: $\alpha>0$ shape (real), $\beta>0$ shape (real); $\mathcal{W}(X)$: $x\in[0,\infty)$; $\mathbb{E}[X]$: $\frac{\alpha}{\beta-1}$ if $\beta>1$; $Var[X]$: $\frac{\alpha(\alpha+\beta-1)}{(\beta-2)(\beta-1)^2}$ if $\beta>2$; f_x: $f(x)=\frac{x^{\alpha-1}(1+x)^{-\alpha-\beta}}{B(\alpha,\beta)}$ F_x: $I_{\frac{x}{1+x}(\alpha,\beta)}$ where $I_x(\alpha,\beta)$ is the incomplete beta function

4.17 Benktander type II distribution

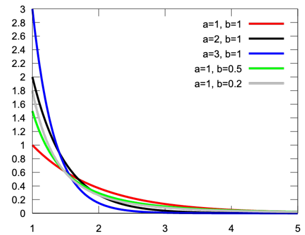

Abbildung 32: 325px

Params.: $a>0$ (real), $0<b\leq 1$ (real); $\mathcal{W}(X)$: $x\geq 1$; $\mathbb{E}[X]$: $1+\frac{1}{a}$; $Var[X]$: $\frac{-b+2ae^{\frac{a}{b}}\mathbf{E}_{1-\frac{1}{b}}\left(\frac{a}{b}\right)}{a^2b}$, Where $\mathbf{E}_n(x)$ is the generalized Exponential integral; f_x: $e^{\frac{a}{b}(1-x^b)}x^{b-2}\left(ax^b-b+1\right)$ F_x: $1-x^{b-1}e^{\frac{a}{b}(1-x^b)}$

4.18 Inverse-gamma distribution

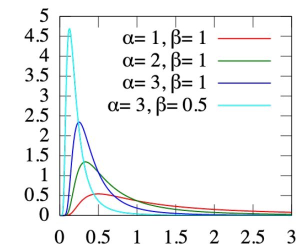

Abbildung 33: 325px

Params.: $\alpha>0$ shape (real), $\beta>0$ scale (real); $\mathcal{W}(X)$: $x\in(0,\infty)$; $\mathbb{E}[X]$: $\frac{\beta}{\alpha-1}$ for $\alpha>1$; $Var[X]$: $\frac{\beta^2}{(\alpha-1)^2(\alpha-2)}$ for $\alpha>2$; f_x: $\frac{\beta^\alpha}{\Gamma(\alpha)}x^{-\alpha-1}\exp\left(-\frac{\beta}{x}\right)$ F_x: $\frac{\Gamma(\alpha,\beta/x)}{\Gamma(\alpha)}$

4.19 Burr distribution

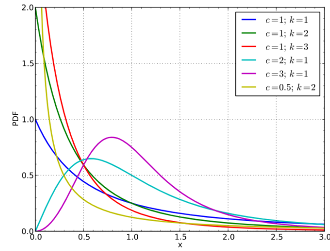

Abbildung 34: 325px

Params.: $c>0$, $k>0$; $\mathcal{W}(X)$: $x>0$; $\mathbb{E}[X]$: $\mu_1=k\,\mathrm{B}(k-1/c,\,1+1/c)$ where () is the beta function; $Var[X]$: $-\mu_1^2+\mu_2$; f_x: $ck\frac{x^{c-1}}{(1+x^c)^{k+1}}$ F_x: $1-(1+x^c)^{-k}$

4.20 Chi distribution

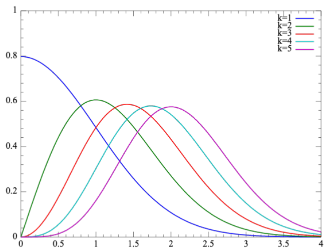

Abbildung 35: Plot of the Chi PMF

Params.: $k>0$ (degrees of freedom); $\mathcal{W}(X)$: $x\in[0,\infty)$; $\mathbb{E}[X]$: $\mu=\sqrt{2}\,\frac{\Gamma((k+1)/2)}{\Gamma(k/2)}$; $Var[X]$: $\sigma^2=k-\mu^2$; f_x: $\frac{1}{2^{(k/2)-1}\Gamma(k/2)}x^{k-1}e^{-x^2/2}$ F_x: $P(k/2,x^2/2)$

4.21 Generalized inverse Gaussian distribution

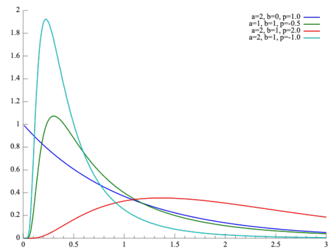

Abbildung 36: Probability density plots of GIG distributions

Params.: a ¿ 0, b ¿ 0, p real; $\mathcal{W}(X)$: x ¿ 0; $\mathbb{E}[X]$: $\mathrm{E}[x]=\frac{\sqrt{b}\,K_{p+1}(\sqrt{ab})}{\sqrt{a}\,K_p(\sqrt{ab})}$, $\mathrm{E}[x^{-1}]=\frac{\sqrt{a}\,K_{p+1}(\sqrt{ab})}{\sqrt{b}\,K_p(\sqrt{ab})}-\frac{2p}{b}$, $\mathrm{E}[\ln x]=\ln\frac{\sqrt{b}}{\sqrt{a}}+\frac{\partial}{\partial p}\ln K_p(\sqrt{ab})$; $Var[X]$: $\left(\frac{b}{a}\right)\left[\frac{K_{p+2}(\sqrt{ab})}{K_p(\sqrt{ab})}-\left(\frac{K_{p+1}(\sqrt{ab})}{K_p(\sqrt{ab})}\right)^2\right]$; f_x: $f(x)=\frac{(a/b)^{p/2}}{2K_p(\sqrt{ab})}x^{(p-1)}e^{-(ax+b/x)/2}$

4.22 Log-normal distribution

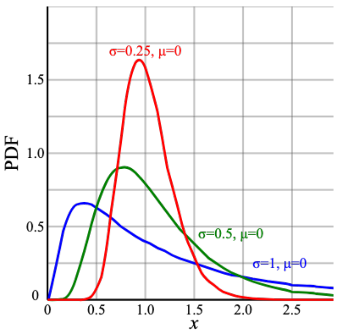

Abbildung 37: Plot of the Lognormal PDF

Params.: $\mu \in (-\infty,+\infty)$, , $\sigma > 0$; **Not.**: Lognormal(μ, σ^2); $\mathcal{W}(X)$: $x \in (0,+\infty)$; $\mathbb{E}[X]$: $\exp\left(\mu+\frac{\sigma^2}{2}\right)$; $Var[X]$: $[\exp(\sigma^2)-1]\exp(2\mu+\sigma^2)$; f_x: $\frac{1}{x\sigma\sqrt{2\pi}}\ \exp\left(-\frac{(\ln x-\mu)^2}{2\sigma^2}\right)$ F_x: $\frac{1}{2}+\frac{1}{2}\operatorname{erf}\left[\frac{\ln x-\mu}{\sqrt{2}\sigma}\right]$

4.23 Half-logistic distribution

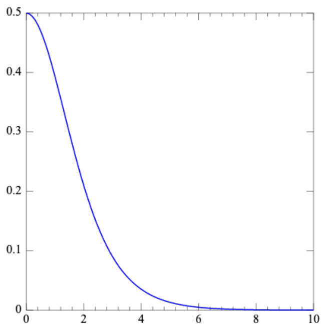

Abbildung 38: Probability density plots of half-logistic distribution

$\mathcal{W}(X)$: $k \in [0;\infty)$; $\mathbb{E}[X]$: $\log_e(4)=1.386\ldots$; $Var[X]$: $\pi^2/3-(\log_e(4))^2=1.368\ldots$; f_x: $\frac{2e^{-k}}{(1+e^{-k})^2}$ F_x: $\frac{1-e^{-k}}{1+e^{-k}}$

4.24 Fréchet distribution

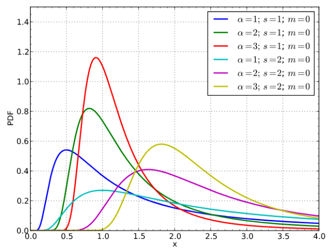

Abbildung 39: PDF of the Fréchet distribution

Params.: $\alpha \in (0,\infty)$ shape. , (Optionally, two more parameters) , $s \in (0,\infty)$ scale (default: $s=1$) , $m \in (-\infty,\infty)$ location of minimum (default: $m=0$); $\mathcal{W}(X)$: $x>m$; $\mathbb{E}[X]$:

$$\begin{cases} m+s\Gamma\left(1-\frac{1}{\alpha}\right) & \text{for } \alpha>1 \\ \infty & \text{otherwise} \end{cases}$$

; $Var[X]$:

$$\begin{cases} s^2\left(\Gamma\left(1-\frac{2}{\alpha}\right)-\left(\Gamma\left(1-\frac{1}{\alpha}\right)\right)^2\right) & \text{for } \alpha>2 \\ \infty & \text{otherwise} \end{cases}$$

; f_x: $\frac{\alpha}{s}\ \left(\frac{x-m}{s}\right)^{-1-\alpha}\ e^{-\left(\frac{x-m}{s}\right)^{-\alpha}}$ F_x: $e^{-\left(\frac{x-m}{s}\right)^{-\alpha}}$

4.25 Gompertz distribution

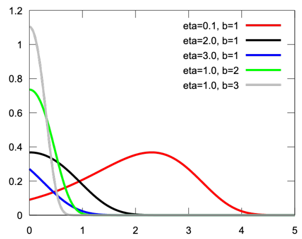

Abbildung 40: 325px

Params.: shape $\eta>0$, scale $b>0$; $\mathcal{W}(X)$: $x \in [0,\infty)$; $\mathbb{E}[X]$: $(1/b)e^{\eta}\text{Ei}\left(-\eta\right)$, where $\text{Ei}\left(z\right)=\int\limits_{-z}^{\infty}\left(e^{-v}/v\right)dv$; $Var[X]$: $(1/b)^2\,e^{\eta}\{-\gamma^2$

$$+\left(\pi^2/6\right)+2\gamma\ln\left(\eta\right)+\left[\ln\left(\eta\right)\right]^2-e^{\eta}\left[\text{Ei}\left(-\eta\right)\right]^2\}$$

,

$$\text{where } \gamma \text{ is the Euler constant:} \quad (1)$$
$$\gamma=-\psi\left(1\right)=0.577215\ldots \quad (2)$$
$$\text{and } {}_3\text{F}_3\left(1,1,1;2,2,2;-z\right)= \quad (3)$$
$$\sum_{k=0}^{\infty}\left[1/\left(k+1\right)^3\right]\left(-1\right)^k\left(z^k/k!\right) \quad (4)$$

; f_x: $b\eta\exp\left(\eta+bx-\eta e^{bx}\right)$ F_x: $1-\exp\left(-\eta\left(e^{bx}-1\right)\right)$

4.26 Lévy distribution

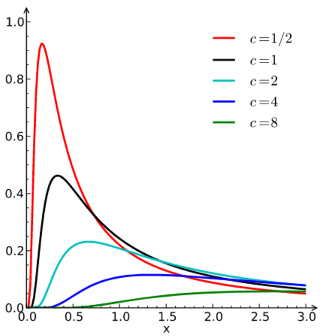

Abbildung 41: Levy distribution PDF

Params.: μ location; $c>0$ scale; $\mathcal{W}(X)$: $x \in [\mu,\infty)$; $\mathbb{E}[X]$: ∞; $Var[X]$: ∞; f_x: $\sqrt{\frac{c}{2\pi}}\ \frac{e^{-\frac{c}{2(x-\mu)}}}{(x-\mu)^{3/2}}$ F_x: $\operatorname{erfc}\left(\sqrt{\frac{c}{2(x-\mu)}}\right)$

4.27 Pareto distribution

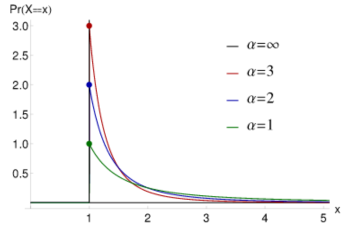

Abbildung 42: Pareto Type I probability density functions for various

Params.: $x_\mathrm{m}>0$ scale (real), $\alpha>0$ shape (real); $\mathcal{W}(X)$: $x \in [x_\mathrm{m},\infty)$; $\mathbb{E}[X]$:

$$\begin{cases} \infty & \text{for } \alpha\le 1 \\ \dfrac{\alpha x_\mathrm{m}}{\alpha-1} & \text{for } \alpha>1 \end{cases}$$

; $Var[X]$:

$$\begin{cases} \infty & \text{for } \alpha\le 2 \\ \dfrac{x_\mathrm{m}^2\alpha}{(\alpha-1)^2(\alpha-2)} & \text{for } \alpha>2 \end{cases}$$

; f_x: $\frac{\alpha x_\mathrm{m}^\alpha}{x^{\alpha+1}}$ F_x: $1-\left(\frac{x_\mathrm{m}}{x}\right)^\alpha$

4.28 Nakagami distribution

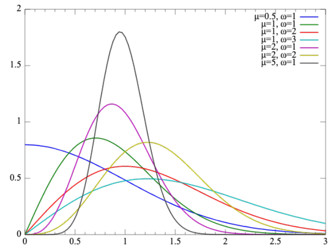

Abbildung 43: 325px

Params.: m or $\mu\ge 0.5$ shape (real), Ω or $\omega>0$ spread (real); $\mathcal{W}(X)$: $x>0$; $\mathbb{E}[X]$: $\frac{\Gamma(m+\frac{1}{2})}{\Gamma(m)}\left(\frac{\Omega}{m}\right)^{1/2}$; $Var[X]$: $\Omega\left(1-\frac{1}{m}\left(\frac{\Gamma(m+\frac{1}{2})}{\Gamma(m)}\right.\right.$ f_x: $\frac{2m^m}{\Gamma(m)\Omega^m}x^{2m-1}\exp\left(-\frac{m}{\Omega}x^2\right)$ F_x: $\frac{\gamma\left(m,\frac{m}{\Omega}x^2\right)}{\Gamma(m)}$

4.29 Exponential distribution

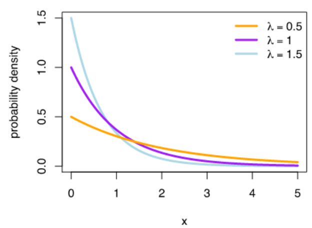

Abbildung 44: plot of the probability density function of the exponential distribution

Params.: $\lambda>0$, rate, or inverse scale; $\mathcal{W}(X)$: $x \in [0,\infty)$; $\mathbb{E}[X]$: $\frac{1}{\lambda}$; $Var[X]$: $\frac{1}{\lambda^2}$; f_x: $\lambda e^{-\lambda x}$ F_x: $1-e^{-\lambda x}$

4.30 Erlang distribution

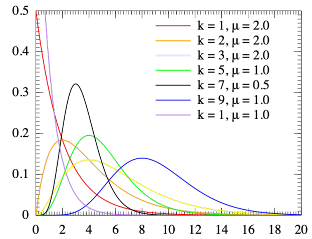

Abbildung 45: Probability density plots of Erlang distributions

Params.: $k \in \{1,2,3,\ldots\}$, shape , $\lambda \in (0,\infty)$, rate , alt.: $\mu=1/\lambda$, scale; $\mathcal{W}(X)$: $x \in [0,\infty)$; $\mathbb{E}[X]$: $\frac{k}{\lambda}$; $Var[X]$: $\frac{k}{\lambda^2}$; f_x: $\frac{\lambda^k x^{k-1}e^{-\lambda x}}{(k-1)!}$ F_x: $P(k,\lambda x)=\frac{\gamma(k,\lambda x)}{(k-1)!}=1-\sum_{n=0}^{k-1}\frac{1}{n!}e^{-\lambda x}(\lambda x)^n$

4.31 Shifted Gompertz distribution

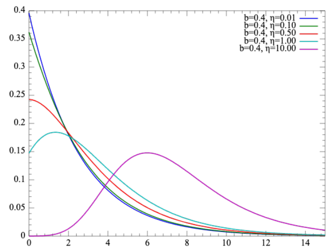

Abbildung 46: Probability density plots of shifted Gompertz distributions

Params.: $b\ge 0$ scale (real), $\eta\ge 0$ shape (real); $\mathcal{W}(X)$: $x \in [0,\infty)$; $\mathbb{E}[X]$: $(-1/b)\{\text{E}[\ln(X)]-\ln(\eta)\}$ where $X=\eta e^{-bx}$ and

$$\text{E}[\ln(X)]=[1+1/\eta]\int_0^{\eta}e^{-X}[\ln(X)]dX \quad (5)$$
$$-1/\eta\int_0^{\eta}Xe^{-X}[\ln(X)]dX \quad (6)$$

; $Var[X]$: $(1/b^2)(\text{E}\{[\ln(X)]^2\}-(\text{E}[\ln(X)])^2)$ where $X=\eta e^{-bx}$ and

$$\text{E}\{[\ln(X)]^2\}=[1+1/\eta]\int_0^{\eta}e^{-X}[\ln(X)]^2dX \quad (7)$$
$$-1/\eta\int_0^{\eta}Xe^{-X}[\ln(X)]^2dX \quad (8)$$

; f_x: $be^{-bx}e^{-\eta e^{-bx}}\left[1+\eta\left(1-e^{-bx}\right)\right]$ F_x: $\left(1-e^{-bx}\right)e^{-\eta e^{-bx}}$

4.32 Gompertz distribution

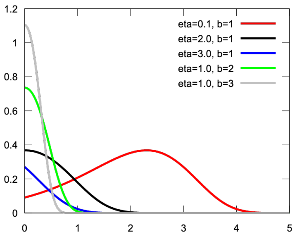

Abbildung 47: 325px

Params.: shape $\eta > 0$, scale $b > 0$; $\mathcal{W}(X)$: $x \in [0,\infty)$; $\mathbb{E}[X]$: $(1/b)e^{\eta}\text{Ei}\,(-\eta)$, where $\text{Ei}\,(z) = \int_{-z}^{\infty} (e^{-v}/v)\,dv$; $Var[X]$: $(1/b)^2\, e^{\eta}\{-$
γ^2

$$+ \left(\pi^2/6\right) + 2\gamma \ln(\eta) + [\ln(\eta)]^2 - e^{\eta}[\text{Ei}\,(-\eta)]^2\}$$

,

$$\text{where } \gamma \text{ is the Euler constant:} \tag{9}$$
$$\gamma = -\psi(1) = 0.577215... \tag{10}$$

$$\text{and } {}_3\text{F}_3\,(1,1,1;2,2,2;-z) = \tag{11}$$
$$\sum_{k=0}^{\infty} \left[1/(k+1)^3\right] (-1)^k \left(z^k/k!\right) \tag{12}$$

;
f_x: $b\eta \exp\left(\eta + bx - \eta e^{bx}\right)$ F_x: $1 - \exp\left(-\eta\left(e^{bx} - 1\right)\right)$

4.33 Inverse Gaussian distribution

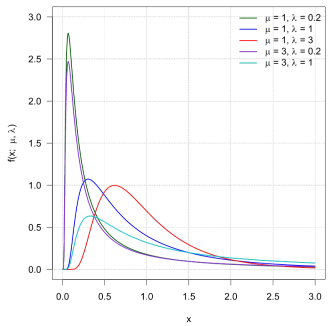

Abbildung 48: 325px

Params.: $\mu > 0$, $\lambda > 0$; **Not.**: IG (μ, λ); $\mathcal{W}(X)$: $x \in (0,\infty)$; $\mathbb{E}[X]$: $\text{E}[X] = \mu$, $\text{E}[\frac{1}{X}] = \frac{1}{\mu} + \frac{1}{\lambda}$; $Var[X]$: $\text{Var}[X] = \frac{\mu^3}{\lambda}$, $\text{Var}[\frac{1}{X}] = \frac{1}{\mu\lambda} + \frac{2}{\lambda^2}$;
f_x: $\sqrt{\frac{\lambda}{2\pi x^3}} \exp\left[-\frac{\lambda(x-\mu)^2}{2\mu^2 x}\right]$ F_x: $\Phi\left(\sqrt{\frac{\lambda}{x}}\left(\frac{x}{\mu} - 1\right)\right)$ $+\exp\left(\frac{2\lambda}{\mu}\right)\Phi\left(-\sqrt{\right.$
where Φ is the standard normal (standard Gaussian) distribution c.d.f.

4.34 Rayleigh distribution

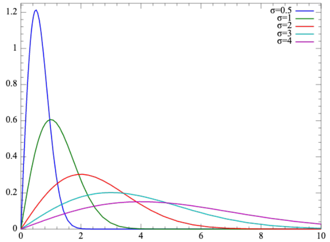

Abbildung 49: Plot of the Rayleigh PDF

Params.: scale: $\sigma > 0$; $\mathcal{W}(X)$: $x \in [0,\infty)$; $\mathbb{E}[X]$: $\sigma\sqrt{\frac{\pi}{2}}$; $Var[$
$\frac{4-\pi}{2}\sigma^2$;
f_x: $\frac{x}{\sigma^2}e^{-x^2/(2\sigma^2)}$ F_x: $1 - e^{-x^2/(2\sigma^2)}$

4.35 Weibull distribution

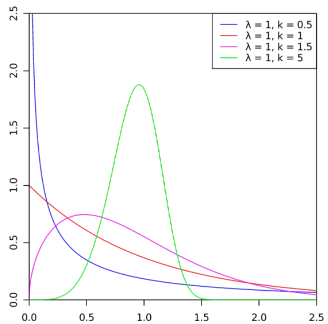

Abbildung 50: Probability distribution function

Params.: $\lambda \in (0,+\infty)$ scale , $k \in (0,+\infty)$ shape; $\mathcal{W}(X)$: $x \in [0,+\infty)$; $\mathbb{E}[X]$: $\lambda\,\Gamma(1+1/k)$; $Var[X]$: $\lambda^2\left[\Gamma\left(1+\frac{2}{k}\right) - \left(\Gamma\left(1+\frac{1}{k}\right.\right.\right.$
f_x:

$$f(x) = \begin{cases} \frac{k}{\lambda}\left(\frac{x}{\lambda}\right)^{k-1} e^{-(x/\lambda)^k} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

F_x: $\begin{cases} 1 - e^{-(x/\lambda)^k} & x \geq 0 \\ 0 & x < 0 \end{cases}$

4.36 F-distribution

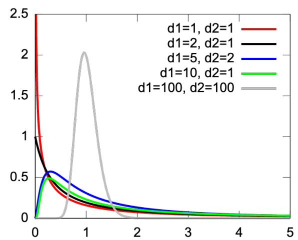

Abbildung 51: 325px

Params.: d_1, d_2 ¿ 0 deg. of freedom; $\mathcal{W}(X)$: $x \in (0,+\infty)$ if $d_1 = 1$, otherwise $x \in [0,+\infty)$; $\mathbb{E}[X]$: $\frac{d_2}{d_2-2}$, for d_2 ¿ 2; $Var[X]$: $\frac{2\,d_2^2\,(d_1+d_2-2)}{d_1(d_2-2)^2(d_2-4)}$, for d_2 ¿ 4;
f_x: $\frac{\sqrt{\frac{(d_1 x)^{d_1}\,d_2^{d_2}}{(d_1 x+d_2)^{d_1+d_2}}}}{x\,\mathrm{B}\left(\frac{d_1}{2},\frac{d_2}{2}\right)}$ F_x: $I_{\frac{d_1 x}{d_1 x+d_2}}\left(\frac{d_1}{2},\frac{d_2}{2}\right)$

4.37 Maxwell–Boltzmann distribution

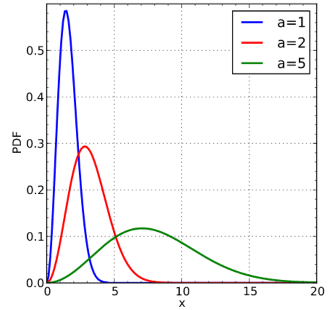

Abbildung 52: 325px

Params.: $a > 0$; $\mathcal{W}(X)$: $x \in (0;\infty)$; $\mathbb{E}[X]$: $\mu = 2a\sqrt{\frac{2}{\pi}}$; $Var[$
$\sigma^2 = \frac{a^2(3\pi-8)}{\pi}$;
f_x: $\sqrt{\frac{2}{\pi}}\frac{x^2 e^{-x^2/\left(2a^2\right)}}{a^3}$ F_x: $\text{erf}\left(\frac{x}{\sqrt{2}a}\right) - \sqrt{\frac{2}{\pi}}\frac{x e^{-x^2/\left(2a^2\right)}}{a}$ where erf is the error function

5 Continuous univariate supported on the whole real line

5.1 Variance-gamma distribution

Params.: μ location (real), α (real), β asymmetry parameter (real), $\lambda > 0$, $\gamma = \sqrt{\alpha^2 - \beta^2} > 0$; $\mathcal{W}(X)$: $x \in (-\infty;+\infty)$; $\mathbb{E}[X]$: $\mu + 2\beta\lambda/\gamma^2$; $Var[X]$: $2\lambda(1+2\beta^2/\gamma^2)/\gamma^2$;
f_x: $\frac{\gamma^{2\lambda}|x-\mu|^{\lambda-1/2}K_{\lambda-1/2}(\alpha|x-\mu|)}{\sqrt{\pi}\Gamma(\lambda)(2\alpha)^{\lambda-1/2}}\; e^{\beta(x-\mu)}$, , K_λ denotes a modified Bessel function of the second kind, Γ denotes the Gamma function

5.2 Generalised hyperbolic distribution

Params.: λ (real), α (real), β asymmetry parameter (real), δ scale parameter (real), μ location (real), $\gamma = \sqrt{\alpha^2 - \beta^2}$; $\mathcal{W}(X)$: $x \in (-\infty;+\infty)$; $\mathbb{E}[X]$: $\mu + \frac{\delta\beta K_{\lambda+1}(\delta\gamma)}{\gamma K_\lambda(\delta\gamma)}$; $Var[X]$:

$$\frac{\delta K_{\lambda+1}(\delta\gamma)}{\gamma K_\lambda(\delta\gamma)} + \frac{\beta^2\delta^2}{\gamma^2}\left(\frac{K_{\lambda+2}(\delta\gamma)}{K_\lambda(\delta\gamma)} - \frac{K_{\lambda+1}^2(\delta\gamma)}{K_\lambda^2(\delta\gamma)}\right)$$

;
f_x: $\frac{(\gamma/\delta)^\lambda}{\sqrt{2\pi}K_\lambda(\delta\gamma)}\; e^{\beta(x-\mu)}$, $\times \frac{K_{\lambda-1/2}\left(\alpha\sqrt{\delta^2+(x-\mu)^2}\right)}{\left(\sqrt{\delta^2+(x-\mu)^2}/\alpha\right)^{1/2-\lambda}}$

5.3 Normal-inverse Gaussian distribution

Params.: μ location (real), α tail heaviness (real), β asymmetry parameter (real), δ scale parameter (real), $\gamma = \sqrt{\alpha^2 - \beta^2}$; $\mathcal{W}(X)$: $x \in (-\infty;+\infty)$; $\mathbb{E}[X]$: $\mu + \delta\beta/\gamma$; $Var[X]$: $\delta\alpha^2/\gamma^3$;
f_x: $\frac{\alpha\delta K_1\left(\alpha\sqrt{\delta^2+(x-\mu)^2}\right)}{\pi\sqrt{\delta^2+(x-\mu)^2}}\; e^{\delta\gamma+\beta(x-\mu)}$, , K_j denotes a modified Bessel function of the third kind

5.4 Holtsmark distribution

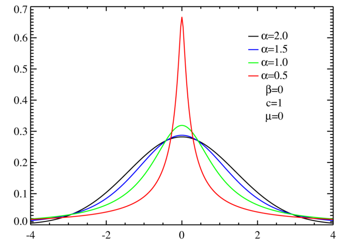

Abbildung 53: Symmetric stable distributions

Params.: $c \in (0,\infty)$ — scale parameter , $\mu \in (-\infty,\infty)$ — location parameter; $\mathcal{W}(X)$: $x \in \mathbf{R}$; $\mathbb{E}[X]$: μ; $Var[X]$: infinite;
f_x: expressible in terms of hypergeometric functions; see text

5.5 Asymmetric Laplace distribution

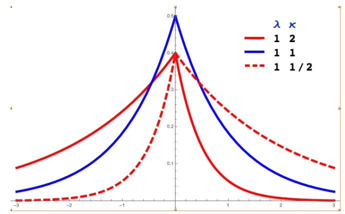

Abbildung 54: 350px

Params.: m location (real), $\lambda > 0$ scale (real), $\kappa > 0$ asymmetry (real); $\mathcal{W}(X)$: $x \in (-\infty;+\infty)$; $\mathbb{E}[X]$: $m + \frac{1-\kappa^2}{\lambda\kappa}$; $Var[X]$: $\frac{1+\kappa^4}{\lambda^2\kappa^2}$;
f_x: (see article) F_x: (see article)

5.6 Johnson's SU-distribution

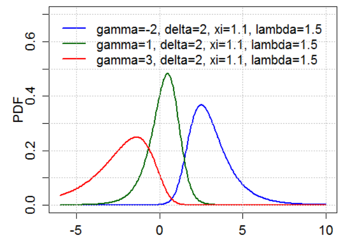

Abbildung 55: JohnsonSU

Params.: $\gamma, \xi, \delta > 0, \lambda > 0$ (real); $\mathcal{W}(X)$: $-\infty$ to $+\infty$; $\mathbb{E}[X]$: $\xi - \lambda \exp\frac{\delta^{-2}}{2}\sinh\left(\frac{\gamma}{\delta}\right)$; $Var[X]$: $\frac{\lambda^2}{2}(\exp(\delta^{-2})-1)\left(\exp(\delta^{-2})\cosh\left(\frac{2\gamma}{\delta}\right.\right.$
f_x: $\frac{\delta}{\lambda\sqrt{2\pi}}\frac{1}{\sqrt{1+\left(\frac{x-\xi}{\lambda}\right)^2}}e^{-\frac{1}{2}\left(\gamma+\delta\sinh^{-1}\left(\frac{x-\xi}{\lambda}\right)\right)^2}$ F_x: $\Phi\left(\gamma + \delta\sinh^{-1}\left(\frac{x-\xi}{\lambda}\right)\right)$

5.7 Normal distribution

Params.: $\mu \in \mathbb{R}$ = mean (location), $\sigma^2 > 0$ = variance (squared scale); **Not.**: $\mathcal{N}(\mu,\sigma^2)$; $\mathcal{W}(X)$: $x \in \mathbb{R}$; $\mathbb{E}[X]$: μ; $Var[X]$: σ^2;
f_x: $\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$ F_x: $\frac{1}{2}\left[1+\text{erf}\left(\frac{x-\mu}{\sigma\sqrt{2}}\right)\right]$

5.8 Gumbel distribution

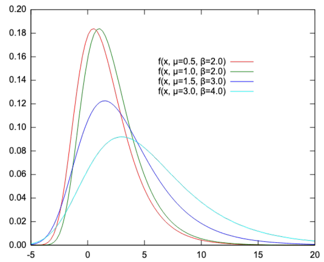

Abbildung 56: Probability distribution function

Params.: μ, location (real), $\beta > 0$, scale (real); $\mathcal{W}(X)$: $x \in \mathbb{R}$; $\mathbb{E}[X]$: $\mu+\beta\gamma$, where γ is the Euler–Mascheroni constant; $Var[X]$: $\frac{\pi^2}{6}\beta^2$;
f_x: $\frac{1}{\beta}e^{-(z+e^{-z})}$, where $z = \frac{x-\mu}{\beta}$ F_x: $e^{-e^{-(x-\mu)/\beta}}$

5.9 Fisher's z-distribution

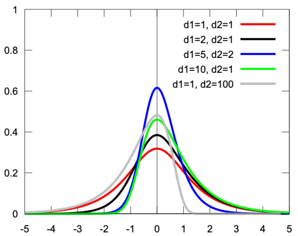

Abbildung 57: 325px

Params.: $d_1 > 0$, $d_2 > 0$ deg. of freedom; $\mathcal{W}(X)$: $x \in (-\infty;+\infty)$;
f_x: $\frac{2d_1^{d_1/2}d_2^{d_2/2}}{B(d_1/2,d_2/2)}\frac{e^{d_1 x}}{(d_1e^{2x}+d_2)^{(d_1+d_2)/2}}$

5.10 Slash distribution

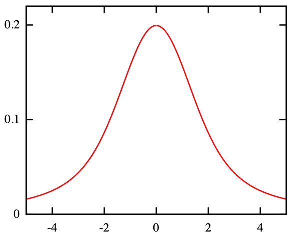

Abbildung 58: center

Params.: none; $\mathcal{W}(X)$: $x \in (-\infty,\infty)$; $\mathbb{E}[X]$: Does not exist; $Var[X]$: Does not exist;
f_x:

$$\begin{cases} \frac{\varphi(0)-\varphi(x)}{x^2} & x \neq 0 \\ \frac{1}{2\sqrt{2\pi}} & x = 0 \end{cases}$$

F_x:

$$\begin{cases} \Phi(x) - [\varphi(0)-\varphi(x)]/x & x \neq 0 \\ 1/2 & x = 0 \end{cases}$$

5.11 Cauchy distribution

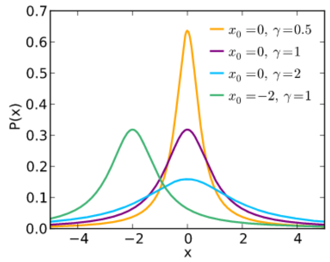

Abbildung 59: Probability density function for the Cauchy distribution

Params.: x_0 location (real), $\gamma > 0$ scale (real); $\mathcal{W}(X)$: $x \in (-\infty,+\infty)$; $\mathbb{E}[X]$: undefined; $Var[X]$: undefined;
f_x: $\frac{1}{\pi\gamma\left[1+\left(\frac{x-x_0}{\gamma}\right)^2\right]}$ F_x: $\frac{1}{\pi}\arctan\left(\frac{x-x_0}{\gamma}\right)+\frac{1}{2}$

5.12 Skew normal distribution

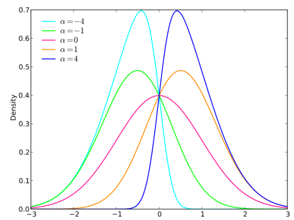

Abbildung 60: Probability density plots of skew normal distributions

Params.: ξ location (real), ω scale (positive, real), α shape (real); $\mathcal{W}(X)$: $x \in (-\infty;+\infty)$; $\mathbb{E}[X]$: $\xi+\omega\delta\sqrt{\frac{2}{\pi}}$ where $\delta = \frac{\alpha}{\sqrt{1+\alpha^2}}$; $Var[X]$: $\omega^2\left(1-\frac{2\delta^2}{\pi}\right)$;
f_x: $\frac{2}{\omega\sqrt{2\pi}}e^{-\frac{(x-\xi)^2}{2\omega^2}}\int_{-\infty}^{\alpha\left(\frac{x-\xi}{\omega}\right)}\frac{1}{\sqrt{2\pi}}e^{-\frac{t^2}{2}}\,dt$ F_x: $\Phi\left(\frac{x-\xi}{\omega}\right)-2T\left(\frac{x-\xi}{\omega},\alpha\right)$, $T(h,a)$ is Owen's T function

5.13 Hyperbolic secant distribution

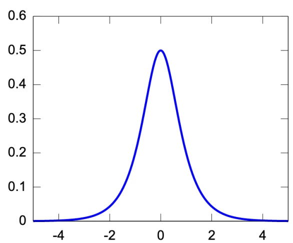

Abbildung 61: Plot of the hyperbolic secant PDF

Params.: *none*; $\mathcal{W}(X)$: $x \in (-\infty;+\infty)$; $\mathbb{E}[X]$: 0; $Var[X]$: 1;
f_x: $\frac{1}{2}\operatorname{sech}(\frac{\pi}{2}x)$ F_x: $\frac{2}{\pi}\arctan[\exp(\frac{\pi}{2}x)]$

5.14 Logistic distribution

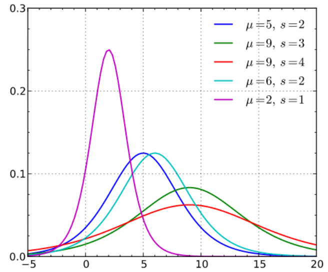

Abbildung 62: Standard logistic PDF

Params.: μ, location (real), $s > 0$, scale (real); $\mathcal{W}(X)$: $x \in (-\infty,\infty)$; $\mathbb{E}[X]$: μ; $Var[X]$: $\frac{s^2\pi^2}{3}$;
f_x: $\frac{e^{-(x-\mu)/s}}{s\left(1+e^{-(x-\mu)/s}\right)^2}$ F_x: $\frac{1}{1+e^{-(x-\mu)/s}}$

5.15 Noncentral t-distribution

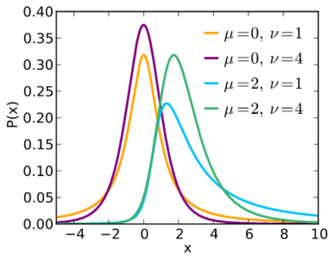

Abbildung 63: 325px

Params.: ¿ 0 degrees of freedom, $\mu \in \Re$ noncentrality parameter; $\mathcal{W}(X)$: $x \in (-\infty;+\infty)$; $\mathbb{E}[X]$: see text; $Var[X]$: see text;
f_x: see text

5.16 Landau distribution

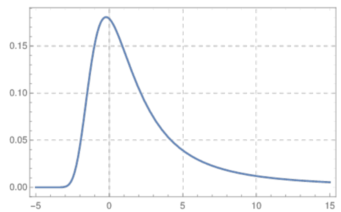

Abbildung 64: 350px

Params.: $c \in (0,\infty)$ — scale parameter , $\mu \in (-\infty,\infty)$ — location parameter; $\mathcal{W}(X)$: $\mathbb{R}$; $\mathbb{E}[X]$: Undefined; $Var[X]$: Undefined;
f_x: $\frac{1}{\pi c}\int_0^\infty e^{-t}\cos\left(t\left(\frac{x-\mu}{c}\right)+\frac{2t}{\pi}\log\left(\frac{t}{c}\right)\right)\,dt$

5.17 Generalized normal distribution

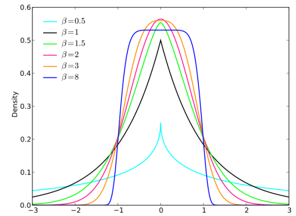

Abbildung 65: Probability density plots of generalized normal distributions

Params.: μ location (real), α scale (positive, real), β shape (positive, real); $\mathcal{W}(X)$: $x \in (-\infty;+\infty)$; $\mathbb{E}[X]$: μ; $Var[X]$: $\frac{\alpha^2\Gamma(3/\beta)}{\Gamma(1/\beta)}$;
f_x: $\frac{\beta}{2\alpha\Gamma(1/\beta)}\,e^{-(|x-\mu|/\alpha)^\beta}$, , Γ denotes the gamma function F_x: $\frac{1}{2}+\frac{\text{sign}(x-\mu)}{2}\frac{1}{\Gamma\left(\frac{1}{\beta}\right)}\gamma\left(\frac{1}{\beta},x\alpha^\beta\right)$.

5.18 Generalized normal distribution

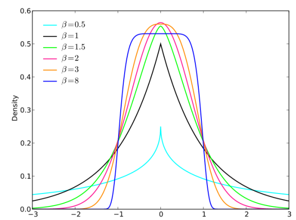

Abbildung 66: Probability density plots of generalized normal distributions

Params.: μ location (real), α scale (positive, real), β shape (positive, real); $\mathcal{W}(X)$: $x \in (-\infty;+\infty)$; $\mathbb{E}[X]$: μ; $Var[X]$: $\frac{\alpha^2\Gamma(3/\beta)}{\Gamma(1/\beta)}$;
f_x: $\frac{\beta}{2\alpha\Gamma(1/\beta)}\,e^{-(|x-\mu|/\alpha)^\beta}$, , Γ denotes the gamma function F_x: $\frac{1}{2}+\frac{\text{sign}(x-\mu)}{2}\frac{1}{\Gamma\left(\frac{1}{\beta}\right)}\gamma\left(\frac{1}{\beta},x\alpha^\beta\right)$.

5.19 Student's t-distribution

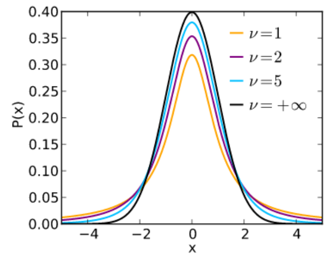

Abbildung 67: 325px

Params.: $\nu > 0$ degrees of freedom (real); $\mathcal{W}(X)$: $x \in (-\infty,\infty)$; $\mathbb{E}[X]$: 0 for $\nu > 1$, otherwise undefined; $Var[X]$: $\frac{\nu}{\nu-2}$ for $\nu > 2$, ∞ for $1 < \nu \leq 2$, otherwise undefined;
f_x: $\frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\nu\pi}\,\Gamma\left(\frac{\nu}{2}\right)}\left(1+\frac{x^2}{\nu}\right)^{-\frac{\nu+1}{2}}$

5.20 Laplace distribution

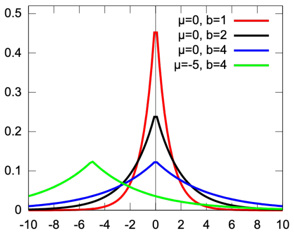

Abbildung 68: Probability density plots of Laplace distributions

Params.: μ location (real), $b > 0$ scale (real); $\mathcal{W}(X)$: $\mathbb{R}$; $\mathbb{E}[X]$: μ; $Var[X]$: $2b^2$;
f_x: $\frac{1}{2b}\exp\left(-\frac{|x-\mu|}{b}\right)$ F_x:

$$\begin{cases} \frac{1}{2}\exp\left(\frac{x-\mu}{b}\right) & \text{if } x \leq \mu \\ 1-\frac{1}{2}\exp\left(-\frac{x-\mu}{b}\right) & \text{if } x \geq \mu \end{cases}$$

5.21 Voigt profile

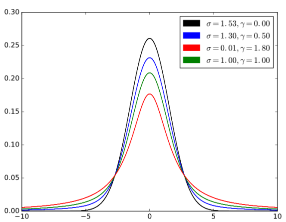

Abbildung 69: Plot of the centered Voigt profile for four cases

Params.: $\gamma, \sigma > 0$; $\mathcal{W}(X)$: $x \in (-\infty, \infty)$; $\mathbb{E}[X]$: (not defined); $Var[X]$: (not defined);
f_x:

$$\frac{\Re[w(z)]}{\sigma\sqrt{2\pi}}, \quad z = \frac{x+i\gamma}{\sigma\sqrt{2}}$$

F_x: (complicated - see text)

6 Continuous univariate with support wh se type varies

6.1 Shifted log-logistic distribution

Params.: $\mu \in (-\infty, +\infty)$ location (real), $\sigma \in (0, +\infty)$ scale (real), $\xi \in (-\infty, +\infty)$ shape (real); $\mathcal{W}(X)$: $x \geqslant \mu - \sigma/\xi$ $(\xi > 0)$, $x \leqslant \mu - \sigma/\xi$ $(\xi < 0)$, $x \in (-\infty, +\infty)$ $(\xi = 0)$; $\mathbb{E}[X]$: $\mu + \frac{\sigma}{\xi}(\alpha\csc(\alpha) - 1)$, where $\alpha = \pi\xi$; $Var[X]$: $\frac{\sigma^2}{\xi^2}[2\alpha\csc(2\alpha) - (\alpha\csc(\alpha))^2]$, where $\alpha = \pi\xi$;
f_x: $\frac{(1+\xi z)^{-(1/\xi+1)}}{\sigma\left(1+(1+\xi z)^{-1/\xi}\right)^2}$, where $z = (x-\mu)/\sigma$ F_x: $\left(1+(1+\xi z)^{-1/\xi}\right)^{-1}$, where $z = (x-\mu)/\sigma$

6.2 Generalized extreme value distribution

Params.: $\mu \in \mathbf{R}$ — location,, ¿ 0 — scale,, $\in \mathbf{R}$ — shape.; **Not.**: GEV(μ, σ, ξ); $\mathcal{W}(X)$: $x \in [\mu - /, +\infty)$ when ¿ 0,, $x \in (-\infty, +\infty)$ when $= 0$,, $x \in (-\infty, \mu - /]$ when ¡ 0.; $\mathbb{E}[X]$: $\begin{cases} \mu + \sigma(g_1 - 1)/\xi & \text{if } \xi \neq 0, \xi < 1, \\ \mu + \sigma\gamma & \text{if } \xi = 0, \\ \infty & \text{if } \xi \geq 1, \end{cases}$ where $g_k = (1-k)$, , and γ is Euler's constant.; $Var[X]$: $\begin{cases} \sigma^2(g_2 - g_1^2)/\xi^2 & \text{if } \xi \neq 0, \xi < \frac{1}{2}, \\ \sigma^2\frac{\pi^2}{6} & \text{if } \xi = 0, \\ \infty & \text{if } \xi \geq \frac{1}{2}, \end{cases}$.;
f_x: $\frac{1}{\sigma}t(x)^{\xi+1}e^{-t(x)}$, where $t(x) = \begin{cases} \left(1+\xi\left(\frac{x-\mu}{\sigma}\right)\right)^{-1/\xi} & \text{if } \xi \neq 0 \\ e^{-(x-\mu)/\sigma} & \text{if } \xi = 0 \end{cases}$ F_x: $e^{-t(x)}$, for $x \in$ support

6.3 Q-Weibull distribution

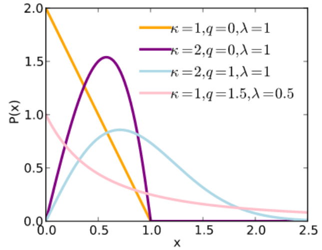

Abbildung 70: Graph of the q-Weibull pdf

Params.: $q < 2$ shape (real), $\lambda > 0$ rate (real), $\kappa > 0$ shape (real); $\mathcal{W}(X)$: $x \in [0; +\infty)$ for $q \geq 1$, $x \in [0; \frac{\lambda}{(1-q)^{1/\kappa}})$ for $q < 1$; $\mathbb{E}[X]$: (see article);
f_x:

$$\begin{cases} (2-q)\frac{\kappa}{\lambda}\left(\frac{x}{\lambda}\right)^{\kappa-1}e_q^{-(x/\lambda)^\kappa} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

F_x: $\begin{cases} 1 - e_{q'}^{-(x/\lambda')^\kappa} & x \geq 0 \\ 0 & x < 0 \end{cases}$

6.4 Q-Gaussian distribution

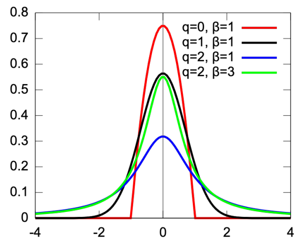

Abbildung 71: Probability density plots of q-Gaussian distributions

Params.: $q < 3$ shape (real), $\beta > 0$ (real); $\mathcal{W}(X)$: $x \in (-\infty; +\infty)$ for $1 \leq q < 3$, $x \in \left[\pm\frac{1}{\sqrt{\beta(1-q)}}\right]$ for $q < 1$; $\mathbb{E}[X]$: 0 for $q < 2$, otherwise undefined; $Var[X]$: $\frac{1}{\beta(5-3q)}$ for $q < \frac{5}{3}$,

$$\infty \text{ for } \frac{5}{3} \leq q < 2$$

, Undefined for $2 \leq q < 3$;
f_x: $\frac{\sqrt{\beta}}{C_q}e_q(-\beta x^2)$

6.5 Generalized chi-squared distribution

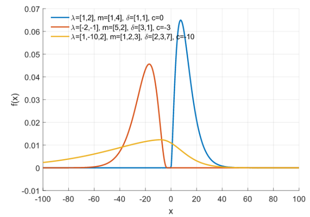

Abbildung 72: Generalized chi-square probability density function

Params.: $\boldsymbol{\lambda}$, vector of weights of chi-square components, $\boldsymbol{m}$, vector of degrees of freedom of chi-square components, $\boldsymbol{\delta}$, vector of non-centrality parameters of chi-square components, σ, scale of normal term; $\mathcal{W}(X)$: $x \in \mathbb{R}$; $\mathbb{E}[X]$: $\sum\lambda_j(m_j + \delta_j^2)$; $Var[X]$: $2\sum\lambda_j^2(m_j + 2\delta_j^2) + \sigma^2$;

6.6 Tukey lambda distribution

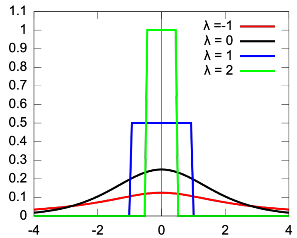

Abbildung 73: Probability density plots of Tukey lambda distributions

Params.: $\in \mathbf{R}$ — shape parameter; **Not.**: Tukey(); $\mathcal{W}(X)$: $x \in [-1/, 1/]$ for ¿ 0,, $x \in \mathbf{R}$ for ≤ 0; $\mathbb{E}[X]$: 0, $\lambda > -1$; $Var[X]$: $\frac{2}{\lambda^2}\left(\frac{1}{1+2\lambda} - \frac{\Gamma(\lambda+1)^2}{\Gamma(2\lambda+2)}\right)$, $\lambda > -1/2$, $\frac{\pi^2}{3}$, $\lambda = 0$;
f_x: $(Q(p;\lambda), q(p;\lambda)^{-1})$, $0 \leq p \leq 1$ F_x: $(e^{-x}+1)^{-1}$, $\lambda = 0$ (special case), $(Q(p;\lambda), p)$, $0 \leq p \leq 1$ (general case)

6.7 Generalized Pareto distribution

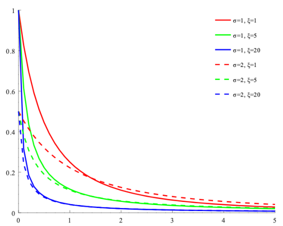

Abbildung 74: Gpdpdf

Params.: $\mu \in (-\infty, \infty)$ location (real), $\sigma \in (0, \infty)$ scale (real), $\xi \in (-\infty, \infty)$ shape (real); $\mathcal{W}(X)$: $x \geqslant \mu$ $(\xi \geqslant 0)$, $\mu \leqslant x \leqslant \mu - \sigma/\xi$ $(\xi < 0)$; $\mathbb{E}[X]$: $\mu + \frac{\sigma}{1-\xi}$ $(\xi < 1)$; $Var[X]$: $\frac{\sigma^2}{(1-\xi)^2(1-2\xi)}$ $(\xi < 1/2)$;
f_x: $\frac{1}{\sigma}(1+\xi z)^{-(1/\xi+1)}$, where $z = \frac{x-\mu}{\sigma}$ F_x: $1 - (1+\xi z)^{-1/\xi}$

6.8 Q-exponential distribution

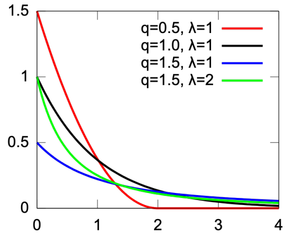

Abbildung 75: Probability density plots of q-exponential distributions

Params.: $q < 2$ shape (real), $\lambda > 0$ rate (real); $\mathcal{W}(X)$: $x \in [0, \infty)$ for $q \geq 1$, $x \in \left[0, \frac{1}{\lambda(1-q)}\right)$ for $q < 1$; $\mathbb{E}[X]$: $\frac{1}{\lambda(3-2q)}$ for $q < \frac{3}{2}$, Otherwise undefined; $Var[X]$: $\frac{q-2}{(2q-3)^2(3q-4)\lambda^2}$ for $q < \frac{4}{3}$;
f_x: $(2-q)\lambda e_q^{-\lambda x}$ F_x: $1 - e_{q'}^{-\lambda x/q'}$ where $q' = \frac{1}{2-q}$

7 Mixed continuous-discrete univariate

8 Multivariate (joint)

8.1 Discrete

8.1.1 Negative multinomial distribution

Params.: $x_0 \in \mathbf{N}_0$ — the number of failures before the experiment is stopped,, $p \in \mathbf{R}^m$ — m-vector of ßuccessprobabilities,, ¡hr¿$p_0 = 1 - (p_1 + \ldots + p_m)$ — the probability of a failure".; **Not.**: NM(x_0, p); $\mathcal{W}(X)$: $x_i \in \{0, 1, 2, \ldots\}, 1 \leq i \leq m$; $\mathbb{E}[X]$: $\frac{x_0}{p_0}p$; $Var[X]$: $\frac{x_0}{p_0^2}pp' + \frac{x_0}{p_0}\operatorname{diag}(p)$;
f_x: $\Gamma(\sum_{i=0}^m x_i)\frac{p_0^{x_0}}{\Gamma(x_0)}\prod_{i=1}^m \frac{p_i^{x_i}}{x_i!}$, , where (x) is the Gamma function.

8.1.2 Multinomial distribution

Params.: $n > 0$ number of trials (integer), $p_1, \ldots, p_k$ event probabilities ($\Sigma p_i = 1$); $\mathcal{W}(X)$: $x_i \in \{0, \ldots, n\}$, $i \in \{1, \ldots, k\}$, $\Sigma x_i = n$; $\mathbb{E}[X]$: $\mathrm{E}(X_i) = np_i$; $Var[X]$: $\mathrm{Var}(X_i) = np_i(1-p_i)$, $\mathrm{Cov}(X_i, X_j) = -np_ip_j$ $(i \neq j)$;
f_x: $\frac{n!}{x_1! \cdots x_k!}p_1^{x_1} \cdots p_k^{x_k}$

8.1.3 Dirichlet-multinomial distribution

Params.: $n > 0$ number of trials (positive integer), $\alpha_1, \ldots, \alpha_K > 0$; $\mathcal{W}(X)$: $x_i \in \{0, \ldots, n\}$, $\Sigma x_i = n$; $\mathbb{E}[X]$: $E(X_i) = n\frac{\alpha_i}{\sum\alpha_k}$; $Var[X]$: $Var(X_i) = n\frac{\alpha_i}{\sum\alpha_k}\left(1 - \frac{\alpha_i}{\sum\alpha_k}\right)\left(\frac{n+\sum\alpha_k}{1+\sum\alpha_k}\right)$, $\mathrm{Cov}(X_i, X_j) = -n\frac{\alpha_i\alpha_j}{(\sum\alpha_k)^2}\left(\frac{n+\sum\alpha_k}{1+\sum\alpha_k}\right)$ $(i \neq j)$;
f_x:

$$\frac{(n!)\,\Gamma\left(\sum\alpha_k\right)}{\Gamma\left(n+\sum\alpha_k\right)}\prod_{k=1}^K \frac{\Gamma(x_k+\alpha_k)}{(x_k!)\,\Gamma(\alpha_k)}$$

8.2 Continuous

8.2.1 Multivariate Laplace distribution

Params.: $\boldsymbol{\mu} \in \mathbf{R}^k$ — location, $\in \mathbf{R}^{k\times k}$ — covariance (positive-definite matrix); $\mathcal{W}(X)$: $\boldsymbol{x} \in \boldsymbol{\mu} + \operatorname{span}() \subseteq \mathbf{R}^k$; $\mathbb{E}[X]$: $\boldsymbol{\mu}$; $Var[$;
f_x: :If $\boldsymbol{\mu} = \mathbf{0}$,, $\frac{2}{(2\pi)^{k/2}|\boldsymbol{\Sigma}|^{0.5}}\left(\frac{\mathbf{x}'\boldsymbol{\Sigma}^{-1}\mathbf{x}}{2}\right)^{v/2}K_v\left(\sqrt{2\mathbf{x}'\boldsymbol{\Sigma}^{-1}\mathbf{x}}\right)$, , where $v = (2-k)/2$ and K_v is the modified Bessel function of the second kind.

8.2.2 Normal-gamma distribution

Params.: μ location (real), $\lambda > 0$ (real), $\alpha > 0$ (real), $\beta > 0$ (real); $\mathcal{W}(X)$: $x \in (-\infty, \infty)$, $\tau \in (0, \infty)$; $\mathbb{E}[X]$: $\mathrm{E}(X) = \mu$, $\mathrm{E}() = \alpha\beta^{-1}$; $Var[X]$:

$$\operatorname{var}(X) = \Big(\frac{\beta}{\lambda(\alpha-1)}\Big), \quad \operatorname{var}() = \alpha\beta^{-2}$$

;

f_x: $f(x, \tau \mid \mu, \lambda, \alpha, \beta) = \frac{\beta^\alpha \sqrt{\lambda}}{\Gamma(\alpha)\sqrt{2\pi}}\, \tau^{\alpha - \frac{1}{2}}\, e^{-\beta\tau}\, e^{-\frac{\lambda\tau(x-\mu)^2}{2}}$

8.2.3 Multivariate t-distribution

Params.: $\boldsymbol{\mu} = [\mu_1, \dots, \mu_p]^T$ location (real $p \times 1$ vector), $\boldsymbol{\Sigma}$ scale matrix (positive-definite real $p \times p$ matrix) , ν is the degrees of freedom; **Not.**: $t_\nu(\boldsymbol{\mu}, \boldsymbol{\Sigma})$; $\mathcal{W}(X)$: $\mathbf{x} \in \mathbb{R}^p$; $\mathbb{E}[X]$: $\boldsymbol{\mu}$ if $\nu > 1$; else undefined; $Var[X]$: $\frac{\nu}{\nu-2}\boldsymbol{\Sigma}$ if $\nu > 2$; else undefined;
f_x:

$$\frac{\Gamma\left[(\nu+p)/2\right]}{\Gamma(\nu/2)\nu^{p/2}\pi^{p/2}\left|\boldsymbol{\Sigma}\right|^{1/2}}\left[1+\frac{1}{\nu}(\mathbf{x}-\boldsymbol{\mu})^{\mathrm{T}}\boldsymbol{\Sigma}^{-1}(\mathbf{x}-\boldsymbol{\mu})\right]^{-(\nu+p)/2}$$

F_x: No analytic expression, but see text for approximations

8.2.4 Multivariate normal distribution

Params.: $\boldsymbol{\mu} \in \mathbf{R}^k$ — location, $\in \mathbf{R}^{k\times k}$ — covariance (positive semi-definite matrix); **Not.**: $\mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$; $\mathcal{W}(X)$: $\boldsymbol{x} \in \boldsymbol{\mu} + \operatorname{span}() \subseteq \mathbf{R}^k$; $\mathbb{E}[X]$: $\boldsymbol{\mu}$; $Var[X]$: ;
f_x: $(2\pi)^{-\frac{k}{2}} \det(\boldsymbol{\Sigma})^{-\frac{1}{2}}\, e^{-\frac{1}{2}(\mathbf{x}-\boldsymbol{\mu})^{\mathrm{T}}\boldsymbol{\Sigma}^{-1}(\mathbf{x}-\boldsymbol{\mu})}$, , exists only when is positiv definite

8.2.5 Multivariate stable distribution

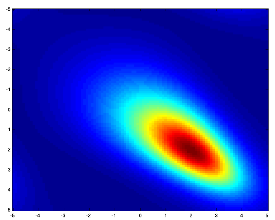

Abbildung 76: 220px

Params.: $\alpha \in (0, 2]$ — exponent, $\delta \in \mathbb{R}^d$ - shift/location vector, $\Lambda(s)$ - a spectral finite measure on the sphere; $\mathcal{W}(X)$: $u \in \mathbb{R}^d$; $Var[X]$: Infinite when $\alpha < 2$;
f_x: (no analytic expression)F_x: (no analytic expression)

8.2.6 Dirichlet distribution

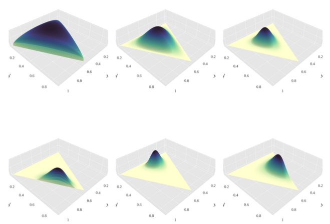

Abbildung 77: A panel illustrating probability density functions of a few Dirichlet distributions over a 2-simplex, for the following vectors (clockwise, starting from the upper left corner): (1.3, 1.3, 1.3), (3,3,3), (7,7,7), (2,6,11), (14, 9, 5), (6,2,6).

Params.: $K \geq 2$ number of categories (integer), $\alpha_1, \dots, \alpha_K$ concentration parameters, where $\alpha_i > 0$; $\mathcal{W}(X)$: $x_1, \dots, x_K$ where $x_i \in (0,1)$ and $\sum_{i=1}^K x_i = 1$; $\mathbb{E}[X]$: $\mathrm{E}[X_i] = \frac{\alpha_i}{\sum_{k=1}^K \alpha_k}$, $\mathrm{E}[\ln X_i] = \psi(\alpha_i) - \psi(\sum_k \alpha_k)$, (see digamma function); $Var[X]$: $\operatorname{Var}[X_i] = \frac{\tilde{\alpha}_i(1-\tilde{\alpha}_i)}{\alpha_0+1}$, $\operatorname{Cov}[X_i, X_j] = \frac{-\alpha_i\alpha_j}{\alpha_0^2(\alpha_0+1)}$ $(i \neq j)$, where $\tilde{\alpha}_i = \frac{\alpha_i}{\alpha_0}$ and $\alpha_0 = \sum_{i=1}^K \alpha_i$;
f_x: $\frac{1}{\mathrm{B}(\boldsymbol{\alpha})}\prod_{i=1}^K x_i^{\alpha_i - 1}$, where $\mathrm{B}(\boldsymbol{\alpha}) = \frac{\prod_{i=1}^K \Gamma(\alpha_i)}{\Gamma\left(\sum_{i=1}^K \alpha_i\right)}$, where $\boldsymbol{\alpha} = (\alpha_1, \dots,$

8.2.7 Normal-inverse-gamma distribution

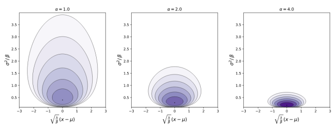

Abbildung 78: Probability density function of normal-inverse-gamma distribution for = 1.0, 2.0 and 4.0, plotted in shifted and scaled coordinates.

Params.: μ location (real), $\lambda > 0$ (real), $\alpha > 0$ (real), $\beta > 0$ (real); $\mathcal{W}(X)$: $x \in (-\infty, \infty)$, $\sigma^2 \in (0, \infty)$; $\mathbb{E}[X]$: $\mathrm{E}[x] = \mu$, $\mathrm{E}[\sigma^2] = \frac{\beta}{\alpha-1}$, for $\alpha > 1$,; $Var[X]$: $\operatorname{Var}[x] = \frac{\beta}{(\alpha-1)\lambda}$, for $\alpha > 1$, $\operatorname{Var}[\sigma^2] = \frac{\beta^2}{(\alpha-1)^2(\alpha-2)}$, for $\alpha > 2$, $\operatorname{Cov}[x, \sigma^2] = 0$, for $\alpha > 1$;
f_x:

$$\frac{\sqrt{\lambda}}{\sqrt{2\pi\sigma^2}}\frac{\beta^\alpha}{\Gamma(\alpha)}\left(\frac{1}{\sigma^2}\right)^{\alpha+1}\exp\left(-\frac{2\beta+\lambda(x-\mu)^2}{2\sigma^2}\right)$$

8.3 Matrix-valued

8.3.1 Normal-Wishart distribution

Params.: $\boldsymbol{\mu}_0 \in \mathbb{R}^D$ location (vector of real), $\lambda > 0$ (real), $\mathbf{W} \in \mathbb{R}^{D\times D}$ scale matrix (pos. def.), $\nu > D - 1$ (real); **Not.**: $(\boldsymbol{\mu}, \boldsymbol{\Lambda}) \sim \mathrm{NW}(\boldsymbol{\mu}_0, \lambda, \mathbf{W}, \nu)$; $\mathcal{W}(X)$: $\boldsymbol{\mu} \in \mathbb{R}^D; \boldsymbol{\Lambda} \in \mathbb{R}^{D\times D}$ covariance matrix (pos. def.);
f_x: $f(\boldsymbol{\mu}, \boldsymbol{\Lambda}|\boldsymbol{\mu}_0, \lambda, \mathbf{W}, \nu) = \mathcal{N}(\boldsymbol{\mu}|\boldsymbol{\mu}_0, (\lambda\boldsymbol{\Lambda})^{-1})\ \mathcal{W}(\boldsymbol{\Lambda}|\mathbf{W}, \nu)$

8.3.2 Inverse matrix gamma distribution

Params.: $\alpha > (p-1)/2$ shape parameter , $\beta > 0$ scale parameter, $\boldsymbol{\Psi}$ scale (positive-definite real $p \times p$ matrix); **Not.**: $\mathrm{IMG}_p(\alpha, \beta, \boldsymbol{\Psi})$; $\mathcal{W}(X)$: $\mathbf{X}$ positive-definite real $p \times p$ matrix;
f_x: $\frac{|\boldsymbol{\Psi}|^\alpha}{\beta^{p\alpha}\Gamma_p(\alpha)}|\mathbf{X}|^{-\alpha-(p+1)/2}\exp\left(-\frac{1}{\beta}\operatorname{tr}\left(\boldsymbol{\Psi}\mathbf{X}^{-1}\right)\right)$ * Γ_p is the multivariate gamma function.

8.3.3 Normal-inverse-Wishart distribution

Params.: $\boldsymbol{\mu}_0 \in \mathbb{R}^D$ location (vector of real), $\lambda > 0$ (real), $\boldsymbol{\Psi} \in \mathbb{R}^{D\times D}$ inverse scale matrix (pos. def.), $\nu > D - 1$ (real); **Not.**: $(\boldsymbol{\mu}, \boldsymbol{\Sigma}) \sim \mathrm{NIW}(\boldsymbol{\mu}_0, \lambda, \boldsymbol{\Psi}, \nu)$; $\mathcal{W}(X)$: $\boldsymbol{\mu} \in \mathbb{R}^D; \boldsymbol{\Sigma} \in \mathbb{R}^{D\times D}$ covariance matrix (pos. def.);
f_x: $f(\boldsymbol{\mu}, \boldsymbol{\Sigma}|\boldsymbol{\mu}_0, \lambda, \boldsymbol{\Psi}, \nu) = \mathcal{N}(\boldsymbol{\mu}|\boldsymbol{\mu}_0, \frac{1}{\lambda}\boldsymbol{\Sigma})\ \mathcal{W}^{-1}(\boldsymbol{\Sigma}|\boldsymbol{\Psi}, \nu)$

8.3.4 Matrix normal distribution

Params.: $\mathbf{M}$ location (real $n \times p$ matrix), $\mathbf{U}$ scale (positive-definite real $n \times n$ matrix), $\mathbf{V}$ scale (positive-definite real $p \times p$ matrix); **Not.**: $\mathcal{MN}_{n,p}(\mathbf{M}, \mathbf{U}, \mathbf{V})$; $\mathcal{W}(X)$: $\mathbf{X} \in \mathbb{R}^{n\times p}$; $\mathbb{E}[X]$: $\mathbf{M}$; $Var[$
$\mathbf{U}$ (among-row) and $\mathbf{V}$ (among-column);
f_x: $\frac{\exp\left(-\frac{1}{2}\operatorname{tr}\left[\mathbf{V}^{-1}(\mathbf{X}-\mathbf{M})^T\mathbf{U}^{-1}(\mathbf{X}-\mathbf{M})\right]\right)}{(2\pi)^{np/2}|\mathbf{V}|^{n/2}|\mathbf{U}|^{p/2}}$

8.3.5 Matrix gamma distribution

Params.: $\alpha > 0$ shape parameter (real), $\beta > 0$ scale parameter, $\boldsymbol{\Sigma}$ scale (positive-definite real $p \times p$ matrix); **Not.**: $\mathrm{MG}_p(\alpha, \beta, \boldsymbol{\Sigma})$; $\mathcal{W}(X)$: $\mathbf{X}$ positive-definite real $p \times p$ matrix;
f_x: $\frac{|\boldsymbol{\Sigma}|^{-\alpha}}{\beta^{p\alpha}\Gamma_p(\alpha)}|\mathbf{X}|^{\alpha-(p+1)/2}\exp\left(\operatorname{tr}\left(-\frac{1}{\beta}\boldsymbol{\Sigma}^{-1}\mathbf{X}\right)\right)$ * Γ_p is the multivariate gamma function.

8.3.6 Matrix t-distribution

Params.: $\mathbf{M}$ location (real $n \times p$ matrix), $\boldsymbol{\Omega}$ scale (positive-definite real $p \times p$ matrix), $\boldsymbol{\Sigma}$ scale (positive-definite real $n \times n$ matrix) , ν degrees of freedom; **Not.**: $\mathrm{T}_{n,p}(\nu, \mathbf{M}, \boldsymbol{\Sigma}, \boldsymbol{\Omega})$; $\mathcal{W}(X)$: $\mathbf{X} \in \mathbb{R}^{n\times p}$; $\mathbb{E}[X]$: $\mathbf{M}$ if $\nu + p - n > 1$, else undefined; $Var[X]$: $\frac{\boldsymbol{\Sigma}\otimes\boldsymbol{\Omega}}{\nu-2}$ if $\nu > 2$, else undefined;
f_x:

$$\frac{\Gamma_p\left(\frac{\nu+n+p-1}{2}\right)}{(\pi)^{\frac{np}{2}}\Gamma_p\left(\frac{\nu+p-1}{2}\right)}|\boldsymbol{\Omega}|^{-\frac{n}{2}}|\boldsymbol{\Sigma}|^{-\frac{p}{2}}$$

:

$$\times\left|\mathbf{I}_n + \boldsymbol{\Sigma}^{-1}(\mathbf{X}-\mathbf{M})\boldsymbol{\Omega}^{-1}(\mathbf{X}-\mathbf{M})^{\mathrm{T}}\right|^{-\frac{\nu+n+p-1}{2}}$$

F_x: No analytic expression

9 Directional

9.1 Univariate (circular) directional

9.1.1 Wrapped Cauchy distribution

Params.: μ Real, $\gamma > 0$; $\mathcal{W}(X)$: $-\pi \leq \theta < \pi$; $\mathbb{E}[X]$: μ (circular); $Var[X]$: $1 - e^{-\gamma}$ (circular);
f_x: $\frac{1}{2\pi}\,\frac{\sinh(\gamma)}{\cosh(\gamma)-\cos(\theta-\mu)}$$F_x$:

9.1.2 Wrapped asymmetric Laplace distribution

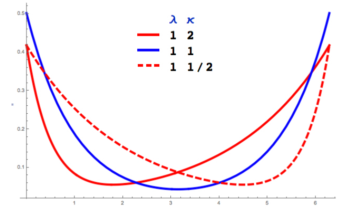

Abbildung 79: 350px

Params.: m location $(0 \leq m < 2\pi)$, $\lambda > 0$ scale (real), $\kappa > 0$ asymmetry (real); $\mathcal{W}(X)$: $0 \leq \theta < 2\pi$; $\mathbb{E}[X]$: m (circular); $Var[X]$: $1 - \frac{\lambda^2}{\sqrt{\left(\frac{1}{\kappa^2}+\lambda^2\right)(\kappa^2+\lambda^2)}}$ (circular);
f_x: (see article)

9.1.3 Wrapped exponential distribution

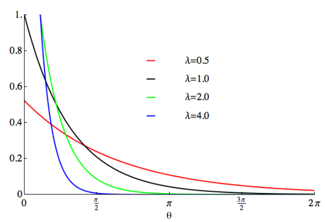

Abbildung 80: Plot of the wrapped exponential PDF

Params.: $\lambda > 0$; $\mathcal{W}(X)$: $0 \leq \theta < 2\pi$; $\mathbb{E}[X]$: $\arctan(1/\lambda)$ (circular); $Var[X]$: $1 - \frac{\lambda}{\sqrt{1+\lambda^2}}$ (circular);
f_x: $\frac{\lambda e^{-\lambda\theta}}{1-e^{-2\pi\lambda}}$$F_x$: $\frac{1-e^{-\lambda\theta}}{1-e^{-2\pi\lambda}}$

9.1.4 Wrapped normal distribution

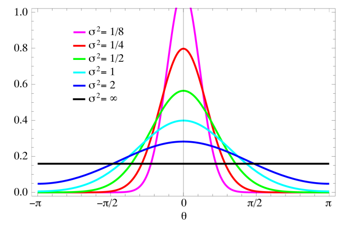

Abbildung 81: Plot of the von Mises PMF

Params.: μ real, $\sigma > 0$; $\mathcal{W}(X)$: $\theta \in$ any interval of length 2; $\mathbb{E}[X]$: μ if support is on interval $\mu \pm \pi$; $Var[X]$: $1 - e^{-\sigma^2/2}$ (circular);
f_x: $\frac{1}{2\pi}\vartheta\left(\frac{\theta-\mu}{2\pi}, \frac{i\sigma^2}{2\pi}\right)$

9.2 Bivariate (spherical)

9.3 Bivariate (toroidal)

9.4 Multivariate

10 Degenerate and singular

10.1 Degenerate

10.2 Singular

10.2.1 Cantor distribution

Params.: none; $\mathcal{W}(X)$: Cantor set; $\mathbb{E}[X]$: 1/2; $Var[X]$: 1/8;
f_x: noneF_x: Cantor function

11 Families